Reply to "Comment on 'Fast-slow mode coupling instability for coasting beams in the presence of detuning impedance'"

N. Biancacci,[1] E. Métral,[1] and M. Migliorati[2]

[1]*CERN, Geneva, Switzerland*

[2]*University of Rome, "La Sapienza" and INFN—Roma 1, Rome, Italy*

(Received 17 June 2021; accepted 8 July 2021; published 29 July 2021)

DOI: 10.1103/PhysRevAccelBeams.24.078002

In this reply, we provide a response to the comment of A. Burov and V. Lebedev [1] (referred to below as the comment's authors) about the PRAB paper "Fast-slow mode coupling instability for coasting beams in the presence of detuning impedance" [2].

Our proposed mode coupling instability for coasting-beams was never predicted/discussed in the past (see for instance Chao's textbook [3]) and it is considered to be impossible by the comment's authors.

We fully disagree with them and we have provided our full derivation of Eq. (23) [or Eq. (24)] of [2] using the Vlasov's formalism in the paper "Self-consistent derivation of the transverse mode coupling instability for coasting beams using the linearized Vlasov's equation" we recently uploaded to arXiv [4] and submitted to PRAB. We therefore confirm what we wrote on [2], i.e., that mode coupling can take place between 2 modes for coasting beams (from pyHEADTAIL simulations, simple theory from the single-particle formalism and self-consistent theory from Vlasov) and we believe that the reason why it was not discovered in the past is twofold: (1) to derive it analytically from the linearized Vlasov's equation, one should not make the usual approximation $\sin(\varphi) \sim e^{j\varphi}/(2j)$ (as discussed for instance at the bottom of page 335 in [3] for a bunched beam) but really consider the 2 terms $\sin(\varphi) = (e^{j\varphi} - e^{-j\varphi})/(2j)$ as the second term is the one responsible for the mode coupling in coasting beams. It should be stressed here that mode coupling is found already with driving impedance only. By the way, as mentioned above, this approximation is also made for bunched beams and this case should also be reviewed in the future. (2) by including the detuning impedance, the coupling is much stronger and this is what we found also in our pyHEADTAIL simulations.

Before answering, point by point, to the comment's authors, we would like to mention clearly what the different steps were in the study we presented in the PRAB paper: (1) We extended the pyHEADTAIL code to simulate transverse (and longitudinal) coherent instabilities for coasting beams. (2) To gain confidence in what had been done, we first benchmarked our new simulations with the classical transverse coasting-beam approach (e.g., from Laclare, see [5]) and an excellent agreement was reached, as can be seen in Fig. 3 of [2], for both the real and imaginary parts of the complex tune shift. (3) We introduced the detuning impedance in our pyHEADTAIL simulations and found that the results did not agree anymore with the classical approach. Equation (23) was then proposed as an extension of Chao's ansatz (see Eq. (5.71) of page 243 of [3]). This equation was found to be in excellent agreement with the new pyHEADTAIL simulations, as can be observed in Fig. 6, nicely reproducing, in particular, the plane exchange of the most critical instability vs intensity (which is one of the important findings of this study). (4) As this new Eq. (23) was not derived (yet) self-consistently from the Vlasov's equation, we mentioned in [2] that "This result is presently being investigated following also the Vlasov's formalism and will allow future studies on the effect of a finite momentum spread and chromaticity". The study of the effect of a finite momentum spread and chromaticity is still work in progress, but we can show our full derivation of Eq. (23) [or Eq. (24)] using the Vlasov's approach in [4].

We now answer the two points raised by the comment's authors (whose comments appear in italic below):

> *From a conventional smooth Eq. (22), the authors derive a new Eq. (23) that has nonzero cross-terms, or mode-coupling terms. This derivation is certainly incorrect: when the Fourier transformation over s is applied to the smooth Eq. (22), all the cross-terms necessarily cancel. This cancellation reflects the translational symmetry, provided by the smooth approximation. […]*

In the context of [2], we did not provide Eq. (23) as a derivation of Eq. (22). Equation (23) was proposed as an ansatz to describe the coupling of two nearby waves. The comment's authors provide a derivation that does not

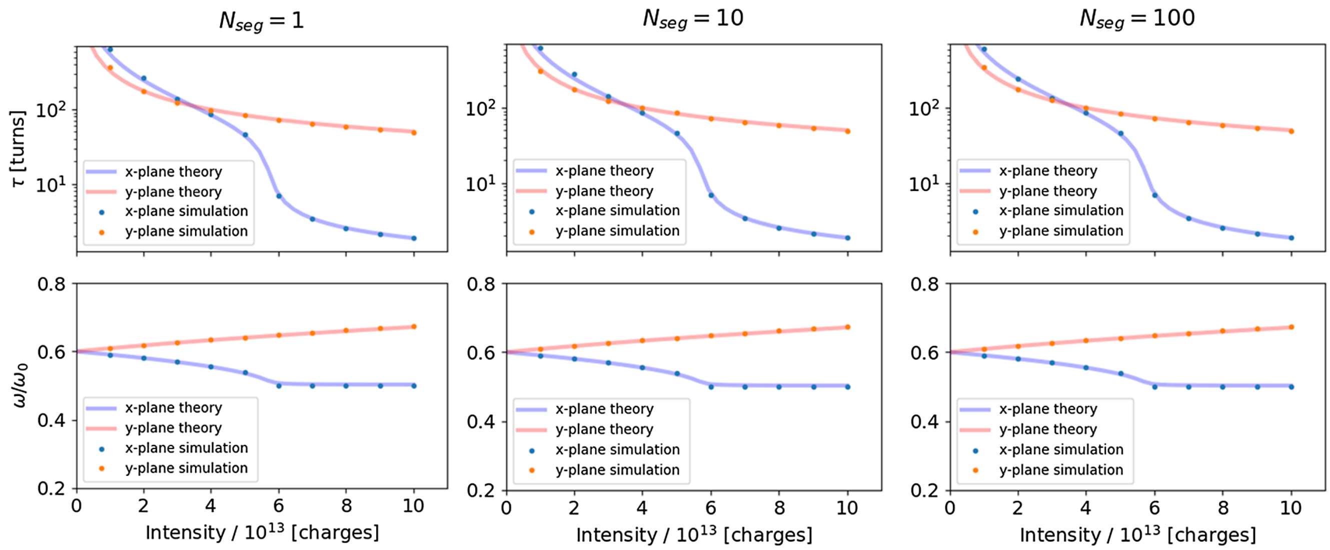

FIG. 1. Same figure as Fig. 6 of [2], using a logarithmic scale for the vertical axis of the instability rise times, and dividing the lattice in 1, 10 and 100 segments (N_{seg}). pyHEADTAIL simulations (with dots) are compared to theory accounting for coupling between two waves (with full lines) for the horizontal (blue) and the vertical (red) planes. The rise time, at the top, and the normalized frequency shift, at the bottom, are shown. No effect of the lattice segmentation is observed.

account for the possible coupling of two nearby waves. This is in line with the approach developed in [3,5] in which the usual approximation $\sin(\varphi) \sim e^{j\varphi}/(2j)$ is done, therefore limiting the analysis to uncoupled waves. In the conclusions of [2] we wrote "This result is presently being investigated following also the Vlasov's formalism" to give a fully consistent justification of Eq. (23). This is at present available in [4].

Another point of our comment relates to the claimed excellent agreement between the theory of Eq. (23) and pyHEADTAIL simulations. […] The Authors should explain how the lattice smoothness is implemented, what the phase advance per cell is, how many cells per ring and how many impedance kicks per turn there are, and how the claimed agreement depends on these parameters. If the code is correct, the smoother the lattice is and the denser the impedance kicks are, the weaker the mode coupling must be. […]

The code implements a one-turn-map, i.e., all the particles experience a one-turn rotation in transverse phase space with a phase advance given by the machine tune parameter. Accordingly, only one impedance kick per turn is applied. In this context, the smooth approximation is applied, i.e., the particles travel in a constant focusing channel with average beta function given by $\bar{\beta}_{x,y} = R/Q_{x,y}$, with R the machine radius and $Q_{x,y}$ the machine tune in the horizontal and vertical plane respectively. Figure 1 reproduces Fig. 6 of [2] using a logarithmic scale for the vertical axis of the instability rise times to better reveal the excellent agreement between the new theory and the pyHEADTAIL simulations over all the intensity range. From left to right, we show the effect of lattice segmentation on the obtained complex mode shift for the horizontal and vertical plane. The lattice one-turn map has been divided in 1, 10 and 100 segments, between which the impedance kick is given (scaled down by the corresponding factor): no difference is noted between the three cases.

[1] A. Burov and V. Lebedev, preceding Comment, Comment on "Fast-slow mode coupling instability for coasting beams in the presence of detuning impedance", Phys. Rev. Accel. Beams **24**, 078001 (2021).

[2] N. Biancacci, E. Métral, and M. Migliorati, Fast-slow mode coupling instability for coasting beams in the presence of detuning impedance, Phys. Rev. Accel. Beams **23**, 124402 (2020).

[3] A. Chao, Physics of collective beam instabilities in high-energy accelerators, 1993. https://www.slac.stanford.edu/~achao/wileybook.html.

[4] N. Biancacci, E. Métral, and M. Migliorati, Self-consistent derivation of the transverse mode coupling instability for coasting beams using the linearized Vlasov equation, 2021. https://arxiv.org/abs/2106.05557.

[5] J. L. Laclare, Introduction to coherent instabilities: Coasting beam case, in *CERN Accelerator School: Course on General Accelerator Physics*, 1985, pp. 377–414, https://cds.cern.ch/record/620151.