Reply to "Comment on 'Electric Power Generation from Earth's Rotation through its Own Magnetic Field' "

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In our original paper [Phys. Rev. Applied **6**, 014017 (2016)], we considered electric power generation from Earth's rotation through its own magnetic field. Jeener claims to prove that no such effect is possible [Phys. Rev. Applied **13**, 028001 (2020)]. But this conclusion results from a failure to recognize a distinction between the corotation with Earth of the nonaxially symmetric components of Earth's magnetic field and the nonrotation with Earth of the axially symmetric components. We use Earth's geomagnetic field to demonstrate the important consequences of this distinction. Because the distinction has considerable empirical support, Jeener's argument is likely incorrect.

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I. INTRODUCTION

In our original paper [1] (C&H), we examined a seemingly general argument that it is impossible to generate electricity from Earth's rotation through its own magnetic field. We identified a loophole in the argument that could be exploited by an appropriate system and showed that the continuous production of at least very low electric power appears possible. Jeener [2] claims to prove instead that no such system can succeed, then relies on this general proof to argue that our own derivation should also yield a null result.

But Jeener's claims result from a failure to recognize a distinction between the corotation with Earth of the nonaxially symmetric components of Earth's magnetic field and the nonrotation with Earth of the axially symmetric components. In Sec. II, we review experiments implying that the axisymmetric components of Earth's magnetic field *do not* rotate with Earth, even while the nonaxisymmetric components *do*. In Sec. III, we summarize the lowest-order standard model for Earth's geomagnetic field and in Sec. IV, show that the axisymmetric and nonaxisymmetric field components have different effects on charged particles corotating with Earth. In Sec. V, we show that this difference negates Jeener's argument.

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II. NONROTATING AXISYMMETRIC FIELDS

In 1912, Barnett reported experiments in which he placed a cylindrical capacitor axially in the field of a solenoid (or two flat-pole electromagnets), with a wire connecting the concentric cylinders of the capacitor [3]. Corotation of the cylinders and their connecting wire while holding the solenoid stationary charged the capacitor due to the $q\mathbf{v} \times \mathbf{B}$ force on charges q in the wire. The wire was then disconnected, the system despun, and an opposite charge on the cylinders measured. But rotating the solenoid (or electromagnets) while holding the capacitor and connecting wire stationary did not charge the capacitor. These results were independently reproduced by Kennard [4] and Pegram [5], thereby proving that the field of a rotating axially symmetric electromagnet does not itself rotate with the electromagnet. These experiments have never been refuted and are a standard (albeit unemphasized) part of the electromagnetics literature [6–9]. (An attempted theoretical refutation has been proven to be incorrect due to calculational error [10–12].)

Some have argued that an axisymmetric field nevertheless rotates with a *permanent* magnet [13,14]. Experiments are more challenging because if the field does not rotate with the magnet, a rotating conducting permanent magnet produces an *electric* field due to charge redistribution within itself as it rotates through its own field [15] and this effect must be distinguished from the putative rotating-magnetic-field effect. But Earth's deeply generated magnetic field cannot be due to permanent magnetism: the Curie temperature is reached at a depth of only about 30 km [16] and magnetic field reversals are inconsistent with a permanent magnet [17]. Various studies in the geophysics literature therefore treat Earth as rotating through its own nonrotating axisymmetric field and examine the consequences of this behavior [18–20].

Historically, the idea of the field "rotating with the magnet" was understood to mean that a $q\mathbf{v}\times\mathbf{B}$ force would be experienced by an electric charge q if q had a velocity $\mathbf{v}$ relative to axes corotating with the magnet (see, e.g., Ref. [4]). As C&H stated, this differs from the current usual understanding of the $q\mathbf{v}\times\mathbf{B}$ force, in which $\mathbf{v}$ is the velocity of q in the frame in which the magnetic flux density is $\mathbf{B}$ [21,22]. Nevertheless (see Sec. IV), even in a modern interpretation, an axisymmetric nonrotating field produces a Lorentz force on a charge q corotating with Earth, whereas a nonaxisymmetric field corotating with q produces no Lorentz force on such a charge.

III. EARTH'S GEOMAGNETIC FIELD

To discuss the implications of the geomagnetic field, we work with Earth's field complete through first order. We make use of two reference frames, identical to those used by Jeener. Frame Q is an inertial frame with origin at the center of the Earth and the usual spherical coordinates (r,θ,φ); it moves with Earth in its orbit but does not rotate with Earth's polar (z-axis) rotation. Frame Q' shares its origin with Q but rotates with Earth at angular frequency $\boldsymbol{\omega}=\omega\hat{\mathbf{z}}$, so that a particular point rotating with Earth does not change its coordinates in Q' over time. Q' has coordinates (r,θ,φ'), where $\varphi'=\varphi-\omega t$. C&H cited the standard literature [7] showing that at Earth-rotation velocities, electromagnetism in Q' behaves like that in an inertial frame to order $(v/c)^2\sim 10^{-12}$.

Detailed models of Earth's field derive it from a magnetic potential written in terms of surface harmonics and Schmidt-normalized associated Legendre polynomials with coefficients g_l^m and h_l^m of degree l and order m [16,17]. The g_1^0 term corresponds to Earth's dipole axisymmetric about (and antiparallel to) Earth's rotation axis and g_1^1 and h_1^1 are the leading off-axis terms, corresponding to orthogonal dipoles lying in the equatorial plane. The units are those of magnetic flux density. We ignore small long-term secular corrections. Explicitly, Earth's axisymmetric dipole is given by

$$B_r^{m=0}=2g_1^0(a/r)^3\cos\theta, \tag{1a}$$

$$B_\theta^{m=0}=g_1^0(a/r)^3\sin\theta, \tag{1b}$$

$$B_\varphi^{m=0}=0, \tag{1c}$$

where $a=6371.2$ km is a reference radius close to Earth's mean radius, $g_1^0=-24496.5$ nT [23], and the superscript "$m=0$" labels these as components of the axisymmetric field. Obviously, $\mathbf{B}^{m=0}$ has no φ dependence.

The lowest-order nonaxisymmetric field is given by

$$B_r^{m\neq 0}=2(a/r)^3(g_1^1\cos\varphi'+h_1^1\sin\varphi')\sin\theta, \tag{2a}$$

$$B_\theta^{m\neq 0}=-(a/r)^3(g_1^1\cos\varphi'+h_1^1\sin\varphi')\cos\theta, \tag{2b}$$

$$B_\varphi^{m\neq 0}=(a/r)^3(g_1^1\sin\varphi'-h_1^1\cos\varphi'), \tag{2c}$$

where $g_1^1=-1585.9$ nT, $h_1^1=4945.1$ nT [23], and the superscript "$m\neq 0$" labels these as components of the nonaxisymmetric field. Because φ' denotes longitude in Q', $\mathbf{B}^{m\neq 0}$ is rotating in Q at angular speed ω.

The source of the fields in Eqs. (1) and (2) lies in Earth's liquid-iron outer core. The material generating these fields of course shares Earth's rotation but dynamo models also require this material to undergo convective flows with radial motions the angular velocities of which are not coaxial with $\boldsymbol{\omega}$ [24,25]. The motion of the sources generating the fields observed at Earth's surface, given in Eqs. (1) and (2), is likely quite complex.

IV. THE LORENTZ FORCE

To focus our initial discussion on the implications of the fields in Eqs. (1) and (2) themselves, consider two idealized thought experiments. First, consider a frame Q in which $\mathbf{E}=\mathbf{0}$ and the only $\mathbf{B}$ field is that of an axially symmetric dipole field formally identical to $\mathbf{B}^{m=0}$ in Eq. (1). [These idealized thought experiments allow us initially to ignore complications arising from the presence of conductors, as well as whatever deep fluid flows and currents are needed to maintain the geomagnetic dynamo producing the ultimately observed fields of Eqs. (1) and (2).] Imagine a test charge q at colatitude θ and longitude φ, rotating about the z axis with velocity

$$\mathbf{v}=r\omega\sin\theta\hat{\boldsymbol{\varphi}}. \tag{3}$$

We ignore the tiny current associated with the motion of the test charge q. Of course, we have $\nabla\times\mathbf{E}=-\partial\mathbf{B}^{m=0}/\partial t$ but by Eq. (1), $\partial\mathbf{B}^{m=0}/\partial t=\mathbf{0}$, consistent with $\mathbf{E}=\mathbf{0}$. The Lorentz force in Q on q is

$$\mathbf{F_L}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) \tag{4}$$

and we have the physically expected result that in Q, q experiences a Lorentz force

$$\mathbf{F_L}=q\mathbf{v}\times\mathbf{B}^{m=0}. \tag{5}$$

The Lorentz force must be frame independent; in Q', it is

$$\mathbf{F'_L}=q\mathbf{E}', \tag{6}$$

since q has zero velocity in Q'. Because $v/c\ll 1$,

$$\mathbf{E}'=\mathbf{E}+\mathbf{v}\times\mathbf{B}, \tag{7}$$

so $\mathbf{E}' = \mathbf{v} \times \mathbf{B}^{m=0}$ since $\mathbf{E} = 0$ and

$$\mathbf{F'}_\mathbf{L} = q\mathbf{v} \times \mathbf{B}^{m=0} \tag{8}$$

by Eq. (6), giving $\mathbf{F'}_\mathbf{L} = \mathbf{F}_\mathbf{L}$ as required.

Consider an analogous second thought experiment but in which the only $\mathbf{B}$ field present in Q is an axially asymmetric field formally identical to $\mathbf{B}^{m\neq 0}$ in Eq. (2). In strong contrast to Eq. (8) for $\mathbf{B}^{m=0}$, we must have

$$\mathbf{F'}_\mathbf{L} \neq q\mathbf{v} \times \mathbf{B}^{m\neq 0}. \tag{9}$$

This may be proven by contradiction: assume instead that $\mathbf{F'}_\mathbf{L} = q\mathbf{v} \times \mathbf{B}^{m\neq 0}$. Then Eqs. (6) and (7) would require $\mathbf{E} = \mathbf{0}$. But this is impossible given the definition of $\mathbf{B}^{m\neq 0}$ in Eq. (2), because

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}^{m\neq 0}/\partial t \tag{10}$$

and $\partial \mathbf{B}^{m\neq 0}/\partial t \neq \mathbf{0}$ since $\mathbf{B}^{m\neq 0}$ depends on $\varphi' = \varphi - \omega t$. Therefore, the inequality in Eq. (9) must hold. In fact, from Eqs. (2) and (3) one may explicitly show that

$$\partial \mathbf{B}^{m\neq 0}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B}^{m\neq 0}). \tag{11}$$

We conclude that while q experiences a Lorentz force of $q\mathbf{v} \times \mathbf{B}$ for the nonrotating axisymmetric $\mathbf{B}^{m=0}$, q does *not* experience a corresponding Lorentz force for the case of a rotating nonaxisymmetric $\mathbf{B}^{m\neq 0}$. What force does q experience in this case? By Eqs. (10) and (11), the simplest choice for $\mathbf{E}$ is $\mathbf{E} = -\mathbf{v} \times \mathbf{B}^{m\neq 0}$. Then, by Eqs. (7) and (6), $\mathbf{E}' = \mathbf{0}$ and $\mathbf{F}_{\mathbf{L}'} = \mathbf{0}$. The charge q experiences no Lorentz force when corotating with $\mathbf{B}$, contrary to the situation for a nonrotating axially symmetric $\mathbf{B}$. When $\mathbf{F}_\mathbf{L} = \mathbf{F}_{\mathbf{L}'} = \mathbf{0}$, no work can be done on q, the electromotive force (emf) must equal 0, and for this case Jeener's conclusion holds.

Formally, Eqs. (10) and (11) admit a solution $\mathbf{E} = -\mathbf{v} \times \mathbf{B}^{m\neq 0} - \nabla\psi$, or $\mathbf{E}' = -\nabla\psi$. But one cannot use ψ to establish the Lorentz force. To attempt to do so, one would choose $\nabla\psi = -\mathbf{v} \times \mathbf{B}^{m\neq 0}$ to give $\mathbf{E} = 0$ and thereby $\mathbf{F'}_\mathbf{L} = q\mathbf{v} \times \mathbf{B}^{m\neq 0}$. But this choice is impossible, since $\nabla \times \nabla\psi = \mathbf{0}$, whereas $\nabla \times (\mathbf{v} \times \mathbf{B}^{m\neq 0}) \neq \mathbf{0}$. One cannot make the rotating nonaxisymmetric field produce a Lorentz force analogous to that produced by the nonrotating axisymmetric field. We show in Sec. V that this implies that an emf could be generated with the proper material and topology in the latter case but not the former. Jeener's analysis misses this difference by failing to recognize the distinction between rotating and nonrotating fields. The distinction is not a "legacy notion."

V. THE POSSIBILITY OF emf GENERATION

Now consider relaxing some of our thought experiments' idealizations. In particular, imagine q to be embedded in a conductor at rest in Q'. In our second thought experiment, there is still no Lorentz force on any charge q within the conductor. With $\mathbf{F}_\mathbf{L} = \mathbf{0}$, we have $\mathbf{E} = -\mathbf{v} \times \mathbf{B}^{m\neq 0}$ and again therefore have Eq. (11), which is identical to Jeener's Eq. (3):

$$-[\partial \mathbf{B}^s/\partial t]_Q + \nabla \times [\mathbf{v} \times \mathbf{B}^s] = \mathbf{0}, \tag{J3}$$

where his superscript s denotes "steady-state" and his subscript Q is a reminder that Eq. (12), like Eq. (11), applies in the frame Q. Jeener derives this equation in his Appendix A for any vector field that "is immobile as seen by Earth bound observers" (that is, by an observer in Q') and has zero divergence. In our derivation of Eq. (11), we have shown that this conclusion is correct for a field $\mathbf{B}^{m\neq 0}$ that is corotating with q. In this case, Jeener's argument holds and no emf can be generated.

But this conclusion does not necessarily hold for a field such as $\mathbf{B}^{m=0}$ in our first thought experiment. This field is not corotating with Q' but, rather, objects in Q' are rotating through it. Take q to lie within a conductor at rest in Q'. In Q, as the conductor moves through $\mathbf{B}^{m=0}$, its charge carriers experience a force $q\mathbf{v} \times \mathbf{B}^{m=0}$ and they redistribute over a time scale of approximately 10^{-11} s [1] until they establish an electrostatic field

$$\mathbf{E} = -\nabla V = -\mathbf{v} \times \mathbf{B}^{m=0}, \tag{12}$$

after which Jeener's Eq. (12) again holds. Then $\mathbf{F}_\mathbf{L} = \mathbf{0}$ within the object, no current flows are possible, and emf $= 0$. But this conclusion cannot hold for a conductor such as a magnetically permeable cylindrical shell, the topology of which yields $\nabla \times (\mathbf{v} \times \mathbf{B}) \neq \mathbf{0}$ [1]. In this case, because $\nabla \times \nabla \mathbf{V} = \mathbf{0}$ always, Eq. (13) *cannot* be valid and continuous currents become possible.

Determining whether these currents could be more than negligible requires an examination of Ohm's law. Ohm's law in Q for a conductor of conductivity σ moving at velocity $\mathbf{v}$ is:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{J}/\sigma, \tag{13}$$

where $\mathbf{J}$ is the current density. Taking the curl and applying Maxwell's equations gives

$$-\partial \mathbf{B}/\partial t + \nabla \times (\mathbf{v} \times \mathbf{B}) = -\eta \nabla^2 \mathbf{B}, \tag{14}$$

for constant magnetic diffusivity $\eta = (\sigma\mu)^{-1}$ and magnetic permeability $\mu = \mu_r\mu_0$, with relative and vacuum permeabilities μ_r and μ_0. The left-hand side of Eq. (15) integrated over the relevant surface S is just the emf around the corresponding path C, provided that there are no jump discontinuities on S [26]. Equation (15) implies that the emf need not be negligible for a permeable cylindrical shell with magnetic Reynold's number $R_m \ll 1$ [1]. Jeener's Eq. (12) holds for a field that is simply advecting in Q; it is straightforward to show for Eq. (3) that $\nabla \times (\mathbf{v} \times \mathbf{B}) = -\omega\partial\mathbf{B}/\partial\varphi$ [1], which makes this interpretation obvious. However, Eq. (15) shows that for a field not corotating in Q', $\mathbf{B}$ changes due both to diffusion within the conductor as well as advection, as the conductor moves through $\mathbf{B}$.

In particular, as a magnetically permeable conducting cylindrical shell with $R_m \ll 1$ moves through $\mathbf{B}^{m=0}$, $\mathbf{B}$ within the shell both advects and diffuses. Orient the long axis of the shell to be orthogonal to both $\mathbf{v}$ and the local direction of $\mathbf{B}^{m=0}$ (C&H, Fig. 1). The magnetic flux density $\mathbf{B_0}$ within such a shell, were it motionless in a frame Q' with $\mathbf{E}' = \mathbf{0}$, is well known [27,28]. But in Sec. IV, we show that for a *nonrotating field* in Q, $\mathbf{E}' = \mathbf{v} \times \mathbf{B}$ in Q'. Far away from the cylindrical shell, $\mathbf{E}' = \mathbf{v} \times \mathbf{B}^{m=0}$. The shell is immersed in this background electric field, the direction of which is parallel to the shell's long axis. The magnetic flux density (and correspondingly $\mathbf{E}'$) near the shell is distorted from $\mathbf{B}^{m=0}$ [see C&H, Eq. (9)] and within the shell the fields become $\mathbf{B}_s$ and $\mathbf{E}' = \mathbf{v} \times \mathbf{B}_s$. In Q' with $(v/c)^2 \ll 1$, $\mathbf{B'_s} = \mathbf{B_s}$ [21] and Ohm's law $\mathbf{E}' = \mathbf{J}'/\sigma$ becomes

$$\mathbf{v} \times \mathbf{B}_s = \eta\nabla \times \mathbf{B}_s, \qquad (15)$$

where the subscript "s" is a reminder that Ohm's law, and therefore Eq. (16), applies only within the conducting shell. Written in terms of the vector potential $\mathbf{A}_s$, Eq. (16) is just the equation solved by C&H for the fields $\mathbf{A}_s$ and $\mathbf{B}_s$ for $R_m \ll 1$. In particular, $\mathbf{B_0}$ is not a solution to Eq. (16) but the solution $\mathbf{B}_s$ equals $\mathbf{B_0}$ plus a series of terms scaled by successive powers of R_m. This solution yields an emf [1].

VI. CONCLUSIONS

This rebuttal of Jeener's argument does not, of course, guarantee that our original prediction [1] is correct. For example, perhaps Earth's axisymmetric field does rotate with Earth [29], despite the seeming inconsistency of this with laboratory results for electromagnets. Further experiments are needed to test our prediction and its assumptions.

This Reply responds only to Jeener's Comment [2] and we hold our response to the experimental paper by Veltkamp and Wijngaarden [29] for another occasion. But it may be helpful to offer a few comments for any group attempting to test the hypothesis of our original paper. First, we have found in the laboratory that Mn-Zn ferrites—a practical $\mu_r \gg 1$, $R_m \ll 1$ Ohm's-law material—appear to be photoactive, so that experiments must be protected from light. Worse, Mn-Zn ferrites have very high Seebeck coefficients [30], and laboratory temperature gradients of $0.5\,^{\circ}\mathrm{C\,m^{-1}}$ easily overwhelm or cancel (depending on orientation) the sought-after effect. Reliable results therefore require either isothermal conditions or that temperatures at both ends of the Mn-Zn cylindrical shell be measured simultaneously with the emf, in order to control for the Seebeck voltage.

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