Feasibility of a strong focusing synchrotron for cold neutron beams

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We study the feasibility of storing cold neutron beams in a strong focusing synchrotron. We also propose an alternating-current (ac) dipole magnet to be used as a neutron acceleration device and a pulsed quadrupole as a neutron beam kicker for injection and extraction. The ac acceleration device can provide adiabatic capture, acceleration, or deceleration of neutron beams. It seems feasible to design a high-quality neutron synchrotron for many future neutron beam applications, such as neutron life time measurements, monoenergetic neutron scattering experiments, and pencil neutron beam applications. This synchrotron can also be used as an injector for a neutron accumulator ring.

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I. INTRODUCTION

Measurements of the neutron lifetime continue to be of great importance [1]. Measurement methods of the neutron lifetime include magnetic field confinement, the gravitational bottle, and neutron-beam storage ring method [2,3]. At this time, there are many active neutron beam lifetime measurements in preparation or ongoing.

The storage of neutrons in a storage ring has been demonstrated in Ref. [3]. There is also a proposal to store neutrons in a compact toroidal superconducting magnetic ring [4]. All earlier efforts are weak focusing magnetic storage rings without the possibility of neutron bunching, acceleration, and bunch profile manipulations. This paper addresses the feasibility of a strong focusing synchrotron for neutron beams and introduces a neutron beam kicker for beam injection or extraction. We further propose an ac dipole magnet as a "neutron-beam" accelerating device. The neutron acceleration device can be used for the adiabatic capture, acceleration, or deceleration of neutron beams.

In a neutron storage ring, the magnetic dipole field is used to maintain the stability of neutron beams of a single spin state, magnetic quadrupoles are used to provide the closed orbit of the neutron storage ring, and magnetic sextupoles are used to provide focusing of the neutron beams. For demonstration, we discuss the storage of very-cold neutron beams with a kinetic energy of $E = 1\ \mu\text{eV}$.

II. EQUATION OF MOTION OF NEUTRONS IN MAGNETIC FIELD

In the presence of a magnetic field, spin particles will be quantized along the direction of the magnetic field direction. The potential energy of the particle is

$$H = -\vec{\mu} \cdot B, \tag{1}$$

where μ is the magnetic moment of the spin particle. For neutrons, $\mu = g_n \mu_N \vec{s}$, where $g_n = -1.9134$ is the neutron g factor, $\mu_N = 3.152 \times 10^{-2}\ \mu\text{eV/T}$, and $\vec{s}$ stands for the neutron spin vector. The equation of particle motion is governed by the force $\vec{F} = -\nabla H$. We consider neutrons quantized in the magnetic field in the vertical direction. The magnetic flux density in the vertical direction is

$$B_z = B_0 + B_1 x + \frac{1}{2} B_2 (x^2 - z^2), \tag{2}$$

where B_0 is the dipole flux density, B_1 is the quadrupole field strength, and B_2 is the sextupole field strength. The Hamiltonian for particle motion becomes $H = \mp \frac{1}{2} g_n \mu_N B_z$, where the upper sign corresponds to particles with spin aligned with the dipole magnetic field and the lower sign to particles antialigned with the dipole magnetic field. To be specific, we consider the equations of motion for spin-up particles:

$$\frac{dp_x}{dt} = \frac{1}{2} g_n \mu_N B_1 + \frac{1}{2} g_n \mu_N B_2 x, \tag{3}$$

$$\frac{dp_z}{dt} = -\frac{1}{2} g_n \mu_N B_2 z. \tag{4}$$

Note that the quadrupole field provides a uniform force to neutrons, while the sextupolar field provides a focusing or defocusing force. Since g_n is negative, the uniform field points in the $-\hat{x}$ direction if $B_1 > 0$. In the presence of a constant quadrupole field, we consider a curvilinear coordinate system or the Frenet-Serret coordinate system, where $\hat{x}$ points in the outward direction, $\hat{s}$ points along the beam direction, and $\hat{z}$ points upward. Let ρ represents the bending radius. A particle located at distance r from the center can be expressed as $r = \rho + x$. The equations of motion in the Frenet-Serret coordinate system become [5]

$$m\frac{d^2x}{dt^2} - mr\dot{\theta}^2 = \frac{1}{2}g_n\mu_N B_1 + \frac{1}{2}g_n\mu_N B_2 x, \tag{5}$$

$$m\frac{d^2z}{dt^2} = -\frac{1}{2}g_n\mu_N B_2 z - mg, \tag{6}$$

where we add a uniform gravitational force on the vertical plane with the gravitational constant g. We will neglect the gravitational force for the moment and discuss it in the Appendix. The quadrupole force provides the centripetal force for the curvilinear coordinate system so that

$$-m\rho\dot{\theta}^2 = -\frac{mv^2}{\rho} = \frac{1}{2}g_n\mu_N B_1 \tag{7}$$

for a reference particle. The bending radius ρ depends on the energy of neutrons and the quadrupolar field strength. Figure 1 shows the bending radius ρ in meters vs B_1 in T/m for neutrons with a kinetic energy $E = 1$ μeV. The minimum size of a storage ring is $2\pi\rho$. For example, the minimum circumference of the neutron storage ring is about 21 m for $B_1 = 20$ T/m for neutrons with $E = 1$ μeV. We can divide the quadrupole into many segments to provide drift sections for the synchrotron. These drift sections can provide spaces for beam manipulations and detection.

For $B_1 > 0$, the centripetal force points toward the $-\hat{x}$ direction for spin-up particles. If the B_1 field is reversed, the same storage ring can store spin-down particles. Expanding the equations of motion to the first order of x and z and using the longitudinal path length coordinate $s = vt$, our equations of motion become [5]

$$\frac{d^2x}{ds^2} + \left(\frac{1}{\rho^2} + K(s)\right)x = 0, \tag{8}$$

$$\frac{d^2z}{ds^2} - K(s)z = -\frac{g}{v^2}, \tag{9}$$

where the focusing function is

$$K(s) = -\frac{g_n\mu_N B_2}{2mv^2}. \tag{10}$$

Figure 2 shows the focusing function $K(s)$ in (m^{-2}) vs the sextupolar field strength B_2 in (T/m^2) for neutrons with $E = 1$ μeV. For $B_2 > 0$, the sextupole field is focusing in the horizontal plane and defocusing in the vertical plane.

The dipole field in the entire storage ring is used to provide stability of neutron spin orientation. Piece-wise constant quadrupoles can be used to bend neutron beams. The sextupolar field is used to maintain beam focusing in the storage ring. Figure 2 shows that sextupoles are quite effective in providing neutron beam focusing, i.e., a reasonably weak sextupole field is sufficient to provide neutron beam focusing. There are two options in a storage ring design: the weak focusing approach and the strong alternating focusing approach. If $K(s)$ is negative and $|K(s)| < 1/\rho^2$, both the horizontal and vertical planes are stable, and the storage ring is a weak focusing one. The design used in the neutron storage ring in Ref. [3] is a weak focusing one. A strong focusing synchrotron has smaller betatron amplitude functions and dispersion function so that the dynamic aperture and the momentum acceptance will be larger for magnets at a similar magnetic aperture. Thus, the neutron beam will have a higher intensity than that from a weak focusing synchrotron. Alternating the signs of B_2 along the ring, one can design a storage ring like those of charged particle beam synchrotrons [5].

In a drift section, one can design a neutron beam kicker by using a pulsed quadrupole. This device can be used for beam injection and extraction. Using Eq. (5), one finds the kicker angle

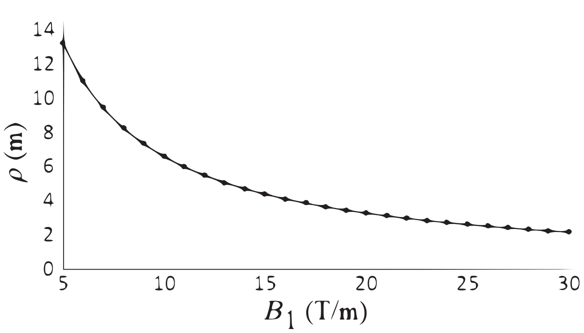

FIG. 1. Bending radius ρ in meters vs the quadrupole gradient B_1 in T/m is shown for neutrons with a kinetic energy $E = 1$ μeV.

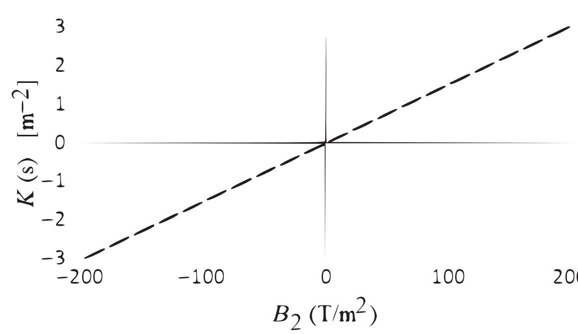

FIG. 2. Focusing function $K(s)$ is plotted vs the sextupolar field strength B_2 (T/m^2) for neutrons with a kinetic energy $E = 1$ μeV.

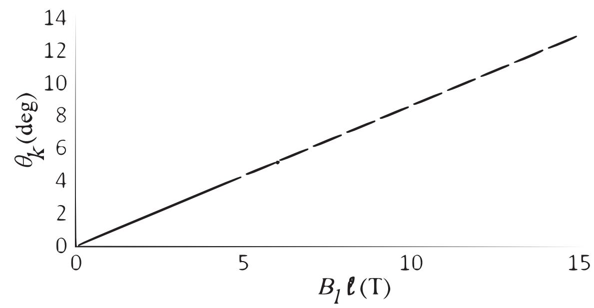

FIG. 3. Kicker angle of neutron beam θ_k in degree vs the integrated quadrupole field strength $B_1\ell$ in (T) for neutrons with $E = 1$ μeV.

$$\theta_k = \frac{g_n \mu_N B_1 \ell}{2mv^2}. \tag{11}$$

Figure 3 shows the kicker angle of a neutron beam passing through a quadrupole kicker vs the integrated quadrupole field strength $B_1\ell$ in (T) for neutrons with $E = 1$ μeV. A reasonable kicker angle can be obtained by a reasonable pulsed quadrupole field.

Neutron acceleration is a difficult subject [6,7]. In a drift section, we consider a device consisting of a time-varying vertical dipole, called "ac dipole." Let the dipole magnetic flux density be

$$B(s,t) = B_0 + G(s)\sin(\omega_{ac}t), \tag{12}$$

where B_0 is the uniform dipole field for the entire storage ring, $s \in (0, L)$ is the path length along the time-varying dipole, and the functions $G(s)$ and ω_{ac} are, respectively, the profile of the dipole strength and the angular frequency of the ac dipole. When a particle, with velocity v and a phase factor ϕ with respective to the sinusoidal wave, passes through the ac dipole, its energy change is

$$\begin{aligned}\delta E &= g_n\mu_N \int_0^L \frac{\partial}{\partial s}[B_0 + G(s)\sin(\omega_{ac}t + \phi)]ds \\ &= g_n\mu_N\left(G(L)\sin\left(\frac{\omega_{ac}L}{v} + \phi\right) - G(0)\sin(\phi)\right), \end{aligned}\tag{13}$$

where L is the length of the ac dipole, and L/v is the particle transit time. The ac dipole profile function $G(s)$ can be achieved by varying the number of turns in a dipole design. Similarly, the dipole profile function $G(s)$ can also be reached by varying the dipole gap along the length of the dipole. Note that $\delta E = 0$ if $G(0) = 0$ and $G(L) = 0$, i.e., there is no energy change if both sides are maintained at the same dipole field strength B_0. The ac dipole scheme proposed in Ref. [6] will not work. If one chooses the time-varying dipole magnetic field

$$B(s) = B_0 + B_g s\,\sin(\omega_{ac}t), \tag{14}$$

the energy change of a particle passing through the ac dipole with phase ϕ with respect to the ac dipole and with velocity v is

$$\delta E = g_n\mu_N B_g L\,\sin\left(\phi + \frac{\omega_{ac}L}{v}\right). \tag{15}$$

Unfortunately, just right passing through the ac dipole, the energy change is completely canceled by the existence of the uniform dipole field B_0. There is no energy gain or loss through the ac dipole embedded in the uniform dipole field B_0.

The solution is to remove the uniform field in the background similar to the insulation of rf cavities from the rest of the vacuum chambers in charged particle beam synchrotrons. In fact, for maintaining the stability of the spin direction, dipole field is needed only in regions of bending quadrupoles and focusing sextupoles.

We thus choose an ac dipole in a zero-field drift section of Eq. (14) with $B_0 = 0$. The energy change for a synchronous particle with phase ϕ_s and the velocity v_0 is

$$\delta E_0 = g_n\mu_N B_g L\,\sin\left(\phi_s + \frac{\omega_{ac}L}{v_0}\right). \tag{16}$$

The maximum energy gain is $\delta E_{\max} = |g_n\mu_N B_g L|$. Figure 4 shows the maximum $\delta E_{\max}$ vs $B_g L$ in Tesla. The utility of such a neutron beam acceleration device can be tested in neutron beam lines [8].

The frequency ω_{ac} must be an integer multiple of the revolution frequency of the synchronous particle $\omega_0 = C_0/v_0$, where C_0 is the circumference of the storage ring and v_0 is the speed of the synchronous particle. For example, a neutron with $E = 1$ μeV travels at about 13.8 m/s. For a storage ring of 30 m circumference, the revolution frequency is 0.46 Hz. The neutron acceleration dipole should oscillate at an integer multiple of 0.46 Hz, i.e., $\omega_{ac} = h\omega_0$.

From Eqs. (15) and (16), we find that the energy difference between a nonsynchronous particle and the synchronous particle per passage through the ac dipole is

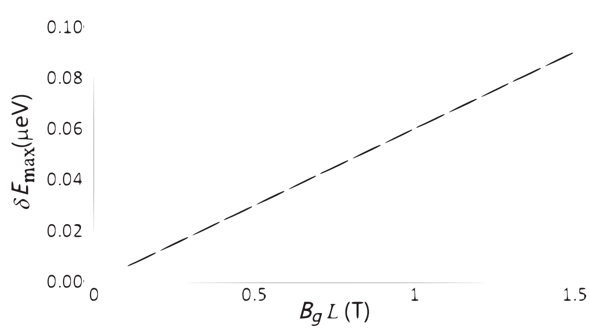

FIG. 4. Maximum energy gain for a neutron beam in μeV vs $B_g L$ in Tesla.

$$\Delta E = \delta E_{\max}\left(\sin\left(\phi + \frac{\omega_{ac} L}{v}\right) - \sin\left(\phi_s + \frac{\omega_{ac} L}{v_0}\right)\right). \quad (17)$$

The synchrotron phase-space parameters between a nonsynchronous particle and a synchronous particle are $\omega = \omega_0 + \Delta\omega$, $\phi = \phi_s + \Delta\phi$, and $E = E_0 + \Delta E$. Here ϕ_s, ω_0, and E_0 are the ac dipole phase angle, angular revolution frequency, and the *kinetic energy* of a synchronous particle, respectively, and ϕ, ω, and E are the corresponding parameters for an off-energy particle, respectively. The azimuthal orbital angle θ is related to the ac dipole phase angle by $\phi = -h\theta$.

The evolution of the energy difference between the nonsynchronous and synchronous particles becomes [5]

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_0}\right) = \frac{\delta E_{\max}}{2\pi}\left(\sin\left(\phi + \frac{\omega_{ac} L}{v}\right) - \sin\left(\phi_s + \frac{\omega_{ac} L}{v_0}\right)\right). \quad (18)$$

From Eq. (5), the horizontal equation of motion for an off-energy particle is

$$\frac{d^2 x}{ds^2} + \left(\frac{1}{\rho^2} + K(s)\right)x = \frac{1}{\rho}\frac{\Delta E}{E_0}. \quad (19)$$

The solution of this inhomogeneous equation can be decomposed into a betatron motion around an off-energy closed orbit similar to that of charged particle accelerators [5].

The time evolution of the phase angle variable ϕ can easily be derived as

$$\frac{d\phi}{dt} = -h(\omega - \omega_0) = -h\Delta\omega = \frac{h\eta\omega_0^2}{E_0}\frac{\Delta E}{\omega_0}, \quad (20)$$

where we have used the identity

$$\frac{\Delta\omega}{\omega_0} = \frac{vR_0}{v_0 R} - 1 \equiv -\eta\frac{\Delta E}{E_0} \quad (21)$$

to obtain Eq. (20). Here R and R_0 are, respectively, the mean radii for the nonsynchronous and synchronous particles, $\eta \approx \alpha_c - \frac{1}{2}$ is the phase slip factor, and α_c is the energy compaction factor with $(R - R_0)/R_0 = \alpha_c \frac{\Delta E}{E_0}$. For a strong focusing synchrotron, we $\alpha_c \ll 1$, and thus $\eta \approx -1/2$.

Eq. (18) has a phase difference passing through the ac dipole for a nonsynchronous particle and a synchronous particle:

$$\Delta\Phi = \frac{\omega_{ac} L}{v} - \frac{\omega_{ac} L}{v_0} = h\omega_0 L\left(\frac{1}{v} - \frac{1}{v_0}\right) \approx -\frac{hL}{2R_0}\frac{\Delta E}{E_0}. \quad (22)$$

This phase shift is on the order of $|\Delta\Phi| < 10^{-3}$. Neglecting this phase shift, Eqs. (18) and (20) form a coupled synchrotron equation of motion for the conjugate phase-space coordinates $\frac{\Delta E}{\omega_0}$ and ϕ. An ac dipole serves as an accelerating cavity for the neutron beam. This device can provide neutron beam manipulations, such as adiabatic capture at beam injection, beam acceleration, and beam deceleration. The additional phase shift $\Delta\Phi$ produces rf phase modulation. Its effects on beam dynamics have been experimentally and theoretically explored [5,9].

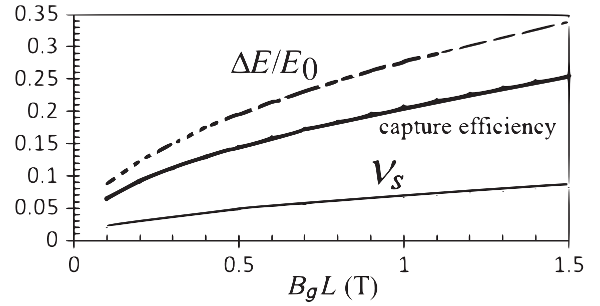

FIG. 5. Fractional energy bucket height ($h = 1$) and the capture efficiency for very-cold neutrons at a temperature of 1 μeV vs the ac dipole field strength $B_g L$ in Tesla.

The fractional energy acceptance of a stationary bucket is [5]

$$\frac{\widehat{\Delta E}}{E_0} = \frac{2}{\sqrt{2\pi h|\eta|}}\sqrt{\frac{\delta E_{\max}}{E_0}}. \quad (23)$$

Figure 5 shows the fractional energy acceptance, the synchrotron tune, and the capture efficiency of a very-cold neutron beam at a temperature of 1 μeV kinetic energy vs $B_g L$ in Tesla. The actual energy acceptance and capture efficiency depend also on the dynamic aperture of a realistically designed storage ring. A comprehensive study provides numerical simulations on the capture and storage of neutron beams [7].

Since the transverse dynamic aperture (DA) is expected to be about $\Delta E/E_0 \sim \pm 3\%$, we expect $B_g L < 0.25$ T to be sufficient to capture all particles within the DA. The capture efficiency is expected to be about 5% of the neutron beam from the source.

III. SPIN MOTION

Neutrons are quantized in magnetic field, and the neutron spin precesses in a magnetic field. The equation of spin motion for very-cold neutrons in the storage ring is the Thomas-BMT equation [10,11]

$$\frac{d\vec{S}}{dt} = g_N \frac{e}{2m_N}\vec{S} \times \vec{B}_\perp, \quad (24)$$

where $\vec{B}_\perp$ is the transverse components of the magnetic field with respect to the velocity vector of the neutron beam. In the Frenet-Serret coordinate system, we have

$$\vec{B}_\perp = B_z\hat{z} + B_x\hat{x}. \tag{25}$$

We note that the main dipole field component B_0 of Eq. (2) is not related to the orbital motion, and yet it is the main source of spin precession. The precession frequency is 14.5 MHz/T. Depending on B_0 strength, the neutron precession frequencies in the storage ring will vary. Depending on B_1 strength for neutron bending, the precession frequencies will also vary around the orbit. This spin precession frequency differs significantly from that of the betatron and synchrotron frequencies of the storage ring; there is no possibility of spin resonances in the neutron storage ring [12]. The neutron spin resonances in the lattice is of the order of a few Hz or tens of Hz, neutron loss rate due to depolarization should be small.

IV. CONCLUSION

In conclusion, we have explored the beam dynamics of neutrons in a strong focusing synchrotron. We find that a pulsed quadrupole at reasonable quadrupole field strength can be used as a neutron beam kicker. An isolated ac dipole can be used to accelerate, decelerate, or store neutron beams in a strong focusing accelerator. This device is equivalent to an “rf cavity” in charged-particle beam accelerators.

In the neutron beam synchrotron, we need a uniform dipole field B_0 to maintain the spin orientation in quadrupole or sextupole locations. Since the strong focusing accelerator can have piece-wise constant quadrupoles for neutron beam bending and piece-wise sextupoles for beam focusing, it is tempting to leave out the dipole field in drift spaces. However, such a design will have quantum fluctuations arising from the dispersion function and induce horizontal emittance dilution. Thus, it is important to maintain the entire ring with a uniform B_0 field in order to eliminate quantum fluctuations associated with horizontal emittance dilution.

On the other hand, one cannot have a uniform B_0 field extending through the ac dipole in order to achieve a net energy change. In order to eliminate horizontal emittance dilution due to quantum fluctuations, the ac dipole must be located in a dispersion-free section of the synchrotron.

One can also reverse the quadrupole and sextupole magnetic field directions of the storage ring to store neutrons of the opposite spin direction. The storage ring is not expected to encounter any spin resonances. However, one needs to study the effect of spin tune oscillation in passing through quadrupoles, sextupoles, and the ac dipole.

The synchrotron discussed in this paper can be used in many neutron physics applications, such as neutron life time measurements and focused monoenergetic neutron beam for neutron scattering. This accelerator can also be used as an injector for a neutron accumulator ring. The neutron-rf-system can be used to accelerate or decelerate neutron beams. It would be of interest to carry out experiments with ac dipole in currently available neutron beam lines. Combining the ac dipole with quadrupoles and sextupoles in a neutron beam line, one can also design a neutron beam buncher.

Finally, this neutral beam synchrotron can also be used to store and accelerate neutral atomic or molecular beams at keV energy range for the study of possible atomic and molecular interactions.

ACKNOWLEDGMENTS

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APPENDIX: EFFECT OF EARTH’S GRAVITY

In the presence of the gravitational force, the solution of Eq. (6) gives an off-center vertical closed orbit

$$z_{\rm co} = \frac{2mg}{-g_n\mu_N B_2}. \tag{A1}$$

One can choose a proper B_2 value for the stability of the storage ring. For a strong focusing neutron storage ring, Eq. (9) can be solved by carrying out the Floquet transformation to obtain an off-centered closed orbit [5]. It is an integral part of the lattice design.

Although the Earth’s rotation speed is about 460 m/s on the Earth’s surface around the equator and the storage ring sits on the Earth surface, with the neutron beam moving with respect to the Earth’s moving reference frame, the neutron motion should not be affected by the Earth’s motion. The centripetal force due to the Earth’s rotation is only about 0.33% that of the gravity, which is a small correction to the neutron beam closed orbit. Furthermore, the neutron lifetime is only about 887.7 ± 2.2 s. Consequently, the targeted neutron beam storage time is expected to be less than 1–2 h, and the effect of the Earth’s rotation on the neutron beam storage and acceleration is not expected to be significant.

[1] See e.g., https://www.energy.gov/science/articles/mystery-neutron-lifetime.

[2] See e.g., J. T. Wilson, D. J. Lawrence, P. N. Peplowski, V. R. Eke, and J. A. Kegerreis, Phys. Rev. C **104**, 045501 (2021); UCN Collaboration, Phys. Rev. Lett. **127**, 162501 (2021).

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[8] The magnetic flux density of a dipole is $B = \mu_0 NI/a$, where a is the dipole aperture gap, N is the number of coil turns, and I is the electric current. Making N increases along the length of the dipole and $I = I_0 \sin(\omega_{ac} t)$, one can produce an ac dipole for neutron beam acceleration. It would be of great interests to test the utility of such an ac dipole in neutron beam lines.

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