Environmentally Induced Photon Blockade via Two-Photon Absorption

Yan-Hui Zhou,[1] Xing-Yuan Zhang,[2] Tong Liu,[1] Qi-Cheng Wu,[1,*] Zhi-Cheng Shi,[3] Hong-Zhi Shen,[4,†] and Chui-Ping Yang[1,5,‡]

[1] *Quantum Information Research Center, Shangrao Normal University, Shangrao 334001, China*
[2] *Department of Physics, Dalian Maritime University, Dalian 116026, China*
[3] *Department of Physics, Fuzhou University, Fuzhou 350116, China*
[4] *Center for Quantum Sciences and School of Physics, Northeast Normal University, Changchun 130024, China*
[5] *Department of Physics, Hangzhou Normal University, Hangzhou 311121, China*

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We propose that photon blockade can be enabled by a two-photon absorption environment. The validity of this conclusion is confirmed by studying the blockade in a driven single-mode cavity. The feature of this environmentally induced photon blockade is that the nonlinear interaction in the system is not needed, which is entirely different from the conventional and the unconventional blockade mechanisms. We clearly give the physics behind this blockade mechanism. And the advantages of this environmentally induced photon blockade are discovered by comparing it with the conventional mechanism in a weakly driven region. In addition, we show that the proposal can be generalized straightforwardly to realize environmentally induced photon blockade in two coupled single-mode cavities.

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I. INTRODUCTION

The study of generation and application of nonclassical optical fields is an international advanced research topic [1–11]. In the past two decades, there has been a great interest in photon blockade (PB), because it can generate typical nonclassical light with antibunching photons [12–20]. This PB effect can be described as: the existence of a single photon in a nonlinear cavity blocking the creation of a second photon under certain conditions [21–23]. The significance of this subject is mainly motivated by the considerable applications of correlated photons to foundations in quantum theory as well as in quantum information science [24].

PB, also known as conventional PB (CPB), enabled by a large nonlinearity changing the energy-level structure of a system, can suppress the production of multiple photons [25–31]. This is one of the mechanisms for creating strong antibunching photons. Under this conventional mechanism, the presence of photon-photon interaction in a nonlinear cavity modifies the energy ladder of a harmonic oscillator. And the photon-number probability distribution is significantly changed if the driving field is tuned to be resonant with the cavity.

CPB was experimentally observed using an optical cavity with a trapped atom [32]. Subsequently, a sequence of experimental results were reported by various experimental groups using different systems, such as a quantum dot in a photonic crystal [33] and a circuit QED system [34,35]. Recently, two-PB and multi-PB have been proposed with the conventional mechanism. Two-PB was studied in various nonlinear systems [36–38], and was experimentally observed in an optical cavity strongly coupled to a single atom [39]. Meanwhile, multi-PB was also studied [40–44].

In the earlier studies, it was believed that PB necessarily requires a large nonlinearity with respect to the decay rate of the system. However, Liew and Savona found that strong photon antibunching can also be obtained even with a nonlinearity much smaller than the decay rate of the cavity modes [45,46], which is called unconventional PB (UPB) [47–50]. The physics behind this mechanism is the destructive interference between two different transition paths, which means that the system cannot occupy the two-photon state [51–53]. UPB has been experimentally observed in a quantum dot cavity QED system [54] and two coupled superconducting resonators [55].

In this paper, we find that PB can also be displayed with a two-photon absorption environment. To confirm the validity of the conclusion, this environmentally induced PB (EPB) is investigated by considering a driven single-mode cavity. The numerical results show that strong photon antibunching can be achieved, and the physical mechanism is discussed by analyzing the energy-level

*wuqi.cheng@163.com
†shenhz458@nenu.edu.cn
‡yangcp@hznu.edu.cn

diagram and the two-photon absorption process. To show the advantage of EPB, the present proposal is compared with CPB. We find that EPB has a higher brightness and a larger single-photon probability under the weak drive approximation. In addition, the existence of EPB is confirmed even with a single-photon dissipation. Finally, we show that EPB can also be obtained in two coupled single-mode cavities.

This paper is organized as follows. In Sec. II, we introduce the two-photon absorption environment and the blockade description. In Sec. III, we study EPB in a driven single-mode cavity. Specifically, the physical mechanism is analyzed, EPB is compared with CPB via numerical results, EPB is discussed in terms of undergoing a single-photon dissipation, and the experimental possibility is also considered. In Sec. IV, we study EPB in two coupled single-mode cavities. Conclusion is given in Sec. V.

II. TWO-PHOTON ABSORPTION ENVIRONMENT AND BLOCKADE DESCRIPTION

The two-photon absorption master equation, described by the two-photon absorption environment, has been derived for light traveling through a two-photon absorbing medium [56–59]. Here we give a brief review of the derivation of this master equation starting from the system of a single-mode cavity with frequency ω. The system is described by the Hamiltonian (hereafter setting $\hbar = 1$)

$$\hat{H}_s = \omega \hat{a}^\dagger \hat{a}, \tag{1}$$

where $\hat{a}$ is the photon annihilation operator for the system. The system is coupled to a thermal reservoir via the second-order nonlinear process. The Hamiltonian of the thermal reservoir is

$$\hat{H}_e = \sum_k v_k \hat{o}_k^\dagger \hat{o}_k, \tag{2}$$

where $\hat{o}_k$ is the photon annihilation operator for the reservoir with frequency v_k. In addition, the environment can also be a beam of two-level atoms; in this case, $\hat{o}_k$ is the lowering operator for the atoms. The interaction Hamiltonian of the system and the environment is

$$\hat{H}_i = \sum_k g_k (\hat{o}_k \hat{a}^{\dagger 2} + \hat{a}^2 \hat{o}_k^\dagger), \tag{3}$$

where the second-order nonlinearity mediates the conversion of two photons in the cavity with one photon in the reservoir.

The master equation can be derived and written as the Lindblad form by assuming zero temperature of the reservoir, which governs the dynamics of the density matrix ρ [56–59]:

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}_s, \hat{\rho}] + \kappa \ell(\hat{a}^2)\hat{\rho}, \tag{4}$$

where we define the super operator $\ell(\hat{a})\hat{\rho} = \hat{a}\hat{\rho}\hat{a}^\dagger - \frac{1}{2}\hat{a}^\dagger\hat{a}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{a}^\dagger\hat{a}$ and κ is the two-photon absorption constant. The two-photon absorption master equation indicates that two photons get away from the cavity simultaneously, which has many potential applications [60–62].

The statistical properties of photons are described by the zero-delay-time correlation function $g^{(2)}(0)$ in the steady state, which is applied to observe PB. Here $g^{(2)}(0)$ is defined by

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}. \tag{5}$$

The second-order correlation function $g^{(2)}(0) < 1$ corresponds to sub-Poissonian statistics.

The Fock-state basis of the system is denoted by $|m\rangle$, with m denoting the photon number in the cavity. The photon-number probability P_m denotes the probability that the system occupies the state $|m\rangle$. The value of P_m is obtained by analyzing the diagonal elements of the steady-state density matrix. We define the multiphoton probability as

$$P_{\text{multi}} = \sum_{m=2}^{\infty} P_m. \tag{6}$$

And the brightness is defined as the average photon number:

$$N = \langle \hat{a}^\dagger \hat{a} \rangle. \tag{7}$$

PB can also be confirmed by $P_{\text{multi}} \ll 1$ and enough brightness.

III. MODEL I: DRIVEN SINGLE-MODE CAVITY

We first investigate PB in a driven single-mode cavity with frequency ω_a, which interacts with a two-photon absorption environment. In the frame rotating at the pump frequency ω_L, the Hamiltonian of this model reads

$$\hat{H}_s = \Delta \hat{a}^\dagger \hat{a} + \varepsilon(\hat{a} + \hat{a}^\dagger), \tag{8}$$

where ε is the coupling strength of the laser and $\Delta = \omega_a - \omega_L$ is the detuning of the cavity with respect to the driving frequency.

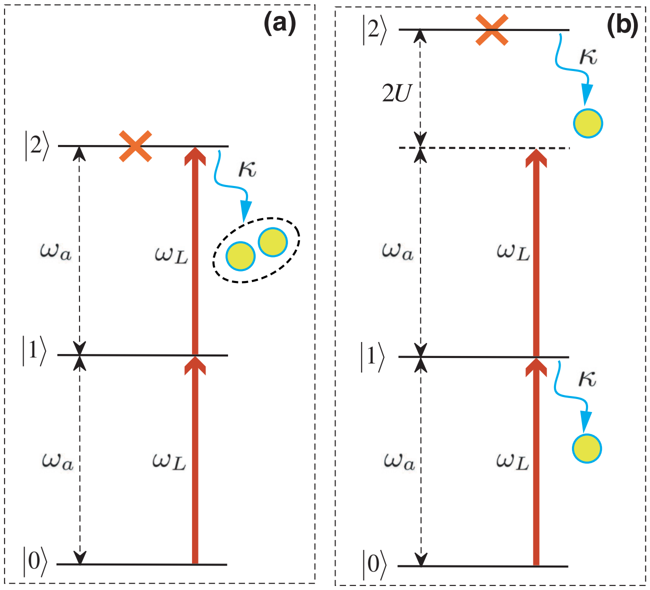

FIG. 1. (a) Energy spectrum of a single-mode cavity explaining the occurrence of EPB. (b) Energy spectrum of a single cavity with a Kerr nonlinearity explaining the occurrence of CPB, where U is the Kerr nonlinear strength. In both (a),(b), ω_a is the cavity resonance frequency, ω_L is the driving frequency, and κ is the damping constant of the cavity.

A. Physical mechanism

We aim at PB induced by the two-photon absorption environment. To clearly explain this blockade mechanism under the weak driving approximation, we show the energy-level diagram of the system in Fig. 1(a) by restricting the system within the zero-, one-, and two-photon subspaces. There is no nonlinearity in the system, and thus the energy levels are evenly spaced with energies $E_m = m\omega_a$. If we adjust the driving frequency ω_L to the cavity resonance frequency ω_a, the single-photon probability increases greatly due to the resonance. Generally, the two-photon probability will also increase dramatically with the equally spaced energy levels. However, once there are two photons in the cavity, they leak out from the cavity simultaneously due to the two-photon absorption environment, which means that the two-photon probability tends to zero. Under the weak driving limit, the second-order correlation function can be written as

$$g^{(2)}(0) \simeq 2P_2/P_1^2. \tag{9}$$

With the zero detuning condition $\Delta = 0$, the greatly increased P_1 and decreased P_2 give rise to an important result, $g^{(2)}(0) \simeq 0$, and strong PB is triggered.

We now consider the similarities and differences of EPB, CPB, and UPB from the viewpoint of physical mechanisms. According to the requirements of CPB and UPB: (i) we add a Kerr nonlinear term $U\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}$ in Eq. (8), and the Hamiltonian is

$$\hat{H}_{\rm CPB(UPB)} = \Delta\hat{a}^\dagger\hat{a} + U\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + \varepsilon(\hat{a} + \hat{a}^\dagger); \tag{10}$$

(ii) we replace $\ell(\hat{a}^2)$ with $\ell(\hat{a})$ in Eq. (4), and the master equation is

$$\frac{\partial\hat{\rho}}{\partial t} = -i[\hat{H}_s, \hat{\rho}] + \kappa\ell(\hat{a})\hat{\rho}. \tag{11}$$

The comparison of EPB with CPB and UPB is shown in Table I.

TABLE I. Comparison of EPB, CPB, and UPB in a driven single-mode cavity.

	Nonlinearity	Mechanism	Blockade condition
EPB	Not required	Environment	$\Delta = 0$
CPB	Large	Energy-level shift	$\Delta = 0$
UPB	Small	Interference	None

(i) The requirements for the nonlinearity in the system are different. A large nonlinearity is required for CPB, and a small nonlinearity is required for UPB, but the nonlinearity is not required for EPB.

(ii) The mechanisms are different. Under the CPB mechanism, the energy levels are unevenly spaced with energies $E_m = m\omega_a + m(m-1)U$ shown in Fig. 1(b). The two-photon probability tends to zero due to the shift of energy levels (induced by the large nonlinearity) for CPB. While for EPB, two photons leak out from the cavity simultaneously due to the two-photon absorption environment, which means that the two-photon probability tends to zero. For UPB, the mechanism of the blockade is quantum interference.

(iii) For both EPB and CPB, when the driving frequency ω_L is equal to the cavity resonance frequency ω_a, the single-photon probability increases greatly due to the resonance. So EPB and CPB have the same blockade condition $\Delta = 0$. For the UPB mechanism, two interference paths are needed, and the photons in these two paths interfere destructively, which means that the system cannot occupy the two-photon state. However, for the driven single-mode cavity, there is only one path for the system to reach the two-photon state $|0\rangle \xrightarrow{\varepsilon} |1\rangle \xrightarrow{\varepsilon} |2\rangle$. Thus, blockade does not exist in this system under the UPB mechanism.

B. Comparison between EPB and CPB via numerical results

We numerically study EPB and the results are compared with those for CPB in Fig. 2, where we plot second-order correlation function $g^{(2)}(0)$ as a function of the detuning

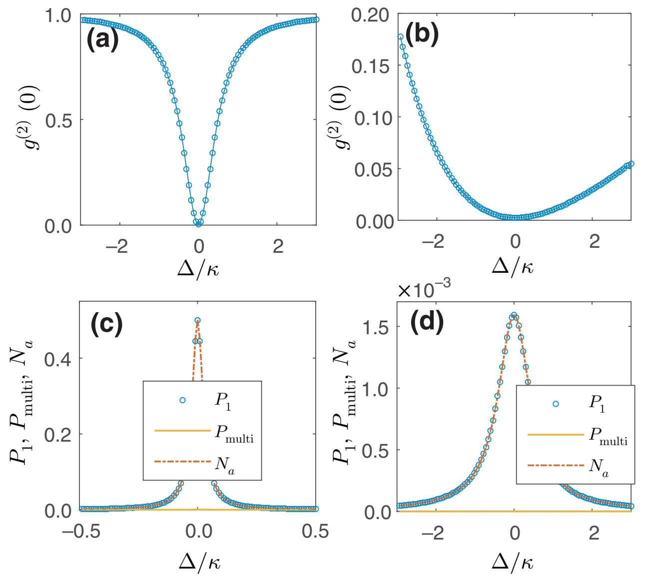

FIG. 2. (a) Second-order correlation function $g^{(2)}(0)$ as a function of detuning Δ/κ for EPB. (b) $g^{(2)}(0)$ as a function of Δ/κ for CPB with $U/\kappa = 10$. (c) Single-photon probability P_1, average photon number N_a, and multiphoton probability $P_{\rm multi}$ as a function of Δ/κ for EPB. (d) P_1, N_a, and $P_{\rm multi}$ as a function of Δ/κ for CPB with $U/\kappa = 10$. In all plots, the driving strength is $\varepsilon/\kappa = 0.02$.

Δ/κ for both EPB and CPB. Clearly, PB appears at $\Delta/\kappa = 0$ as predicted, shown in Figs. 2(a) and 2(b), exhibiting strong photon antibunching $g^{(2)}(0) \ll 1$ for both EPB and CPB.

The average photon number N_a and the single-photon probability P_1 are also used to evaluate the blockade effect. In Figs. 2(c) and 2(d), we plot single-photon probability P_1, average photon number N_a, and multiphoton probability $P_{\rm multi}$ as a function of Δ/κ for both EPB and CPB. We note that the average photon number agrees well with the single-photon probability for both EPB and CPB. The average photon number can be written as

$$N_a = \sum_{i=0}^{\infty} iP_i, \tag{12}$$

while under the blockade mechanism, multiphoton probability $P_{\rm multi}$ tends to zero, so the average photon number is determined mainly by the single photon, which makes $N_a \simeq P_1$. The combination of this condition and a large P_1 can also be used as a criterion for the existence of PB, which can guarantee a large brightness and a small multiphoton probability.

In Figs. 2(c) and 2(d), we show $P_1 \simeq 0.5$ for EPB, while $P_1 \simeq 0.0016$ for CPB. The proposal based on the EPB mechanism has a larger single-photon probability P_1 and a larger average photon number N_a than the proposal based on the CPB mechanism. We try to analyze the reason leading to this conclusion. For the EPB mechanism, only if two photons are in the cavity will they leave the cavity simultaneously, which ensures a large single-photon probability and a small two-photon probability. While for the CPB mechanism, the photons are allowed to leave the cavity even if there is only one photon [see Fig. 1(b)], which decreases both the single-photon probability and the two-photon probability. Thus EPB has a higher brightness and a larger single-photon probability than CPB in the weak drive region.

C. EPB with single-photon dissipation

In the previous section, we illustrate that EPB takes place with a two-photon absorption environment. A question is whether EPB still exists when the two-photon absorption environment comes with a single-photon dissipation. To answer this question, we describe the dynamics of the system in Eq. (8), which undergoes two-photon absorption and single-photon dissipation, by the following master equation:

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}_s, \hat{\rho}] + \gamma \ell(\hat{a})\hat{\rho} + \kappa \ell(\hat{a}^2)\hat{\rho}, \tag{13}$$

where γ is the single-photon decay rate.

The existence of EPB is confirmed in Fig. 3(a) by the numerical calculation, where we show $g^{(2)}(0)$ and N_a as a function of Δ/κ for $\gamma/\kappa = 0.1$. And blockade appears for $\Delta/\kappa = 0$ even with single-photon dissipation. The existence of single-photon dissipation will reduce the single-photon probability, but EPB still occurs due to the two-photon absorption.

Now we examine the effect of the single-photon decay rate γ on EPB. To this end, we plot $g^{(2)}(0)$ and N_a as a function of γ/κ under the blockade condition $\Delta/\kappa = 0$. The results are shown in Fig. 3(b). We find that $g^{(2)}(0)$ gradually increases with increasing γ/κ. Meanwhile, the average photon number N_a decreases. The key role of the two-photon absorption is the suppression of the production of two photons in the cavity, which means that the two-photon probability P_2 tends to zero. While the single photon can also leak out of the cavity due to the existence of the single-photon dissipation. An increase of γ/κ leads to a decrease of single-photon probability P_1. Reduction of both P_1 and P_2 leads to a decrease of N_a according to Eq. (12). At the same time, $g^{(2)}(0)$ increases with increasing γ/κ according to Eq. (9).

We give a brief discussion on the experimental possibility. Here we design a scheme based on photonic crystal cavities to realize the system-environment interaction. The schematic setup is illustrated in Fig. 3(c), where the system of a driven single-mode cavity couples to an environment of a multimode cavity via second-order nonlinearity. The Hamiltonians for the system, the thermal reservoir, and the system-reservoir interaction shown in Eqs. (1)–(3) are

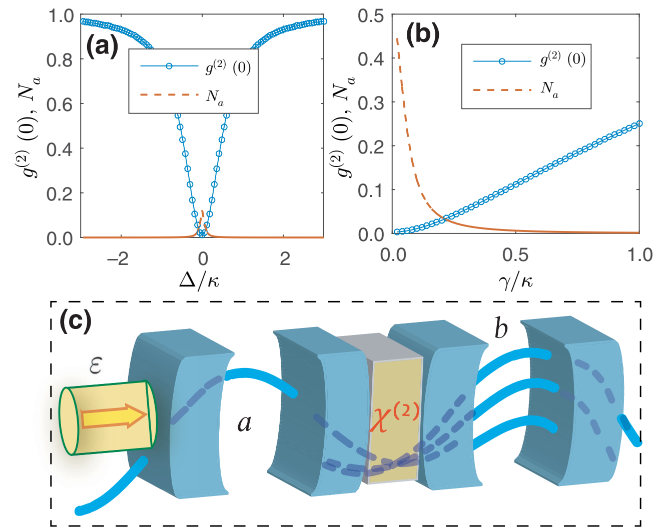

FIG. 3. (a) $g^{(2)}(0)$ and N_a as a function of Δ/κ for $\gamma/\kappa = 0.1$ and $\varepsilon/\kappa = 0.02$. (b) $g^{(2)}(0)$ and N_a as a function of γ/κ for $\Delta/\kappa = 0$ and $\varepsilon/\kappa = 0.02$. (c) Schematic diagram for a driven single-mode cavity interacting with a multimode cavity via second-order nonlinearity ($\chi^{(2)}$ nonlinearity), where the multimode cavity b is treated as a multimode environment, and the second-order nonlinearity mediates the conversion of two photons in cavity a with one photon in cavity b.

obtained with this design. And the master equation in Eq. (4) can be derived with the methods in Refs. [56–59].

It is worth evaluating the actual experimental parameters of environmentally induced photon blockade. The second-order nonlinearity can be obtained by employing main-group III-V materials, such as GaAs [63,64], GaN, and AlN [65], where the nonlinear susceptibility can be of the order of 10–200 pm/V in optoelectronics. A realistic estimation of the two-photon absorption constant κ is $\kappa \sim 50$ μeV with engineered photonic crystal cavities. Working in the typical telecommunication band, ω_a is given by $\omega_a \sim 0.8$ eV. Without loss of generality, we assume that two-photon absorption comes with single-photon dissipation. The required loss rate is 10 μeV corresponding to cavity a, and the Q factor is $Q \sim 80\,000$. And the driving strength is $\varepsilon \sim 1$ μeV. These values can be routinely achieved in photonic crystal cavities made of III-V semiconductor materials [30,31,66].

IV. MODEL II: TWO COUPLED SINGLE-MODE CAVITIES

We consider two coupled single-mode cavities as the second example to study EPB, where one of the cavities is driven by a laser with frequency ω_L. The Hamiltonian reads

$$\hat{H}_s = \Delta_a \hat{a}^\dagger \hat{a} + \Delta_b \hat{b}^\dagger \hat{b} + J(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}) + \varepsilon(\hat{a} + \hat{a}^\dagger), \tag{14}$$

where $\hat{a}$ ($\hat{b}$) is the photon annihilation operator for cavity a (cavity b), J is the coupling strength of the two cavities, ε is the coupling strength of the laser with cavity a, and $\Delta_a = \omega_a - \omega_L$ ($\Delta_b = \omega_b - \omega_L$) is the detuning of cavity a (cavity b) with respect to the driving frequency ω_L. Here we assume $\Delta_a = \Delta_a = \Delta$ ($\omega_a = \omega_b = \omega$) for convenience.

We use $|mn\rangle$ to denote the Fock-state basis of the system, where m denotes the photon number in cavity a and n denotes the photon number in cavity b. By expanding the Hamiltonian in a single-photon-excitation subspace, two energy eigenfrequencies $E_1^{1,2}$ are created by solving the expanded Hamiltonian, which are

$$E_1^{1,2} = \omega \mp J, \tag{15}$$

where the subscript denotes single-photon excitation. The corresponding eigenvectors $|\Psi_1^{1,2}\rangle$ are

$$|\Psi_1^{1,2}\rangle = \frac{1}{\sqrt{2}}(|10\rangle \mp |01\rangle), \tag{16}$$

where minus in the expression corresponds to $|\Psi_1^1\rangle$, while plus corresponds to $|\Psi_1^2\rangle$. When the drive frequency ω_L is tuned to $E_1^{1,2}$, the system occupies the state $|\Psi_1^{1,2}\rangle$. We deduce the following conditions for PB:

$$J = \pm\Delta. \tag{17}$$

When one of the conditions is satisfied, PB is triggered in both cavities under the two-photon absorption environment.

The two-photon absorption master equation is

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}_s, \hat{\rho}] + \kappa_a \ell(\hat{a}^2)\hat{\rho} + \kappa_b \ell(\hat{b}^2)\hat{\rho}, \tag{18}$$

where κ_a and κ_b are the two-photon absorption constants of the cavities. Without loss of generality, the two-photon absorption constants are assumed to be equal, i.e., $\kappa_a = \kappa_b = \kappa$.

The photon-number probability is expressed as P_{mn}, which denotes the probability that the system occupies the state $|mn\rangle$. In the weak drive approximation, the single-photon probability of cavity a (cavity b) is largely determined by P_{10} (P_{01}). And the average photon number in cavity a (cavity b) is represented by $N_a = \langle \hat{a}^\dagger \hat{a}\rangle$ ($N_b = \langle \hat{b}^\dagger \hat{b}\rangle$).

We plot P_{10} (P_{01}) and N_a (N_b) as a function of Δ/κ, as shown in Fig. 4(a)[Fig. 4(b)]. The results show that the blockades can be obtained at $\Delta/\kappa = \pm 4$ for both cavity a and cavity b as predicted in Eq. (17). And the average photon number agrees well with the single-photon probability, which indicates that the average photon number is

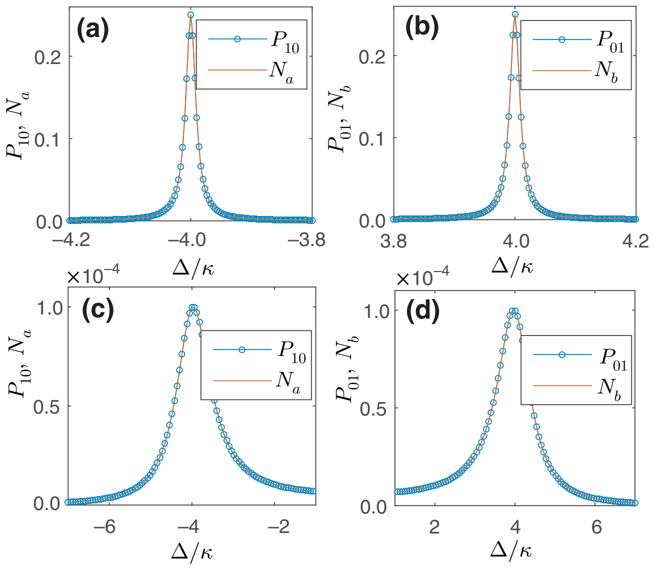

FIG. 4. Single-photon probability and average photon number as a function of the detuning Δ/κ for EPB and CPB. (a) EPB for cavity a. (b) EPB for cavity b. In both (a),(b), $g/\kappa = 4$ and $\varepsilon/\kappa = 0.01$. (c) CPB for cavity a. (d) CPB for cavity b. For both (c),(d), $U/\kappa = 10$, $J/\kappa = 4$, and $\varepsilon/\kappa = 0.01$.

determined mainly by the single photon, and the existence of the single photon blocks the creation of the subsequent photons due to two-photon absorption.

We compare EPB with CPB in the weak drive region as before. Two Kerr nonlinear terms $U(\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + \hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b})$ are added in the Hamiltonian (14) and a general master equation is used with $\ell(\hat{a})$ and $\ell(\hat{b})$ in Eq. (18). The single-photon probabilities and the average photon numbers for both cavities are plotted in Figs. 4(c) and 4(d) with the CPB mechanism. We show that $P_{10} = N_a = P_{01} = N_b \simeq 0.25$ for EPB, while $P_{10} = N_a = P_{01} = N_b \simeq 0.0001$ for CPB. The EPB mechanism leads to a larger single-photon probability and a larger average photon number than the CPB mechanism.

V. CONCLUSION

We study PB caused by the two-photon absorption environment. By analyzing a single-mode cavity and two coupled cavities with two-photon absorption environments, we confirm that strong photon antibunching can be obtained in these systems. The numerical results are obtained by numerically solving the master equation in the steady-state limit, which confirms our theoretical analysis. We compare the proposal with CPB and find advantages. Specifically, EPB has a higher brightness and a larger single-photon probability under the weak drive approximation. It is worth noting that a state-dependent photon blockade was studied in Ref. [67], where the two-photon absorption master equation had been used. However, there are fundamental differences between Ref. [67] and our proposal, which mainly manifest in the following points. (i) Besides the two-photon absorption master equation, Ref. [67] studies blockade under the conventional mechanism, where a large nonlinearity is required to change the energy-level structure. While for our work, blockade is induced by only the two-photon absorption environment, and nonlinearity is not required in the system. (ii) In Ref. [67], photon blockade depends on the initial state; the steady state of the cavity field can be the single-photon Fock state or a partially incoherent superposition of several Fock states with photon numbers, e.g., (0,2), (1,3), (0,1,2), or (0,2,4). While for our proposal, the blockade is entirely unrelated to the initial state. The steady state of the cavity field, with an arbitrary initial state, is the superposition of only the single-photon state and the vacuum state, i.e., (0,1). Actually, our proposal might be extended to study multi-PB by using a multiple-photon absorption master equation derived in Ref. [57]. Our proposal paves an avenue towards the study of photon blockades.

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