Reply to "Comment on 'Scattering Cancellation-Based Cloaking for the Maxwell-Cattaneo Heat Waves'"

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I. INTRODUCTION

Comment [1] points out possible inconsistencies in the notations of our paper [2] and, based on these remarks, it questions the validity of our conclusions. In this Reply, we demonstrate the general validity of all conclusions in Ref. [2], and we take the opportunity to clarify our notation and our results and to discuss their domain of validity.

II. EQUATION DERIVATION

The remarks in Comment [1] are rooted in a normalization that we implicitly apply in the definition of Eq. (3) of Ref. [2]. More explicitly, the author of Ref. [1] points out that Eq. (3) should read $\rho c(\partial T/\partial t) = -\nabla\Phi$, whereas in our notation the coefficient ρc is missing. In our paper [2], for simplicity of notation, we normalize the term κ_0, using the normalized quantity $\tilde{\kappa}_0 = \kappa_0/(\rho c)$, which represents the thermal diffusivity, with units of $\mathrm{m}^2\mathrm{s}^{-1}$. This normalization explains the reason for the missing term in Eq. (3). Also, subsequently, Eq. (5) and the paragraph below it, as well as Eq. (13), are consistent with this normalization, where κ in those equations needs to be read again as the thermal diffusivity $\tilde{\kappa}$. In Secs. III B, III C, and IV, similarly, κ denotes the diffusivity $\tilde{\kappa}$, including in Eq. (16), which makes its units consistent. Along with this normalization, σ_0 also has units of $\mathrm{m}^2\mathrm{s}^{-1}$. Given the form of Eq. (3), we had assumed that this normalization would be straightforward for the average reader, and the reviewers of our paper had no problem with it. However, it is clear that the author of Ref. [1] was confused by it, and we are glad to have the opportunity to clarify it for all readers. It is important to stress, however, that this normalization does not change any of the results in our paper, since all our simulations and derivations properly treat κ as a thermal diffusivity term.

The source term that the author of Ref. [1] would like us to include in our equation is not needed in our theory, since our problem considers the scattering of a passive object from an external excitation, without the presence of sources in the scattering region of interest. Similarly, in Eq. (4), the source term δ is not necessary for the problem at hand.

Finally, we may have introduced a potential ambiguity in our paper [2] by using the same symbol, *T*, for the temperature in Eqs. (4) and (5), where these two values are proportional by a phase $e^{-i\omega t}$; this simplified notation is very common in the broad scientific literature to lighten the formulas, in the many instances, in which the analysis is carried out in the frequency domain, so we are confident that the text is sufficiently clear for the average reader.

III. DIFFUSIVE TERM

Classically, the Maxwell-Cattaneo (MC) equations are derived without a diffusive term [σ_0 in Eq. (3) of Ref. [2]], and we agree with the author of Ref. [1] that the implications of our theory in the special case $\sigma_0 = 0$ are interesting for future exploration. Our work also considers, more generally, a diffusive term in these equations. Inspired by the Comment [1], in Fig. 1, we numerically compare our cloaking condition derived in Ref. [2] against the scenarios in which σ_0 is zero and that in which σ_0 is 10 times smaller than the value used in Ref. [2], keeping all other parameters the same. We note that these results agree very well with the results in the paper. This numerical comparison confirms that our findings also apply in the limit $\sigma_0 = 0$, which is consistent with classical MC equations, as seen in Figs. 1(a) and 1(b). To avoid any source of confusion, we reiterate here that, in our paper [2], we work with heat pseudowaves that possess complex wave numbers or diffuse photon density waves DPDW, corresponding to the generalized form of MC equations used in the paper.

IV. FLUX DEFINITION

We use the expression $\kappa\nabla T$ to express the flux for continuity at the boundaries between different layers.

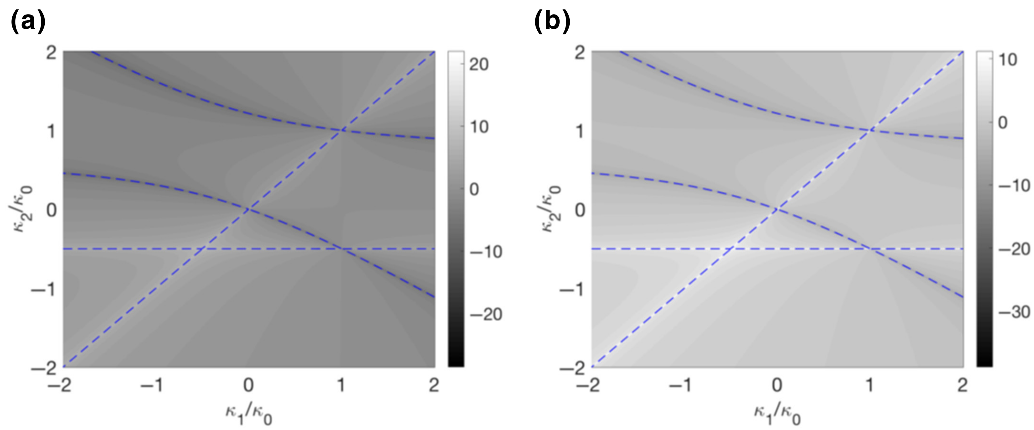

FIG. 1. Landscape of scattering cross section on the logarithmic scale, i.e., $10\log_{10}|\Sigma_2^{sc}/\Sigma_1^{sc}|$ for (a) $\sigma = 0.1\sigma_0$ and (b) $\sigma = 0$, with σ_0 being the value used in Ref. [2]. Dashed blue line corresponds to cloaking condition derived from our model in Ref. [2].

As outlined in Ref. [1], a more general expression for the flux is $(1 + i\omega\tau_0)/(1 + \omega^2\tau_0^2)\kappa\nabla T$. We are well aware of this, but we make this approximation rightfully, because in Ref. [2] we deal with the quasistatic regime. Indeed, the term $\omega\tau_0$ in the regime considered throughout our paper is lower than 10^{-3}, so no calculations are affected by using either expression. The suggested modification starts to have an impact for much larger frequencies, but this is not the subject of our paper [2], since, in such regimes, many other scattering harmonics would contribute to the overall response, and our theory would not be applicable.

To validate our claims, we compare our simulations using both expressions for flux in Fig. 2(a) (solid lines correspond to the results in Ref. [2], whereas dashed lines correspond to the dispersive definition of the flux proposed in Ref. [1]. Over the frequency range of interest for Ref. [2], the results are identical for $\omega\tau_0 = 0.1$ and nearly identical for $\omega\tau_0 = 0.4$ (there is a tenuous mismatch between the curves at specific conductivities), as seen in Fig. 2. Also, in this case, no results or conclusions in Ref. [2] are affected at all, since we operate in the quasistatic limit ($\omega\tau_0 \ll 1$), as clearly spelled out in the paper [2]. In Fig. 2(b), we consider a larger frequency, for which we start observing some slight deviations from our results. Again, our results are very accurate, even in this regime, which, in any case, starts to deviate from the quasistatic assumption at the basis of our work.

We hope that these clarifications may help to clarify the results of our paper for the author of Ref. [1] and all other interested readers. We stand by all conclusions and remarks in our work, which are indeed correct and accurate, as shown in this Reply.

[1] I. C. Christov, Comment on “Scattering Cancellation-Based Cloaking for the Maxwell-Cattaneo Heat Waves”, Phys. Rev. Appl. 15, 058001 (2021).

[2] M. Farhat, S. Guenneau, P.-Y. Chen, A. Alù, and K. N. Salama, Scattering Cancellation Based Cloaking for the Maxwell-Cattaneo Heat Waves, Phys. Rev. Appl. 11, 044089 (2019).

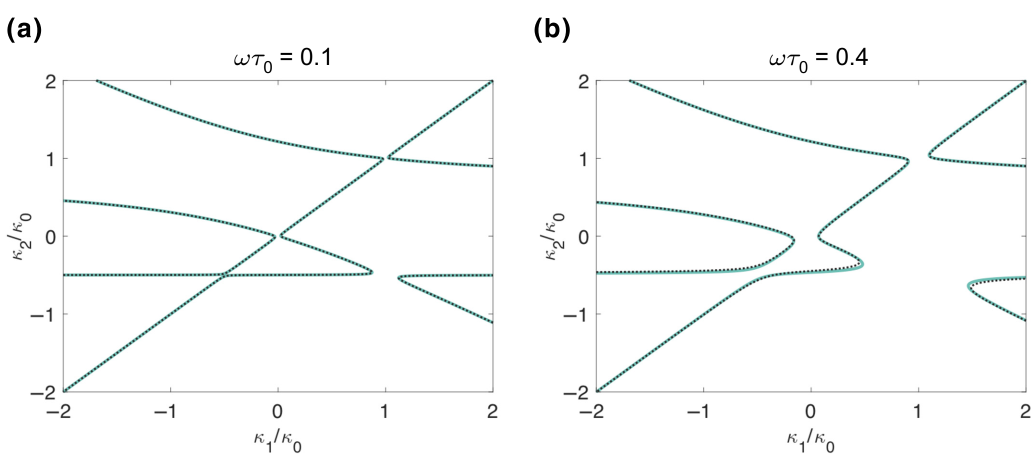

FIG. 2. Contour plot of conductivities that make cloaking possible for (a) $\omega\tau_0 = 0.1$ and (b) $\omega\tau_0 = 0.4$. Dashed curves correspond to flux expression in Ref. [1], while solid lines correspond to flux expression used in Ref. [2].