Letter

Enhanced Longitudinal Relaxation of Magnetic Solitons in Ultrathin Films

Ivan A. Yastremsky,[1,2,*] Jürgen Fassbender,[2] Boris A. Ivanov,[1,3,4] and Denys Makarov[2,†]

[1] *Taras Shevchenko National University of Kyiv, 01601 Kyiv, Ukraine*
[2] *Helmholtz-Zentrum Dresden–Rossendorf e.V., Institute of Ion Beam Physics and Materials Research, Bautzner Landstrasse 400, 01328 Dresden, Germany*
[3] *Institute of Magnetism, National Academy of Sciences and Ministry of Education and Science, 03142 Kyiv, Ukraine*
[4] *Institute for Molecules and Materials, Radboud University Nijmegen, Nijmegen, Netherlands*

(Received 11 January 2022; revised 20 April 2022; accepted 2 May 2022; published 17 June 2022)

Relaxation fundamentally determines the operation speed and energy efficiency of spintronic and spinorbitronic devices. We develop a theory of the longitudinal contribution to the relaxation of domain walls in ferromagnetic films of any thickness with the Dzyaloshinskii-Moriya interaction, which allows quantitative comparison with experiments. We show that the longitudinal contribution increases with a decrease of the transversal relaxation (e.g., the Gilbert constant). We predict a substantial enhancement of the contribution of the longitudinal relaxation to the damping of magnetic solitons with a decrease of the film thickness. We demonstrate that for ultrathin ferromagnetic films, the contribution of the longitudinal relaxation to the damping of domain walls is comparable to or stronger than any other traditional transversal mechanisms, including spin pumping. Although in this work we focus on the analysis of longitudinal relaxation for domain walls, in ultrathin samples it should be taken into account also for other magnetic solitons including skyrmions. This work adds to the fundamental understanding of the design and optimization of spintronic and spinorbitronic devices based on moving solitons in ultrathin films.

DOI: 10.1103/PhysRevApplied.17.L061002

The phenomenological theory of magnetization dynamics was formulated in 1935 [1]. The famous Landau-Lifshitz equation in the original form or with Gilbert damping term conserves the magnetization length, allowing only its transversal evolution. Since then, the transversal picture of magnetization relaxation has become overwhelmingly successful when describing linear magnetic excitations (spin waves) and motion of topologically protected magnetic excitations (magnetic solitons) including domain walls (DWs), bubbles, droplets, and recently skyrmions.

Still, in any magnetic system, a change of the magnetization length is allowed [2–7]. The prominent example of longitudinal evolution is laser-induced ultrafast magnetization dynamics [4–7]. The transversal and longitudinal evolution of magnetization are fundamentally different [3,7]. In particular, they have different relaxation times. Furthermore, while the longitudinal evolution of magnetization can be only dissipative, the transversal one can be also nondissipative.

For solitons, the change of the magnetization length, M, occurs due to anisotropic magnetic interactions [2,3]. Arriving at a given location, solitons spend energy to create a deviation (dip) of the magnetization length by the longitudinal (with respect to the magnetization) component of the effective field (Fig. 1). The magnetization length at this location is restored as soon as the soliton moves away. The process of changing the magnetization length is irreversible and leads to the dissipation of energy. This is a generic mechanism of energy dissipation, which is valid when describing dynamics of solitons in any ordered medium. It is known to be dominant for the dynamics of vortices in superfluid helium [8] and type II superconductors [9]. Investigations performed for bulk ferromagnets with the Landau-Lifshitz-Bar'yakhtar (LLBar) [3,10–13] and the Landau-Lifshitz-Bloch (LLB) [14] equations for solitons without structural suppression of M (e.g. DWs) sufficiently far from the Curie temperature have not identified any significant contribution from longitudinal relaxation. In bulk materials, up to now only two examples are known where the transversal relaxation picture fails to describe the dynamics of magnetic solitons properly. The longitudinal relaxation dominates the dynamics of solitons at temperatures close to the Curie temperature [2,15–17]. The motion of Bloch points (with strong structural suppression of M) is another example where the longitudinal dynamics is important for bulk ferromagnets even far from the Curie temperature [18].

It is established that in magnetic films with thicknesses of the order of 1 nm, which are in contact with efficient

*Corresponding author. yastremsky@ukr.net
†Corresponding author. d.makarov@hzdr.de

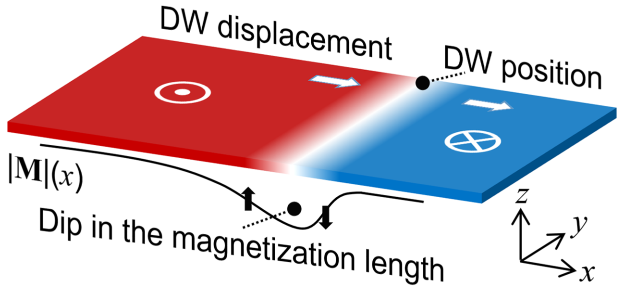

FIG. 1. Longitudinal relaxation is related to the appearance of the dip in the magnetization length, |**M**|, at the location of a DW. This dip is in the range of $\chi_{\parallel} H_K$, where $\chi_{\parallel}$ is the longitudinal magnetic susceptibility and H_K is the anisotropy field. The dip follows the moving DW. Black arrows indicate the evolution of |**M**| for a moving DW. In ultrathin films, $\chi_{\parallel}$ is enhanced.

spin sinks, the *transversal* damping constant is dominated by the spin pumping mechanism [19–21]. However, the damping constant extracted from DW mobility experiments, where the moving dip of M exists, for a prototypical system of Pt/Co/oxide with Co thickness in the range of 1 nm [22,23] is about five times larger than from the spin pumping mechanism [24,25]. This discrepancy cannot be explained in the framework of the transversal relaxation mechanism and calls for an exploration of the longitudinal dynamics. In contrast to bulk magnets, there is no theory available for the description of the *longitudinal* magnetization dynamics of nonlinear magnetic excitations like DWs and skyrmions in ultrathin films. This theory is highly demanded for the communities working in spintronic and spinorbitronics given the primary relevance of moving magnetic solitons for prospective memory and logic devices.

Here, we develop a theory of the longitudinal contribution to the relaxation of DWs in ferromagnets, which allows quantitative comparison with experiments. We describe how the longitudinal relaxation changes with the crossover from bulk (3D) to ultrathin films (2D limit). In this respect, our theory can be used for the analysis of thin films of any thickness. We demonstrate that in ultrathin films, the longitudinal magnetic susceptibility, and consequently the longitudinal relaxation, is enhanced. We show that for ultrathin Co films at a broad range of temperatures the longitudinal relaxation for magnetic solitons is much stronger than the standard transversal Gilbert or nonlocal contributions and is comparable to or even stronger than the spin pumping contribution, which is traditionally considered to be dominant. The enhancement is related to the two-dimensional behavior of the thermal gas of magnons in ultrathin magnetic films, which is more sensitive to external stimuli compared with its bulk counterpart [26–30]. The enhancement of the longitudinal relaxation mechanism discussed here is generic and should be valid for other magnetic solitons including skyrmions.

We demonstrate that properties of the transversal and longitudinal contributions to damping of solitons are fundamentally different concerning the response of solitons to external stimuli.

We consider a ferromagnetic film with perpendicular anisotropy and Dzyaloshinskii-Moriya interaction (DMI) stemming from the asymmetry at interfaces. The film accommodates a DW and its position can be manipulated with external out-of-plane magnetic fields H and (or) spin currents, flowing perpendicular to the DW. We calculate the influence of the longitudinal relaxation mechanisms on the energy dissipation, mobility of DWs μ, Walker field H_W, and response of DWs to the spin current. The free energy density, Φ, which accounts for the change of the magnetization length, can be written as [3]

$$\Phi = A\,(\nabla \mathbf{M})^2 + (M - M_0)^2/2\chi_{\parallel} - KM_z^2 + D\,(M_z \mathbf{div}\mathbf{M} - (\mathbf{M}\cdot\nabla)\,M_z) - HM_z, \qquad (1)$$

where A is the exchange stiffness, $\mathbf{M}$ is the magnetization vector, $M = |\mathbf{M}|$ is the length of $\mathbf{M}$, M_0 is the equilibrium value of the magnetization at given temperature T, $\chi_{\parallel}$ is the longitudinal magnetic susceptibility at thermodynamic equilibrium at zero magnetic field, K is the easy-axis anisotropy constant [the easy axis is oriented along the z axis (Fig. 1)], and D is the DMI constant.

To describe local and nonlocal relaxations, transversal and longitudinal dynamics within a unified phenomenological approach, the LLBar equation has no alternatives. Therefore, we describe the evolution of $\mathbf{M}$ in the frame of the LLBar equation [3], additionally accounting for a spin current [14]:

$$\dot{\mathbf{M}} = -\gamma[\mathbf{M}\times\mathbf{H}_{\rm eff}] + \gamma M\hat{\alpha}_l\mathbf{H}_{\rm eff} - \gamma M\nabla\cdot(\hat{\lambda}_{\rm nl}\nabla\mathbf{H}_{\rm eff}) - (\mathbf{u}\cdot\nabla)\mathbf{M} + (1/M)\beta\mathbf{M}\times[(\mathbf{u}\cdot\nabla)\mathbf{M}], \qquad (2)$$

where γ is the gyromagnetic ratio, $\mathbf{H}_{\rm eff} = -\delta\Phi/\delta\mathbf{M}$ is the effective magnetic field, $\hat{\alpha}_l$ is the local relaxation tensor, $\hat{\lambda}_{\rm nl}$ is the nonlocal relaxation tensor, β is the ratio of nonadiabatic to adiabatic spin torque, $\mathbf{u} = (g\mu_B P/2eM)\mathbf{j}$, g is the Landé factor, μ_B is the Bohr magneton, P is the polarization, and $\mathbf{j}$ is the current density. Following Garanin and co-workers and Evans and co-workers, we choose $\hat{\alpha}_l$ in a way that the longitudinal evolution is determined by the longitudinal relaxation constant $\alpha_{\parallel}$ [31–33], whereas the transversal dissipative evolution is determined by the transversal local relaxation constant (the enhanced Gilbert damping), which includes the Gilbert α_G and the spin-pumping $\alpha_{\rm sp}$ contributions. The nonlocal relaxation is supposed to be isotropic [34]; therefore $\hat{\lambda}_{\rm nl}$ is replaced by the nonlocal relaxation constant $\lambda_{\rm nl}$. When $\chi_{\parallel} = 0$ and $\lambda_{\rm nl} = 0$ (the longitudinal and nonlocal relaxations are disregarded), the LLBar equation in the form of Eq. (2) reduces to the Landau-Lifshitz equation with the standard

Gilbert relaxation term (LLG equation). When $\lambda_{nl} = 0$, Eq. (2) reduces to the LLB equation.

We restrict our analysis [34] to the case when the change of M at the DW location is small (Fig. 1), $\chi_{\|} H_K \ll M$, where $H_K = 2MK$ is the anisotropy field. Describing the dynamics of $\mathbf{M}$ in the frame of the LLBar equation, the magnetic-field- and spin-current-induced velocity of the DW in a flow regime can be written in a standard form [35]:

$$v = \mu H + |\mathbf{u}|\beta/\alpha_{\text{eff}}, \quad \mu = \gamma\Delta/\alpha_{\text{eff}}, \tag{3}$$

where $\Delta \approx \sqrt{A/K}$ is a DW's thickness. Still, the standard transversal contributions to damping in this equation for v are replaced by the effective relaxation constant $\alpha_{\text{eff}} = \alpha_l^{\perp} + \alpha_{\text{nl}}^{\perp} + \alpha_l^{\|} + \alpha_{\text{nl}}^{\|}$. Here, the enhanced Gilbert damping $\alpha_l^{\perp} = \alpha_G + \alpha_{\text{sp}}$ and $\alpha_{\text{nl}}^{\perp} = \lambda_{\text{nl}}/3\Delta^2$ [3,10–13,36] are well-known *transversal* local and nonlocal relaxation contributions, and $\alpha_l^{\|}$ is the longitudinal local and $\alpha_{\text{nl}}^{\|}$ the longitudinal nonlocal contributions to the damping of DWs. The *longitudinal* contributions are the focus of our study and originate from the evolution of M. The presence of $\alpha_l^{\|}$ and $\alpha_{\text{nl}}^{\|}$ leads to a decrease of the mobility, an increase of the Walker field, $H_W = v_W/\mu$, where v_W is the Walker velocity, and a lowering of the efficiency of the interaction of the spin current with the DW.

For the case of a dominant local transversal relaxation $\alpha_{\text{nl}}^{\perp} \ll \alpha_l^{\perp}$, $\alpha_l^{\|}$ is larger than $\alpha_{\text{nl}}^{\|}$, and the effective relaxation constant is $\alpha_{\text{eff}} \approx \alpha_l^{\perp} + \alpha_l^{\|}$, where for the DMI field $H_D = 2MD/\Delta$ lower than the critical DMI field $H_D^c = 4H_K/\pi$ [37], the expression for $\alpha_l^{\|}$ takes the form [34]

$$\alpha_l^{\|} = \frac{32}{15}\frac{\mathcal{F}(v)\,\chi_{\|}^2 H_K^2}{\alpha_{\|} M^2}\left(1 \pm \frac{15}{16}\frac{|H_D|}{H_D^c}\sqrt{1-\frac{v^2}{v_W^2}} + \frac{5}{2\pi^2}\left[\frac{H_D}{H_D^c}\right]^2\left[1-\frac{v^2}{v_W^2}\right]\right). \tag{4}$$

Here, $\alpha_{\|}$ is linked with the enhanced Gilbert damping. In the frame of the model of a classical spin interacting with a bath, $\alpha_{\|} = 2T\alpha_l^{\perp}/(3T_C - T)$ at any temperature below T_C [31–33], where T_C is the Curie temperature. Therefore, in contrast to previous studies of the longitudinal relaxation, the longitudinal contribution to the damping of DWs is rigorously connected with the experimentally measured transversal relaxation constant. Note that, in contrast to the transversal picture, the longitudinal contribution to the damping of DWs increases as the enhanced Gilbert damping decreases.

We note that a nonzero H_D leads to the dependence of $\alpha_l^{\|}$ on the velocity. The plus and minus signs in Eq. (4) correspond to the upper and lower branches of DWs [38], which have different structure and energy, and, as is seen from Fig. 2(a), different $\alpha_l^{\|}$. In the flow regime, only the minus sign is realized and a nonzero H_D results in a decrease of $\alpha_l^{\|}$. In the precessional regime (Walker breakdown, $H > H_W$), $\mathbf{M}$ rotates around the z axis, leading to the oscillatory motion of the DW with interchanges of the signs of type "−" ⇒ "+" ⇒ "−" and so on. When the DW is on the upper branch in Fig. 2(a), a nonzero DMI results in an increase of $\alpha_l^{\|}$.

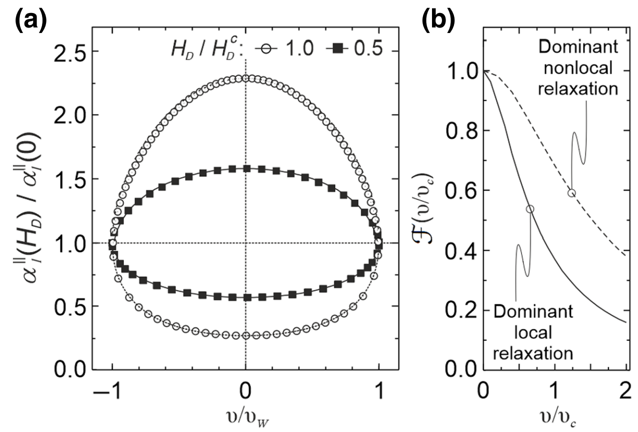

FIG. 2. (a) Influence of the DMI field, H_D, on the velocity dependence of $\alpha_l^{\|}$ [Eq. (4)]. (b) Numerically calculated $\mathcal{F}(v)$, describing the decrease of the contribution of the longitudinal relaxation mechanism due to the effect related to the lagging of M. Two limiting cases are presented: the dominant local ($\alpha_l^{\perp} \gg \alpha_{\text{nl}}^{\perp}$) and nonlocal ($\alpha_l^{\perp} \ll \alpha_{\text{nl}}^{\perp}$) relaxation processes.

In addition to the discussion above, there is another effect that leads to the dependence of $\alpha_l^{\|}$ on v. It is related to the process of lagging of M from its equilibrium value. This effect is described by the velocity-dependent function $\mathcal{F}(v)$ and characterized by the critical velocity $v_c = \Delta/\tau_0$, where $\tau_0 = \chi_{\|}/\gamma M\left(\alpha_{\|} + 3\alpha_{\text{nl}}^{\perp}\right)$ is the characteristic relaxation time for M to its equilibrium value [7]. When the DW moves with velocity v, the longitudinal component of the effective field acts on $\mathbf{M}$ at a given point during time Δ/v. If this time is comparable to or shorter than τ_0, M lags behind its equilibrium value. This leads to a decrease of the dip in M within the DW. Consequently, the longitudinal relaxation mechanism becomes less effective. The numerically calculated $\mathcal{F}(v)$ for the case of dominant local relaxation processes is presented in Fig. 2(b) with a solid line [34]. $\mathcal{F}(v)$ decreases with an increase of v. For $v \ll v_c$, this decrease is not pronounced and $\mathcal{F} \lesssim 1$. For $v \gg v_c$, the longitudinal relaxation mechanism is switched off as $\mathcal{F}$ tends to 0.

A similar discussion is valid for the case of the dominant nonlocal relaxation processes [34].

The dissipation mechanism related to the appearance of the dip in the magnetization length is generic and applicable to any magnetic soliton. This dip at a location of both DWs and solitons is in the range of $\chi_{\|} H_K$ and is a characteristic of a system (but not of a soliton).

Analogously to DWs, this dip follows a moving soliton. Therefore, when the longitudinal relaxation is relevant for DW dynamics, we predict that the longitudinal relaxation should be relevant also for solitons, including skyrmions.

In this respect, for the description of soliton dynamics, the LLG equation, which accounts only for the transversal relaxation, is physically incomplete. Still, when the change of the magnetization length at the domain wall location is small, $\chi_\parallel H_K \ll M$, the LLG equation can be used for the description of the dynamics of domain walls. However, to recover the physical consistency of the LLG equation, instead of standard transversal contributions to the damping, the effective relaxation constant (a sum of transversal and longitudinal contributions) should be taken. This effective relaxation constant enters the expressions for every standard parameter describing the dynamics of domain walls, e.g., a mobility, a response of domain walls to spin current, a Walker field. For topologically protected solitons with a finite topological charge (like skyrmions), the effective relaxation constant calculated for this soliton enters to the dissipation tensor in the Thiele equation. Dependent on the regime, the Dzyaloshinskii-Moriya interaction can enhance or suppress the longitudinal relaxation.

The key parameter for the analysis of the longitudinal contributions is the longitudinal magnetic susceptibility, $\chi_\parallel$. We calculate the temperature dependence $\chi_\parallel(T) = dM(H,T)/dH$ at $H = 0$ within the spin wave approach, which is proven to describe thermodynamic properties of ultrathin ferromagnetic films sufficiently far from the Curie temperature [26–30]. The external magnetic field H and the anisotropy field H_K enter the expression for $M(H,T)$ via the spin wave gap $\omega_0 = \gamma(H_K + H)$. For the limiting cases of two-dimensional [30] and three-dimensional [39] ferromagnets and for temperatures higher than the excitation temperature of the spin wave gap (of the order of 1 K), $\chi_\parallel$ reduces to

$$\chi_\parallel^{2D} = \frac{\mu_B}{4\pi AMt}\frac{k_B T}{\hbar\omega_0}, \quad \chi_\parallel^{3D} = \frac{0.028\mu_B k_B T}{(AM)^{3/2}\hbar\sqrt{\gamma\omega_0}}, \tag{5}$$

where k_B is the Boltzmann constant and t is the film thickness. $\chi_\parallel^{2D}$ depends on t, which can be explained as follows. The change of M due to H is determined by the change of the magnon's density [26–30]. The number of magnons in one quantum state depends only on their energy. Thus, the number of magnons in two-dimensional films is independent of t. Consequently, the magnon's density and $\chi_\parallel$ both increase with a decrease of the film thickness t. For typical parameters of ultrathin Co films with $t = 0.6$ nm and perpendicular anisotropy at room temperature ($A = 4.8 \times 10^{-13}$ cm^2, $H_K = 10$ kOe, $M = 800$ emu/cm^3 [30]), $\chi_\parallel^{2D} \sim 7.7 \times 10^{-3}$, which is about 10 times larger than $\chi_\parallel^{3D} \sim 8.4 \times 10^{-4}$. This enhancement of $\chi_\parallel$ for the two-dimensional case is in accordance with the Mermin-Wagner theorem, stating that for two-dimensional magnets

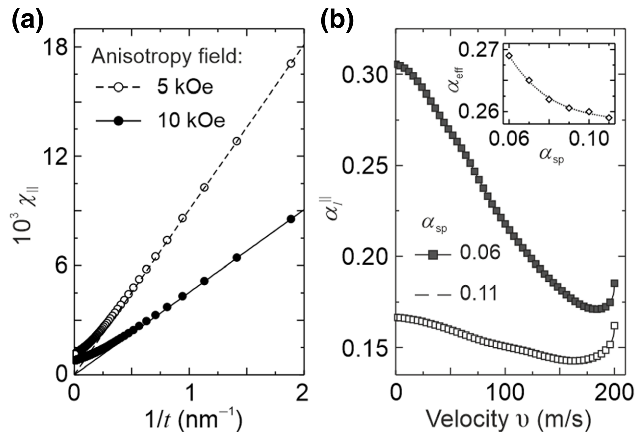

FIG. 3. (a) The dependence of the longitudinal magnetic susceptibility $\chi_\parallel$ on the thickness of the magnetic film t. Solid (dashed) lines are calculated according to $\chi_\parallel^{2D}$ [Eq. (5)]. (b) Velocity dependence of the local longitudinal relaxation constant $\alpha_l^\parallel$ for the case of ultrathin Pt/Co(0.6 nm)/oxide film for $\alpha_{sp} \approx 0.06, 0.11$. The inset of (b) shows the dependence of the effective relaxation constant $\alpha_{eff} \approx \alpha_{sp} + \alpha_l^\parallel$ on α_{sp}.

with zero spin wave gap a long-range order is absent [27]. Additionally, we model $\chi_\parallel$ for $H_K = 5, 10$ kOe and different t [34]. Figure 3(a) demonstrates that $\chi_\parallel$ increases for films with t smaller than 10 nm. For t smaller than about 2 nm, mainly spin wave modes with the wave vector parallel to the sample plane are excited. In this case, $\chi_\parallel$ reduces to $\chi_\parallel^{2D}$ and the system can be considered as two-dimensional.

In the following, we apply our theory to analyze the experimental data on asymmetric Co sandwiches. For a prototypical system of Pt/Co/oxide with Co thickness in the range of $t = 0.6$ nm, the effective damping constant estimated based on the magnetic-field- and spin-current-driven DW dynamics experiments is $\alpha_{eff} \approx 0.32$ [22] (see discussion in the Supplemental Material [34]). The spin pumping mechanism alone cannot explain this high value as for these samples its contribution is about 5 times smaller, $\alpha_{sp} \approx 0.06$ [24,25]. Other transversal contributions to the damping of DWs, α_G and $\alpha_{nl}^\perp$, for metallic thin films are comparable [36,40–44] and substantially smaller than α_{sp}. Therefore, this higher damping for the case of DWs moving in ultrathin ferromagnets cannot be explained in the framework of the transversal relaxation mechanism. The following estimations are performed for the typical magnetic parameters of Pt/Co/oxide with $t = 0.6$ nm at room temperature: $A = 4.8 \times 10^{-13}$ cm^2, $H_K = 10$ kOe, $M_0 = 800$ emu/cm^3, $H_D/H_D^c = 0.4$, $T_C = 380$ K [34]. We demonstrate that the longitudinal relaxation constant $\alpha_l^\parallel$ can be as large as 0.3 for small velocities [Fig. 3(b)]. This results in the effective relaxation constant $\alpha_{eff} = \alpha_{sp} + \alpha_l^\parallel$ of about 0.36. The contribution of the longitudinal relaxation decreases with an increase of the velocity of the moving DW, but remains larger than the spin pumping contribution. For a typical velocity of DWs of 100 m/s

($v_W \approx 200$ m/s, $v_c \approx 220$ m/s), $\alpha_{\rm eff}$ is about 0.27. Interestingly, this estimation is insensitive to the change of the $\alpha_{\rm sp}$. Even when $\alpha_{\rm sp}$ is increased to 0.11 (typical for asymmetric Co sandwiches with Co thickness of 0.3 nm [25]), Fig. 3(b) shows that $\alpha_l^{\|} \approx 0.15$, which results in $\alpha_{\rm eff} \approx 0.26$. The inset in Fig. 3(b) shows the dependence of $\alpha_{\rm eff} = \alpha_{\rm sp} + \alpha_l^{\|}$ on $\alpha_{\rm sp}$ calculated at a typical velocity of the DWs of 100 m/s. The reason for the observation that $\alpha_{\rm eff}$ weakly decays with $\alpha_{\rm sp}$ is that the critical velocity $v_c \propto \alpha_{\rm sp}$. Therefore, for larger $\alpha_{\rm sp}$ the decrease of $\alpha_l^{\|}$ due to the lagging of M is less pronounced. We note that the longitudinal contribution to the DW damping remains larger than $\alpha_{\rm sp}$, highlighting the relevance of the longitudinal relaxation mechanism in explaining the relaxation of DWs in ultrathin magnetic films with DMI. Assuming $\alpha_{\rm sp}$ to be temperature independent, the analysis of $\alpha_l^{\|}$ for different T demonstrates that $\alpha_l^{\|}$ is larger than $\alpha_{\rm sp}$ from $T \approx 100$ K up to T_C. To compare, for typical parameters of bulk Co and $\alpha_G = 0.01$ at room temperature, $\alpha_l^{\|}$ is negligible (of the order of 1% of α_G). As $\chi_{\|}^{\rm 2D} \propto 1/t$ and $\alpha_{\rm sp} \propto 1/t$, the longitudinal contribution decays with an increase of the film thickness, $\alpha_l^{\|} \propto 1/t$. This prediction is in line with a smaller effective relaxation constant $\alpha_{\rm eff} \approx 0.13$ extracted from DW motion experiments for thicker Pt/Co/oxide samples with Co thickness $t = 1$ nm [23]. When applying our model to the experimental parameters relevant for these samples [23], we predict $\alpha_{\rm eff} \sim 0.1$, which is close to the experimental estimate.

In conclusion, we identify that the contribution of the so far neglected longitudinal relaxation to the damping of solitons without strong structural suppression of the magnetization length (like DWs, skyrmions) in ultrathin (2D) magnetic films sufficiently far from the Curie temperature is comparable to or even stronger than that of any other transversal mechanisms discussed before, including spin pumping. This enhancement is associated with the Mermin-Wagner theorem and caused by a higher sensitivity of 2D ferromagnets to an effective field compared with their 3D counterparts.

While the transversal evolution of the magnetization can be nondissipative, the longitudinal one can be only dissipative. This fundamental difference leads to a counterintuitive situation: the longitudinal contribution to the damping of a soliton increases as the enhanced Gilbert constant (measured, e.g., from the ferromagnetic resonance technique) decreases. Thus, a smaller value of the enhanced Gilbert constant does not necessarily lead to faster motion of a soliton. There is an optimal value of the enhanced Gilbert constant, for which the response of a soliton to external stimuli is maximized.

Our results are also valid for technologically relevant multilayer stacks consisting of ultrathin magnetic films, when the interaction between magnetic layers is sufficiently small.

Acknowledgments.—We thank Prof. Kulagin for fruitful discussions on the domain wall profile in samples with DMI. This work is financed in part via German Research Foundation (DFG) Grants No. MA5144/14-1, No. MA5144/22-1, and No. MA5144/24-1. I.A.Y. is supported financially in the frame of the DAAD program "Research Stays for University Academics and Scientists, 2019." B.A.I. acknowledges support by National Research Foundation of Ukraine Grant No. 2020.02/0261.

[1] L. D. Landau and E. M. Lifshits, On the theory of the dispersion of magnetic permeability in ferromagnetic bodies, Sov. Phys. **8**, 153 (1935).

[2] L. N. Bulaevskii and V. L. Ginzburg, Temperature dependence of the shape of the domain wall in ferromagnetics and ferroelectrics, Soviet Phys. JETP **18**, 530 (1964).

[3] V. G. Bar'yakhtar, Phenomenological description of relaxation processes in magnetic materials, Zh. Eksp. Theor. Fiz. **87**, 1501 (1984). [Sov. Phys. JETP **60**, 863, (1984)].

[4] E. Beaurepaire, J.-C. Merle, A. Daunois, and J.-Y. Bigot, Ultrafast Spin Dynamics in Ferromagnetic Nickel, Phys. Rev. Lett. **76**, 4250 (1996).

[5] I. Radu, *et al.*, Transient ferromagnetic-like state mediating ultrafast reversal of antiferromagnetically coupled spins, Nature **472**, 205 (2011).

[6] J. H. Mentink, *et al.*, Ultrafast Spin Dynamics in Multisublattice Magnets, Phys. Rev. Lett. **108**, 057202 (2012).

[7] I. A. Yastremsky, P. M. Oppeneer, and B. A. Ivanov, Theory of fast time evolution of nonequilibrium spin states in magnetic heterostructures, Phys. Rev. B **90**, 024409 (2014).

[8] R. J. Donnely, *Quantized Vortices in Helium II*, Ed. by A. M. Goldman, P. V. E. McClintock, and M. Springford (Cambridge Univ. Press, Cambridge, 1991).

[9] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966; Mir, Moscow, 1968).

[10] V. G. Bar'yakhtar, B. A. Ivanov, T. K. Sobolyeva, and A. L. Sukstanskii, Theory of dynamic soliton relaxation in ferromagnets, Zh. Eksp. Teor. Fiz. **91**, 1454 (1986).

[11] E. G. Galkina, B. A. Ivanov, and K. A. Safaryan, Theory of retardation of magnetic domain walls in rhombic magnetic materials, JETP **84**, 87 (1997).

[12] V. G. Baryakhtar, B. A. Ivanov, A. L. Sukstanskii, and E. Yu. Melikhov, Soliton relaxation in magnets, Phys. Rev. B **56**, 619 (1997).

[13] W. Wang, *et al.*, Phenomenological description of the nonlocal magnetization relaxation in magnonics, spintronics, and domain wall dynamics, Phys. Rev. B **92**, 054430 (2015).

[14] C. Schieback, D. Hinzke, M. Kläui, U. Nowak, and P. Nielaba, Temperature dependence of the current-induced domain wall motion from a modified Landau-Lifshitz-Bloch equation, Phys. Rev. B **80**, 214403 (2009).

[15] M. Grahl and J. Kötzler, Speeding-up and scaling of domain-wall relaxation near T_c of a uniaxial ferromagnet, Z. Phys. B **75**, 527 (1989).

[16] J. Kötzler, M. Grahl, I. Sessler, and J. Ferre, Size and Fluctuation Effects on the Dynamics of Linear Domain Walls in an Ising Ferromagnet, Phys. Rev. Lett. **64**, 2446 (1990).

[17] J. Kötzler, D. A. Garanin, M. Hartl, and L. Jahn, Evidence for Critical Fluctuations in Bloch Walls Near Their Disordering Temperature, Phys. Rev. Lett. **71**, 177 (1993).

[18] E. G. Galkina, B. A. Ivanov, and V. A. Stephanovich, Phenomenological theory of Bloch point relaxation, JMMM **118**, 373 (1993).

[19] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Enhanced Gilbert Damping in Thin Ferromagnetic Films, Phys. Rev. Lett. **88**, 117601 (2002).

[20] Y. Tserkovnyak, A. Brataas, Gerrit E. W. Bauer, and B. I. Halperin, Nonlocal magnetization dynamics in ferromagnetic heterostructures, Rev. Mod. Phys. **77**, 1375 (2005).

[21] A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Scattering Theory of Gilbert Damping, Phys. Rev. Lett. **101**, 037207 (2008).

[22] I. M. Miron, *et al.*, Fast current-induced domain-wall motion controlled by the Rashba effect, Nat. Mater. **10**, 419 (2011).

[23] Oleksii M. Volkov, *et al.*, Domain-Wall Damping in Ultrathin Nanostripes with Dzyaloshinskii-Moriya Interaction, Phys. Rev. Appl. **15**, 034038 (2021).

[24] J-M. L. Beaujour, J. H. Lee, A. D. Kent, K. Krycka, and C.-C. Kao, Magnetization damping in ultrathin polycrystalline Co films: Evidence for nonlocal effects, Phys. Rev. B **74**, 214405 (2006).

[25] A. J. Schellekens, *et al.*, Determining the Gilbert damping in perpendicularly magnetized Pt/Co/AlOx films, Appl. Phys. Lett. **102**, 082405 (2013).

[26] F. Bloch, Zur Theorie des Ferromagnetismus, Z. Phys. **61**, 206 (1930).

[27] N. D. Mermin and H. Wagner, Absence of Ferromagnetism or Antiferromagnetism in One- or Two-Dimensional Isotropic Heisenberg Models, Phys. Rev. Lett. **17**, 1133 (1966).

[28] R. P. Erickson and D. I. Mills, Thermodynamics of thin ferromagnetic films in the presence of anisotropy and dipolar coupling, Phys. Rev. B **44**, 11825 (1991).

[29] P. Bruno, Magnetization and Curie temperature of ferromagnetic ultrathin films: The influence of magnetic anisotropy and dipolar interactions, Mater. Res. Soc. Symp. Proc. **231**, 299 (1992).

[30] I. A. Yastremsky, *et al.*, Thermodynamics and Exchange Stiffness of Asymmetrically Sandwiched Ultrathin Ferromagnetic Films with Perpendicular Anisotropy, Phys. Rev. Appl. **12**, 064038 (2019).

[31] D. A. Garanin, Fokker-Planck and Landau-Lifshitz-Bloch equations for classical ferromagnets, Phys. Rev. B **55**, 3050 (1997).

[32] D. A. Garanin and O. Chubykalo-Fesenko, Thermal fluctuations and longitudinal relaxation of single-domain magnetic particles at elevated temperatures, Phys. Rev. B **70**, 212409 (2004).

[33] R. F. L. Evans, *et al.*, Stochastic form of the Landau-Lifshitz-Bloch equation, Phys. Rev. B **85**, 014433 (2012).

[34] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevApplied.17.L061002 for further details of the derivation of the equation of the main text, analysis of the different relaxation contributions, analysis of the structure of the moving domain wall, and estimation of parameters of ultrathin films used for comparison with the experiment, which includes Refs. [45–54].

[35] A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Micromagnetic understanding of current-driven domain wall motion in patterned nanowires, Europhys. Lett. **69**, 990 (2005).

[36] H. Y. Yuan, Z. Yuan, K. Xia, and X. R. Wang, Influence of nonlocal damping on the field-driven domain wall motion, Phys. Rev. B **94**, 064415 (2016).

[37] A. Thiaville, S. Rohart, E. Jue, V. Cros, and A. Fert, Dynamics of Dzyaloshinskii domain walls in ultrathin magnetic films, Europhys. Lett. **100**, 57002 (2012).

[38] N. L. Schryer and L. R. Walker, The motion of 180° domain walls in uniform dc magnetic fields, J. Appl. Phys. **45**, 5406 (1974).

[39] B. E. Argyle, S. H. Charap, and E. W. Pugh, Deviations from $T^3/2$ Law for Magnetization of Ferrometals: Ni, Fe, and Fe+3% Si, Phys. Rev. **132**, 1538 (1963).

[40] H. T. Nembach, J. M. Shaw, C. T. Boone, and T. J. Silva, Mode- and Size-Dependent Landau-Lifshitz Damping in Magnetic Nanostructures: Evidence for Nonlocal Damping, Phys. Rev. Lett. **110**, 117201 (2013).

[41] S. Zhang, Steven S.-L. Zhang, Generalization of the Landau-Lifshitz-Gilbert Equation for Conducting Ferromagnets, Phys. Rev. Lett. **102**, 086601 (2009).

[42] A. A. Starikov, P. J. Kelly, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Unified First-Principles Study of Gilbert Damping, Spin-Flip Diffusion, and Resistivity in Transition Metal Alloys, Phys. Rev. Lett. **105**, 236601 (2010).

[43] T. Weindler, *et al.*, Magnetic Damping: Domain Wall Dynamics versus Local Ferromagnetic Resonance, Phys. Rev. Lett. **113**, 237204 (2014).

[44] Z. Yuan, *et al.*, Gilbert Damping in Noncollinear Ferromagnets, Phys. Rev. Lett. **113**, 266603 (2014).

[45] V. V. Slastikov, C. B. Muratov, J. M. Robbins, and O. A. Tretiakov, Walker solution for Dzyaloshinskii domain wall in ultrathin ferromagnetic films, Phys. Rev. B **99**, 100403(R) (2019).

[46] Stavros Komineas, C. Melcher, and S. Venakides, Traveling domain walls in chiral ferromagnets, Nonlinearity **32**, 2392 (2019).

[47] O. Boulle, *et al.*, Domain Wall Tilting in the Presence of the Dzyaloshinskii-Moriya Interaction in Out-of-Plane Magnetized Magnetic Nanotracks, Phys. Rev. Lett. **111**, 217203 (2013).

[48] E. G. Galkina, B. A. Ivanov, N. E. Kulagin, L. M. Lerman, and I. A. Yastremsky, Dynamics of Domain Walls in Chiral Magnets, JETP **132**, 572 (2021).

[49] V. P. Kravchuk, Infuence of Dzialoshinskii-Moriya interaction on static and dynamic properties of a transverse domain wall, J. Magn. Magn. Mater. **367**, 9 (2014).

[50] A. A. Thiele, On the momentum of ferromagnetic domains, J. Appl. Phys. **47**, 2759 (1976).

[51] E. G. Galkina and B. A. Ivanov, On the dispersion relation for kink - type solitons in one - dimensional ferromagnets, JETP Lett. **71**, 259 (2000).

[52] I. A. Yastremsky and V. E. Kireev, The evolution of the total magnetization of a Ni-Fe heterostructure after exposure to a femtosecond laser pulse, Low. Tem. Phys. **42**, 290 (2016).

[53] L. J. de Jongh and A. R. Miedema, Experiments on simple magnetic model systems, Adv. Phys. **50**, 947 (2001).

[54] N. Kazantseva, *et al.*, Towards multiscale modeling of magnetic materials: Simulations of FePt, Phys. Rev. B **77**, 184428 (2008).