Tuning the Skyrmion Hall Effect via Engineering of Spin-Orbit Interaction

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We demonstrate that the Magnus force acting on magnetic skyrmions can be efficiently tuned via modulation of the strength of spin-orbit interactions. We show that the skyrmion Hall effect, which is a direct consequence of the nonvanishing Magnus force on the magnetic structure, can be suppressed in certain limits. Our calculations show that the emergent magnetic fields in the presence of spin-orbit coupling (SOC) renormalize the Lorentz force on itinerant electrons, and thus, influence topological transport. In particular, we show that, for a Néel-type skyrmion and Bloch-type antiskyrmion, the skyrmion Hall effect (SkHE) can vanish by tuning appropriately the strength of Rashba and Dresselhaus SOCs, respectively. Our results open up alternative directions to explore in a bid to overcome the parasitic and undesirable SkHE for spintronic applications.

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I. INTRODUCTION

Over the last decade, spintronic research interest has switched towards a different direction called spin-orbitronics that exploits the relativistic coupling of the electron's spin to its orbit to create intriguing effects and materials [1–3]. It turns out that the spin-orbit interaction is crucial in effects related to the efficient conversion of charge to spin current [4,5], which is essential for spintronic applications. The former has been largely exploited for the creation of a novel class of topological materials, such as chiral domain walls and magnetic skyrmions with enhanced thermal stability, low critical currents, and smaller sizes. Therefore, spin-orbit related effects open up promising directions to create, manipulate, and detect spin currents for spintronic applications.

In magnetic materials with broken inversion symmetry, an atom with a strong spin-orbit coupling (SOC) can mediate an antisymmetric exchange interaction called the Dzyaloshinskii-Moriya interaction (DMI), which favors the noncollinear alignments of atomic spins [6,7]. In such materials, competition between the DMI and other magnetic interactions, notably, exchange (which favors collinear alignment of atomic spins), is essential for the stabilization of exotic magnetic states, such as helimagnets [8] and magnetic skyrmions [9]. The latter are widely projected to be viable contenders for information carriers in next-generation data storage and spin-logic devices due to their small spatial extent, high topological protection, and efficient current-induced manipulation that allows for robust, energy-efficient, and ultrahigh density spintronic applications [10,11]. However, the integration of ferromagnetic skyrmions in such applications is hindered by the undesirable skyrmion Hall effect (SkHE), i.e., a transverse motion of skyrmions to the direction of current flow [12,13].

To overcome this parasitic effect, several proposals have been put forward, such as the use of antiferromagnetic skyrmions [14–17], edge repulsion [18], magnetic bilayer skyrmion [19], skyrmionium [20–25], antiskyrmions [26,27], and via spin current partially polarized along the direction of applied current [28]. These proposals focus on suppressing the inherent Magnus force in these systems, while little attention has been paid to understanding its source, i.e., the nature of the texture-induced emergent magnetic field. Moreover, since the stabilization of topological magnetic textures, such as skyrmions, usually requires a strong SOC, it is important to investigate the effect of the latter on these fictitious electromagnetic fields. Indeed, recent studies show that the SOC induces an additional fictitious electric field that manifests itself as a spin-motive force [29,30] on the itinerant electrons and gives rise to a charge current [31] and chiral damping

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[32]. However, the effect of the SOC-induced emergent magnetic field on conduction electrons has essentially been overlooked.

Here, we provide a theoretical framework that takes into account these fictitious magnetic fields and elucidate their impact on the topological transport inherent to magnetic skyrmions. We demonstrate that the SkHE, which is a direct consequence of a nonvanishing Magnus force on the magnetic structure can be efficiently tuned via modulation of the strength of the spin-orbit interaction. In particular, we show that for a Néel-type skyrmion and Bloch-type antiskyrmion, the SkHE can be tuned to zero via the modulation of the strength of the Rashba SOC (RSOC) and Dresselhaus SOC (DSOC), respectively. Our results open up alternative directions to overcome the parasitic and undesirable SkHE in ferromagnetic skyrmions.

II. THEORETICAL MODEL

It is known that the DSOC stabilizes Bloch-type skyrmions [33–35], while the RSOC stabilizes Néel-type skyrmions [36,37]. However, the recent realization of Bloch-type skyrmions in Rashba metals [38,39] motivates us here to consider an interplay of both types of SOC in a two-dimensional skyrmionic system described by the Hamiltonian,

$$\hat{\mathcal{H}} = \frac{\hat{\boldsymbol{p}}^2}{2m^*} + J\boldsymbol{m}(\boldsymbol{r}) \cdot \hat{\boldsymbol{\sigma}} + \hat{\mathcal{H}}_R + \hat{\mathcal{H}}_D, \tag{1}$$

where m^* is the effective mass of electrons, $\hat{\boldsymbol{p}}$ is the momentum operator, and J is the exchange interaction between the local moments in the direction of the unit vector $\boldsymbol{m}$ and spins of itinerant electrons given by the vector of Pauli matrices $\hat{\boldsymbol{\sigma}}$. The terms $\mathcal{H}_R = \beta_R(\sigma_y p_x - \sigma_x p_y)/\hbar$ and $\mathcal{H}_D = \beta_D(\sigma_x p_x - \sigma_y p_y)/\hbar$ describe RSOC and DSOC, respectively. Furthermore, we consider J to be the dominant interaction relative to the SOCs and, since our considerations are based on two-dimensional systems with strong confinement along $\boldsymbol{e}_z$, the cubic DSOC contribution is assumed to be small and negligible [40,41]. To keep our analysis trackable, we consider an isolated skyrmion (antiskyrmion) with analytical ansatz without loss of generality given in spherical coordinates,

$$\boldsymbol{m}(\boldsymbol{r}) = \left(\cos\Phi\sin\theta, \sin\Phi\sin\theta, \cos\theta\right), \tag{2}$$

where the azimuthal angle is given as

$$\Phi(x, y) = q\,\mathrm{Arg}\,(x, y) + \gamma_c, \tag{3}$$

where $q = \pm 1$ is the vorticity, i.e., $q = +1$ for skyrmions and $q = -1$ for antiskyrmions, and c is the helicity, such that $\gamma_c = 0$ or π for Néel-type skyrmions and antiskyrmions and $\gamma_c = \pm\pi/2$ for Bloch-type skyrmions and antiskyrmions. To provide a very general analysis, we consider the radial angle, $\theta(r)$, with properties [42]

$$\cos\theta\left(r\right)_{r\to 0} = -\cos\theta\left(r\right)_{r\to R} = p, \tag{4}$$

and

$$\sin\theta\left(r\right)_{r\to 0} = \sin\theta\left(r\right)_{r\to R} = 0, \tag{5}$$

where $R \gg r_s$, r_s is the skyrmion radius, and $p = \pm 1$ is the polarity that defines the orientation of the skyrmion's core. In this representation, the topological charge, Q, of the magnetic soliton is given by $Q = pq$ [43]. Before we proceed, we note that our theoretical analysis is general and does not depend on any particular ansatz that satisfies Eqs. (3)–(5). However, for our numerical calculations, we consider the radial angle to be $\theta(r) = \pi(3-p)/2 + 4\tan^{-1}(e^{r/r_s})$.

III. ANALYTICAL RESULTS

It is well established that, when itinerant electrons traverse a smooth and slowly varying magnetic texture, $\boldsymbol{m}(\boldsymbol{r})$, there is a reorientation of their spins to follow the direction of local magnetization. This process gives rise to fictitious electromagnetic fields that act on the itinerant electrons [44–47]. The emergent electrodynamics resulting from the system described by Eq. (1) is derived following the standard approach, i.e., the exchange term is diagonalized via a unitary transformation, $\hat{U} = \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}}$, where $\boldsymbol{n} = \left[\cos\Phi\sin(\theta/2), \sin\Phi\sin(\theta/2), \cos(\theta/2)\right]$ in the spin space [48,49]. The end result is that itinerant electrons in the transformed frame are subjected to a uniform ferromagnetic state and weakly coupled to the spin-gauge fields,

$$\mathcal{A}_{\eta,\mu} = A^{\alpha}_{\eta,\mu} \cdot \sigma^{\alpha} = A^{z}_{\eta,\mu} \cdot \sigma^{z} + \boldsymbol{A}^{\perp}_{\eta,\mu} \cdot \boldsymbol{\sigma}^{\perp}, \tag{6}$$

where $\eta = s$, R, and D for the texture-, Rashba-, and Dresselhaus-induced gauge fields, respectively; α and μ represent the spin and real-space indices, respectively; $\boldsymbol{A}^{\perp}_{\eta,\mu} = (A^{x}_{\eta,\mu}, A^{y}_{\eta,\mu}, 0)$; and $\boldsymbol{\sigma}^{\perp} = (\sigma^x, \sigma^y, 0)$. We ignore the off-diagonal component, $\boldsymbol{A}^{\perp}_{\eta,\mu}$, that describes nonadiabatic processes, which is important in the nonadiabatic regime of weak exchange and/or sharp magnetic textures [50–53] typical in dilute magnetic semiconductors [54]. In this study, we focus on the $A^{z}_{\eta,\mu}$ component, which is diagonal and describes adiabatic processes that preserve the spin state in all parameter space considered. The origin of the spin-gauge fields given by Eq. (6) includes contributions: (i) due to the magnetic texture, $\boldsymbol{A}^{z}_{s}$; (ii) that result from an interplay between RSOC and the magnetic texture, $\boldsymbol{A}^{z}_{R}$; and (iii) interplay between the DSOC and magnetic texture,

$\boldsymbol{A}_D^z$, given by [32]

$$\boldsymbol{A}_s^z = \mp\frac{\hbar}{2e}(1-\cos\theta)\nabla\Phi, \tag{7a}$$

$$\boldsymbol{A}_R^z = \mp\frac{\hbar}{2e}\frac{(m_y\boldsymbol{e}_x - m_x\boldsymbol{e}_y)}{\lambda_R}, \tag{7b}$$

$$\boldsymbol{A}_D^z = \mp\frac{\hbar}{2e}\frac{(m_x\boldsymbol{e}_x - m_y\boldsymbol{e}_y)}{\lambda_D}, \tag{7c}$$

where $\mp$ represents the spin projections, i.e., $-1(+1)$ for spin-up(down) [55] and $\lambda_{R(D)} = \hbar^2/2m^*\beta_{R(D)}$ is the characteristic length scale of the RSOC (DSOC). While previous studies have focused on the effect of the SOC-induced emergent electric field on itinerant electrons (spin-motive force), here we focus on the effect of the corresponding emergent magnetic field on the itinerant electrons (Lorentz force), which are calculated from the spin-gauge fields,

$$\boldsymbol{B}_\eta = \nabla\times\boldsymbol{A}_\eta^z, \tag{8}$$

to obtain

$$\boldsymbol{B}_s = \mp\frac{\hbar}{2e}([\partial_x\boldsymbol{m}\times\partial_y\boldsymbol{m}]\cdot\boldsymbol{m}), \tag{9a}$$

$$\boldsymbol{B}_R = \pm\frac{\hbar}{2e}\frac{(\partial_x m_x + \partial_y m_y)}{\lambda_R} + \mathcal{O}(\beta_R^2), \tag{9b}$$

$$\boldsymbol{B}_D = \pm\frac{\hbar}{2e}\frac{(\partial_x m_y + \partial_y m_x)}{\lambda_D} + \mathcal{O}(\beta_D^2). \tag{9c}$$

It turns out that $\boldsymbol{B}_s$ and $\boldsymbol{B}_R$ in Eq. (9) act in opposite directions for Néel-type skyrmions, as illustrated in Fig. 1. The direct consequence of this fact is that the electrons traversing an array of Néel-type skyrmions in the positive x direction, experience two opposite fictitious magnetic fields: $\boldsymbol{B}_s$ in the positive z direction that tends to deflect electrons in the positive y direction, and $\boldsymbol{B}_R$ in the negative z direction that tends to deflect electrons in the negative y direction. Because $\boldsymbol{B}_R$ has two free parameters, r_s and λ_R, by tuning them, one can realize a condition where $\boldsymbol{B}_R$ completely cancels out $\boldsymbol{B}_s$ [i.e., $\lambda_R \approx r_s/2$, as given in Eq. (12)]. In this case, the transverse Lorentz force acting on electrons transversing the skyrmions is completely suppressed, or equivalently the Magnus force on the magnetic structure vanishes.

To gain more insight into the physics originating from the interplay of different contributions to emergent magnetic fields given by Eq. (9), we consider a discrete square system of size $101 \times 101a_0^2$, with an equilibrium skyrmion radius $r_s = 12a_0$, where $a_0 = 0.3$ nm is the lattice constant. Furthermore, we subtract the effective mass of electrons, $m^* = 0.4m_0$, where m_0 is the bare mass of electrons, and the RSOC strength $\beta_R = 2.5\times10^{-11}$ eV m (equivalent to $\lambda_R = 3.8$ nm). Then we calculate the corresponding magnetic fields for different vorticities and helicities. In Fig. 2, the results for Néel-type skyrmions ($q = +1$) and antiskyrmions ($q = -1$) are shown with the corresponding magnetization profiles [Figs. 2(a) and 2(d), respectively]. We see that, for Néel-type skyrmions, indeed, the fictitious magnetic fields $\boldsymbol{B}_s$ [cf. Fig. 2(b)] and $\boldsymbol{B}_R$ [cf. Fig. 2(c)] act in opposite directions, such that it is possible to achieve a current-driven motion without SkHE by tuning the strength of the RSOC. However, in the case of a Néel antiskyrmion, as shown in Figs. 2(e) and 2(f), even though the transversing electrons experience these fictitious magnetic fields, the effect of $\boldsymbol{B}_R$ on its trajectory cancels out by symmetry [cf. Fig. 2(f)]. As such, only $\boldsymbol{B}_s$ influences its trajectory, leading to SkHE for current-driven motion.

Similar arguments can be used to explain the characteristics of Bloch-type skyrmions and antiskyrmions, as shown in Fig. 3, with the corresponding magnetization profiles depicted in Figs. 3(a) and 3(d), respectively. It turns out that, unlike in the Néel-type case, the SOC-induced fictitious magnetic field, $\boldsymbol{B}_D$, does not influence the trajectory of electrons traversing Bloch-type skyrmions, since the latter cancels out by symmetry [cf. Fig. 3(c)]. As such, the trajectory of traversing electrons are detected by $\boldsymbol{B}_s$ [cf. Fig. 3(b)], leading to SkHE for current-driven motion. However, in the case of Bloch-type antiskyrmions, $\boldsymbol{B}_s$ [cf. Fig. 3(e)] and $\boldsymbol{B}_D$ [cf. Fig. 3(f)] act in opposite directions and, as a result, by tuning the strength of DSOC and/or the r_s via material engineering, it is possible to achieve a SkHE-free current-driven motion of Bloch antiskyrmions

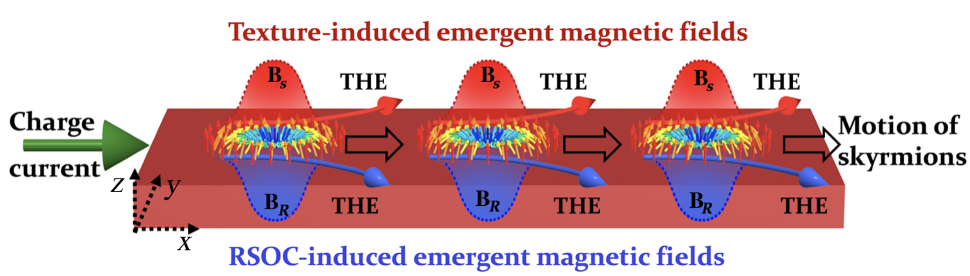

FIG. 1. Schematic illustration of the current-driven motion of the Néel-type skyrmions array in the presence of SOC. Fields $\boldsymbol{B}_s$ (red) and $\boldsymbol{B}_R$ (blue) act in opposite directions, leading to the topological Hall effect (THE) on traversing electrons in opposite directions. As such, tuning $\boldsymbol{B}_R$ through the strength of the SOC can produce current-driven motion without the skyrmion Hall effect (black arrows).

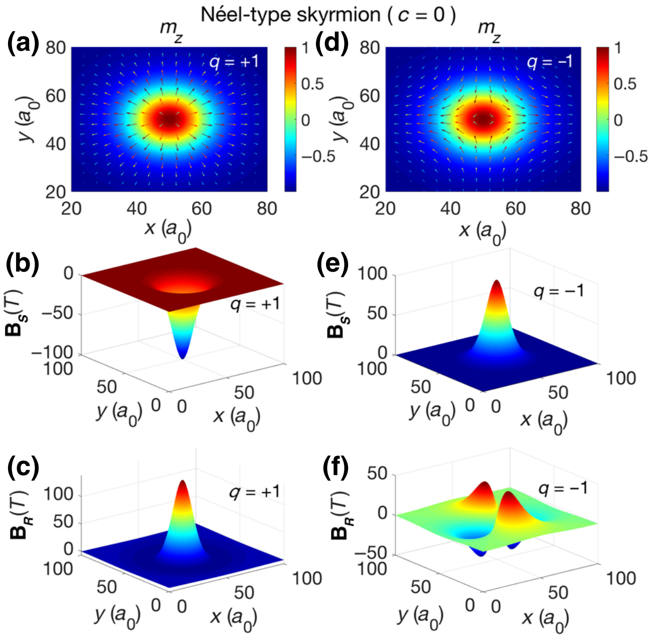

FIG. 2. Schematic diagram of the spatial magnetization profile for a Néel-type skyrmion (a) and antiskyrmion (b), and their corresponding emergent magnetic fields (c),(e) and (d),(f), respectively, in the presence of RSOC.

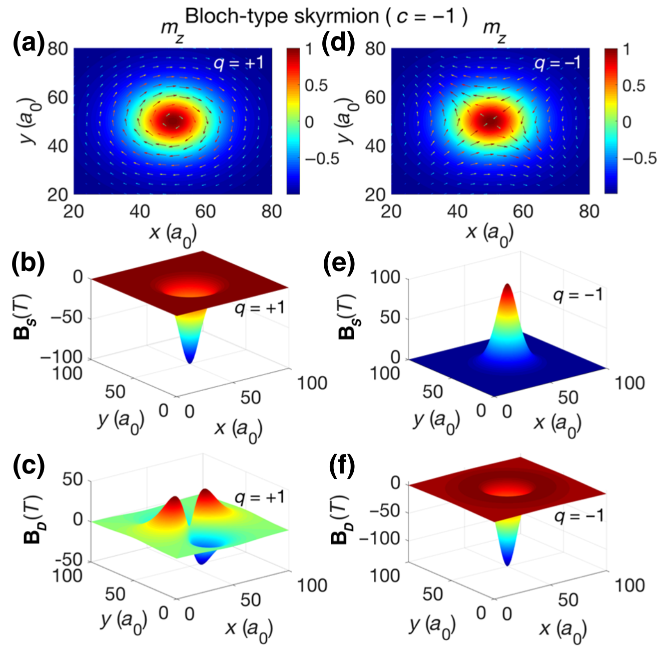

FIG. 3. Schematic diagram of the spatial magnetization profile for a Bloch-type skyrmion (a) and antiskyrmion (b), and their corresponding emergent magnetic fields (c),(e) and (d),(f), respectively, in the presence of DSOC.

[i.e. $\lambda_D \approx r_s/2$ as given in Eq. (12)]. Therefore, our analysis shows that, depending on the vorticity and chirality of ferromagnetic solitons, it is possible to achieve a current-driven motion of the latter without SkHE via the engineering of the spin-orbit interaction in the system. Although the heuristic analysis presented above captures the important physics, in what follows, we provide a more rigorous argument based on the average emergent magnetic field that electrons traversing magnetic skyrmions experience.

A straightforward calculation of emergent magnetic fields given in Eq. (9), using the general ansatz in Eqs. (3)–(5), yields

$$\boldsymbol{B}_s = \mp\frac{q\hbar}{2e}\frac{\mathrm{d}\theta}{\mathrm{d}r}\frac{\sin\theta}{r}, \tag{10a}$$

$$\boldsymbol{B}_R = \pm\frac{\hbar}{2e\lambda_R}\left(\frac{\mathrm{d}\theta}{\mathrm{d}r}\cos\theta + q\frac{\sin\theta}{r}\right)\cos\left(\Phi - q\Phi - \gamma_c\right) + \mathcal{O}\left(\beta_R^2\right), \tag{10b}$$

$$\boldsymbol{B}_D = \pm\frac{\hbar}{2e\lambda_D}\left(\frac{\mathrm{d}\theta}{\mathrm{d}r}\cos\theta - q\frac{\sin\theta}{r}\right)\sin\left(\Phi + q\Phi - q\gamma_c\right) + \mathcal{O}\left(\beta_D^2\right), \tag{10c}$$

from which the following inference is immediately drawn: (i) a Néel ($\gamma_c = 0$ or π) skyrmion ($q = +1$) in the presence of the RSOC experiences an equal but opposite emergent magnetic field, similar to a Bloch ($\gamma_c = -\pi/2$ or $\pi/2$) antiskyrmion ($q = -1$) in the presence of the DSOC; (ii) due to the symmetry of Φ, as defined by Eq. (3), the spatial average of the RSOC-induced emergent magnetic field for Néel antiskyrmions [cf. Eq. (10b)] and the DSOC-induced emergent magnetic field for Bloch antiskyrmions [cf. Eq. (10c)] vanish. Since we focus on the half-metallic and strong adiabatic regimes, in which spin-flip processes are not relevant, the THE can be quantified by the average fictitious magnetic fields. However, we note that such a description in the ballistic regime, especially in the weak exchange limit, is subtle [50,53]. The average fictitious magnetic fields are calculated as $\langle B_\eta\rangle_{\rm av} = \int B_\eta \mathrm{d}^2\boldsymbol{r}/\int \mathrm{d}^2\boldsymbol{r}$ from Eq. (10), so that

$$\langle B_s\rangle_{\rm av} = \mp\frac{2pq\hbar}{eR^2}, \tag{11a}$$

$$\langle B_R\rangle_{\rm av} = \pm\frac{p\hbar(1+q)r_s}{2e\lambda_R R^2}\cos\gamma_c + \mathcal{O}(\beta_R^2), \tag{11b}$$

$$\langle B_D\rangle_{\rm av} = \pm\frac{p\hbar(1-q)r_s}{2e\lambda_D R^2}\sin\gamma_c + \mathcal{O}(\beta_D^2). \tag{11c}$$

We immediately deduce from Eq. (11) that, up to the linear order in SOC, the average SOC-emergent magnetic field (i) vanishes for Néel antiskyrmions [cf. $q = -1$, in Eq. (11b)] and Bloch skyrmions [cf. $q = +1$, in Eq. (11c)];

and (ii) is finite and opposite to $\boldsymbol{B}_s$ for Néel skyrmions [cf. $q = +1$, in Eq. (11b)] and Bloch antiskyrmions [cf. $q = -1$, in Eq. (11c)]. As a result, since the topological Hall effect of itinerant electrons as they traverse magnetic skyrmions is governed by these average fictitious magnetic fields, we recover the conclusions discussed in our heuristic analysis above. Interestingly, these average additional fictitious magnetic fields are proportional to $\beta_{R,D} r_s$ and, since the DMI has a subtle dependence on r_s [56,57], but is proportional to the strength of the SOC [58,59], one expects that the former should be at least dependent on the strength of the DMI in these systems. This theoretical prediction provides an interesting direction to explore, tune, and potentially completely overcome this undesirable SkHE for spintronic applications.

To conclude this section, we deduce the critical value of the SOC ($\alpha^c_{R(D)}$) [or equivalently $\lambda^c_{R(D)}$] for the complete cancellation of the emergent magnetic field for the Néel (Bloch) skyrmion (antiskyrmion) [cf. calculated from Eq (11) by setting $\langle B_{R(D)}\rangle_{\text{av}} = \langle B_s\rangle_{\text{av}}$] as

$$\lambda^c_{R,D} \approx r_s/2. \tag{12}$$

Equation (12) is very important and should act as a guide for material engineering of topological transport in magnetic solitons.

An interesting extension, which is, however, out of the scope of the present work, would be to directly investigate this effect via micromagnetic simulations. This is achievable, for example, by taking into account the effect of the spin torques induced by the fictitious magnetic fields in Eq. (9). Indeed, previous studies have incorporated textured-induced magnetic and electric fields and shown that this gives rise to the so-called topological torque and topological damping that directly influence the mobility of skyrmions [60,61].

IV. NUMERICAL RESULTS

We corroborate our analytical predictions via numerical calculations of the THE of electrons as they traverse an isolated skyrmion in the presence of the RSOC or DSOC. Our considerations are based on a two-dimensional tight-binding model on a square lattice, as described by the Hamiltonian

$$\mathcal{H} = \sum_{\boldsymbol{i}} \hat{c}^\dagger_{\boldsymbol{i}} (\epsilon_{\boldsymbol{i}} + J \boldsymbol{m}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{\sigma}}) \hat{c}_{\boldsymbol{i}} - \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} \hat{c}^\dagger_{\boldsymbol{i}} t_{\boldsymbol{i}\boldsymbol{j}} \hat{c}_{\boldsymbol{j}}, \tag{13}$$

where $\epsilon_{\boldsymbol{i}}$ and $\hat{c}^\dagger_{\boldsymbol{i}}(\hat{c}_{\boldsymbol{i}})$ are the on-site energy and spinor creation (annihilation) operators on site $\boldsymbol{i} = (i_x, i_y)$, respectively. J is the exchange energy that couples the spin of electrons $\hat{\boldsymbol{\sigma}}$ to local magnetization $\boldsymbol{m}_{\boldsymbol{i}}$, and $t_{\boldsymbol{i}\boldsymbol{j}}$ is the nearest neighbor hopping that incorporates the spin-orbit interaction and is given by

$$t_{\boldsymbol{i}\boldsymbol{j}} = \begin{array}{ll} t_0 + i t_R \sigma_y + i t_D \sigma_x, & \boldsymbol{j} = \boldsymbol{i} \pm (1,0), \\ t_0 - i t_R \sigma_x - i t_D \sigma_y, & \boldsymbol{j} = \boldsymbol{i} \pm (0,1). \end{array} \tag{14}$$

Here, t_0 is hopping in the absence of the SOC, $t_{R(D)} = \beta_{R(D)}/a_0$, and a_0 is the lattice constant. We note that, at low band filling, there is a direct correspondence between the continuous and discrete Hamiltonians given in Eqs. (1) and (13), respectively, for $t_0 = \hbar^2/2m^* a_0^2$ [62]. An isolated skyrmion of radius $r_s = 10a_0$ in embedded in a ferromagnetic background, to which four ferromagnetic leads are attached, as depicted in Fig. 4. We employ the Landauer-Büttiker formalism [63] to investigate coherent charge transport in our system, which we subject to a longitudinal bias voltage across the left (L) and right (R) leads and measure the transverse responds via the top (T) and bottom (B) leads. The terminal current of the μ lead is calculated as

$$I_\mu = \frac{e^2}{2\pi\hbar} \sum_{\mu \neq \nu} (T_{\nu\mu} V_\mu - T_{\mu\nu} V_\nu), \tag{15}$$

where V_μ is the voltage at the μ lead and $T_{\nu\mu}$ is the transmission coefficient for electrons from the μ lead to the ν lead, which is calculated via the use of the KWANT software package [64]. The terminal voltages are calculated following Ref. [42], from which the THE is quantified via the topological Hall angle, which is defined as

$$\theta_{\text{TH}} = \frac{V_T - V_B}{V_R - V_L}. \tag{16}$$

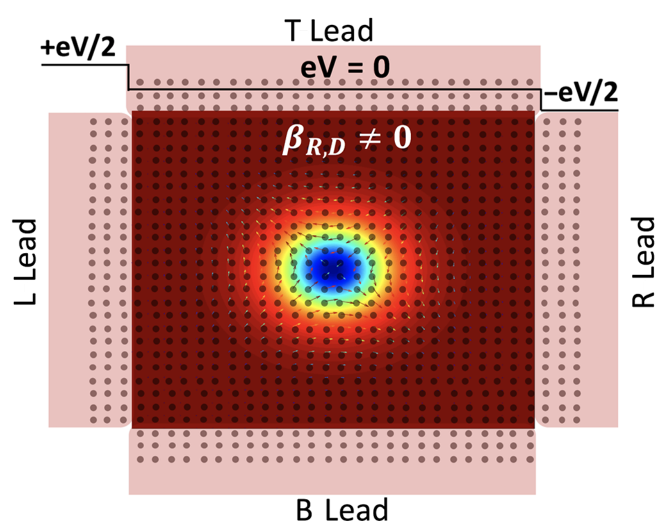

FIG. 4. Schematic diagram of four-terminal setup made up of a central region, containing a magnetic skyrmion in the presence of SOC, attached to four ferromagnetic leads: left (L), top (T), right (R), and bottom (B). The system is subjected to a longitudinal voltage bias of eV, while the transverse leads measure the Hall response of the system.

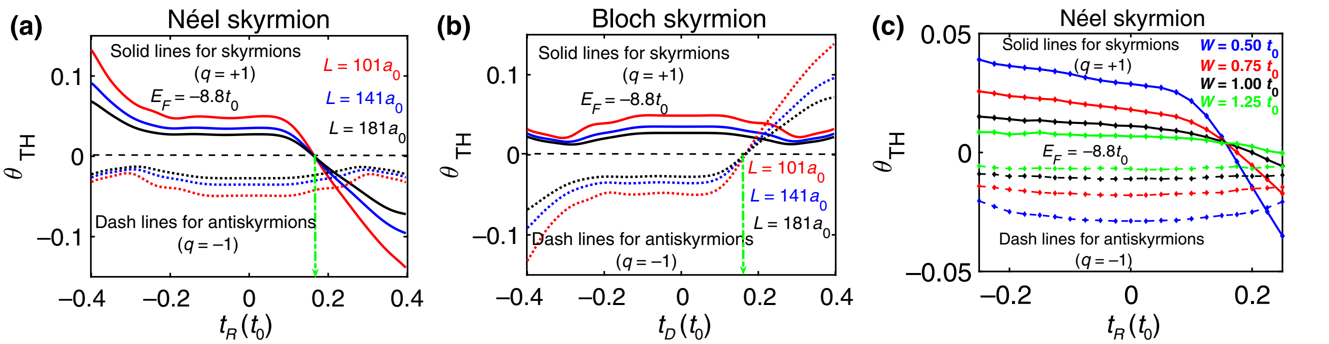

FIG. 5. Dependence of the THE on the strength of (a) RSOC (t_R) for Néel and (b) DSOC (t_D) for Bloch skyrmions and antiskyrmions, for different system sizes. The value of t_R for the Néel skyrmion and t_D for the Bloch antiskyrmion at which the THE vanishes is independent of the system size. (c) Dependence of THE on t_R for different impurity strengths for the Néel skyrmion and antiskyrmion.

We consider the strong exchange limit with $J = 5t_0$, and investigate the dependence of θ_{TH} on the strength of RSOC (t_R) for Néel-type and DSOC (t_D) for Bloch-type skyrmions and antiskyrmions. Furthermore, to rule out any possibility that our results stem from a size effect, we perform a systematic study with different system sizes $L \times L$, for $L = 101a_0$, $161a_0$, $181a_0$, and an optimal (to ensure smooth enough magnetization variation from the system to leads [61]) skyrmion radius of $10a_0$. Our numerical results, as shown in Fig. 5, confirm the physics underscored by our analytical derivations, i.e., the existence an optimal strength of RSOC (DSOC) at which the topological Hall effect vanishes in Néel skyrmions (Bloch antiskyrmions). Furthermore, the green arrows in Figs. 5(a) and 5(b) show that the value of t_R for the Néel skyrmion and t_D for the Bloch antiskyrmion at which the THE vanishes (green arrow) is independent of the system size, and thus, we rule out the possibilities that the observed results are artifacts of a size effect.

Finally, based on our analytical prediction given by Eq. (12), the estimate for the strength of the RSOC and DSOC at which the THE vanishes ($t^*_{R(D)}$) for a skyrmion of radius $r_s = 10a_0$ is predicted to be $t^*_{R/D} = 0.2t_0$. This value is very close to what we obtain numerically ($t^*_{R/D} = 0.17t_0$) using similar sets of parameters. We attribute the small discrepancy to nonlinear corrections and, most importantly, to the fact that our model does not take into account the explicit dependence of the skyrmion size on the SOC. The latter, which is out of the scope of this work, can be investigated, for example, by using micromagnetic simulations.

We conclude this study by investigating the effect of disorder, which is unavoidable in real materials. We model nonmagnetic impurity scattering via the randomization of the on-site energy given in Eq. (13), i.e., $\epsilon_i \to \epsilon_i + \delta V_i$, where $\delta V_i \in [-(W/2), (W/2)]$; W is the strength of the disorder and we take an average over 50 000 disorder configurations. We calculated the THE in the presence of nonmagnetic impurity scattering for different impurity strengths, for which the transport of electrons remains adiabatic (the adiabaticity condition might be lost in the diffusive regime), since our analysis is based on the adiabatic limit of the spin of electrons following the direction of local magnetization. Our result, as depicted in Fig. 5(c), shows that, for an impurity strength of $W = 0.5t_0, 0.75t_0, 1.0t_0$, and $1.25t_0$, which correspond to the mean free paths (λ) of $\lambda = 140a_0, 66a_0, 38a_0$, and $21a_0$, respectively, disorder scattering does not affect our conclusions in Sec. III. Therefore, our proposal of tuning the THE in magnetic solitons is robust against impurity scattering, if the transport of electrons remains adiabatic.

V. CONCLUSIONS

Magnetic skyrmions are considered as contenders for information carriers in future spintronic applications. However, the parasitic SkHE constitutes a technological challenge for the integration of the former in such applications. Several theoretical proposals, which focus on suppressing the Magnus force that gives rise to this detrimental SkHE, have been put forward. Here, we focus on exploring the possibilities of overcoming the SkHE via tuning spin-orbit interactions that are inherent in skyrmionic systems. Starting from the emergent electrodynamics in the latter in the presence of SOC, we demonstrate that the additional SOC-induced emergent fictitious magnetic fields can be used to tune the SkHE. Our calculations show that, by tuning the strength of the RSOC in Néel skyrmions or the DSOC in Bloch antiskyrmions, it is possible to achieve a current-driven motion without SkHE in these systems. Our findings open up a promising perspective on overcoming the SkHE.

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