Nonlinear Dynamical Behavior of the Deep White Matter during Head Impact

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Traumatic brain injury (TBI) is a major public health concern, affecting as many as 3 million people each year in the U.S. Despite substantial research efforts in recent years, our physical understanding of the cause of injury remains rather limited. In this paper, we characterize the nonlinear dynamical behavior of the brain-skull system through modal analysis and advanced finite-element (FE) simulations. We observe nonlinear behavior in the deep-white-matter (WM) structures near the dural folds, with an energy redistribution of around 30% between the dominant modes. We find evidence of shear-wave redirection near the falx and the tentorium (approximately 15° in the axial and 8° in the coronal plane) as a result of geometric nonlinearities. The shift in the falx mode shape, which is perpendicular to the deformation of the brain, causes geometrical nonlinear effects at the falx-brain tissue boundary. This is accompanied by a lateral sliding of the tentorium below the brain tissue, which induces higher local strains at its interface with deep regions of the brain. We observe that deep regions of the brain with high principal strains coincide with the identified nonlinear regions. The onset of nonlinear behavior in brain tissue is closely associated with the previously reported concussion thresholds, suggesting a possible link between the damage mechanism and the underlying nonlinear brain biomechanics.

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I. INTRODUCTION

Traumatic brain injury (TBI) is a major cause of death in the U.S., with at least 2.8 million people diagnosed annually [1] and about twice as many unreported cases, especially among athletes [2–4]. In order to develop diagnostic and preventive strategies against this growing epidemic, it is imperative to develop a deep physical understanding of the mechanisms of brain injury. Evaluation of the patterns of brain movement and deformation during a head impact has long been one of the most powerful strategies for attempting to predict brain damage. The first efforts to understand the underlying mechanics of the brain date back to 1943, when Holbourn proposed to model the brain as a mechanical system and studied the relation between different head kinematic inputs and deformation metrics. He hypothesized that rotational rather than translational acceleration is the dominant cause of larger strains in the brain [5,6]. Kornhauser proposed a second-order mass-spring system to analyze the brain mechanically and suggested that as the brain deformation surpasses a specific threshold, it can result in injury [7].

One direct benefit of a better understanding of the dynamics of the brain-skull system would be to come up with useful clinical-injury metrics for defining the severity of TBI. Many scientists have focused on developing such criteria [8], based either on kinematics [e.g., the head injury criterion (HIC) [9,10]] or on brain FE models [e.g., the cumulative strain damage measure (CSDM) [11]]. However, these scalar measures treat the whole brain as a single unit and lack a sufficient mechanical and dynamical understanding of the characteristics of the brain-skull interface. Hence, the question remains as to why specific anatomical structures, notably the brain stem and the corpus callosum (CC), reveal a higher susceptibility to strain [12–14].

A crucial point to consider is that the brain is a complex biomechanical system with an intricate geometry, nonuniform interfacial boundary conditions, and significantly inhomogeneous and nonlinear material properties [15,16]. This complexity can result in nonlinear effects, which are a common feature in dynamical systems. Previous studies,

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especially in the structural dynamics community, have shown that such nonlinearities can lead to many interesting phenomena, such as energy localization [17,18], targeted energy transfer [19–21], and nonlinear modal interactions [22,23]. To address the lack of physical understanding of the brain, a recent study by Laksari *et al.* analyzed the sensitivity of the human brain to deformation and found the localized modes and multimodal behavior—common characteristics of a nonlinear system—in deep regions of this tissue [15]. Others have also hinted at nonlinear behavior around the gray-white-matter junction and the CC. Sabet *et al.* reported disrupted strain fields at those regions through tagged magnetic resonance imaging (MRI), when they evaluated the deformation of the brain during a rotational acceleration [13]. Further analysis of the brain showed that localized strain concentrations occur in different regions of a linear and nonlinear viscoelastic medium during impact to the head [24,25]. Gurdjian *et al.* found that the concentration of the shear strains is in the vicinity of the CC and the brain stem, which interface with stiff ventricular or membranous structures [14] such as the falx cerebri and the tentorium cerebelli; these membranes also play a role in the mechanical response of the brain, by reflecting or redirecting shear waves [26].

In this paper, we use a brain FE model to simulate the brain's response under various rotational accelerations. By using a modal-decomposition technique, we characterize the dynamical response of the brain and identify its patterns of regional and overall nonlinearity. We then identify potential anatomical features that contribute to the observed nonlinear behavior. Finally, we investigate how these distinct responses correlate with a brain-injury metric, i.e., CSDM, and its implications for mild TBI (mTBI).

II. METHODS

Brain responses are simulated using the Worcester Head Injury Model (WHIM; [16,27]). The WHIM is a FE brain-injury model, in which the brain tissue is modeled as an isotropic homogeneous material using a second-order Ogden hyperelastic model [Fig. 1(a); see also the Supplemental Material [28]]. We analyze the deformation of the brain under a range of coronal rotational accelerations that contain kinematics shown to cause concussive injury [29,30]. We choose the coronal plane since it has been shown that rotations in this direction can cause large strains in the CC [12,31], which is one of the structures often used to predict mTBI [8,32,33]. We apply a half-sine acceleration impulse [Fig. 1(b)] to the center of gravity of the head by varying the amplitude α between 1 and 10 krad/s^2 and the duration Δt between 5 and 25 ms (with increments of 1 krad/s^2 and 5 ms, respectively).

Having simulated the brain responses for various accelerations, we implement modal-analysis decomposition to understand the important features of the brain dynamics. Modal analysis is a widely used method in structural dynamics, with applications in damage detection [34–36], dynamic characterization [37,38], and model-order reduction [39,40]. In this study, we use proper orthogonal decomposition (POD), a modal-analysis technique that extracts the modes from a system by performing an order reduction to transform a high-dimensional data set into a lower-degree-of-freedom one to obtain its relevant but unexpected hidden behaviors [41–43].

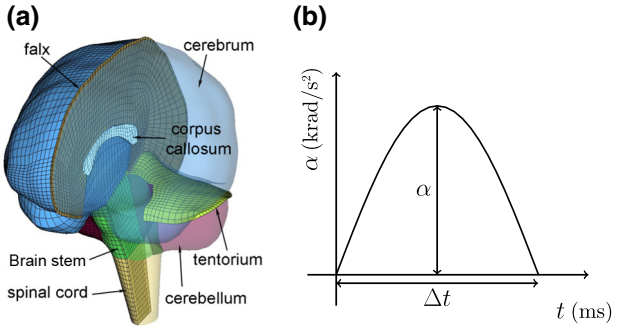

FIG. 1. Finite-element simulation of the brain using the WHIM. (a) The brain anatomical features in the WHIM model [27]. (b) A half-sine rotational acceleration pulse with varying amplitude ($\alpha = 1-10$ krad/s^2) and duration ($\Delta t = 5-25$ ms), imposed at the center of gravity of the head.

In order to apply POD, we consider a temporal sequence of the brain's nodal relative displacement fields $u_{\text{brain}}(x, y, z, t)$, where x, y, z is the spatial location of each node considered at time t. For any real u_{brain} in the form of an $m \times n$ matrix, there exists a factorization called singular-value decomposition (SVD) that can be written as follows [41,44]:

$$u_{\text{brain}} = USV^T, \tag{1}$$

where $U = [v_1, v_2, \ldots, v_m]$ is an $m \times m$ orthonormal matrix containing the left singular vectors v_i that correspond to proper orthogonal modes (POMs); V is an $n \times n$ orthonormal matrix containing the right singular vectors. S is an $m \times n$ diagonal matrix of real and non-negative diagonal entries, containing the singular values σ_i that correspond to the portions of the energy of the POD modes (see the Supplemental Material [28]).

In order to analyze how the mode shapes of various structures of the brain tissue change with increasing acceleration, the first nondimensional mode shape corresponding to $\alpha = 1$ krad/s^2 and $\Delta t = 5$ ms (the minimum rotational acceleration of the simulations) is defined as the linear baseline of the system. To quantify the percentage change of the ith POM at various α and Δt compared to the ith POM of the linear baseline, the following formulation

is introduced:

$$\Delta v_i = \left| \frac{v_i(x,y,z,\alpha,\Delta t) - v_i(x,y,z,\alpha_{\mathrm{ref}},\Delta t_{\mathrm{ref}})}{v_i(x,y,z,\alpha_{\mathrm{ref}},\Delta t_{\mathrm{ref}})} \right| \times 100, \tag{2}$$

where $v_i(x,y,z,\alpha,\Delta t)$ is the POM of the system at $\alpha = 1-10$ krad/s^2 and $\Delta t = 5-25$ ms, and $v_i(x,y,z,\alpha_{\mathrm{ref}}, \Delta t_{\mathrm{ref}})$ corresponds to the linear baseline POM of the system.

III. RESULTS

Having simulated the relative displacement of the global brain-skull system for each coronal rotational acceleration, we perform POD on these data to find the corresponding POMs of each acceleration as well as their contribution to the energy of the system (Fig. 2). With changing duration and amplitude, we find a minor change of 2.7% in the energy contribution of the dominant POM of the whole brain [Fig. 2(a)], which indicates weak global nonlinear dynamical behavior. This is also confirmed by the high Pearson's correlation coefficient (PCC = 0.93) between the dominant mode shapes of the brain corresponding to $\alpha = 1$ krad/s^2, $\Delta t = 5$ ms and $\alpha = 10$ krad/s^2, $\Delta t = 25$ ms [Fig. 2(b)]. The negligible variation in the energy contribution and the high PCC between the mode shapes of the brain at various accelerations indicate that, in spite of the brain's intricate geometry and material nonlinearity, this system *as a whole* behaves linearly in the aforementioned acceleration levels.

One possible explanation behind this globally linear dynamical behavior despite many sources of nonlinearities could be the localization of nonlinear effects in certain brain regions. In order to test this hypothesis, we conduct POD on various brain substructures. We compare the change of the dominant mode shape (v_1) at different α and Δt with the linear baseline to find the local nonlinear dynamics in the brain. We observe that as the acceleration level increases, the dominant mode shape of the regions surrounded by deep white matter (WM), near the falx cerebri and the tentorium cerebelli, varies by more than 50% [Fig. 3(a)]. To further analyze these particular regions, we also evaluate the change in the energy contribution of the first three mode shapes [Fig. 3(b)]. We find that as the acceleration increases to $\alpha = 10$ krad/s^2 and $\Delta t = 25$ ms, the energy level of the dominant POM of the local nonlinear regions in the deep WM decreases by approximately 30%. These results hint at the dependence of the mode shape and its energy contribution on the input excitation level, which is a common indicator of nonlinear effects in a system [41]. We also compare the volume of the identified nonlinear region at various acceleration levels with the volume of the WM and observe that at the maximum simulated acceleration, the nonlinear region covers approximately 25% of the WM [Fig. 3(c)]. Furthermore, we analyze this nonlinearity ratio with respect to the concussion threshold suggested by Margulies and Thibault [45] (a criterion developed based on the Kelvin-Voigt

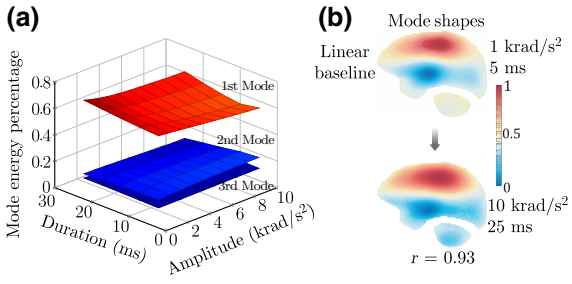

FIG. 2. Brain tissue does not exhibit global nonlinear behavior. (a) The energy contribution of the first three POMs corresponding to varying amplitudes and durations in the coronal plane: the negligible change of 2.7% in the energy redistribution of the first POM is an indication of weak global nonlinearity. (b) The high correlation between the brain's dominant mode shape at minimum and maximum acceleration levels. A midsagittal section is shown as a reference.

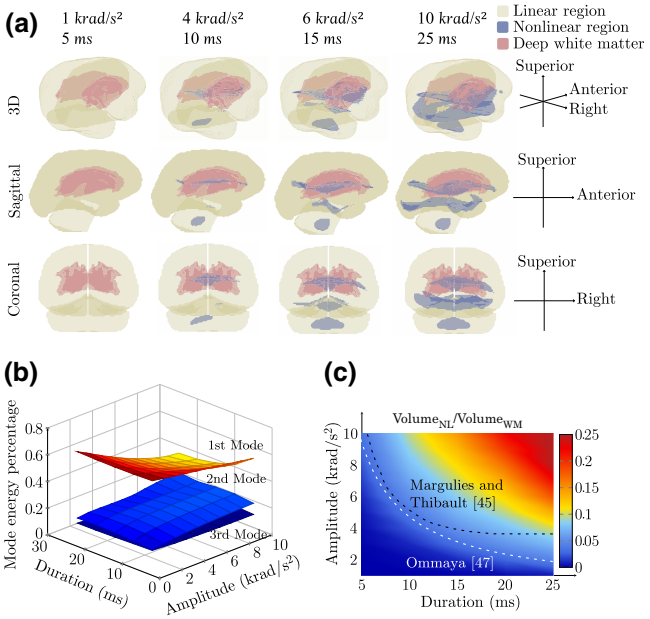

FIG. 3. The existence of local nonlinearity in the human brain. (a) By varying the acceleration amplitude and duration in the coronal plane, the normalized dominant mode shape $[v_1(x,y,z,\alpha,\Delta t)]$ corresponding to each acceleration level is compared with the linear baseline $[v_1(x,y,z,\alpha_{\mathrm{ref}},\Delta t_{\mathrm{ref}})]$ and a nonlinear region is identified ($\Delta v_1 > 50\%$). (b) The energy redistribution of approximately 30% in the first three POMs of the nonlinear regions in the deep WM is an indication of local nonlinearity. (c) The nonlinear region covers approximately 25% of the WM. A comparison of concussion thresholds with the volume of the nonlinear region suggests a possible link with the injury mechanism. In scale-bar legend, NL: nonlinear, WM: white matter.

model, which was later confirmed as a tolerance curve for mTBI [46]) and by Ommaya [47] (who extrapolated this threshold from primate experiments). These concussion thresholds, as adapted from their original form in velocity-acceleration curves, seem to lie within the transition range from the linear to the nonlinear regime. Intriguingly, this could suggest a possible correlation of brain nonlinearity with the onset of mTBI.

Having identified the structures that exhibit nonlinear dynamical effects in the brain tissue, in the next step we apply POD to the dural folds in order to individually analyze the dominant mode shapes of the falx cerebri and the tentorium cerebelli (Fig. 4). We observe that at the maximum acceleration level (compared to the linear baseline), the dominant mode shapes of these structures change their direction in the axial and coronal plane, by $\Delta\theta \approx 15^\circ$ and $\Delta\varphi \approx 8^\circ$, respectively (see the Supplemental Material [28, Table S1]). Such a variance in the direction of the mode shape of the dural folds could be a possible indication of their involvement in causing local nonlinear effects in the brain tissue.

Due to the idealized inputs used in our simulations, a natural follow-up question is whether such nonlinear effects are present in more complicated impact scenarios. To test this, we simulate a 6-degree-of-freedom (DOF) real-world head impact with a dominant rotational acceleration in the coronal direction [8]. We implement the previously published head kinematics [Fig. 5(a)] of a football athlete who was equipped with an instrumented mouth guard [48] during a game in which he suffered from loss of consciousness (LOC). After applying POD, we find that the regions located near the CC and the dural folds exhibit nonlinear effects, as in the case of idealized simulations [Fig. 5(b)].

Having observed such a phenomenon in the human brain's dynamical response, it is essential to examine the implications of these nonlinear effects in the existing injury

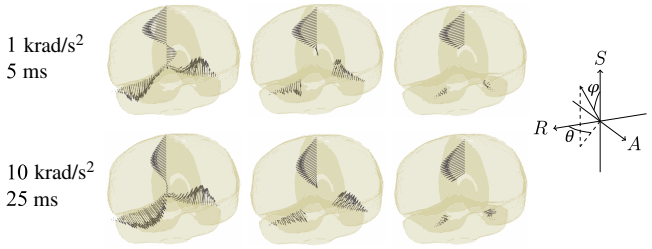

FIG. 4. The effect of the falx and the tentorium on shear-wave redirection. The dominant mode shape of the falx and the tentorium for $\alpha = 1$ krad/s^2, $\Delta t = 5$ ms and $\alpha = 10$ krad/s^2, $\Delta t = 25$ ms shows that as the rotational acceleration increases, there is a change in the direction of these two structures of the brain, which in turn can cause shear-wave reflection in their surrounding areas. Three different sections of the falx and the tentorium are shown for reference.

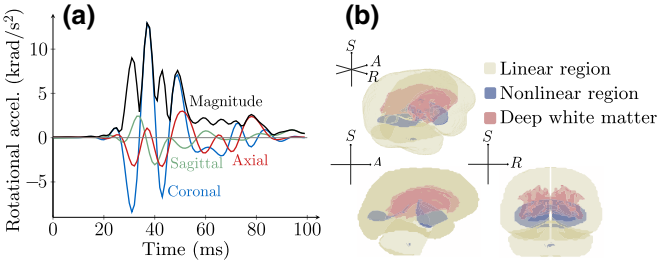

FIG. 5. The nonlinear regions of the human brain due to a real-world football head impact that led to LOC. (a) The recorded 3D rotational acceleration of a football athlete who suffered a LOC injury [8]. (b) The POD of the brain deformation showed nonlinear regions similar to those in an ideal coronal rotation.

metrics. To do so, we study the correlation of the nonlinear regions with CSDM$_{0.35}$, a FE injury criterion that describes the total volume fraction of the brain that experiences strain levels larger than 35% [11,49]. Interestingly, we observe that other than the expected high strain levels near the cortex, the brain tissue volumes that undergo such high strains are the regions connected to the nonlinear area [Fig. 6(a)]. This could indicate a physical correspondence between the nonlinear regions and high-strain areas in this

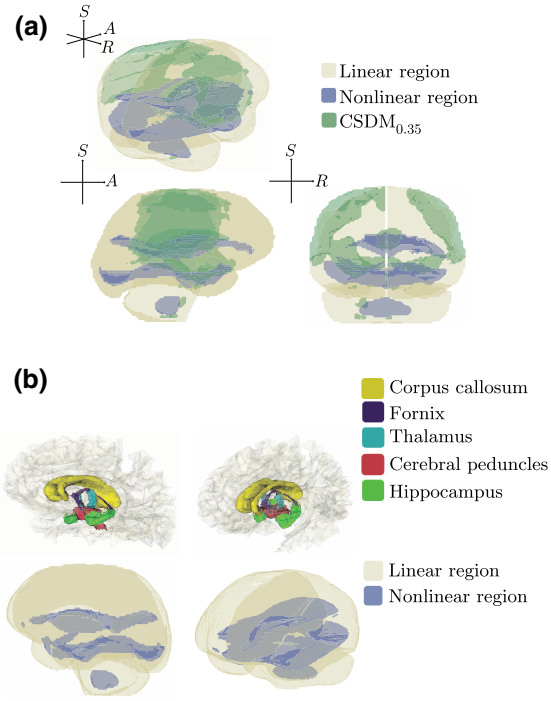

FIG. 6. The correspondence of the nonlinear regions with the tissue-level TBI metrics. (a) We observe that high-strain regions (CSDM$_{0.35}$) of the brain follow the nonlinear regions over the tentorium, signaling a physical correspondence. (b) The nonlinear regions entail several WM tracts of interest in TBI pathology.

soft tissue. Following that, we analyze whether the nonlinear regions correlate with the fiber tracts in the brain known to be the most vulnerable to injury [50,51]. We observe that the identified nonlinear regions overlap with the several commonly implicated WM tracts to TBI [Fig. 6(b), [50]]. These results could suggest a correlation between nonlinearity in the human brain tissue and TBI pathology.

IV. DISCUSSION

A comprehensive study of the mechanistic behavior of the brain is a necessity in order to better understand the underlying mechanisms of TBI. In this study, we find that with increasing excitation levels in the form of rotational accelerations to the center of the head, brain-tissue structures within the deep WM, as well as regions near the falx and the tentorium, exhibit nonlinear dynamics. Such local nonlinearities take shape due to the brain's complex geometry, inhomogeneous material properties, and, especially, its boundary condition with the skull and the dural folds [15,16,52,53]. Previous studies have shown how these stiff membranous structures (i.e., the falx and the tentorium) affect the deformation of the brain during head impacts by inducing large strains to the surrounding regions, such as the CC and the brain stem, by constraining the motion of the cerebellum [54,55]. Zhang *et al.* hypothesized that in a coronal rotation, the regions above the brain stem experience high shear strains during lateral movement of the brain over the tentorium [56]. Lu *et al.* also used tagged MRI to measure the regional deformation of the brain *in vivo* as a result of head rotation. They observed that the propagated shear waves from the exterior boundary (i.e., the skull) are reflected as they reach the falx and the tentorium [57]. Given the potential influence of these anatomical features on the brain response, we further evaluate their effects on the initiation of the identified nonlinear regions. It becomes apparent that, as the input acceleration increases, there is a significant change in the direction of the dominant mode shape of the falx and the tentorium, which is due to their nonlinear geometry. Here, we observe that the shift in the falx mode shape is toward the superior-inferior direction, which is perpendicular to the right-left deformation of its surrounding tissue. This can cause geometric nonlinearity at the boundary of the falx and the brain tissue. The tentorium mode shape, on the other hand, has a change along the right-left direction, which can result in its lateral sliding below the soft tissue. This imposes higher strains at its interface with the brain compared to the surrounding regions of the tissue. Others have found similar results near the dural folds. Recently, Okamoto *et al.*, induced external harmonic vibrations to the human skull to obtain the dynamics of the brain *in vivo*. In this study, they used magnetic resonance elastography (MRE) [58], a technique based on the imaging of shear-wave propagation in tissue as a result of external actuation [59,60]. They analyzed 15 human subjects and observed shear-wave reflection and/or redirection near the falx and the tentorium. They also found approximately 9% difference between the cerebrum and cerebellum motion in the anterior-posterior direction, which could induce higher strains around the tentorium [58]. Such high strains at the boundary of these membranes have also been observed in other TBI studies, where bleeding as the result of subdural hematoma is frequently present along the falx and above the tentorium [61].

This observed nonlinear behavior in the human brain might have important clinical relevance. The onset of nonlinear behavior [Fig. 3(c)] seems to have coincided with the concussion thresholds previously reported by Margulies and Thibault [45] and by Ommaya [47], suggesting a possible injury mechanism. Similar to their observations [45], we find that in the low-duration regime the nonlinear region is less sensitive to the impact amplitude and varies mostly with the duration; whereas in the high-duration regime, this region is more sensitive to the input amplitude and less dependent on the increase of the duration. Another clinical implication of our results is demonstrated in a study by Hulkower *et al.*, who implemented diffusion tensor imaging (DTI) to identify fractional-anisotropy (FA) abnormalities in the brains of patients who suffered TBI. They showed that WM tracts such as the CC (including the genu, splenium, and body) as well as others such as the fornix, the thalamus, the cerebral peduncles, and the hippocampus are among the most common brain regions with abnormal FA in TBI patients [50]. Hernandez *et al.*, also identified the peak principal strain in the CC as the best indicator of mTBI after a head impact [8].

Given the intricacies involved in the brain-skull system, our study inevitably comes with several limitations that should be addressed. The inclusion of more anatomical details, such as the cortical gyri, could potentially affect the modal behavior of the tissue. This could change the energy contribution of the dominant POMs of the whole brain and make the global nonlinear behavior more pronounced. In addition, here we utilize an isotropic and homogeneous version of the WHIM, which could substantially limit the energy transfer between compression and shear waves, especially at the gray and white matter junction. This approximation can also affect the shear-wave redirection found at the interface with the dural folds and alter the patterns of nonlinearity. Nevertheless, an upgraded anisotropic version of the WHIM [62] is already available to differentiate the tissue properties of gray and white matter, which could be applied to mitigate some of these limitations in the future. Another limitation of this study is lack of comparison with experimental data. In spite of the existence of promising imaging techniques to study brain motion, including magnetic resonance elastography (MRE) [59,63], amplified MRI (aMRI) [64], tagged MRI [65], and displacement-encoded imaging with stimulated

echoes (DENSE) MRI [66], the measurement of brain deformation *in vivo* is still challenging. In a clinical setting, the range of acceptable actuation levels is rather narrow; therefore, only the linear response of the brain can be assessed, making a direct experimental comparison out of reach. An alternative approach is to study a human cadaver head *in situ* under high excitation levels [67–69]. Such data sets provide valuable validation for the development of brain FE models (including the WHIM [16]), which can therefore be used to simulate brain responses even at high acceleration levels, causing potential nonlinearity in the brain. Finally, it should be noted that although this study does not identify a conclusive link between the nonlinear behavior of brain tissue and the corresponding injury mechanism, we see our nonlinear analysis tool as an initial step in deciphering this link.

Considering the high number of TBI incidents [70,71] and the importance of proper assessment of the severity of an injury, this study hints at the dire need to reevaluate how current brain-injury criteria are developed. Existing deterministic tools, such as the brain injury criterion "BrIC," obtain injury-risk curves based on the linear relationship that is present between BrIC and strain measures such as CSDM and maximum principal strain, respectively [72]. However, such criteria fail to take into account the nonlinear effects present in the brain, which can cause higher strains in certain anatomical features. This is especially true in higher excitation levels, where regions within the deep WM and in the vicinity of the dural folds exhibit characteristics that are commonly found in nonlinear systems. Therefore, we hope that this paper will encourage further studies on the contribution of certain brain substructures to the injury mechanism and their corresponding nonlinear dynamics.

ACKNOWLEDGMENTS

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