

## Nonzero Skyrmion Hall Effect in Topologically Trivial Structures


Robin Msiska<sup>1</sup>,<sup>2</sup>,<sup>3</sup>,<sup>4</sup> Davi R. Rodrigues<sup>2,\*</sup>, Jonathan Leliaert<sup>3</sup>, and Karin Everschor-Sitte<sup>1,4</sup>

<sup>1</sup>*Faculty of Physics, University of Duisburg-Essen, 47057 Duisburg, Germany*

<sup>2</sup>*Department of Electrical and Information Engineering, Politecnico di Bari, 70126 Bari, Italy*

<sup>3</sup>*Department of Solid State Sciences, Ghent University, 9000 Ghent, Belgium*

<sup>4</sup>*Center for Nanointegration Duisburg-Essen (CENIDE), 47057 Duisburg, Germany*

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It is widely believed that the skyrmion Hall effect, often disruptive for device applications, vanishes for overall topologically neutral structures such as (synthetic) antiferromagnetic skyrmions and skyrmioniums due to a compensation of Magnus forces. In this paper, however, we report that in contrast to the case of spin-transfer-torque-driven skyrmion motion, this notion is generally false for spin-orbit-torque-driven objects. We show that the skyrmion Hall angle is directly related to their helicity and imposes an unexpected roadblock for developing faster and lower input racetrack memories based on spin-orbit torques.

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Magnetic skyrmions are localized whirl-like magnetic textures with a nontrivial topology [1]. Following their early theoretical prediction [2], they were first experimentally observed in the form of a skyrmion lattice [3]. Skyrmions driven by electric currents have been shown to exhibit a significant transverse component in addition to their longitudinal current-induced motion along a track. This deviation in skyrmion motion has been termed the skyrmion Hall effect [4–8]. While the physics of the skyrmion Hall effect is fascinating, and allows skyrmions to evade defects [9–12], it often imposes a challenge for skyrmion-based devices [1,13–16]. This is particularly important for shift-register magnetic storage devices such as skyrmion racetrack proposals [17–19]. Here, the presence-absence of a skyrmion is used to store information on a nanowire. For such racetracks, the driving speed of magnetic skyrmions is limited by the skyrmion Hall effect, as beyond a certain drive threshold skyrmions get pushed to the boundary of the sample.

Numerous suggestions have been made to suppress or eliminate the skyrmion Hall effect [20–27]. Prominent among them is the idea of using combined skyrmion structures with opposite winding numbers. Note that it is not the topological charge of the individual skyrmion components alone that is important: the relevant quantity is actually the winding number scaled with the saturation magnetization. Therefore, we introduce the term “topologically neutral.” This notion refers to magnetic structures with an

effective net zero winding number, defined by adding the two opposite individual skyrmion numbers scaled by their corresponding saturation magnetizations. Among topologically neutral magnetic textures are skyrmions structures in antiferromagnetic and synthetic antiferromagnetic (SAF) materials, which have the additional advantages of obeying faster dynamics as well as small stray fields, and being insensitive to external fields [28,29]. Another proposal for the elimination of the skyrmion Hall effect is the use of skyrmioniums [23,30–32], which are both topologically neutral and topologically trivial. In these systems there is the wide belief that the opposite topological charges of the two substructures lead to a cancelation of the acting Magnus forces [23,29,31–36].

In this work, we show that this picture is generally not correct for spin-orbit-torque (SOT) driven skyrmions in (synthetic) antiferromagnets and skyrmioniums. This effect occurs as the Magnus forces acting on the different skyrmionic structures do not cancel (see Fig. 1). By computing the Hall angle, we reveal that there is typically a nonzero skyrmion Hall effect originating in the structure’s helicity, that is, the azimuthal angle of the skyrmionlike structures [see Fig. 1(d)]. The helicity of a skyrmionic structure is typically determined by the twisting interactions, such as Dzyaloshinskii-Moriya interactions (DMI) and dipolar fields [21,37,38], and can be measured via resonant elastic X-ray scattering techniques [39].

First, we present the SOT-driven magnetization dynamics for topologically neutral structures, that is, the skyrmionium and the (synthetic) antiferromagnetic skyrmion. Then we derive the helicity dependence of the skyrmion Hall angle for such structures. Our analytical results were

\*davi.rodrigues@poliba.it

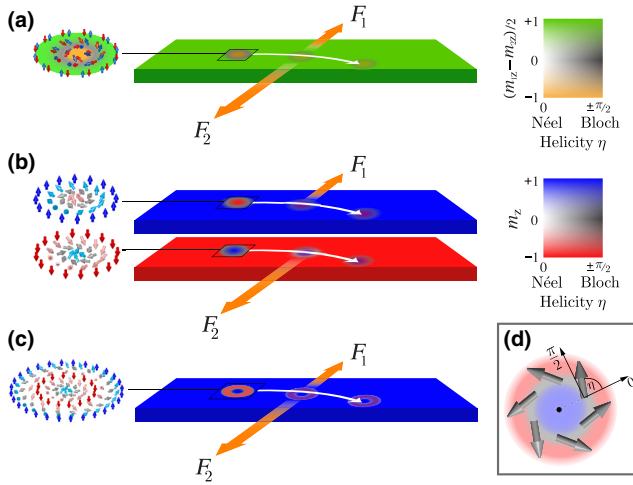


FIG. 1. Net skyrmion Hall effect in topologically neutral skyrmionic structures. The Magnus forces acting on (a) the different sublattices for an antiferromagnetic skyrmion, (b) the different layers for a synthetic antiferromagnetic skyrmion, and (c) the different cocentric skyrmions for the skyrmionium do not compensate each other. The gray scale encodes the helicity and the color shows the out-of-plane component of (a) the Néel order parameter, and (b),(c) the magnetization. In (d) we show the definition of the helicity as the azimuthal angle of the magnetization in a skyrmionic structure. Notice that  $\eta = 0$  corresponds to an outward pointing Néel skyrmion, whereas  $\eta = \pi/2$  represents a Bloch skyrmion as shown.

verified with micromagnetic simulations. Finally, we summarize our results and discuss implications.

The current-driven magnetization dynamics of a ferromagnetic material is well described by the Landau-Lifshitz-Gilbert equation [40,41]

$$\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H}^{\text{eff}} + \alpha \mathbf{m} \times \partial_t \mathbf{m} + \mathbf{T}(\mathbf{m}), \quad (1)$$

where  $\gamma$  is the gyromagnetic ratio and  $\alpha$  is the Gilbert damping parameter. The effective magnetic field is given by  $\mathbf{H}^{\text{eff}} = -(1/M_s) \delta E[\mathbf{m}] / \delta \mathbf{m}$ , where  $M_s$  is the magnetization saturation and  $E$  is the total free energy of the system. The term  $\mathbf{T}(\mathbf{m})$  represents torques which are acting on the system. In the case of SOTs it takes the form [42–44]

$$\mathbf{T}^{\text{SOT}}(\mathbf{m}) = \xi \mathbf{m} \times (\hat{\mathbf{z}} \times \mathbf{v}^{\text{eff}}) + \mathbf{m} \times [\mathbf{m} \times (\hat{\mathbf{z}} \times \mathbf{v}^{\text{eff}})], \quad (2)$$

where  $\xi$  is the field- to dampinglike torque ratio,  $\mathbf{v}^{\text{eff}} = \gamma \hbar \theta_{\text{Hall}} \mathbf{j} / (2M_s e l)$  is the effective spin velocity with  $\hbar$  being the Planck constant,  $\theta_{\text{Hall}}$  is the spin Hall ratio,  $\mathbf{j}$  is the applied current density,  $e$  is the electronic charge, and  $l$  the thickness of the sample [8,18].

Assuming that the applied torques are weak compared to the magnetic interactions, we consider an ansatz of a rigid magnetic texture moving with drift velocity  $\mathbf{v}^d$  as  $\mathbf{m} = \mathbf{m}(\mathbf{r} - \mathbf{v}^d t)$ . The skyrmionium and antiferromagnetic skyrmions can be described in terms of coupled skyrmions,

where the index  $i = 1, 2$  labels the corresponding skyrmion in what follows. In particular, we can describe a skyrmionium as two concentric skyrmions with different radii  $R_1$  and  $R_2$ , and antiferromagnetic skyrmions as a pair of skyrmions belonging to two different layers/sublattices with same radius  $R_1 = R_2$ . The current-driven dynamics of these objects can be described by the coupled Thiele equations for each skyrmion structure described by  $\mathbf{m}_i(\mathbf{r} - \mathbf{v}^d t)$  [45,46] (for details, see Sec. I of the Supplemental Material [47]):

$$-\mathbf{G}_i \times \mathbf{v}^d - \alpha \mathbf{D}_i \mathbf{v}^d + \gamma \mathbf{F}_i^{\text{int}} + (\xi \mathbf{T}_{\text{FL } i} + \mathbf{T}_{\text{DL } i}) \mathbf{v}^{\text{eff}} = \mathbf{0}. \quad (3)$$

Here  $\mathbf{G}_i = -4\pi Q_i \mathbf{e}_z$  is the gyrocoupling vector with the topological magnetic charge

$$Q_i = \frac{1}{4\pi} \int dx dy \mathbf{m}_i \cdot (\partial_x \mathbf{m}_i \times \partial_y \mathbf{m}_i). \quad (4)$$

Furthermore,

$$(\mathbf{D}_i)_{ab} = \int dx dy (\partial_a \mathbf{m}_i \cdot \partial_b \mathbf{m}_i) \quad (5)$$

is the dissipative tensor. The force  $\mathbf{F}_i^{\text{int}}$  captures the interaction between the two skyrmions. In the case of skyrmioniums it is mostly due to the exchange coupling between the skyrmions, while for the antiferromagnetic skyrmion it is due to the antiferromagnetic exchange between the two layers/sublattices. We notice that  $\mathbf{F}_1^{\text{int}} = -\mathbf{F}_2^{\text{int}}$ , since the net force acting on the coupled skyrmions due to the mutual interaction vanishes.

The tensors  $\mathbf{T}_{\text{FL}}$  and  $\mathbf{T}_{\text{DL}}$  represent the fieldlike and dampinglike spin torques. For SOTs they are given as

$$(\mathbf{T}_{\text{FL } i}^{\text{SOT}} \mathbf{v}^{\text{eff}})_a = \int dx dy (\hat{\mathbf{z}} \times \mathbf{v}^{\text{eff}}) \cdot \partial_a \mathbf{m}_i, \quad (6a)$$

$$(\mathbf{T}_{\text{DL } i}^{\text{SOT}} \mathbf{v}^{\text{eff}})_a = \int dx dy [\mathbf{m}_i \times (\hat{\mathbf{z}} \times \mathbf{v}^{\text{eff}})] \cdot \partial_a \mathbf{m}_i. \quad (6b)$$

For the skyrmionium as well as the (synthetic) antiferromagnetic skyrmion we obtain  $Q_1 = -Q_2$ ,  $\mathbf{D}_1/\mathbf{D}_2 \approx f_D(R_1/R_2)$ ,  $(\mathbf{T}_{\text{FL } 1}^{\text{SOT}})/(\mathbf{T}_{\text{FL } 2}^{\text{SOT}}) \approx -f_{\text{FL}}(R_1/R_2)$ ,  $(\mathbf{T}_{\text{DL } 1}^{\text{SOT}})/(\mathbf{T}_{\text{DL } 2}^{\text{SOT}}) \approx f_{\text{DL}}(R_1/R_2)$ , with positive functions  $f_D, f_{\text{DL}}$ , and  $f_{\text{FL}}$  that satisfy  $f_D(1) = f_{\text{FL}}(1) = f_{\text{DL}}(1) = 1$ . Thus, the net motion of the coupled skyrmions is given by the sum of the Thiele equations for each skyrmion and simplifies approximately to

$$\alpha \left[ 1 + f_D \left( \frac{R_2}{R_1} \right) \right] \mathbf{D}_1 \mathbf{v}^d + \left[ 1 + f_{\text{DL}} \left( \frac{R_2}{R_1} \right) \right] \mathbf{T}_{\text{DL } 1}^{\text{SOT}} \mathbf{v}^{\text{eff}} + \xi \left[ 1 - f_{\text{FL}} \left( \frac{R_2}{R_1} \right) \right] \mathbf{T}_{\text{FL } 1}^{\text{SOT}} \mathbf{v}^{\text{eff}} = \mathbf{0}. \quad (7)$$

For a radially symmetric skyrmionic structure  $\mathbf{D}$  is diagonal and independent of the helicity with

$(\mathbf{D}_1)_{xx} = (\mathbf{D}_1)_{yy} \equiv D$ . The spin torque tensors  $\mathbf{T}_{\text{FL}}^{\text{SOT}}$  and  $\mathbf{T}_{\text{DL}}^{\text{SOT}}$ , however, have helicity-dependent off-diagonal components,

$$(\mathbf{T}_{\text{FL}}^{\text{SOT}})_{ab} = \tau_{\text{FL}} (-\cos \eta \epsilon_{zab} + \sin \eta \delta_{ab}), \quad (8a)$$

$$(\mathbf{T}_{\text{DL}}^{\text{SOT}})_{ab} = \tau_{\text{DL}} (\sin \eta \epsilon_{zab} + \cos \eta \delta_{ab}). \quad (8b)$$

For example, for a Néel skyrmion,  $\eta = 0$  (Bloch skyrmion,  $\eta = \pi/2$ ) the dampinglike torques are aligned only (parallel) perpendicular to the effective spin velocity. The constants  $D$  and  $\tau_{\text{DL}}$  are determined by the specific radial profile of the skyrmionlike structure.

By leveraging all contributions in Eq. (7) we derive the drift velocity  $\mathbf{v}^d$  for the SOT-driven topologically neutral skyrmionic structures as

$$\mathbf{v}^d = \frac{1}{\alpha D} \left[ 1 + f_D \left( \frac{R_2}{R_1} \right) \right]^{-1} (v_{\parallel} \mathbf{v}^{\text{eff}} + v_{\perp} (\hat{\mathbf{z}} \times \mathbf{v}^{\text{eff}})), \quad (9)$$

with the parallel and perpendicular components being

$$v_{\parallel} = \tau_{\text{FL}} \xi \left[ 1 - f_{\text{FL}} \left( \frac{R_2}{R_1} \right) \right] \sin \eta + \tau_{\text{DL}} \left[ 1 + f_{\text{DL}} \left( \frac{R_2}{R_1} \right) \right] \cos \eta, \quad (10a)$$

$$v_{\perp} = \tau_{\text{FL}} \xi \left[ 1 - f_{\text{FL}} \left( \frac{R_2}{R_1} \right) \right] \cos \eta - \tau_{\text{DL}} \left[ 1 + f_{\text{DL}} \left( \frac{R_2}{R_1} \right) \right] \sin \eta. \quad (10b)$$

Equation (9) represents the main results of this manuscript. (i) Topologically neutral and even topologically trivial structures (skyrmionium) can experience a skyrmion Hall effect. Although Eq. (9) is independent of the topological charge, the magnetic structure does not move along the SOT spin velocity ( $\mathbf{v}^d \nparallel \mathbf{v}^{\text{eff}}$ ). (ii) In the limit of vanishing fieldlike torques or for  $R_1 = R_2$ , within the rigid particle ansatz the skyrmion Hall angle is independent of the strength of the SOTs as well as of the specific radial profile and shape; it does, however, depend on the helicity degree of freedom. In the limiting cases of a pure Néel (Bloch) type, the antiferromagnetic skyrmion moves along (perpendicular) to  $\mathbf{v}^{\text{eff}}$ .

Typically,  $\tau_{\text{FL}} \ll \tau_{\text{DL}}$ , such that fieldlike contribution to the skyrmion Hall angle is negligible. For this simpler case, the skyrmion Hall angle  $\theta_{\text{sky}} = \arctan(v_{\perp}/v_{\parallel})$ , that is, the angle between  $\mathbf{v}^{\text{eff}}$  and  $\mathbf{v}^d$ , is given by

$$\tan \theta_{\text{sky}} = -\tan \eta. \quad (11)$$

These analytical predictions have been confirmed for skyrmions in synthetic antiferromagnets and skyrmioniums by means of micromagnetic simulations using

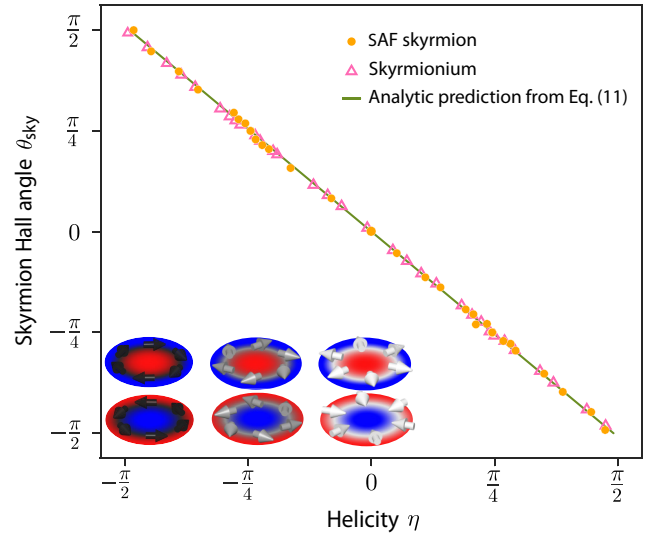


FIG. 2. Dependence of the skyrmion Hall angle  $\theta_{\text{sky}}$  on the helicity degree of freedom of topologically neutral skyrmionic structures.

MuMax<sup>3</sup> [48]. In Fig. 2 we show the skyrmion Hall angle as a function of helicity for various systems in the limit where Eq. (11) applies. Micromagnetic simulation details are in Sec. III of the Supplemental Material [47,49–51].

As a central result we find that topologically neutral magnetic structures, and even topologically trivial magnetic structures such as the skyrmionium, can obey a skyrmion Hall effect when driven by SOTs. The helicity of the topologically neutral skyrmionlike structures is crucial for the direction of motion of their SOT-driven dynamics [52]. This contradicts the usual understanding that associates the skyrmion Hall angle just to the topological charge, as is the case for spin-transfer-torque-driven dynamics (see Sec. II of the Supplemental Material [47]).

Our work motivates a re-evaluation of the experimental results obtained for skyrmions in SAFs [53]. In this experiment, the authors claimed to have observed only a nonsignificant skyrmion Hall angle for synthetic antiferromagnetic skyrmions compared to ferromagnetic skyrmions. We would like to mention that an additional source for the lack of a significant skyrmion Hall angle may be the presence of pinning effects [12,54–56], which have been proven to largely affect the skyrmion Hall angle in the ferromagnetic case [57,58]. In particular, the role of impurities in synthetic antiferromagnets needs to be clarified in future studies concerning such topologically neutral composite skyrmion structures. Furthermore, in SAFs, the difference in currents applied to each layer may also be a source of skyrmion Hall angle. For the skyrmionium, the coupling force between the inner and outer skyrmionic structure is bounded by the exchange interaction and may be overcome by current-induced torques leading to the annihilation of the skyrmionium [31].

We emphasize that the predicted skyrmion Hall angle for the topologically neutral magnetic structures is independent of the microscopic details and the different physical mechanisms that determine the helicity. The strongest mechanism is typically associated to chiral interactions such as DMI [21,38]. Weaker influences on the helicity are, for example, given by dipolar fields [37], which in antiferromagnets are typically much smaller compared to ferromagnets. Another source of intermediate helicity are thermal excitations of skyrmions [54] or more generally any type of excitations [59–61]. In particular for the last reason, we point out that tuning the skyrmion Hall angle to zero will always require fine tuning. Moreover, for skyrmionlike structures stabilized in frustrated magnets, the helicity degree of freedom is a Goldstone mode and can be manipulated by electrical fields and currents [30,62–64], thereby changing the skyrmion Hall angle.

An advantage of the skyrmion Hall angle dependence on the helicity is the possibility to control the motion direction of skyrmionic structures by changing the helicity. This can, for example, be done with electrical currents and voltage-controlled DMI [65], providing an additional toolbox for applications.

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