Circular attractors as heating mechanism in coherent electron cooling

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We show that under certain conditions a longitudinal mismatch between an electron and a hadron beam in a coherent electron cooling (CeC) scheme creates a circular attractor in the longitudinal phase space of the cooled hadrons. Formation of an attractor completely stops the cooling and results in anticooling ("heating") causing small synchrotron amplitudes to grow to the attractor's radius, rather than being damped. We present a theory of this effect, compare the analytical results with simulations and derive tolerances to possible sources of the longitudinal mismatch. We further show that under certain conditions a "weak" attractor, affecting hadrons with large synchrotron amplitudes, can appear in the hadrons' longitudinal phase space and explain that formation of such an attractor does not require the presence of any mismatches.

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I. INTRODUCTION

Presently, there are two operational methods of increasing the phase space density of hadron bunches in the collider: stochastic cooling [1] and electron cooling [2,3].

In electron cooling an electron beam is copropagating with the ions with the same average velocity in a straight section of the storage ring called the cooling section. The electrons introduce dynamical friction [4], which reduces both the transverse and the longitudinal momentum spread of the ion bunch over many revolutions.

In stochastic cooling a sensor (called pickup) acquires electrical signals from ions; these signals are amplified and transferred to a "kicker" pushing the ions in directions opposite to their velocities. Since such a system has a limited bandwidth (W), both the pickup and the kicker can act only on an ensemble of ions, which reduces the effectiveness of the cooling. The resulting cooling rate (λ) is directly proportional to the cooler's bandwidth, and inversely proportional to the number of particles in an ion beam (N_i) [1]: $\lambda \propto W/N_i$.

A novel method of coherent electron cooling was proposed [5–7]. Potentially, it has a broader range of applicability and a better performance than the existing cooling methods. A proof of principle coherent electron cooling (CeC) experiment [8] is currently under development at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL). CeC is also the cooler of choice [9,10] for the Electron Ion Collider (EIC), which will be constructed at BNL [11].

In the coherent electron cooling the cotraveling ion and electron bunches interact in the cooling section consisting of a modulator, an amplifier and a kicker. In the modulator each individual ion leaves an "imprint" in the electron bunch by exhibiting a Coulomb pull on the nearby electrons, thus creating a microscopic "wake" in the electron bunch. In the amplifier this wakefield is amplified by a controlled instability of the "electron plasma." Finally, in the kicker section, each ion interacts with its own amplified wake, thus the word "coherent" in the name of this cooling technique. Since the displacement of the ion with respect to its wake is proportional to the ion's relative momentum, the longitudinal momentum spread of the ion bunch is getting reduced, thus providing the cooling.

Conceptually, the coherent electron cooling is a form of the stochastic cooling, with electron bunch playing a role of both a pickup and a kicker, as well as a role of a signal carrier. The promise of CeC is that such coolers have an extremely high bandwidth of about a hundred or even several hundreds of THz. Therefore, the CeC cooling rate is supposed to be much higher than the cooling rate in conventional stochastic coolers.

There are several methods to amplify the ions' imprint in the electron bunch [5]. In a method of radiative instability the ion-induced modulation of the e-bunch is amplified in the high-gain free electron laser [6]. In a microbunched coherent electron cooling [7,9,10] the modulation is amplified via the longitudinal space charge microbunching instability. In a plasma cascade instability method the electron beam's plasma frequency is modulated via variation of its density achieved by a periodic focusing of the e-beam [8].

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In the CeC kicker the "energy kick" acting on an ion can be parametrized [12] as $\Delta E(z, z_1) = -V_0\Phi(z, z_1)$, with the dimensionless wake function:

$$\Phi(z, z_1) = \sin\left(2\pi\frac{z - z_1}{z_0}\right)\exp\left(-\frac{(z - z_1)^2}{\sigma_0^2}\right), \quad (1)$$

where V_0, z_0 and σ_0 are the adjustable parameters defined by the CeC's design, V_0 is the amplitude of the kick, z_0 is its characteristic wavelength, σ_0 is the characteristic width, z is the ion's longitudinal displacement in the kicker with respect to the ion's position in the modulator section of the cooler, and we introduce an additional mismatch z_1 in the modulator-to-kicker path lengths of the electron and the ion beams. It is assumed that z does not change through the kicker and that $z = R_{56}\delta$. Here, $\delta = \delta p/p_0$ is a relative momentum of an ion and R_{56} is the momentum compaction element of the modulator-kicker transfer matrix.

The misalignment z_1 can be caused by a mismatch in relativistic γ factors of the two beams; it also can be the result of errors in the settings of magnetic elements in the cooling section.

Below we will show that if z_1 is constant and is larger than some critical value defined by the parameters of the CeC wake, then the cooling process stops completely and turns into anticooling [13]. This effect sets strict requirements to the matching of the electron's and ion's path lengths through the cooling section, and to the design of coherent electron coolers.

II. EQUATIONS OF MOTION AND CIRCULAR ATTRACTOR

A. Equations of motion

In this paper we use a framework of a so-called "slice model" of the coherent electron cooling [7,9,10,12]. This model considers an ion bunch's slice of a length σ_0 which is defined by the characteristic length of the wake given by Eq. (1). Such a slice contains millions of ions; the exact number N_s of the ions within the slice depends on the ion bunch density. The model assumes that there is no nonlinear mixing of the wakes from the different ions. The interaction of any given ion with its own wake provides the cooling effect, while the effect of all other ions in the slice on the chosen ion is taken into account as a sum of N_s random kicks, which results in the diffusive heating of the ion bunch. The described model allows us to make important conclusions about the ion beam dynamics from considering the equations of motion of an individual ion. Furthermore, averaging a single ion's equations over the ion bunch distribution, we can get expressions for both the cooling rate and the rate of diffusive heating of the ion bunch.

We consider motion of an individual ion in the linear part of the rf bucket using dimensionless variables δ and $\tau = \frac{\omega_s}{\eta}\frac{s}{\beta c} = \frac{\sigma_{\delta 0}}{\sigma_{is0}} s$, where ω_s is a synchrotron frequency, η is a phase slip factor of the ion storage ring, s is a proton's longitudinal position with respect to the center of the rf bucket, β is a relativistic factor (we are setting $\beta = 1$ in the following calculations), c is the speed of light, σ_δ and σ_{is} are respectively the root mean square (rms) momentum spread and the rms length of the ion bunch with the Gaussian distribution, and the index "0" signifies that we chose to use the initial values of σ_δ and σ_{is}.

We introduce the invariant of the undisturbed motion $J = \delta^2 + \tau^2$. For a bunch with the Gaussian distribution the density distribution function for J is $f_J(J) = (1/\tilde{J})e^{-J/\tilde{J}}$, where $\tilde{J} = 2\sigma_\delta^2$ is the average value of J.

Finally, we will assume that the electron bunch length is much smaller than the length of the i-bunch, and that the e-bunch is longitudinally placed at the center of the i-bunch, which is a typical CeC setup. Therefore, the ions interact with the electrons when their synchrotron phase $\phi \approx \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

Under these assumptions, the equations of motion of an individual ion are

$$\begin{aligned} \tau' &= \delta \\ \delta' &= -\tau + \alpha\mathbb{C}(\phi)F(\delta, z_1) + \sqrt{\alpha}\mathbb{C}(\phi)D \\ F(\delta, z_1) &= -\frac{V_0}{E_0}\Phi(R_{56}\delta, z_1) \\ D &= \frac{V_0}{E_0}\sum_n^{N_s}\left[e^{-\frac{\zeta_n^2}{\sigma_0^2}}\sin\left(\frac{2\pi\zeta_n}{z_0}\right)\right]. \end{aligned} \quad (2)$$

Here $\tau' \equiv d\tau/d\phi$, $\delta' \equiv d\delta/d\phi$, E_0 is the ion beam energy, and a Dirac comb function $\mathbb{C}$ is given by

$$\mathbb{C}(\phi) = \sum_{n=0}^{\infty}\delta_D\left(\phi - \frac{\pi(2n+1)}{2}\right), \quad (3)$$

where δ_D is a Dirac delta function. The coefficient $\alpha = \frac{\sqrt{2\ln 2}\sigma_{es}\sigma_{\delta 0}}{\pi Q_s\sigma_{is0}|\delta|}$ is the number of times the ion lands on the e-bunch (having the rms length σ_{es}) when $\tau \approx 0$, here Q_s is the synchrotron tune, and $\sqrt{\alpha}$ in front of the diffusive term D signifies the random walk law. The diffusive term D represents the sum of all the random kicks experienced by the particular ion from the wakes induced by N_s ions in the longitudinal slice (of length σ_0) of the ion bunch. Variables ζ_n are the longitudinal positions of the ions in the slice with respect to the ion under consideration.

From Eq. (2) we get the average difference equation for δ^2 over the synchrotron period T_s:

$$\frac{\Delta\delta^2}{\Delta t} = -\frac{2\alpha_1}{T_s}\frac{V_0}{E_0}[\Phi(R_{56}\delta, z_1) + \Phi(R_{56}\delta, -z_1)] + \frac{2\alpha_1}{T_s|\delta|}\langle D^2\rangle, \quad (4)$$

where $\alpha_1 = \frac{\sqrt{2\ln 2}\sigma_{es}\sigma_{\delta 0}}{\pi Q_s\sigma_{is0}}$.

Squaring the expression for D and bearing in mind that only the terms containing the square of sine will give a nonzero input into the average value, we obtain the following equation for $\langle D^2\rangle$:

$$\langle D^2\rangle \approx \left(\frac{V_0}{E_0}\right)^2 \frac{N_s}{\sigma_0}\int_{-\infty}^{\infty} e^{-\frac{2\zeta^2}{\sigma_0^2}} \sin^2\left(\frac{2\pi\zeta}{z_0}\right) d\zeta$$

$$= \frac{\sqrt{\pi}}{2\sqrt{2}}\left(\frac{V_0}{E_0}\right)^2 N_s\left(1 - e^{-\frac{2\pi\sigma_0^2}{z_0^2}}\right) \approx \frac{\sqrt{\pi}}{2\sqrt{2}}\left(\frac{V_0}{E_0}\right)^2 N_s. \quad (5)$$

For an ion bunch with the Gaussian density distribution the number of ions in the central slice of length σ_0 is $N_s = \frac{\sigma_0 N_i}{\sqrt{2\pi}\sigma_{is}}$, where N_i is the number of ions in the bunch. A detailed discussion of N_s for the general case of a distribution which is circularly symmetric in τ, δ coordinates can be found in [13].

Since in our model the i-e interaction is happening only at $\phi \approx \frac{\pi}{2}, \frac{3\pi}{2}$, at the moment of the interaction $J \approx \delta^2$. Therefore, averaging Eq. (4) over the whole ensemble of ions, we get

$$\frac{1}{\tilde{J}}\frac{d\tilde{J}}{dt} = -\lambda_C + \lambda_D$$

$$\lambda_C = \frac{2\alpha_1 V_0}{T_s E_0}\frac{1}{\tilde{J}}\int_0^{\infty} f_J[\Phi(R_{56}\sqrt{J}, z_1) + \Phi(R_{56}\sqrt{J}, -z_1)]dJ$$

$$\lambda_D = \frac{2\alpha_1}{T_s}\langle D^2\rangle \frac{1}{\tilde{J}}\int_0^{\infty}\frac{f_J}{\sqrt{J}}dJ, \quad (6)$$

where λ_C and λ_D are the instantaneous cooling and heating rates respectively.

For the i-bunch with the Gaussian distribution, for the case of $z_1 = 0$, Eq. (6) gives

$$\lambda_C = \frac{(2\pi)^{\frac{3}{2}}\alpha_1}{T_s\sigma_\delta^2}\frac{V_0}{E_0}\frac{\sigma_z\sigma_0^3}{z_0(2\sigma_z^2 + \sigma_0^2)^{3/2}} e^{-\frac{2(\pi\sigma_0\sigma_z)^2}{z_0^2(2\sigma_z^2+\sigma_0^2)}}$$

$$\lambda_D = \frac{\sqrt{\ln 2}}{4\sqrt{\pi}}\frac{\sigma_{es}\sigma_0}{T_r\sigma_{is}^2\sigma_\delta^2}\frac{V_0^2}{E_0^2}N_i, \quad (7)$$

where $\sigma_z = R_{56}\sigma_\delta$ and T_r is the revolution period in the ion storage ring.

We found the equations defining the longitudinal beam dynamics of the ion bunch with an arbitrary distribution (as long as the distribution stays circularly symmetric in δ, τ phase space). Equations (6) and (7) give the instantaneous cooling and heating rates for the bunch and define the parameters of the equilibrium distribution given by $\lambda_C = \lambda_D$ (it must be noted that the concept of the cooling rate is applicable only while z_1 does not exceed a particular critical value, which is the topic of the following section).

Equation (2) describes longitudinal motion for each ion. For practical simulations one must substitute D in Eq. (2) with the random kicks having the rms amplitude $\sqrt{\langle D^2\rangle}$ given by Eq. (5).

B. Circular attractor and coherent excitations

The longitudinal force F in Eq. (2) acting on the ion is a nonmonotonic function of δ. Hence, a circular attractor in the ions' longitudinal phase space will appear if the systematic mismatch z_1 is larger than a critical value z_c, where z_c is the absolute value of z corresponding to either the maximum or the minimum of the function $\Delta E(z, 0)$. The critical mismatch is given by the closest to $z = 0$ solution of the equation $\frac{d\Delta E(z,0)}{dz} = 0$:

$$\frac{z_c}{\sigma_0^2} = \frac{\pi}{z_0}\cot\left(\frac{2\pi}{z_0}z_c\right). \quad (8)$$

Figure 1 explains the physics underlying the attractor formation. Consider the case of nonzero mismatch z_1. When an ion with such a relative momentum δ that $R_{56}\delta \in [0, z_1]$ interacts with the electron bunch, the force F acting on the ion is codirected with its longitudinal velocity. Hence, the ion receives an energy kick exciting the amplitude of its synchrotron oscillations. The same ion passing the cooling section with negative δ will experience a damping kick. When $z_1 > z_c$, there is a range of $\delta \in [-\delta_A, \delta_A]$ such that the exciting kick is larger than the damping kick. Hence, the average force acting on the ions with relative momentums in this range is an exciting one.

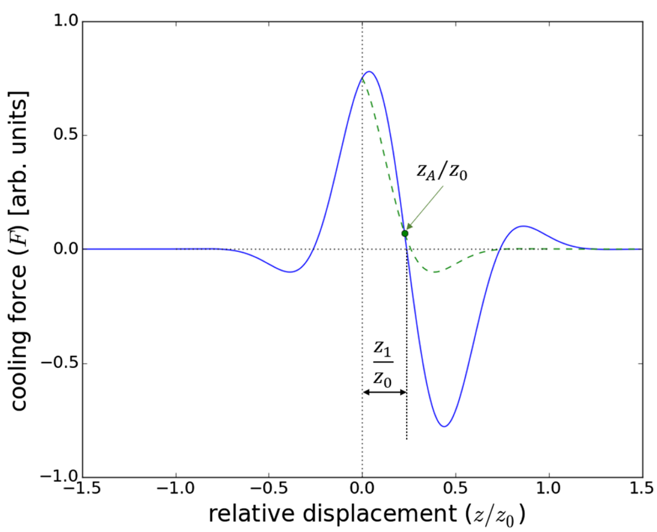

FIG. 1. The force (blue solid line) experienced by an ion on a single pass through the cooling section in the presence of a systematic mismatch $z_1 = 1.6z_c$. The green dashed line represents a reflection (around the vertical axis) of the force for negative z. The green dot shows a "radius" (z_A) of the attractor's projection on the z-space.

For ions with δ outside of this range the average force damps the synchrotron oscillations. Therefore, the force F with a sufficient enough offset makes all the ions to oscillate with the same nonzero amplitude. In other words, for $z_1 > z_c$ the interplay of the rf focusing and the periodic force creates a circular attractor in the longitudinal phase space.

The radius of the attractor ($\sqrt{J_A} = \delta_A = z_A/R_{56}$) is defined by

$$\Phi(R_{56}\delta_A, z_1) + \Phi(R_{56}\delta_A, -z_1) = 0. \tag{9}$$

The presence of the circular attractor stops the cooling of all the ions with $J < J_A$. Instead, such ions experience coherent excitations bringing amplitudes of their oscillations to the attractor's radius.

The described effect is similar to the coherent excitations effect in the regular electron coolers [14,15].

To simulate the ion beam dynamics we integrate Eq. (2) numerically with an explicit, exactly symplectic, third order method [16]. To make the simulations faster, and without a loss of generality, we use force F with a very high peak value of 10^{-5}. Such a force allows to reach the steady state distribution in just about a few hundred synchrotron turns. We simulate diffusion as random energy kicks experienced by each ion. We use the diffusive noise with the normal distribution of the constant rms amplitude 2×10^{-5}. The $\sqrt{\langle D^2 \rangle} = \text{const}$ choice will be justified below. Moreover, since diffusion coefficient is constant, the exact value of the ratio $\max(F)/\sqrt{\langle D^2 \rangle}$ does not matter for the resulting steady state distribution, since the F effect on the ions accumulates linearly with time and the diffusion goes as a square root of time.

Figures 2 and 3 show the initial (Gaussian) distribution of a bunch with 10^4 ions and the same bunch distribution after 200 synchrotron turns under the influence of the force with $z_1 = 1.5 z_c$.

As expected, the main effect observed in simulations is clustering of the ions around the attractor with the radius $\sqrt{J_A} = \delta_A$ defined by Eq. (9). Projection of the resulting phase space on the physical space gives a two-hump longitudinal density distribution, as Fig. 3 demonstrates. For this "hollow" longitudinal distribution the number of particles in the central slice of the i-bunch converges to a constant value:

$$N_{s1} = \frac{N_i \sigma_0 \sigma_{\delta 0}}{\pi \sqrt{J_A} \sigma_{is0}} \tag{10}$$

which gives a constant rms amplitude of the dispersive kick:

$$\langle D_1^2 \rangle = \frac{V_0}{E_0} \frac{N_i \sigma_{\delta 0} \sigma_0}{2\sqrt{2\pi} \delta_A \sigma_{is0}}. \tag{11}$$

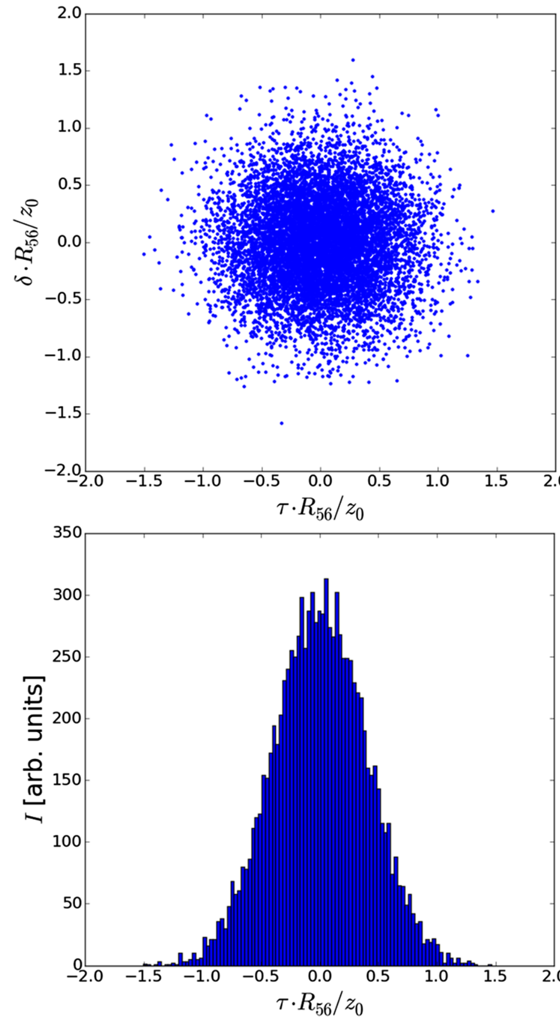

FIG. 2. Initial distribution of the ion bunch in the phase space normalized by R_{56}/z_0.

Thus, the presence of the attractor keeps the diffusive coefficient constant over time. In terms of the initial diffusion ($\langle D_0^2 \rangle$) for the Gaussian bunch we get the following for the steady-state diffusion coefficient:

$$\langle D_1^2 \rangle = \sqrt{\frac{2}{\pi}} \frac{\sigma_{\delta 0}}{\delta_A} \langle D_0^2 \rangle. \tag{12}$$

In addition to the attractor caused by the systematic mismatch, the simulations, clearly show the presence of another attractor of the larger radius. The simulations show [13] that this "weak" attractor has nothing to do with the electron-ion path lengths mismatch and is present even when $z_1 = 0$.

The weak attractor exists because the cooling force has an actual "heating" portion for $z \in (z_0/2, z_0)$ and because the ions interact with the e-bunch only when the longitudinal ion velocity is close to its extremum (when $\phi \approx \frac{\pi}{2}$ or $\frac{3\pi}{2}$). Therefore, in the case of a short e-bunch placed at the

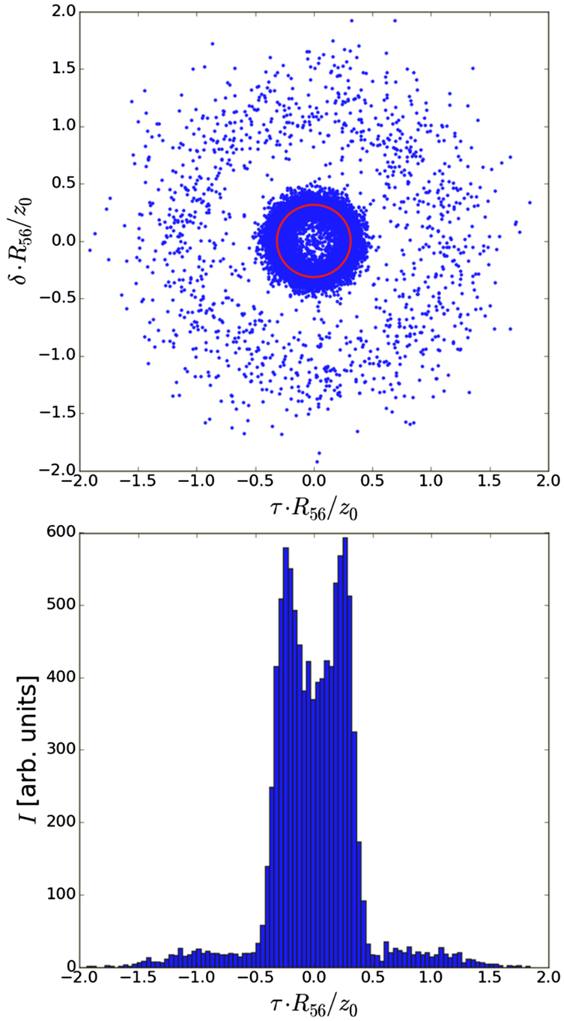

FIG. 3. Phase space of the ion bunch (normalized by R_{56}/z_0) after 200 synchrotron turns under the influence of the force with $z_1 = 1.5z_c$. Blue dots show the simulated i-bunch distribution. The solid red line represents the theoretically predicted attractor with radius $\sqrt{J_A} = \delta_A$ given by Eq. (9).

center of an ion bunch the weak attractor must appear at $z = z_0$, even in the absence of any mismatch.

III. REQUIREMENTS TO CeC DESIGN

The systematic mismatch between the electrons' and the ions' modulator-to-kicker path lengths must be kept less than z_c defined by Eq. (8). Otherwise the cooling will be lost completely.

Bearing in mind the analogy between the CeC and the stochastic cooling discussed in Sec. I, one can get a simplified expression for z_c through the CeC bandwidth W. Since, $z_c \approx \frac{c}{2W}$, Eq. (8) can be substituted for quick estimates by

$$z_1 < \frac{c}{2W}. \tag{13}$$

The need to ensure against the longitudinal mismatch exceeding the critical value z_c also sets requirements to tolerable error in the e-i γ-matching. The following condition must be satisfied for the CeC to work:

$$\frac{\Delta\gamma_i}{\gamma} R_{56} - \frac{\Delta\gamma_e}{\gamma} R^e_{56} < z_c, \tag{14}$$

where γ is the design relativistic factor, $\Delta\gamma_i$ and $\Delta\gamma_e$ are the γ-factor errors of the ion and the electron beams respectively, and R^e_{56} is the modulator-to-kicker momentum compaction for the electron beam.

Avoiding the weak attractor might be not as important as avoiding the attractor caused by the systematic longitudinal mismatch. The weak attractor does not stop cooling completely. It affects only the ions with $\sqrt{J} > \sqrt{J_a}$, where $J_a = (\frac{z_0}{2R_{56}})^2$. The fraction of the ion bunch affected by the weak attractor depends on the CeC parameters, and for the bunch with the Gaussian distribution can be found as $\int_{J_a}^{\infty} (1/\tilde{J}) e^{-J/\tilde{J}} dJ$. For instance, for the EIC parameters, about 8% of the ions are lost to the cooling due to the weak attractor's effect.

To give an example of requirements set by discussed effects we consider parameters of the Strong Hadron Cooler [17] under design to cool protons at 275 GeV in the EIC (Table I).

Substituting the wake parameters into Eq. (8) we get the critical value of the longitudinal mismatch $z_c = 1.3\ \mu$m. In terms of time of flight of the electron and ion beams from the modulator to the kicker the difference must be kept below $\approx$4 fs.

Knowing z_c one can obtain tolerances to errors in setting the absolute energies of each beam from Eq. (14). Assuming that for each beam the error must not exceed $z_c/2$, we obtain for proton beam $\Delta\gamma/\gamma \leq 3 \times 10^{-4}$ and for electrons $\Delta\gamma/\gamma \leq 4 \times 10^{-4}$.

Other possible sources of the path length mismatch are the errors in the magnets' settings. Such errors can result both in the incorrect trajectories of the beams through the cooling section and in the errors in either R_{56} or R^e_{56}. A tolerance to an error in the current of each magnet in the cooling section must be determined at the final stage of the design of the cooler when its lattice is finalized.

TABLE I. EIC cooler parameters.

Kick amplitude (V_0) [eV]	28
Kick characteristic wavelength (z_0) [μm]	6.7
Kick characteristic width (σ_0) [μm]	3
Proton beamline R_{56} [mm]	2.2
Electron beamline R^e_{56} [mm]	1.6
Protons per bunch (N_p)	6.9×10^{10}
Protons relative momentum spread ($\sigma_{\delta 0}$)	6.8×10^{-4}
Proton bunch length (σ_{ps0}) [cm]	6
Electron bunch length (σ_{es}) [mm]	7

IV. CONCLUSION

We showed that in the coherent electron cooling technique a circular attractor appears in the ions' longitudinal phase space if the systematic mismatch between the ion and the electron path lengths through the cooling section is larger than the critical value given by Eq. (8). This attractor causes coherent excitations of the ions with the oscillation amplitudes smaller than the attractor radius, thus turning cooling into anticooling.

The theory of coherent excitations in the CeC was presented and compared to the results of the simulations.

It was further shown that even in the absence of the systematic mismatch another attractor still might be present. This weak attractor results from the diffusive part of the wake and from the fact that the e-bunch used for cooling is much shorter than the i-bunch and is longitudinally placed at the center of the i-bunch.

The requirements to the CeC design were discussed including the requirements to the Strong Hadron Cooler at the Electron Ion Collider.

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