# Multipactor in the end cells of a multicell rf cavity

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A relatively simple method of a guess of multipactor appearance on a cavity cell equator was described earlier in Phys. Rev. Accel. Beams **19**, 102002 (2016). For this guess, evaluation of electric field components in two definite points near the equator is sufficient, it gives the value of a so-called geometrical parameter $p$ of the cell. If this parameter is big enough, for niobium superconducting cavities, it was found as $p > 0.28 \div 0.3$, conditions for multipactor exist. However, this method works, as described, only for the inner cells of a multicell cavity or for single-cell cavities. An explanation is in the breakdown of symmetry of the electric field near the equator when the end cell consists of asymmetric half-cells and has a regular inner cell from one side and a beam pipe from another side. Multipactor in such cells shifts from the equator point. This paper is an addendum to the earlier presented paper, here it is shown how to find a correct value of the geometrical parameter $p$ taking the asymmetry into account.

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## I. INTRODUCTION

The evolution of rf accelerating cavities came to elliptic cavities because this shape is almost free of multipactor. A multipactor is a resonant multiplication of electrons due to secondary emission from the cavity walls at a certain strength of the electromagnetic field. The weak point of these cavities is the area near the equator where multipactor can occur if the shape of the surface at this place is chosen lucklessly [1].

As it was shown in [2], multipactor electrons are attracted to the electric field minimum on the cavity surface. This minimum may shift off the middle plane if the cell is asymmetric, for example, if it has an inner cell from one side and a beam pipe from another side.

This shifted point of the minimum can be treated in the same way as the point on the equator. The changing curvature of the surface near this point can be neglected because the distance at which the fields have to be calculated is usually much smaller than the curvature radius at this point. The only complication is the necessity to draw a perpendicular line to that point on the ellipse, while it is straightforward when drawing this line from its equator point.

## II. DETERMINING POINTS

Sizes of orbits of electrons participating in multipactor near the cavity equator are much less than the curvature radius of the cavity at this place. Within these orbits, with good accuracy, the electric field components depend linearly on the coordinates [1]: $E_z = \alpha r$, $E_r = -\beta z$, where $r$ is measured from the equator to the axis—opposite to $R$, $z$ coincides with $Z$, Fig. 1. The force lines of the electric field near the equator have elliptical arc shapes as follows from

$$\frac{E_r}{E_z} = \frac{dr}{dz} = \frac{-\beta z}{\alpha r}.$$

The solution of this differential equation is

$$\frac{z^2}{\alpha} + \frac{r^2}{\beta} = \text{const}.$$

The parameter $p$ is defined as it is presented in Fig. 2; it was shown in [1] that no multipactor was observed in the niobium superconducting cavities when $p < 0.28$.

If, for any reason, the minimum of the electric field shifts from the equator, for example, to the point 0, Fig. 3, then we need to erect a perpendicular to this point of the ellipse and find the fields at the distance $d$, at point 1, and in the points on a straight line parallel to the tangent at the point 0 at a distance $d$ from the point 1, at points 2 and 3.

Only the upper part of the cell is shown in Fig. 3, it is defined by an ellipse with half-axes $A$ and $B$. We will designate coordinates of the points 0, 1, and so on as $Z_0$, $R_0$, $Z_1$, $R_1$, and so on.

Now we can write a system of equations to find coordinates of the point 1:

$$\frac{R_1 - R_0}{Z_1 - Z_0} = \frac{A \cdot (A^2 - Z_0^2)^{0.5}}{BZ_0},$$
$$d^2 = (R_1 - R_0)^2 + (Z_1 - Z_0)^2,$$

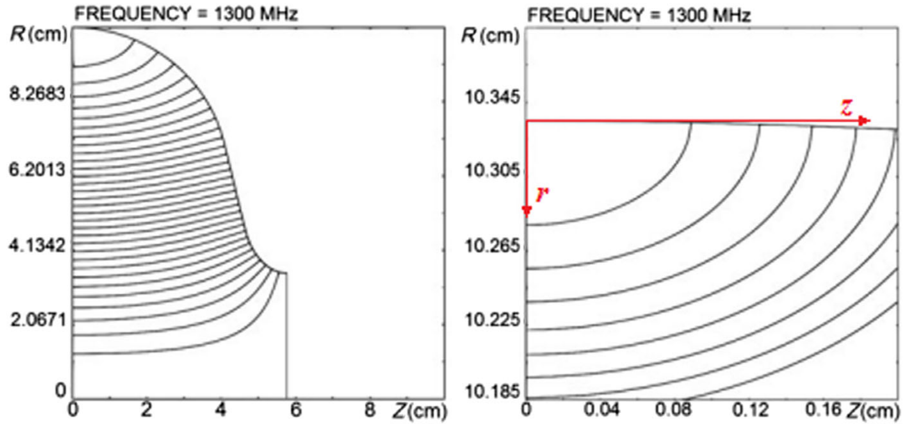


FIG. 1. Electric field force lines for the entire half-cell of the TESLA cavity (left) and near the equator (right).

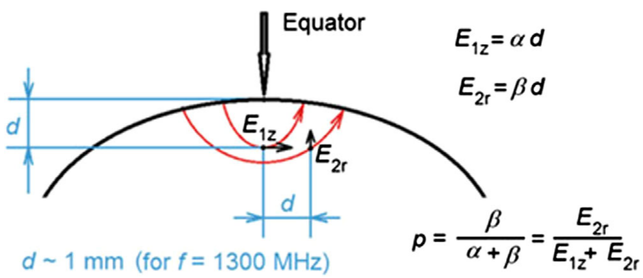


FIG. 2. Definition of the geometrical parameter $p$.

and of the point 2:

$$\frac{R_2 - R_1}{Z_2 - Z_1} = \frac{BZ_0}{A \cdot (A^2 - Z_0^2)^{0.5}},$$
$$d^2 = (R_1 - R_0)^2 + (Z_1 - Z_0)^2.$$

Coordinates of the point 3 can be found if index 2 in the last system changes to 3. The difference in the solutions appears if different guesses are used for the initial values of $Z$ and $R$, for example, in MathCAD.

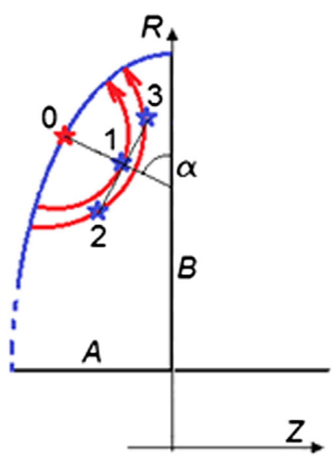


FIG. 3. Geometry for the case when the minimum of the electric field is shifted from the equator.

The software for the calculation of cavities gives usually $E_r$ and $E_z$ components of the electric field. We need to have field components parallel or normal to the tangent at point 0. Let us call them analogously to the fields in the definition of $p$ as $E'_{2r}$ and $E'_{1z}$. This transformation looks as follows:

$$E'_r = E_r \cos\alpha - E_z \sin\alpha,$$
$$E'_z = E_r \sin\alpha + E_z \cos\alpha.$$

## III. EXAMPLE. END CELLS OF THE TESLA CAVITY

Let us, as an example, calculate the geometric parameter $p$ for the end cell of the TESLA cavity. The TESLA cavity is asymmetric, it has different end cells that make it possible to extract some higher order modes otherwise trapped in the inner cells. Let us consider the end cell 1 with dimensions presented in [3], see Fig. 4 and Table I.

Actually, these dimensions correspond to the frequency $f = 1301.369$ MHz instead of 1300 MHz, as was found with SLANS [4], the older program urmel did not guarantee

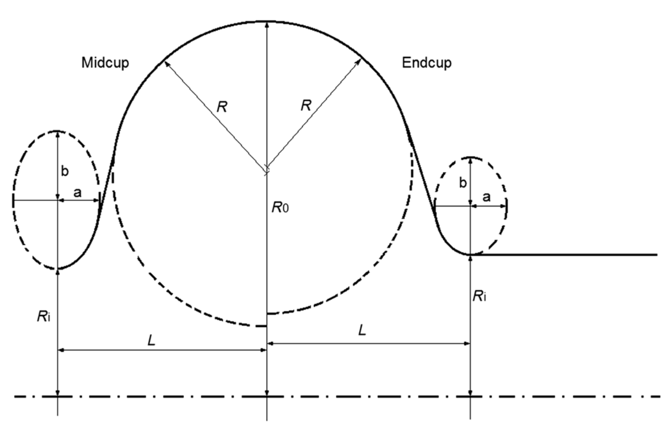


FIG. 4. Dimensions of the TESLA end cells.

TABLE I. Dimensions of the TESLA cavity. All dimensions in mm.

| Cavity shape parameter | Designation | Midcup | Endcup 1 | Endcup 2 |
|---|---|---|---|---|
| Equatorial radius | $R_0$ | 103.3 | 103.3 | 103.3 |
| External curvature radius | $R$ | 42 | 40.34 | 42 |
| Iris radius | $R_\mathrm{i}$ | 35 | 39 | 39 |
| Horizontal half-axis | $a$ | 12 | 10 | 9 |
| Vertical half-axis | $b$ | 19 | 13.5 | 12.8 |
| Length | $L$ | 57.692 | 56 | 57 |

a necessary accuracy but this fact does not substantially change our results.

Calculation of $p$ made with the point 1 (Figs. 2 and 3) located on the normal to the equator gives $p_2 = 0.252$ and $p_3 = 0.317$ where $p_2$ and $p_3$ correspond to points 2 and 3 in Fig. 3, respectively.

The zero of the surface electric field $E_\mathrm{s}$ is shifted to the left from the equator by $\Delta Z = -0.154$ mm so that the angle $\alpha = \mathrm{atan}(Z1 - Z0)/(R0 - R1) = 0.218°$. However, even a regard of this small shift gives us almost equal $p$'s, 0.285 and 0.287 for points 2 and 3, respectively.

Calculations for the end cell 2 give a shift $\Delta Z = -0.195$ mm and $\alpha = 0.266°$ for the min $E_\mathrm{s}$. On the normal to the equator, we can find $p_2 = 0.240$ and $p_3 = 0.327$, but after the correction to the offset point $p_2 = 0.285$ and $p_3 = 0.286$. This calculation was repeated with a twice denser mesh ($224 \times 172$ instead of $112 \times 86$). Now the shift was $\Delta Z = -0.205$ and $p_2 = p_3 = 0.286$ with an accuracy of three decimal places.

For the inner cells consisting of two midcaps, $p = 0.286$ as was found on the straight line in the plane of the equator, Fig. 2. It is known that multipacting in the TESLA cavities is weak and can be easily processed [5].

Note that the fields in formulas for $E'_r$ and $E'_z$ should be taken with the signs as calculated but in formulas for $p$, shown in Fig. 2, should be taken absolute values of vector components.

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