

Optimized Proximity Thermometer for Ultrasensitive Detection

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We present a set of experiments to optimize the performance of a noninvasive thermometer based on proximity superconductivity. Current through a standard tunnel junction between an aluminum superconductor and a copper electrode is controlled by the strength of the proximity induced to this normal metal, which in turn is determined by the position of a direct superconducting contact from the tunnel junction. Several devices with different distances are tested. We develop a theoretical model based on Usadel equations and dynamic Coulomb blockade that reproduces the measured results and yields a tool to calibrate the thermometer and to optimize it further in future experiments. We also propose an analytic formula that reproduces the experimental data for a wide range of temperatures.

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I. INTRODUCTION

Virtually any parameter depending on temperature T , preferably monotonically, can form a basis for thermometry [1,2]. Yet depending on the application, one needs to make a choice of system and technique based on several criteria, including sensitivity, noise, power dissipation, physical size, and speed of response. Besides these criteria, one often needs to consider whether the measured quantity can be obtained theoretically from a well-known, preferably simple physical law without fitting parameters: if this is the case the technique may qualify as “primary thermometry.” However, most of the time, like in the present work, this is not the case, and we deal with “secondary thermometry.” Measurement of the local temperature of nanostructures at very low temperatures (<1 K) has been recently developed with several techniques [3–10]. Here we build on a technique based on temperature-dependent proximity superconductivity yielding sensitive thermometry with ultralow dissipation. The technique is particularly well adaptable to calorimetric detection of tiny heat currents as well as fast thermometry towards the lowest temperatures in mesoscopic on-chip systems. The main goal of the present investigation is to optimize the sensitivity (responsivity) of the sensor and to model its behavior using a well-established theoretical framework. The main results of the current work are (i) an order-of-magnitude increased sensitivity of the device with respect to an earlier

realization [5] and (ii) a full theoretical account of its characteristics.

II. DESCRIPTION OF THE THERMOMETER

The thermometer that we study is schematically shown in the left-hand inset of Fig. 1. The normal lead of a standard normal-metal–insulator–superconductor (N - I - S) junction is connected to another superconducting lead via direct metal-to-metal contact. This lead induces the proximity effect to the N lead and permits a supercurrent via the tunnel junction. The basic characterization of this thermometer is presented in Ref. [5].

This thermometer has been recently operated in a setup that allows one to monitor temperature and its variations on microsecond time scales [11]. The main panel of Figs. 1 presents the dc bias voltage V dependent rf transmission S_{21} at the resonance of the LC circuit loaded by the thermometer junction in parallel, measured at various temperatures. (The rf setup to measure S_{21} including the communication between room temperature and millikelvin temperatures via attenuators, circulators, and cold amplifiers has been described elsewhere [11,12].) In this case, the superconducting contact is at a distance of $L = 450$ nm from the tunnel junction. For low conductance $G = dI/dV$ of the junction, $G = \gamma[S_0 - S_{21}(\omega_0)]$, meaning that the variations of S_{21} are proportional to $-G$. Here γ is a constant that depends on the parameters of the lumped LC circuit with $\omega_0/2\pi$ its resonance frequency and S_0 is a constant offset that includes the attenuation and amplification in the lines [5]. In the figure we thus observe

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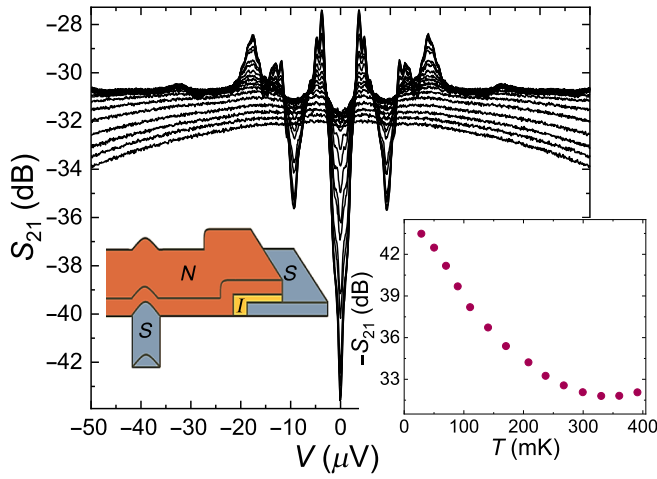


FIG. 1. Principle and basic characteristics of the thermometer. Left-hand inset: schematic illustration of the thermometer tunnel junction. Here N , S , and I stand for normal metal, superconductor, and insulating barrier, respectively. The main panel shows the rf transmission S_{21} of the proximitized junction at various temperatures from approximately 20 mK up to approximately 400 mK, at -140 dBm applied power. This signal is directly related to the conductance of the junction. Right-hand inset: zero-bias conductance ($-S_{21}$) as a function of bath temperature taken from the data in the main panel.

the temperature-dependent conductance of the thermometer junction. The favorable operation of the thermometer is at $V = 0$, where the dependence of S_{21} on temperature T is strongest, and the self-heating IV is minimal.

The right-hand inset of Fig. 1 depicts the temperature dependence of $-S_{21}$ of this junction measured at $V = 0$, using a very small excitation power (-140 dBm) for the measurement. We see that $-S_{21}$ presents an almost linear increase with decreasing temperature well below 300 mK, thus providing a sensitive and noninvasive thermometer. These characteristics are to be compared to the temperature-dependent dc conductance results that will be presented below.

In the current work we limit ourselves to determining the conductance and current voltage characteristics of the junction in a quasi-dc measurement. The samples are fabricated on a commercially available silicon wafer onto which a 300-nm layer of silicon oxide has been grown. In order to have stable tunnel junctions we use a 22-nm suspended germanium hard mask. Moreover all the different kinds of samples are made on the same chip in one fabrication process, meaning that this process for all of them is equal and the dominant difference between them is their geometry. We use electron-beam lithography for writing the patterns on the chip and three-angle electron-beam evaporation of the metal films. All samples have 35 nm copper as normal metal and 20 nm as both superconducting Al leads. The insulator in the tunnel junction is a thin

layer of aluminum oxide formed by letting pure oxygen into the chamber on top of one of the aluminum films. The thermometers are made in a single vacuum cycle allowing fabrication of clean metallic contacts without additional cleaning of the samples.

III. EXPERIMENTAL RESULTS AND OPTIMIZATION OF THE SENSITIVITY

An important figure of merit of a sensor is its responsivity, which for this thermometer reads $\mathcal{R} = |dS_{21}/dT|$. The apparent noise in a temperature measurement is then inversely proportional to $\mathcal{R}$ as long as noise is not intrinsic originating from true temperature fluctuations. Intuitively the responsivity is expected to increase when the proximity is enhanced, by bringing the clean contact closer to the junction. Therefore we fabricate several proximity junctions with nominally equal parameters, apart from the differing distance L . Figure 2 shows scanning electron microscope (SEM) images of three samples ($L = 100$ nm, 250 nm, and 350 nm). Along with the three SEM images, the figure shows in the inset the measured I - V characteristics of the sample with $L = 100$ nm at different bath temperatures (40–240 mK), which indicates the nonvanishing current in the small bias range with maxima at ± 20 μ V, due to the induced proximity effect in the normal-metal island. It is clear that the peak current $I_{\max}$ decreases due to the decrease of proximity effect with increasing bath temperature. We measure the conductance G of the junctions with low-frequency (approximately 10 Hz) lock-in techniques applying typically an excitation voltage of approximately 1 μ V ac. The simplified measurement setup is shown in the inset of Fig. 3(a). The measured conductance of the proximitized junction as a function of applied voltage bias under different conditions is shown in Fig. 3. In Fig. 3(a) this dependence is shown for different distances $L = 50$ –350 nm at 50-nm intervals and fixed bath temperature $T = 40$ mK. A few reproducible features can be observed in the sub-gap regime within $V \sim \pm 50$ μ V $\ll \Delta_0/e$, where Δ_0 is the superconducting gap. The sharpest one of them is our favorable feature at zero-bias voltage, i.e., “zero bias anomaly” (ZBA). All these features get suppressed on increasing the distance L . Figure 3(b) demonstrates the main feature, the temperature dependence of conductance for $L = 50$ nm, i.e., the thermometer with the strongest ZBA feature. Sensitivity of zero-bias conductance down to the lowest temperature is obvious. The overall change of the baseline and its bias dependence are due to quasiparticle current arising at finite temperatures. Figure 3(c) is a wider view of 3(b) that emphasizes conductance character due to quasiparticle current at voltages around the superconducting gap, $V \simeq \pm 200$ μ V. The standard BCS coherence peaks at $eV = \pm \Delta_0$ of a N - I - S junction are now split due to the existence of the minigap in the proximitized normal metal.

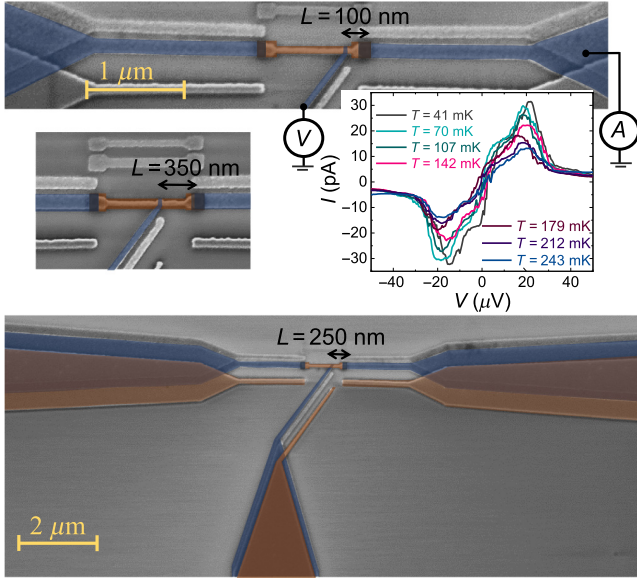


FIG. 2. Scanning electron micrographs of three different samples. The normal metal (brown) is coupled to three superconducting leads (blue): two (left and right) via tunnel barriers (gray) and one via a direct contact. The distance L between the right-hand junction with clean contact varies from 50 to 350 nm in 50-nm intervals (SEM images of three of them are shown: $L = 100$ nm, $L = 350$ nm, and $L = 250$ nm from top to bottom). The bottom panel shows a wider view of one of the three samples; all of them have the same structure outside the actual thermometer details. We present in this paper data on transport between the right-hand and middle contacts. Inset: the I - V characteristics of the sample with $L = 100$ nm at different bath temperatures.

IV. THEORETICAL MODEL

In this section we theoretically analyze an overlap S - N - I - S junction, which is close to the experimental setup

studied above, coupled to an electromagnetic (EM) environment.

A. Numerical calculation

Let us first discuss the S - N - I - S junction with the geometry shown in Fig. 4(a) and described in the corresponding caption. All metallic parts are assumed to be in the dirty limit, in which the elastic mean free path l_e is much smaller than the superconducting coherence length $\xi \sim \sqrt{\hbar D/2\Delta_0}$, where $D = v_F l_e/3$ is the diffusion coefficient with the Fermi velocity v_F . In order to describe such a system we make use of the imaginary time quasiclassical Green's function formalism determined by the Usadel equation of motion [13–15]. If the total thickness of the superconductor and the normal metal of the system depicted in Fig. 4(a) is sufficiently small ($d_N + d_S < \xi$) we can neglect all the derivatives in the y direction and average the Usadel equation over the width reducing it to an effectively one-dimensional problem described by the following ordinary differential equation [14]:

$$\frac{\hbar D}{2} \frac{d^2 \theta_n}{dx^2} = \omega_n \sin[\theta_n(x)] - \Delta(x) \cos[\theta_n(x)]. \quad (1)$$

Here, $\theta_n(x)$ is the proximity angle of the normal metal, D is the diffusion coefficient of the material, $\omega_n = (2n + 1)\pi k_B T$ are the fermionic Matsubara frequencies with temperature T and $n = 0, \pm 1, \pm 2, \dots$, and $\Delta(x)$ is the superconducting order parameter defined as follows:

$$\Delta(x) = \frac{d_S}{d_N + d_S} \Delta \quad (2)$$

for $0 < x < d_1$ and $= 0$ otherwise, where Δ is the order parameter of superconductor S_1 . We note that the normal metal underneath S_1 effectively acts as a superconductor with the reduced superconducting gap, $d_S/(d_S + d_N)\Delta$.

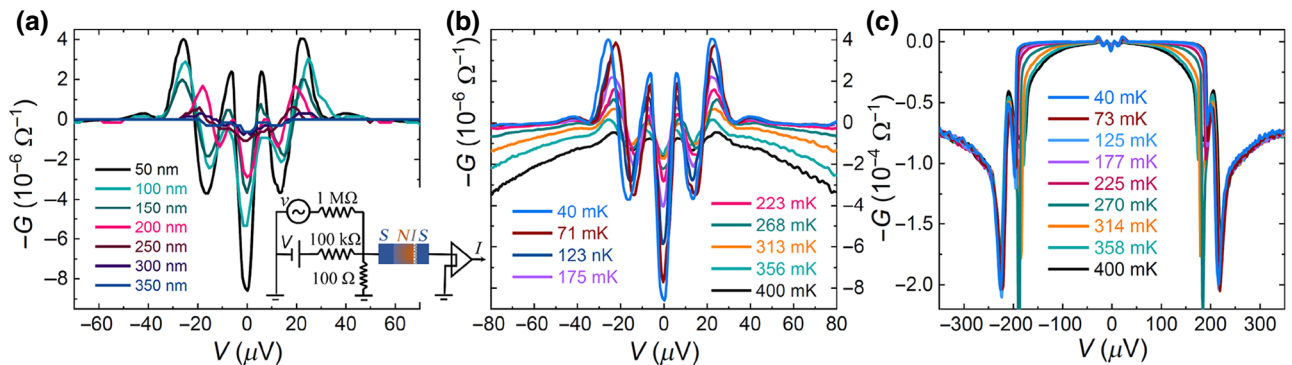


FIG. 3. Bias V dependence of the conductance $G = dI/dV$ of the thermometers under different experimental conditions. (a) $-G$ for various samples with $L = 50$ – 350 nm at temperature $T = 40$ mK. (b) A similar plot as (a) but just for one thermometer with $L = 50$ nm at various temperatures from $T = 40$ up to 400 mK. (c) As in (b) but now for a wider bias range demonstrating the onset of quasiparticle current at $V \simeq \pm \Delta_0/e = 200\ \mu\text{V}$ besides the zero-bias anomalies. Inset: simplified schematic of the measurement setup for the conductance G .

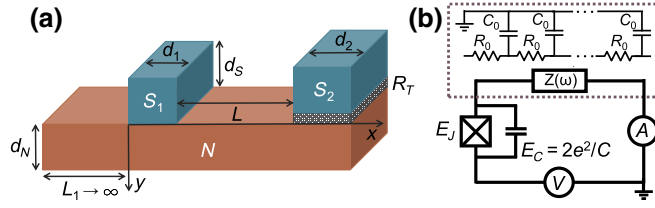


FIG. 4. (a) Scheme of the overlap S - N - I - S junction where two superconductors S_1 and S_2 (blue) are set on top of a semi-infinite normal metal wire (orange). S_1 - N is a clean contact of length d_1 , while S_2 - N is a tunnel contact of length d_2 and a resistance R_T . Thicknesses of the superconductors and the normal metal are d_S and d_N , respectively. (b) Representation of the phenomenological EM environment of the circuit under study. Here the Josephson junction of capacitance C and resistance R_T is coupled to an infinite RC transmission line (see inset) of impedance $Z(\omega) = \sqrt{R_0/i\omega C_0}$, where R_0 and C_0 are the resistance and the capacitance per unit length of the line.

To obtain a general solution, Eq. (1) has to be supplemented by the appropriate boundary conditions at the ends of the normal metal wire: $\theta_n(L_1) = 0$ and $\partial_x \theta_n|_{x=d_1+L+d_2} = 0$. The other boundary conditions come from the continuity of the proximity angle function as well as the current conservation throughout the system: $\theta_n(a-0) = \theta_n(a+0)$ and $\partial_x \theta_n|_{x=a-0} = \partial_x \theta_n|_{x=a+0}$ for $a = 0, d_1$ [see Fig. 4(a)] [14,16]. Since N - S_2 is a tunnel contact with low transparency we neglect the proximity effect in this region. The boundary condition problem described above can be solved numerically by employing the finite difference method where Eq. (1) is rewritten as a system of nonlinear algebraic equations. The normal and the anomalous Green's function components in θ representation read $G_{\omega_n}(x) = \cos[\theta_n(x)]$ and $F_{\omega_n}(x) = \sin[\theta_n(x)]$, respectively [14].

Based on the solution of Eq. (1) for the overlap junction depicted in Fig. 4(a) the critical current through the tunnel N - S_2 interface depends on the anomalous component of the Green's function and, therefore, is given by [17–19]

$$I_c = \frac{2\pi k_B T}{e R_T} \sum_{\omega_n > 0} F_{\omega_n}^S \bar{F}_{\omega_n}^N, \quad (3)$$

where R_T is the resistance of the tunnel junction and $F_{\omega_n}^S = \Delta(T)/\sqrt{\omega_n^2 + \Delta(T)^2}$ is the anomalous Green's function of superconductor S_2 . The temperature dependence of the superconducting gap is assumed to be $\Delta(T) = \Delta_0 \tanh(1.74\sqrt{T_c/T} - 1)$, with T_c as the critical temperature of the superconductor [20]. The proximity angle, $\theta_n(x)$, depends in general on the x coordinate, which means one is supposed to average the solution along the N - S_2

interface of a finite length d_2 obtaining $\bar{F}_{\omega_n}^N$ as follows:

$$\bar{F}_{\omega_n}^N = \frac{1}{d_2} \int_{d_1+L}^{d_1+L+d_2} \sin[\theta_n(x)] dx. \quad (4)$$

Let us now discuss the contribution from the EM environment schematically represented by the phenomenological circuit depicted in Fig. 4(b) and described in the corresponding caption. Due to the dynamical Coulomb blockade the current mediated by the tunneling of a Cooper pair in an ultrasmall Josephson junction of a capacitance C is described by the so-called $P(E)$ function [21,22]

$$I_s(V) = \frac{\pi e E_J^2}{\hbar} [P(2eV) - P(-2eV)]. \quad (5)$$

Here $E_J = \hbar I_c / 2e$ is the Josephson energy of the junction. The $P(E)$ function is the probability for an electron to emit a photon to the environment and it is defined as

$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dt \exp \left[4J(t) + \frac{i}{\hbar} Et \right], \quad (6)$$

where $J(t) = \langle [\varphi(t) - \varphi(0)]\varphi(0) \rangle$ is the equilibrium correlation function of the phase $\varphi(t) = (e/\hbar) \int_{-\infty}^t V(t') dt'$ of the voltage across the junction. This function depends on the total impedance of the system, $Z_t(\omega)$, as follows:

$$J(t) = 2 \int_0^{\infty} \frac{d\omega}{\omega} \frac{\text{Re}[Z_t(\omega)]}{R_K} \times \left\{ \coth \left(\frac{\hbar\omega}{2k_B T} \right) [\cos(\omega t) - 1] - i \sin(\omega t) \right\}. \quad (7)$$

Here $R_K = h/e^2$, the von Klitzing constant, denotes the resistance quantum. The total impedance of the system reads

$$Z_t(\omega) = \frac{1}{i\omega C + Z^{-1}(\omega)}, \quad (8)$$

where C is the capacitance of the junction and $Z(\omega)$ is the impedance of the EM environment. In our model the EM environment is assumed to be an infinite RC transmission line whose impedance is $Z(\omega) = \sqrt{R_0/i\omega C_0}$, where R_0 and C_0 are the resistance and the capacitance per unit length of the line, respectively [see Fig. 4(b)]. Since the impedance of the RC transmission line depends on the ratio between R_0 and C_0 , the appropriate dimensionless parameter that characterizes the line is $\kappa = R_0 C / C_0 R_K$ [22,23]. Here we restrict ourselves to the RC transmission line, which is the limiting case of a general RCL transmission line when the inductance per unit length can be neglected [22]. This can be justified by the fact that the characteristic frequency in the system, $\omega_C = E_C/\hbar \sim 10^{12}$ Hz, is smaller than the ratio $R_0/L_0 \sim 10^{13}$ Hz with $R_0 \sim 10 \Omega/\mu\text{m}$. Here $L_0 \sim 10^{-6}$ H/m is the inductance per unit length of the line.

B. Analytic formula

In order to derive an analytic formula that describes the conductance of the thermometer, we derive a simplified theory that captures the essential physics. We note that based on Eqs. (5) and (6) the tunneling current is expressed as

$$I_s(V, T) = \frac{\pi e E_J^2(T)}{\hbar} \times \frac{i}{\pi \hbar} \int_{-\infty}^{\infty} dt e^{AJ(t)} \sin\left(\frac{2eV}{\hbar} t\right). \quad (9)$$

The first factor is solely determined by the supercurrent at the tunnel junction and depends only on temperature. The second, $P(E)$, contribution depends on the bias voltage leading to the linear conductance in the form

$$G(T) = \frac{\partial I_s}{\partial V} \Big|_{V=0} = \frac{\pi e E_J^2(T)}{\hbar} \times P'(T), \quad (10)$$

with the temperature-dependent factors $E_J(T)$ and $P'(T)$ that we will determine now separately.

The supercurrent through the S - I - N junction can be found using the linear approximation of the Usadel equation. We assume a quasi-one-dimensional S - N - I - S' structure and find the solution of the Usadel equation in the S - N part. The S - N system is coupled to a superconductor S' via a tunnel contact of resistance R_T and the Josephson energy yields

$$E_J(T) = \frac{\hbar k_B T}{2e^2 R_T r} \sum_{n=0}^{\infty} \frac{\Delta(T)^2 \sqrt{\frac{E_{th}}{2\omega_n}}}{[\omega_n^2 + \Delta(T)^2] \sinh\left(\sqrt{\frac{2\omega_n}{E_{th}}}\right)}, \quad (11)$$

where e is the elementary charge, and $E_{th} = \hbar D/L^2$ is the Thouless energy and L as the length of the N wire. The S - N interface itself is characterized by a dimensionless parameter $r \gg 1$ [16,24] taking into account a finite transparency of the interface. We use the following boundary conditions: $\xi r \partial_x F_{\omega_n}|_{x=0} = -\Delta(T)/\sqrt{\omega_n^2 + \Delta(T)^2}$ and $\partial_x F_{\omega_n}|_{x=L} = 0$, where F_{ω_n} is the anomalous Green's function of the proximitized normal metal [14]. At high temperatures and for long junctions, where $E_{th} < k_B T$, one can make use of the single-frequency approximation keeping only the first term ($n = 0$) in the sum of Eq. (11) and arriving at

$$E_J(T) \approx \frac{\hbar k_B T}{e^2 R_T r} \frac{L_T}{L} e^{-L/L_T}, \quad (12)$$

where $L_T = \sqrt{\hbar D/2\pi k_B T}$ is the so-called thermal coherence length.

The second ingredient we need is the conductance due to the EM environment coupled to the junction. We deal with an infinite RC transmission line of $\kappa \ll 1$ and assume that at higher temperatures we can approximate

$\coth(\hbar\omega/2k_B T) \approx 2k_B T/\hbar\omega$. The phase-phase correlation function from Eq. (7) can be evaluated arriving at

$$J(t) \approx -\sqrt{\kappa E_C} \left[\frac{4}{3} k_B T \left(\frac{|t|}{\hbar} \right)^{3/2} + i \operatorname{sgn}(t) \left(\frac{|t|}{\hbar} \right)^{1/2} \right]. \quad (13)$$

The factor in the linear conductance then becomes

$$P'(T) = \frac{2ei}{\pi \hbar^2} \int_{-\infty}^{\infty} t e^{AJ(t)} dt. \quad (14)$$

The latter integral can be approximately evaluated employing the steepest descent method and we arrive at

$$P'(T) = \frac{e}{2\sqrt{2\pi} E_C^2} \left(\frac{1}{\kappa} \right)^{1/4} \left(\frac{E_C}{k_B T} \right)^{7/4} \times \left[1 - \frac{1}{2} \left(\frac{\pi}{8} - \frac{4}{3\sqrt{2}} \sqrt{\frac{E_C}{k_B T}} \kappa \right)^2 \right] \times e^{-4/3\sqrt{2}\sqrt{(E_C/k_B T)\kappa}}. \quad (15)$$

Note the interesting observation that this expression has a characteristic temperature scale given by κE_C , which is substantially smaller than the charging energy $E_C = 2e^2/C$ as is relevant for the experimental situation we have in mind. Finally, combining Eqs. (11) or (12) and (15) the conductance is given by (10).

V. ANALYSIS AND DISCUSSION OF THE DATA

The measured I - V characteristics like the ones shown in Fig. 2 allow us to extract the temperature and length dependences of different samples as shown in Fig. 5. Figure 5(a) demonstrates the T dependence for seven different samples. The main feature of all these datasets is the increase of $I_{\max}$ towards lower T in accordance with the prediction of the theory. Yet, one can observe the saturation of $I_{\max}$ at both high and low T . At high T , this is because of the emergence of a thermal quasiparticle current. More interestingly the current saturates below approximately 100 mK especially for samples with short L , a feature to be discussed below. In Fig. 5(b) we extract $I_{\max}$ for different samples at base $T \simeq 40$ mK. In Fig. 5(b) we also include the theoretically calculated $I_{\max}$, the maximum of I_s , according to the theory presented in the previous section. Assuming $T = 40$ mK, the calculation overestimates $I_{\max}$ by factor of about 3 [brown stars in Fig. 5(b)]. This is very natural basis of overheating of the proximitized normal-metal lead at finite bias voltage of about $V_{\max} = 20 \mu\text{V}$, which is the position where the current is maximized. Quantitatively, writing the heat balance equation $I_{\max} V_{\max} = \Sigma \mathcal{V}(T^5 - T_0^5)$, where $\Sigma = 2 \times 10^9 \text{ W K}^{-5} \text{ m}^{-3}$ is the electron-phonon coupling

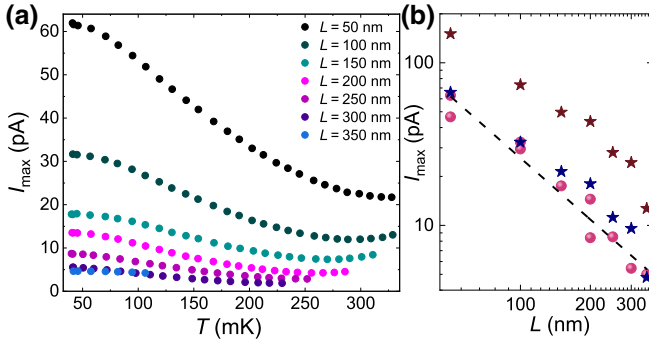


FIG. 5. Peak current $I_{\max}$ of different measured samples. (a) Temperature dependence of $I_{\max}$ for junctions with varying L . (b) Base temperature values of $I_{\max}$. The pink dots are extracted from the measurements in (a). Star symbols are from theory: the brown ones are calculated at $T = 40$ mK, while the blue ones are those with actual overheated temperatures as explained in the text. Dashed line $I_{\max} = 7.9 \times 10^3 L[\text{nm}]^{-1.25}$ pA is a fit through experimental data.

constant of copper, $\mathcal{V} = 1 \times 10^{-21} \text{ m}^3$ is the volume of the copper island [25,26], and $T_0 = 40$ mK is the bath (phonon) temperature, allows us to determine the temperature for each thermometer at this bias point. We obtain $T = 125\text{--}215$ mK for samples with $L = 350\text{--}50$ nm with 50-nm intervals, respectively. Repeating the calculation of $I_{\max}$ at these temperatures for the corresponding samples, we obtain a much better agreement with the measured values of $I_{\max}$ as shown by the blue star symbols in Fig. 5(b). The message of this result and analysis is that it is very important to perform a true zero-bias measurement to avoid overheating. Applying even a very small bias leads to severe self-heating of the thermometer.

Figure 6 shows a comparison between measured (circles) and theoretically predicted (solid lines) zero-bias conductance in the tunnel contact for various lengths of the junction. We obtain an excellent match at low T . As mentioned earlier, due to the tunneling of thermal quasiparticles into N , the thermometer eventually loses its sensitivity at about 300 mK and the back-bending feature appears in this crossover temperature range. The measured zero-bias conductance in this figure shares a similar temperature dependence to that of $I_{\max}$ in Fig. 5. There are important properties worth discussing in these data. First, the overall responsivity $\mathcal{R}$ of the thermometer improves on decreasing the length L by one order of magnitude when L shrinks from 350 to 50 nm. Second, unlike $I_{\max}$ in Fig. 5, the responsivity is not lost even at base temperature; instead the dependence remains more or less linear in T . It is important to mention that the zero-bias conductance is not obtained for exactly $V = 0$. It shows the averaged slope of the I - V curves close to zero-bias voltage ($V \approx 4 \mu\text{V}$) in order to be close to the experimental procedure as G is measured using the lock-in technique with a finite voltage amplitude in the microvolt range.

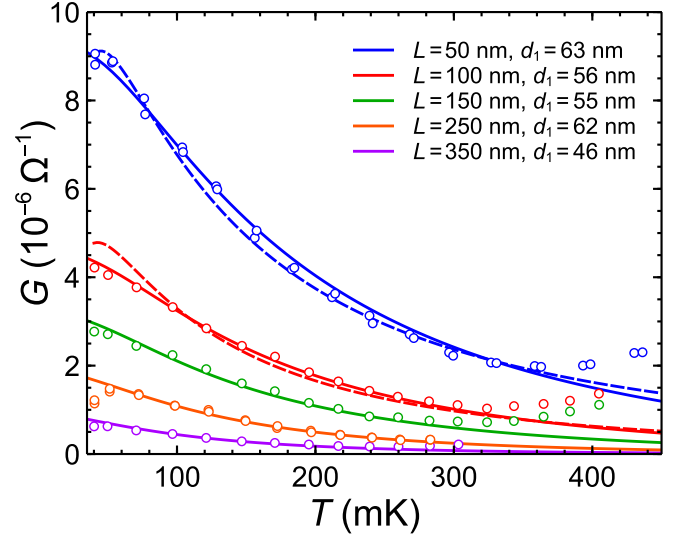


FIG. 6. Comparison between measurement (circles) and numerical calculations (solid lines) for zero-bias conductance as a function of temperature. For the theory of junctions with various normal-metal lengths L , we assume a charging energy $E_C = 3.5\Delta_0 = 700 \mu\text{eV}$, $d_S/(d_N + d_S) = 0.36$, $d_2 = 120$ nm, $d_1 \in [46 \text{ nm}, 63 \text{ nm}]$, and $R_T \in [15 \text{ k}\Omega, 16.1 \text{ k}\Omega]$. The junction is assumed to be coupled to an infinite RC transmission line characterized by $\kappa = 0.0115$ (see the text for the definition of κ). The dashed lines correspond to the analytic formula based on Eqs. (10), (12), and (15) with the following parameters: $\kappa = 0.007$, $R_T = 15 \text{ k}\Omega$, $r = 9.2$, $E_C = 3.5\Delta_0 = 700 \mu\text{eV}$, $d = 0.8\xi = 130$ nm.

We can also use the analytic expression (10) to describe the experiment. For that we take $\Delta_0 = 200 \mu\text{eV}$ as the energy gap of the superconductor at zero temperature, $E_C = 3.5\Delta_0 = 700 \mu\text{eV}$ as the charging energy of the junction, and $\xi = \sqrt{\hbar D/2\Delta_0} = 160$ nm as the superconducting coherence length in the dirty limit. Since the overlap geometry of the junction in the experiment is approximated by a one-dimensional wire for the analytic expression, we introduce an effective length of the junction $d + L$, where the offset d is assumed to be of the order of the overlap regions. The other parameters are $\kappa = 0.007$, $r = 9.2$, $R_T = 15 \text{ k}\Omega$, and $d = 0.8\xi = 130$ nm. For illustration we present the curves calculated from the analytic formula for two different lengths of the junction, $L = 50$ nm and $L = 100$ nm (dashed lines in Fig. 6). The value $r = 9.2$ corresponds roughly to an average transparency of the S - N interface of about 0.1 [16]. We observe that the experimental data are well described by the analytic result, except at the lowest temperature and for the shortest wire.

VI. CONCLUSIONS

We find experimentally that the sensitivity of the S - N - I - S thermometer operated at zero-bias voltage can be enhanced dramatically by bringing the S contact to the very proximity of the tunnel junction, this way increasing

the current through it. Specifically, we demonstrate that the zero-bias conductance measurement outperforms a standard I - V measurement by avoiding self-heating at low temperatures. We develop a theoretical model based on proximity superconductivity and dynamical Coulomb blockade, which captures quantitatively the measured data in their validity range. With this optimization, we increase the responsivity of this thermometer by about one order of magnitude compared to the initial realization of the concept, making it suitable for continuous detection of microwave quanta in the gigahertz range [11,27,28].

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