Letter

Lens-Free Optical Detection of Thermal Motion of a Submillimeter Sphere Diamagnetically Levitated in High Vacuum

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Levitated oscillators of millimeter or submillimeter size are particularly attractive due to their potential role in studying various fundamental problems and practical applications. One of the crucial issues towards these goals is to achieve efficient measurements of oscillator motion, although this remains a challenge. Here we theoretically propose a lens-free optical detection scheme, which can be used to detect the motion of a millimeter or submillimeter levitated oscillator with a measurement efficiency close to the standard quantum limit with a modest optical power. We demonstrate experimentally this scheme on a 0.5-mm-diameter microsphere that is diamagnetically levitated under high vacuum and room temperature, and the thermal motion is detected with high precision. Based on this system, an estimated acceleration sensitivity of 9.7×10^{-10} g/$\sqrt{\text{Hz}}$ is achieved, which is an improvement of more than 1 order of magnitude over the best value reported for a levitated mechanical system. Due to the stability of the system, the minimum resolved acceleration of 3.5×10^{-12} g is reached with measurement times of 10^5 s. This result is expected to have potential applications in the study of exotic interactions in the millimeter or submillimeter range and the realization of compact gravimeters and accelerometers.

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Introduction. A levitated mechanical oscillator in a vacuum is supposed to achieve ultralow mechanical noise due to its isolation from the environment [1]. It has been proposed for studying various fundamental physics such as testing of wave function collapse models [2–7] and investigation of new physics [8–14]. For metrology study, it has also been applied in force detection [15–17], inertia sensing [18–23], thermometry [24,25], and magnetometry [26,27]. According to different applications, the sizes of levitated mechanical oscillators range from 26 nm using a room-temperature optical trap [28] to the centimeter level using superconducting levitation with mass greater than 1 kg [19].

Acceleration sensitivity is a crucial issue in many researches. A centimeter-level superconducting levitated oscillator has been demonstrated to have an ultrahigh acceleration sensitivity of approximately 10^{-10} g/$\sqrt{\text{Hz}}$ [18], and a submillimeter superconducting levitated microsphere has been reported with a potential acceleration sensitivity order of 1.2×10^{-10} g/$\sqrt{\text{Hz}}$, assuming that the thermal noise limit is reached [20]. Smaller oscillators with diameters less than tens of micrometers have been studied in systems based on optical levitation [21], diamagnetical levitation [23,29,30], and superconducting levitation [31–33]. The best reported levitated oscillator acceleration sensitivity of 3.6×10^{-8} g/$\sqrt{\text{Hz}}$ has been achieved at room temperature [23] with a magnetic gravitational system.

Mechanical-oscillator-based acceleration sensing is characterized by the power density of the acceleration noise $S_{aa}^{\text{tot}} = S_{aa}^{\text{th}} + S_{aa}^{\text{mea}}$. Here $S_{aa}^{\text{th}} = 4\gamma k_B T/m$ is the thermal Brownian noise, where m is the system mass, $\gamma/2\pi$ is the dissipation rate of the oscillator, and T is the environment temperature. The thermal Brownian noise sets a low limit of achievable sensitivity when in the thermal noise limit. S_{aa}^{mea} is the measurement noise, which is important in

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practical realization, and in principle is limited by quantum mechanics [34]. Recent experiments using an optically levitated nano-oscillator have shown its motion to be close to the quantum zero-point motion [35,36].

Millimeter or submillimeter mechanical oscillators with milligram masses have recently attracted theoretical interest [37]. One of the main advantages of these systems is that the levitated objects can be manufactured as optical components. Combined with promising methods such as optical radiation pressure suspension [38,39], diamagnetic levitation[22,40], and superconducting levitation [41], it is possible to make the measurement efficiency close to the standard quantum limit (SQL). However, it is still challenging to experimentally achieve a high measurement efficiency of millimeter or submillimeter levitation oscillators under high-vacuum conditions [41]. Especially, the observation of the thermal Brownian motion of millimeter or submillimeter levitation oscillators is still elusive.

In this Letter, we propose a lens-free optical detection scheme that can detect the motion of a diamagnetically levitated transparent microsphere mechanical oscillator that works as an optical function element for motion measurement. Theoretically, this method can achieve a detection efficiency close to the SQL of submillimeter microspheres through moderate laser power. Then we demonstrate the scheme by using a diamagnetically levitated sphere with a diameter of 0.5 mm at room temperature and the thermal Brownian motion is detected. Under high-vacuum conditions, the system's total acceleration noise of $(9.7 \pm 1.1) \times 10^{-10}$ g/$\sqrt{\text{Hz}}$ is experimentally measured, an improvement of more than an order of magnitude over that of the best reported levitation oscillator at room temperature. The estimated minimum resolved acceleration of $(3.5 \pm 1.4) \times 10^{-12}$ g is achieved through a measurement time of about 10^5 s, comparable to the best reported value based on a lab-scale system of cold atom interferometry [42,43]. Further improvements and potential applications of the system are discussed.

Theoretical description of the method. The key idea of our detection method is based on the optical property of a microsphere as a spherical lens, shown in Fig. 1(a), where a levitated transparent microsphere is placed between two fibers. The light comes from the end of an incident fiber with power P_{in}, and the microsphere is used as a spherical lens to refocus the light to the end of another detection fiber. Thus the collected optical power P_{det} depends on the position of the microsphere. The transmission coefficient $\mathcal{T} = P_{\text{det}}/P_{\text{in}}$ is sensitive to the displacement of the microsphere, thus providing a method to measure the motion of the oscillator. Figure 1(b) shows the response of transmission coefficient $\delta\mathcal{T}$ to the detection fiber's position x_{fib} for a given microsphere displacements δx along the x axis. By adjusting the detection fiber's position x_{fib}, an optimal sensitivity can be achieved [44].

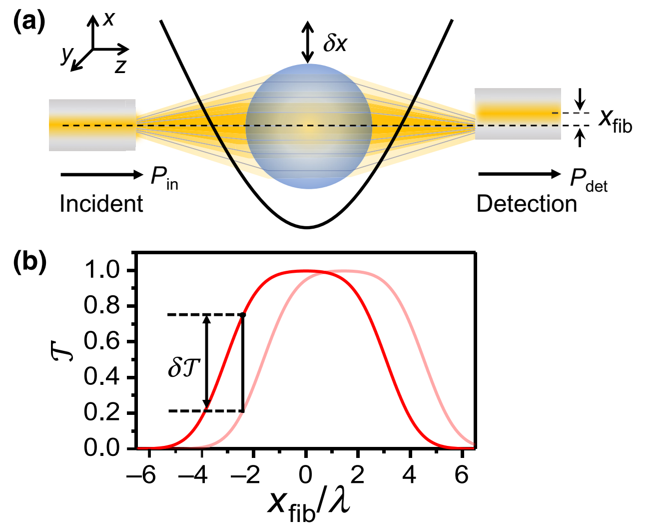

FIG. 1. Schematic of our method. (a) Lens-free optical measurement system. A 1550-nm laser with a power of P_{in} illuminates the levitated microsphere via the left incident fiber. The microsphere works as a spherical lens to refocus the light into the detection fiber on the right. The optical fiber is positioned at x_{fib} relative to the center in the x-axis direction, and the power detected by the detection optical fiber is P_{out}. (b) The calculated transmission coefficient $\mathcal{T} = P_{\text{out}}/P_{\text{in}}$ as a function of the detecting fiber position x_{fib}. The red curve represents where the microsphere is in the central equilibrium position, and the light red curve corresponds to the response curve a small displacement δx along the x axis. The black line represents the response of signal $\delta\mathcal{T}$ of the measured signal to the displacement δx.

Our scheme provides a potential capability to reach a high measurement efficiency of the microsphere motion [44]. The motion equation of a classic oscillator subject to noises is

$$m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = f_{\text{th}}(t) + f_{\text{ba}}(t), \tag{1}$$

where m is the oscillator mass, $\gamma/2\pi$ is the mechanical dissipation rate, $\omega_0/2\pi$ is the resonant frequency, $f_{\text{th}}(t)$ is the thermal Brownian noise, and $f_{\text{ba}}(t)$ is the backaction fluctuation, which comes from the photon shot noise of the incident laser on the oscillator. Thus the detected power spectral density (PSD) of displacement of the oscillator due to Brownian noise and backaction fluctuation is $S_{xx}^{\text{th}}(\omega) = 4\gamma k_B T|\chi(\omega)|^2/m$ and $S_{xx}^{\text{ba}}(\omega) = |\chi(\omega)|^2 S_{ff}^{\text{ba}}(\omega)/m^2$, respectively, where $|\chi(\omega)|^2 = 1/[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]$ is the mechanical susceptibility and $S_{ff}^{\text{ba}}(\omega)$ is the force PSD of backaction fluctuation. The measurement imprecision S_{xx}^{imp} and back action satisfy the relation $S_{xx}^{\text{imp}} S_{ff}^{\text{ba}} = (1/\eta)\hbar^2$. Here $\eta \leq 1$ is the measurement efficiency, and detection reaches the SQL for $\eta = 1$. Considering the measurement imprecision, the PSD of the displacement of the oscillator is $S_{xx}^{\text{tot}}(\omega) = S_{xx}^{\text{th}}(\omega) +$

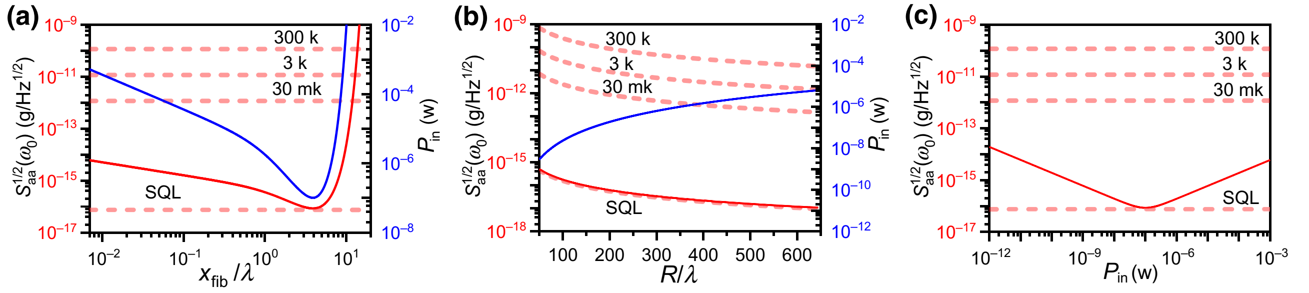

FIG. 2. Estimated measurement noise of acceleration. (a) The red curve (left axis) is the minimum measurement noise of acceleration $\sqrt{S_{aa}^{\rm mea}}$ versus the shifted detection fiber position $x_{\rm fib}$. We assume that the refractive index of the microsphere $n = 1.4$, the diameter $2R = 0.5$ mm, the acceleration noise is estimated using the mode that oscillates along the x axis, the resonant frequency is 10 Hz, and the mechanical dissipation is 10^{-6} Hz. The red dashed lines are the thermal noise of acceleration $\sqrt{S_{aa}^{\rm mea}}$ and SQL as labeled. The blue curve (right axis) is the corresponding optimized laser power $P_{\rm in}^{\rm opt}$. (b) Same as (a) but versus the size of the microsphere (radius R in unit of wavelength) with a fixed $x_{\rm fib} = 5.8$ μm. (c) Same as (a) but versus the incident laser power, and with a fixed $x_{\rm fib} = 5.8$ μm.

$S_{xx}^{\rm imp} + S_{xx}^{\rm ba}(\omega)$. The measurement-related noises, the back action noise $S_{xx}^{\rm ba}$ and the imprecision noise $S_{xx}^{\rm imp}$, can be further optimized. The measurement-related noise $S_{aa}^{\rm mea}$ of the acceleration is [44]

$$S_{aa}^{\rm mea} = \frac{S_{xx}^{\rm imp}}{|\chi(\omega)|^2} + \frac{S_{ff}^{\rm ba}}{m^2}. \tag{2}$$

As an example, we consider a levitated oscillator with a microsphere diameter of 0.5 mm, relative index of refraction $n = 1.4$, resonant frequency $\omega_0/2\pi = 10$ Hz, and mechanical dissipation $\gamma/2\pi = 10^{-6}$ Hz. As shown in Fig. 2(a), the minimum $S_{aa}^{\rm mea}$ can be obtained by optimizing the detection fiber position $x_{\rm fib}$ and the incident power $P_{\rm in}$. Figure 2(b) shows the minimum measurement noise of acceleration as a function of microsphere radius as well as optimized laser power. In practical applications, the laser power may become a crucial issue [21]. Our calculation [Fig. 2(c)] shows that even at ultralow temperature and the laser power around 10^{-12} W, the measurement noise is still much lower than the thermal noise.

Experimental demonstration. There are many transparent diamagnetic materials available, such as quartz, diamond, and most organic compounds like poly(methyl methacrylate) (PMMA). We test our scheme using a 0.5-mm-diameter PMMA sphere as oscillator at room temperature, which is suspended in a magnetic trap [5]. A group of NdFeB magnets of octagonal bilayer geometry are used with gravity-generated stable diamagnetic levitation potential. Especially, two narrow grooves are fabricated on the magnets so fibers can go to the trapping area as shown in Fig. 3(a). After loading the microsphere into the trap, a 1550-nm laser with power of the order of microwatts is applied to the incident fiber while the photon detector is connected to the detection fiber. A pair of three-axis piezoelectric positioners are used to adjust the position of the incident and detection optical fibers. After reaching the optimal position, the optical fiber is fixed to the magnetic trap by UV glues, and the piezoelectric positioner is removed to avoid unnecessary vibration.

According to the design of the magnetogravitational trap, the microsphere oscillator has three orthogonal oscillation modes: one mode oscillates along the direction of gravitational field (mode g) and the other two modes oscillate in the horizontal plane (denoted as mode 1 and mode 2). Oscillation mode 1 is vertical to the optical axis and mode 2 is along the axial direction. In the experiment, as shown in Fig. 3(a), the imperfection of the magnetogravitational trap and the microsphere makes mode g and mode 1 not perfectly vertical and mode 2 not perfectly axial to the optical axis. Without loss of generality, we study the vertical mode (mode 1) whose oscillation direction is close to the vertical direction of the optical axis as shown in Fig. 3(a), and the corresponding resonance frequency is 10.58 Hz.

In the experiment, the pressure of the vacuum chamber is 4×10^{-3} mbar. The motion of the microsphere oscillator is measured by a photon detector, and the voltage signal $V(t)$ is then amplified and recorded. The measurement time is 500 s, which is much longer than the relaxation time of the oscillator. The corresponding power spectral density is $S_V(\omega) = \xi^2 S_{xx}^{\rm tot}(\omega)$, where ξ is the displacement-voltage conversion coefficient. Considering the detection bandwidth b, the detected signal is $\int_{\omega_0-b/2}^{\omega_0+b/2} S_V(\omega)d\omega$. The effective temperature $T_{\rm eff}$ of the oscillator is defined using the measured power spectral density of motion as $T_{\rm eff} = (m\omega_0^2/k_B)\int_{\omega_0-b/2}^{\omega_0+b/2}\left(S_{ff}^{\rm th} + S_{ff}^{\rm ba}\right)|\chi(\omega)|^2/m^2 d\omega$ [44]. Then we obtain a simpler expression as $\int_{\omega_0-b/2}^{\omega_0+b/2} S_V(\omega)d\omega = \xi^2(k_B T_{\rm eff}/m\omega_0^2 + bS_{xx}^{\rm imp})$. Under current pressure (10^{-3} mbar), the oscillator is fully thermalized with the environment, so we take the effective temperature $T_{\rm eff} = T_{\rm en} =$ 298 K measured by a thermometer fixed on the vacuum chamber. So by fitting the measured power spectral

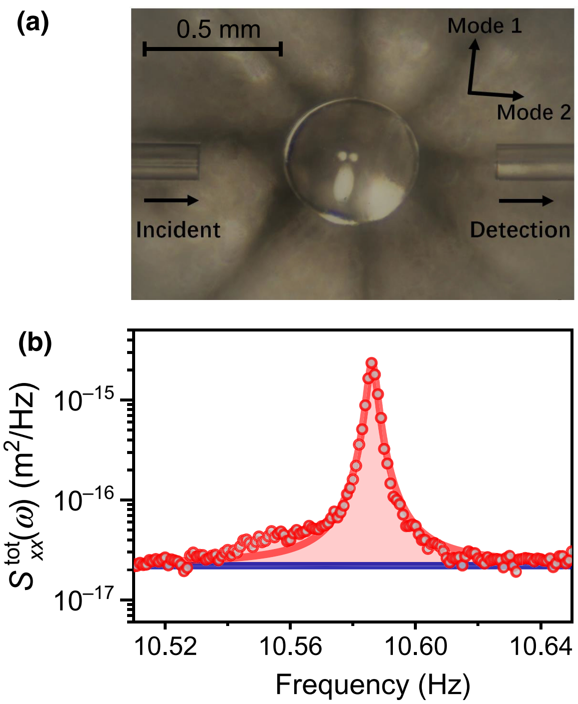

FIG. 3. Experimental measurement of thermal movement of a submillimeter microsphere. (a) An optical image of a microsphere of 0.5 mm in diameter levitated in a magnetic gravitational trap. A 1550-nm laser transmits via the left fiber to the right one passing the submillimeter microsphere. The fibers are fixed on the magnetic trap by UV glues. Vibration mode 1 and mode 2 on the horizontal plane are indicated by arrows in the inset, and mode 1 is used in the current experiment. The angle between mode 1 and the optical axis is about $85^\circ \pm 5^\circ$. (b) The measured power spectral density of motion $S_{xx}^{\text{tot}}(\omega)$ of oscillation mode 1 under a pressure of 4×10^{-3} mbar (red circles). The data are fitted to the Lorentz curve (red line). The blue line shows the fitted imprecision noise power spectral density S_{xx}^{imp}. The light-red area represents the contribution of effective thermal Brownian motion at an ambient temperature of 298 K.

density $S_V(\omega)$ to the Lorentz curve and considering the frequency-independent baseline $\xi^2 b S_{xx}^{\text{imp}}$, conversion coefficient ξ and imprecision noise S_{xx}^{imp} are obtained, and Fig. 3(b) shows the power spectral density of displacement $S_{xx}^{\text{tot}}(\omega) = S_V(\omega)/\xi^2$.

Then we pump the chamber to a lower pressure of 1.2×10^{-5} mbar to study the acceleration sensitivity of the oscillator. We first measure the effective temperature T_{eff} by measuring the power spectral density of the displacement. The mechanical dissipation coefficient obtained from free ringing at lower pressure is as low as $\gamma/2\pi = 71\ \mu$Hz, and the corresponding correlation time is $\tau \approx 2200$ s. Because of the statistical error of the effective temperature $\Delta T_{\text{eff}} \propto \sqrt{1/t_{\text{mea}}\gamma}$ (see Ref. [44]), our measurement time in the experiment is $t_{\text{mea}} = 7 \times 10^5$ s $\gg \tau$ (about 190 h). We obtain the effective temperature $T_{\text{eff}} = 289 \pm 67$ K, which is consistent with the environment temperature. The results show that the oscillator is still in thermal equilibrium, and external noises, such as laser heating and vibration, can be neglected [44]. We estimate the power spectral density of acceleration noise as

$$S_{aa}^{\text{tot}}(\omega) = \frac{4\gamma k_B T_{\text{eff}}}{m} + \frac{S_{xx}^{\text{imp}}}{|\chi(\omega)|^2}. \quad (3)$$

Based on the current experiment, the total acceleration noise at the resonant frequency is obtained as $\sqrt{S_{aa}^{\text{tot}}(\omega_0)} = (9.7 \pm 1.1) \times 10^{-10}$ g/$\sqrt{\text{Hz}}$. The related results are summarized in Table I.

Finally, we estimate the minimum resolvable acceleration $a_{\min}(\omega_0)$ at the resonant frequency. Considering the oscillating acceleration signal at the resonant frequency, Fig. 4 shows the corresponding $a_{\min}(\omega_0)$ as a function of the total measurement time, where $a_{\min}(\omega_0) = (3.5 \pm 1.4) \times 10^{-12}$ g reaches its minimum value when the measuring time is 10^5 s. The measured $a_{\min}(\omega_0)$ is slightly higher than the predicted value $a_{\min}(\omega_0) = \sqrt{S_{aa}^{\text{tot}}(\omega_0)/t}$. A possible reason for this difference is that the temperature fluctuation limits the frequency fluctuation $\delta\omega_0/2\pi \approx 770\ \mu$Hz, about an order of magnitude larger than mechanical dissipation $\gamma/2\pi$ (for a detailed description of the data, see the Supplemental Material [44]). In principle, this can be overcome by better temperature control schemes, such as placing the system in a low-temperature environment [48].

Discussion and summary. We propose a detection scheme for levitated millimeter and submillimeter microspheres that can theoretically approach the standard quantum limit. Our experiments demonstrate the use of a diamagnetic levitated microsphere under high vacuum and the measurement noise is low enough to detect the thermal motion of the microsphere. Our method provides an

TABLE I. Summary of main experiment parameters. Diameter of microsphere is 0.5 mm and the corresponding mass is 78 μg, resonant frequency is $\omega_0/2\pi = 10.58$ Hz, environment temperature is maintained at 298 K, and pressure is 1.2×10^{-5} mbar.

Symbol	Value	Unit
ξ	1.14 ± 0.16	10^{10} V/m
$\gamma/2\pi$	7.1 ± 0.03	10^{-5} Hz
Q	1.49 ± 0.0063	10^5
T_{eff}	289 ± 67	K
$\sqrt{S_x^{\text{imp}}}$	4.70 ± 0.36	10^{-9} m/$\sqrt{\text{Hz}}$
$\sqrt{S_{aa}^{\text{imp}}(\omega_0)}$	2.0 ± 0.2	10^{-12} g/$\sqrt{\text{Hz}}$
$\sqrt{S_{aa}^{\text{tot}}(\omega_0)}$	9.7 ± 1.1	10^{-10} g/$\sqrt{\text{Hz}}$
$a_{\min}(\omega_0)$[a]	3.5 ± 1.4	10^{-12} g

[a]At a measurement time of 10^5 s.

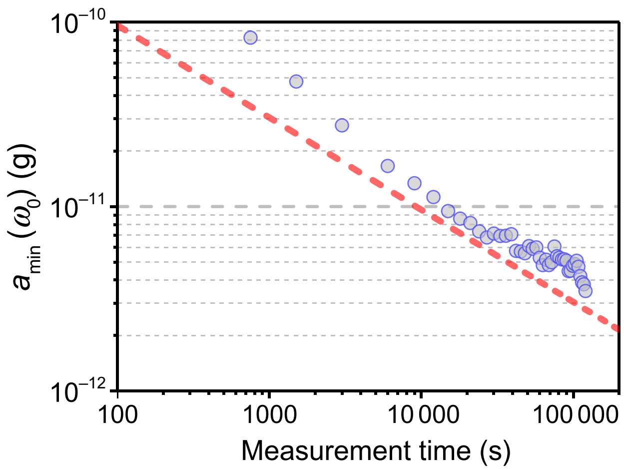

FIG. 4. The minimum resolvable acceleration. The blue circles are the measured acceleration noise as a function of measurement time at pressure 1.2×10^{-5} mbar. To build up the statistics, a total experiment time of about 7×10^5 s (about 190 h) is used. The red curve is the theoretical curve assuming the temperature fluctuations are eliminated.

alternative way to realize quantum-limited displacement measurement for a mechanical oscillator, in comparison with mirror-based systems like Michelson interferometry [44]. At the same time, the measurement noise is still greater than the theoretical estimation, the main causes including the possible misalignment of the incident and detection fibers, the imperfection of the microsphere shape, and the intensity noise of the laser source. These problems are expected to be significantly overcome by further improvement of technology.

Acceleration sensitivity of the current system is also experimentally characterized and a better sensitivity is reached compared with reported mechanical systems at room temperature. The experiment system reported here has further potential applications, such as gravimetric analysis and acceleration measurement. Although our system can only measure the relative acceleration, rather than the absolute acceleration as in a cold-atom system, the lens-free design reduces the size of the device, resulting in a more compact system. Due to the submillimeter scale and the ultralow resolvable acceleration of the system, it can also be used to test some fundamental physical models such as dark energy [49].

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