Reversals of Orbital Angular Momentum Transfer and Radiation Torque

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Our examination of vortex fields of scalar waves illuminating an off-axis-located axisymmetric object shows that the ratio of orbital angular momentum (OAM) transfer to energy transfer is not quantized by the ratio of topological charge l (integer) to wave angular frequency ω. We find that the transfer ratio can even be opposite to the ratio l/ω. The findings reveal how a radiation torque associated with energy absorption can spin an axisymmetric object around its center of mass in a direction reversed with respect to the wave vortex's handedness. We understand the reversal by OAM superposition. The absorption can be either in the absorbing object or in the viscous surrounding fluid. Features of reversed spinning motion of a small particle in a slightly viscous fluid are revealed by use of Bessel-function vortex fields. The results are of interest for the development of acoustic and optical vortex-based tweezers as well as for the field of acoustofluidics.

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I. INTRODUCTION

Wave vortices or vortex beams carry orbital angular momentum (OAM) due to an azimuthal phase dependence $\exp(il\phi)$ with a nonzero topological charge l (an integer) [1]. The OAM carried is quantized by the integer l for both optical and acoustic waves [2–6] and electron beams [7–9]. Transfer of OAM to illuminated objects leads to a radiation torque on the object accompanied by energy transfer. In illumination of an axially centered object of axisymmetry (e.g., a sphere or disk) by vortex beams [10–13], the ratio of transfer of OAM to transfer of energy is given by the ratio of topological charge l to angular frequency ω of the wave [with a time dependence $\exp(-i\omega t)$], implying a relation between the torque T along the propagation axis and the absorption power $P_{\rm abs}$ [4]:

$$T = (l/\omega)P_{\rm abs}. \tag{1}$$

The helicity of the beam (i.e., the sign of l) determines the direction of the OAM carried and consequently the direction of the radiation torque.

When an axisymmetric particle manipulated by vortex waves is not aligned with the vortex core, the particle experiences a radiation torque that rotates the object around axis through its center of mass (i.e., spinning motion) [14,15] as well as an azimuthal force on the object's center that rotates the object in an orbit centered on the vortex core (i.e., orbital revolution) [16–18]. In illumination of an off-axis-located object by vortex waves, it remains unknown how the OAM transfer is related to energy transfer, how the radiation torque is exactly related to the topological charge and absorption, and if Eq. (1) is still applicable in this off-axis scenario. These questions by themselves are important in fundamental physics for quantization of OAM, and are also significant for current interest in vortex-based particle manipulations and the development of vortex-based tweezers [19–26].

This present work addresses the OAM and energy transfers and the radiation torque for spinning an arbitrarily located axisymmetric object around its center of mass in the context of both electromagnetic and acoustic vortices of scalar waves (Fig. 1). We observe via our analysis of illumination of an off-axis-located axisymmetric object by Bessel vortex beams that the OAM-to-energy-transfer ratio is not quantized by the ratio l/ω as Eq. (1). We find that the transfer ratio can even be reversed to the sign of the topological charge l. We understand that the reversal arises from transfer of OAM from beam components of reversed topological charges through the object's center. The reversal reveals a negative radiation torque spinning the object around its own axis in a direction opposite to the handedness of the beam. The results are applied to reveal features of reversed spinning of a small particle and to predict the rotation rate.

II. GENERAL FORMULAS OF BEAM SUPERPOSITION

We start by considering an lth-order Bessel vortex beam of a scalar field propagating along the $\tilde{z}$ axis in the frame centered on $\tilde{O}$ (Fig. 1). The beam is an important family of

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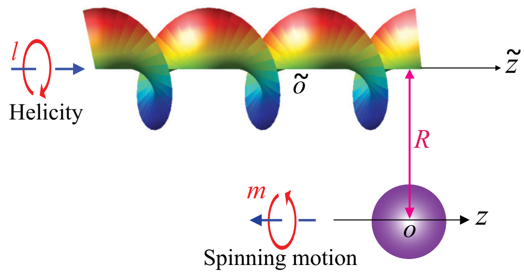

FIG. 1. Ratio of OAM transfer to energy transfer (i.e., torque to absorbed power) is explored for illumination by an lth-order vortex beam of an axisymmetric object (centered on O) that is not aligned with the core of the beam (the $\tilde{z}$ axis), suggesting a negative radiation torque that spins the object around its own axis (the z axis) in a direction opposite to the helicity of the beam. A sphere of finite size and a small sphere in a slightly viscous fluid are examined as examples for practical applications.

solutions of wave equations [27,28] that have an invariant profile along their propagating axis and have found applications in microscopy, imaging, and particle manipulation. The dimensionless profile of the beam in cylindrical coordinates is

$$\psi_l(\tilde{\mathbf{r}}) = i^l J_l(\mu\tilde{\rho}) \exp[i(\kappa\tilde{z} + l\tilde{\phi} - \omega t)], \quad (2)$$

where i^l is a phase included for convenience, J_l is a Bessel function, $\tilde{\mathbf{r}}(\tilde{\rho},\tilde{\phi},\tilde{z})$ is the field point, $\mu = k\sin\beta$ and $\kappa = k\cos\beta$ are the transverse and axial wavenumbers, β is the conic angle of the beam's wave-vector components relative to the propagating axis [29], and the total wavenumber $k = \sqrt{\mu^2+\kappa^2} = \omega/c_0$, with c_0 being speed of the wave. The analysis here includes the Bessel-function vortices in two dimensions as the special case by taking the nonpropagating limit:

$$\beta = 90^\circ, \quad \kappa = 0, \quad \text{and} \quad \mu = k. \quad (3)$$

Let the beam illuminate an object located at an arbitrary location O (Fig. 1). We overcome the off-axis asymmetry by representing the beam in terms of a series of Bessel beams of different orders m whose core is through the object's center O on a shifted, parallel axis z (see the Appendix in Ref. [30]):

$$\psi_l(\tilde{\mathbf{r}}) = \sum_{m=-\infty}^{\infty} \psi_{l-m}(\mathbf{r}_0)\psi_m(\mathbf{r}), \quad (4)$$

where $\psi_m(\mathbf{r})$ are the mth-order Bessel beams propagating along the z axis through the object's center O, with $\mathbf{r}$ being the position vector relative to O, and $\mathbf{r}_0 = \tilde{\mathbf{r}} - \mathbf{r}$ denotes the location of the object from the origin $\tilde{O}$ of the illuminating beam. Let $\mathbf{r}_0 = (R, z_0, \phi_0)$ in cylindrical coordinates. The weighting function $\psi_{l-m}(\mathbf{r}_0)$ has the value of the beam at the object's center O but with the beam order changed to the difference $(l-m)$ [cf. Eq. (2)]: $\psi_{l-m}(\mathbf{r}_0) = i^{l-m}J_{l-m}(\mu R)e^{i\kappa z_0}e^{i(l-m)\phi_0}$. The parallel-axis representation for Bessel beams is a generalization of Graf's addition theorem of Bessel functions.

We implement the parallel-axis superposition equation (4) to explore OAM transfers in scattering of the lth-order Bessel beam by the off-axis object. Given the parallel-axis superposition, the off-axis illumination by the lth-order Bessel beam is described as illumination by the series of Bessel-beam components of different orders m whose core on the z axis is now aligned with the object's center O. Then by orthogonality between the components, the transfers of energy, (axial) linear momentum, and (axial) OAM from the lth-order beam to the off-axis-located object [denoted by $\mathbb{S}_l(R)$] is the sum of transfers from individual mth-order beams to an axially centered object [denoted by $\mathbb{S}_m(0)$]:

$$\mathbb{S}_l(R) = \sum_{m=-\infty}^{\infty} J_{l-m}^2(\mu R)\mathbb{S}_m(0), \quad (5)$$

which follows from the intensity of the weighting function, $|\psi_{l-m}(\mathbf{r}_0)|^2 = J_{l-m}^2(\mu R)$ (illustrated in Fig. 2 in Ref. [30]). The transfer is a function of the transverse location R of the object but not of the axial location z_0 and azimuthal location ϕ_0, which introduce a phase difference in the wave field.

III. REVERSED OAM TRANSFER AND NEGATIVE RADIATION TORQUE

We now relate the OAM transfer (or the torque along the z axis through the object's center, denoted by $\tilde{T}$) to the energy transfer (or total absorption, denoted by $\tilde{P}_{\text{abs}}$). Following from Eq. (5), the total torque and absorption power are now the sum of the on-axis torque T_m and the absorption power $P_{\text{abs},m}$ associated with the individual mth-order beam components through the object's center; that is,

$$\tilde{T} = \sum_{m=-\infty}^{\infty} J_{l-m}^2(\mu R)T_m, \quad (6a)$$

$$\tilde{P}_{\text{abs}} = \sum_{m=-\infty}^{\infty} J_{l-m}^2(\mu R)P_{\text{abs},m}. \quad (6b)$$

Using the proportionality equation [Eq. (1)] relating the torque T_m and the absorption $P_{\text{abs},m}$ for the mth-order beam in the on-axis scenario [4], $T_m = (m/\omega)P_{\text{abs},m}$, we immediately relate the torque to absorption in the off-axis situation:

$$\tilde{T} = (\tilde{l}/\omega)\tilde{P}_{\text{abs}}, \tilde{l} \equiv \frac{\sum_{m=-\infty}^{\infty} mJ_{l-m}^2(\mu R)P_{\text{abs},m}}{\sum_{m=-\infty}^{\infty} J_{l-m}^2(\mu R)P_{\text{abs},m}}, \quad (7)$$

which reveals the proportionality between the total torque $\tilde{T}$ and the total absorption $\tilde{P}_{\text{abs}}$ in arbitrary illumination. The proportional coefficient $\tilde{l}$ is a noninteger number as a function of the object's offset R and can be viewed as an R-dependent effective topological charge. The radiation torque $\tilde{T}$ causes the object to spin around the z axis through the object's center.

Equation (7) is our extension of the fundamental transfer rule [Eq. (1)] to arbitrary illumination. The transfer rule reveals the following:

(i) For the $l = 0$ beam carrying no OAM, $\tilde{l}$ vanishes for arbitrary location R; namely, there is no torque on the absorptive sphere for the ordinary beam.

(ii) The torque exerted by the $l \neq 0$ vortex waves on an arbitrarily located object is all associated with energy absorption (i.e., no torque without absorption even for vortex waves).

(iii) Reversing the hecility (i.e., sign of l) reverses the torque direction (i.e., the sign of $\tilde{l}$) as expected by symmetry.

(iv) In the on-axis limit $R = 0$ [so $J_{l-m}(\mu R)$ reduces to zero when $m \neq l$], the effective topological charge $\tilde{l}$ reduces to the beam's topological charge l, and Eq. (7) reduces to the fundamental transfer rule $\tilde{T} = (l/\omega)\tilde{P}_{\text{abs}}$ in the case of on-axis illumination.

Our transfer rule [Eq. (7)] reveals the presence of the reversal of the radiation torque (or OAM-to-energy-transfer ratio) in terms of the location-dependent effective topological charge $\tilde{l}$. We observe from Eq. (7) that the direction of the radiation torque can be identical or opposite to the helicity $l \neq 0$, depending on the weighting amplitude of the $+m$ and $-m$ beams, $J^2_{l-m}(\mu R)$ versus $J^2_{l+m}(\mu R)$. This amplitude difference relies on the transverse location R of the object and the topological charge l.

IV. A SMALL PARTICLE

We apply the results to analyze radiation torque for a small particle. For convenience, we rewrite the on-axis torque and absorption of the mth-order beam component in Eq. (6) in terms of dimensionless efficiencies as

$$T_m = \frac{m}{\omega}\pi a^2 I_0 Q_{\text{abs},m}, \tag{8a}$$

$$P_{\text{abs},m} = \pi a^2 I_0 Q_{\text{abs},m}, \tag{8b}$$

where a is the characteristic size of the object, I_0 is the time-averaged wave intensity, and the on-axis absorption efficiency $Q_{\text{abs},m}$ for the mth-order Bessel beam is given in terms of a partial-wave expansion for a spherical particle of radius a [Eq. (17) in Ref. [31]]:

$$Q_{\text{abs},m} = \frac{4\pi}{(ka)^2}\sum_{n=|m|}^{\infty}[Y_{nm}(\beta,0)]^2(1-|s_n|^2), \tag{9}$$

where n is the partial-wave index, and the spherical harmonics in terms of the associated Legendre function P_n^m is $Y_{nm}(\beta,0) = \sqrt{[(2n+1)/4\pi][(n-m)!/(n+m)!]}P_n^m$ $(\cos\beta)$.

Here the scattering coefficients $(s_n - 1)/2$ are in the notation of quantum scattering theory [32]: in the case of an ideal sphere causing no dissipation of energy, the complex functions s_n are unimodular: $|s_n| = 1$; otherwise $|s_n| < 1$. That is, the partial-wave absorption factor $(1 - |s_n|^2) \geq 0$ for a passive sphere (no gain). Usually it is convenient to express s_n in terms of phase shifts [32,33]. Note that s_n is a function of ka and material properties. In acoustics, for instance, for a sphere of radius a, the series Eq. (9) converges when n somewhat exceeds ka, and also the series Eqs. (6) and (7) converges when $|m| \leq n$ somewhat exceeds ka.

When a small sphere in the Rayleigh-scattering regime is considered ($ka \ll 1$, i.e., the particle size is much smaller than the wavelength), the monopole ($n = 0$, $m = 0$) and dipoles are dominant ($n = 1$, $|m| \leq 1$) regardless of the order l of the incident beam. Since the axisymmetric component ($m = 0$) has no contribution to the torque [Eq. (8a)], the total torque is the sum of the torques from $m = \pm 1$ beam components:

$$\tilde{T} = J^2_{l-1}(\mu R)T_1 + J^2_{l+1}(\mu R)T_{-1}, \tag{10a}$$

$$= [J^2_{l-1}(\mu R) - J^2_{l+1}(\mu R)]T_1, \tag{10b}$$

$$= \left[\frac{4l}{\mu R}J_l'(\mu R)J_l(\mu R)\right]\frac{P_{\text{abs},1}}{\omega}, \tag{10c}$$

where the torques $T_{\pm 1} = \pm P_{\text{abs},1}/\omega$ are for the dipole sphere centered on the axis of $l = \pm 1$ beams, and Eq. (10c) follows from recurrence relations of Bessel functions.

Equation (10c) reveals that for a small particle:

(i) The torque vanishes at locations where the field has a null, $J_l(\mu R) = 0$, or has a local maximum of intensity, $J_l'(\mu R) = 0$. The directional reversal of the torque occurs in the regimes where $J_l(\mu R)J_l'(\mu R) < 0$. The μR for the vanishing and reversal of torque will depend on ka and β for a particle of finite size [Eq. (9)].

(ii) Even the torque has an explicit proportionality to l herein for a small particle in Eq. (10c). the torque-to-absorption ratio [Eq. (7)] has an effective topological charge $|\tilde{l}| < 1$ since the absorption from the $m = 0$ beam

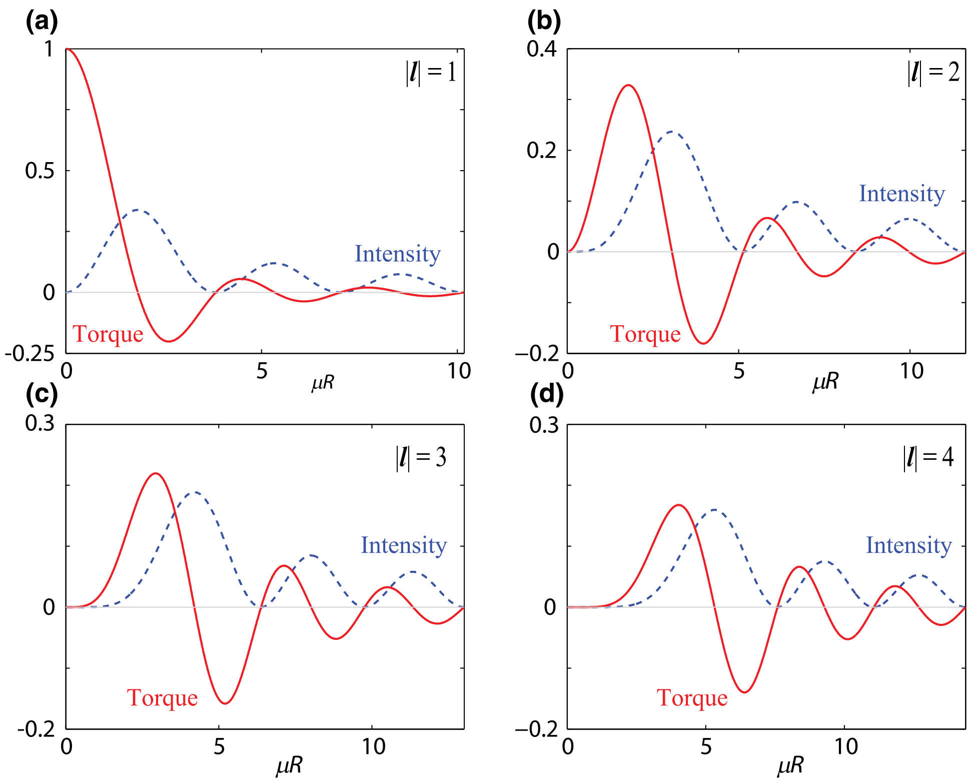

FIG. 2. Negative radiation torque opposite to the handedness of an lth-order Bessel vortex field on a Rayleigh particle (whose size is much smaller than the wavelength) is illustrated by the negative sign of the dimensionless torque $4|l|J_l'(\mu R)J_l(\mu R)/\mu R$ (solid red line) as a function of the object offset R away from the beam axis [see Eq. (10c)]. Also shown is the normalized beam intensity $J_l^2(\mu R)$ (dashed blue line) for reference. The results in (a)–(d) are for $l=\pm1$, ±2, ±3, and ±4, respectively.

has no contribution to the torque and the torques from the $m=\pm1$ beams are opposite.

(iii) The radiation torque on the off-axis Rayleigh sphere exists for arbitrary topological charge $l\neq 0$ since the dipolar fields are always caused by the off-axis asymmetry when μR is large enough, in contrast to on-axis illumination ($R=0$) where the torque exists only for the $l=\pm1$ beam because the dipolar fields are suppressed by high-order $|l|>1$ beam illumination.

These features of negative radiation torque on a small particle are illustrated in Figs. 2(a)–2(d) for vortex fields with topological charge $l=\pm1$, ±2, ±3, and ±4, respectively. These features are applicable to any axisymmetric objects (e.g., a disk or a cylinder, not limited to a sphere) as long as $ka\ll 1$ for contribution to the torque from dipolar fields only.

V. ACOUSTOPHORESIS APPLICATION

For a Rayleigh particle (small ka) of mass density ρ_i immersed in a fluid of mass density ρ_0 in the context of acoustic waves, the dipole-scattering function s_1 [5] is given as

$$s_1-1\simeq\frac{i}{3}(ka)^3f_2, \tag{11}$$

where $f_2=2(\rho_i-\rho_0)/(\rho_0+2\rho_i)$ is a real value if there are no losses (function of the particle-to-fluid density ratio [34]), and $f_2=f_{2r}+if_{2i}$ is a complex value if they are losses (with the subindices r and i denoting real and imaginary parts, respectively). The absorption of dipolar waves is proportional to the imaginary part; that is, $(1-|s_1|^2)\simeq\frac{2}{3}(ka)^3f_{2i}$. The torque on the small sphere [Eq. (10)] then reduces to

$$\tilde{T}=\frac{4l}{\mu R}J_l(\mu R)J_l'(\mu R)\frac{3\pi I_0}{2k^3c_0}\sin^2\beta(1-|s_1|^2), \tag{12a}$$

$$\tilde{T}=\frac{4l}{\mu R}J_l(\mu R)J_l'(\mu R)\frac{\pi a^3I_0}{c_0}\sin^2\beta f_{2i}. \tag{12b}$$

The absorption in f_{2i} can be obtained from scattering theory that accounts for dissipation in the absorbing particle [4,35] and/or in the viscous surrounding fluid (provided that the acoustic streaming is weak) [5,36].

Consider the sphere immersed in a slightly viscous fluid where a boundary layer has a thickness $\delta=\sqrt{2\nu/\omega}\ll a$ (with ν being the kinematic viscosity of the surrounding fluid). The small δ/a approximation of the dipole-scattering coefficient s_1 leads to

$$f_{2i}\simeq(\delta/a)A,\quad A=6[(\rho_i-\rho_0)/(\rho_0+2\rho_i)]^2, \tag{13}$$

where A is a function of the particle-to-fluid density ratio ρ_i/ρ_0 [see Ref. [36] or Eqs. (11) and (12) in Ref. [5]]. The torque [Eq. (12)] reduces to being proportional to the boundary layer thickness δ as

$$\tilde{T} = \frac{4l}{\mu R} J_l'(\mu R) J_l(\mu R) \frac{\pi a^2 I_0}{c_0} \sin^2 \beta A \delta. \tag{14a}$$

The balance of the torque $\tilde{T}$ with the viscous-drag torque $8\pi a^3 \nu \rho_0 \tilde{\Omega}$ on the spinning sphere in the fluid results in a steady spinning rate:

$$\tilde{\Omega} = \frac{\tilde{T}}{8\pi a^3 \nu \rho_0} = \frac{4l}{\mu R} J_l(\mu R) J_l(\mu R) \frac{I_0 \sin^2 \beta A}{4a\rho_0 c_0 \sqrt{2\nu\omega}}. \tag{14b}$$

For illumination by acoustic vortices of pressure field $p = \mathrm{Re}(p_0 \psi_l e^{-i\omega t})$, with the dimensionless field ψ_l given by Eq. (2) and p_0 being a complex amplitude, the sound intensity is $I_0 = |p_0|^2/2\rho_0 c_0$. The heavy-sphere limit ($\rho_i/\rho_0 \to \infty$) is $A = 3/2$. Note that the compressibility or sound speed of the particle does not affect the torque because the monopole scattering ($n = 0$) related to the compressibility is driven by the $m = 0$ beam component, which has no contribution to the torque. The results here in the on-axis limit $R = 0$ reduce to those in Ref. [5] for the $l = \pm 1$ beam [with the factor $(4l/\mu R) J_l(\mu R) J_l'(\mu R)$ reducing to the topological charge l].

VI. AN ARBITRARY-SIZE SPHERE AND THREE-DIMENSIONAL RADIATION FORCES

The mathematical treatment is by beam superposition [Eq. (5)]. For an arbitrarily located sphere of arbitrary size, the total torque $\tilde{T}$ and absorption power $\tilde{P}_{\mathrm{abs}}$ follow from summation of the contributions from all beam components of different orders m. In terms of dimensionless efficiencies $\tilde{T} = \pi a^2 I_0 \tilde{Q}_T/\omega$ and absorption power $\tilde{P}_{\mathrm{abs}} = \pi a^2 I_0 \tilde{Q}_{\mathrm{abs}}$, they are related to the scattering functions s_n [with use of Eq. (9)]:

$$\tilde{Q}_T = \frac{4\pi}{(ka)^2} \sum_{n,m} m J_{l-m}^2(\mu R) [Y_{nm}(\beta, 0)]^2 (1 - |s_n|^2), \tag{15a}$$

$$\tilde{Q}_{\mathrm{abs}} = \frac{4\pi}{(ka)^2} \sum_{n,m} J_{l-m}^2(\mu R) [Y_{nm}(\beta, 0)]^2 (1 - |s_n|^2), \tag{15b}$$

where the notation of the sum is $\sum_{n,m}^{\infty} = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n}$. Replacement of the factor $(1 - |s_n|^2)$ in the absorption efficiency equation (15b) by $|1 - s_n|^2$ gives the scattering efficiency $\tilde{Q}_{\mathrm{sca}}$, and replacement by $2\mathrm{Re}(1 - s_n)$ gives the extinction efficiency $\tilde{Q}_{\mathrm{ext}}$ [4,30]. Together the absorption efficiency and the torque efficiency $\tilde{Q}_T$ give the effective topological charge $\tilde{l} = \tilde{Q}_T/\tilde{Q}_{\mathrm{abs}}$. It would be valuable to compare the value of $\tilde{l}$ with the beam's topological charge l for the interest in torque efficiency of limited absorption.

In addition to the radiation torque addressed herein, there is a three-dimensional radiation force exerted by the lth-order Bessel beam on the center of mass of the arbitrarily located sphere, $\mathbf{F} = \pi a^2 (I_0/c_0) \mathbf{Y}$. The three components in cylindrical coordinates for the dimensionless force efficiency $\mathbf{Y}$ follow from the radiation-force formula [Eq. (20) in Ref. [37] by use of Eq. (6) in Ref. [30]]:

$$Y_z = \frac{2}{(ka)^2} \sum_{n,m} J_{l-m}^2(\mu R) \mathrm{Re}(1 - s_n s_{n+1}^*) \\ P_n^m(\cos\beta) P_{n+1}^m(\cos\beta) \frac{(n-m+1)!}{(n+m)!}, \tag{16a}$$

$$Y_\phi = \frac{1}{(ka)^2} \sum_{n,m} K_m^-(\mu R) \mathrm{Re}(1 - s_n s_{n+1}^*) \\ P_n^m(\cos\beta) P_{n+1}^{m+1}(\cos\beta) \frac{(n-m)!}{(n+m)!}, \tag{16b}$$

$$Y_\rho = \frac{1}{(ka)^2} \sum_{n,m} K_m^+(\mu R) \mathrm{Im}(1 - s_n s_{n+1}^*) \\ P_n^m(\cos\beta) P_{n+1}^{m+1}(\cos\beta) \frac{(n-m)!}{(n+m)!}, \tag{16c}$$

with

$$K_m^{\pm}(\mu R) = \pm J_{l-m}(\mu R) J_{l-m-1}(\mu R) \\ - J_{l+m},(\mu R) J_{l+m+1}(\mu R), \tag{17}$$

giving the analytical formulas of the forces as functions of scattering functions s_n, beam order l, conic angle β, and object location R. The axial component Y_z [Eq. (16a)] recovers the formula following from the treatment by beam superposition [30], while the transverse and azimuthal components Y_ρ, Y_ϕ display coupling of vortex components of adjacent orders m and $m+1$ as illustrated by the factor $K_m^{\pm}(\mu R)$ [Eq. (17)]. It is notable that the negative radiation forces on a sphere (reversed with respect to the propagation direction of the beam) do not necessarily require energy absorption, while the negative radiation torque for spinning motion examined herein does; the negative radiation force arises from asymmetry of the scattered field instead [31,38,39].

VII. SUMMARY

In conclusion, we relate a radiation torque by OAM transfer to absorbed power of energy transfer in illumination of an arbitrarily located axisymmetric object by Bessel vortex beams. We show that the radiation torque is proportional to the absorption power and the ratio of a location-dependent effective topological charge to the wave angular frequency [Eq. (7)], $\tilde{T} = (\tilde{l}/\omega)\tilde{P}_{\text{abs}}$. The results reveal that absorption is the necessary condition to induce the radiation torque for spinning the axisymmetric object, and that the ratio in Eq. (1) is broken (i.e., $\tilde{l} \neq l$) in situations where the axisymmetric object is not aligned with the core of the vortices. The failure of Eq. (1) in the off-axis configuration is generally true for any vortices since the Bessel vortex beam can be used as the basis in cylindrical coordinates to represent arbitrary wave fields.

For a small particle, we find that high-order beams ($|l| > 1$) can exert a radiation torque on a small particle to spin the object around its own axis when the object has a sufficient offset from the beam's axis to drive the dipolar field. Spinning motion of a small particle by high-order vortices ($|l| > 1$) would otherwise be inefficient in on-axis illumination, or even impossible for a Rayleigh particle where the dipolar fields are suppressed by the high-order $|l| > 1$ beam illumination [5]. The flexibility and ability of spinning a Rayleight particle in off-axis illumination advance applications of vortex beams for particle manipulation and the development of vortex-based tweezers. The spinning rate in off-axis illumination shows an explicit proportionality with the topological charge l [Eq. (10)], but again the torque-to-absorption ratio is not equal to the ratio l/ω, or actually $|\tilde{l}| < 1$ in this situation.

We further observe reversals of the OAM-to-energy-transfer ratio and radiation torque for spinning an axisymmetric object around its own axis opposite to the wave vortex's handedness. Application of the results to a spherical particle reveals conditions and features of negative radiation torque on a small particle where the absorption can be in the particle and/or adjacent regimes in the surrounding medium [Eq. (12)]. The results are used to predict torque and spinning rate in a slightly viscous fluid for acoustofluidics applications [Eq. (14)]. The on-axis limit reduces to results in Refs. [4,5]. The standing-wave limit [Eq. (3)] is applicable to Bessel-function fields in acoustophoresis [5].

We reveal that the reversals of the OAM transfer and radiation torque for reversed spin of an axisymmetric object around its own axis (not aligned with the vortex core) originate from OAM transfer and energy absorption from vortex components of opposite topological charges. This negative radiation torque relies on energy absorption for transfers of OAM from vortex components of reversed topological charge. For a Rayleigh particle, the negative radiation torque simply results from the competition between dipole components of opposite topological charges ($m = 1$ versus -1) of the incident beam.

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