Reply to "Comment on 'Empirical condition of betatron resonances with space charge'"

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In the Comment on our paper [Phys. Rev. Accel. Beams **22**, 074201 (2019)], Hofmann claims that our conclusions are based on a questionable interpretation of Vlasov's equation and an over-interpretation of our multiparticle simulations. This assertion, however, comes largely from his misinterpretations of the essence of our work and proposed resonance condition. His criticism based on experimental data from some operating machines has also missed the point; we see no essential conflict between our arguments and experimental observations. While most of the questions raised by Hofmann have already been answered in our original paper above and previous publications, we take this opportunity to provide more information and explanation for clarity.

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I. INTRODUCTION

The "incoherent tune spread" or, in other words, the "necktie" has been employed for years to make a rough estimate of the maximum beam current that can be stored in a high-intensity hadron ring. Many researchers have used this conventional concept as a measure to avoid the resonant instability of the whole beam, especially, the beam's main body (i.e., the "core" defined in *full* phase space) where the incoherent tune shifts of individual particles are large. A good summary on this point is given, for example, in a highly cited review article written by Baartman (Ref. [1; 30]). He says the following: "In many papers, in proceedings of accelerator schools, and even in textbooks on accelerator physics, we read that the linear part of the space charge force is added to the linear equation of motion, leading to a tune shift, which if large enough can place individual particles on low-order betatron resonance lines. This picture, though in some sense compelling, is misleading and inhibits understanding the transverse intensity limit in low energy proton synchrotrons." A similar statement can be found also in a textbook written by Ng (Ref. [1; 65]). We strongly concur with these physicists and believe that historically, this is the most common view regarding the usage of the necktie. If not, what is the purpose of drawing a necktie in the tune diagram or calculating a possible maximum incoherent tune shift? Such information is unnecessary if one cares only about what happens in the Gaussian tail where particles have relatively small tune shifts. In an early stage of accelerator development, everybody is first concerned about any bad effects that may occur even if an ideal beam is injected into his/her machine. Besides, in the recent monograph written by the Comment author himself (Ref. [1; 47]), he has claimed after all that "the distance of the working point to significant resonances in the $\nu_x - \nu_y$ tune diagram must be consistent with the maximum expected tune spread," which obviously follows the conventional view. One of the important points concluded in Ref. [1] is that the maximum tune spread has nothing to do with the stability of the beam's main body.

Any crude estimates, of course, never satisfy professional accelerator designers and will eventually be taken over by a more sophisticated estimate made through advanced, time-consuming calculations. Nonetheless, a simple criterion as described in Ref. [1] is very helpful in establishing conceptual high-intensity lattice designs. It could also offer a useful insight into the cause of beam loss observed in operating machines.

Our theory in Ref. [1] was designed to show very quickly and easily, as the first step toward the best machine performance, which regions in the tune diagram could be dangerous even for a matched beam. We proposed a new type of stability map based on the two-dimensional (2D) semiempirical resonance condition

$$k(\nu_{0x} - C_m \Delta\bar{\nu}_x) + \ell(\nu_{0y} - C_m \Delta\bar{\nu}_y) = \frac{n'}{2}. \qquad (1)$$

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This formula is a natural generalization of the one-dimensional Vlasov prediction in Refs. [1; 27] where the following *coherent resonance* condition has been derived in a purely mathematical manner:

$$\Omega_m \equiv m(\nu_0 - C_m \Delta\bar{\nu}) = \frac{n'}{2}. \tag{2}$$

Equation (1) is reduced to Eq. (2) for noncoupling resonance that occurs in one of the two transverse degrees of freedom.

Most of coherent resonance bands are accompanied by a sort of *incoherent resonance* region (see, e.g., Fig. 18) where tail particles with relatively small incoherent tune shifts may become unstable. The semiempirical rule proposed in Ref. [1] for the construction of a stability chart includes the incoherent aspect of resonances as a part. The two essentially different aspects, i.e., coherent resonances in the core and incoherent resonances in the tail, should suffice to describe the stability of the whole beam initially matched to the lattice.

The coherent core instability has a self-inhibition mechanism; it may be deactivated spontaneously before leading to a well-detectable level of beam loss. That is because the emittance growth reduces the beam density, which results in a shrinkage and a shift of the stop band. Even so, no accelerator designers would dare to choose the operating point within a region where a possible instability is expected from self-consistent calculations, no matter whether it appears to be weak. In our opinion, an emittance growth of a few percent over only 100 alternating-gradient (AG) focusing cells is not a negligible level. The situation will be worsened in actual high-intensity rings where the beam goes through far more AG periods.

The Vlasov formalism was invented to describe the *collective* behavior of particles with long-range Coulomb interaction. It is self-consistent and includes all relevant physical processes except for interparticle Coulomb collisions. Solving the Vlasov-Poisson equations is physically almost equivalent to performing particle-in-cell (PIC) simulations. There are two important questions we should ask here: First, is it really possible for Vlasov's equation to predict the incoherent resonance conditions of individual particles in a matched beam core? Second, do all nonlinear coherent modes really become inactive in any actual beams because of the Landau damping mechanism? These two issues can be addressed separately.

The first question above has already been answered in Ref. [1] and past Vlasov theories. There is no serious incoherent response activated in a matched beam core. The tune spread does not matter; even the single-particle resonance has a finite width depending on the strength of the resonance driving term, but no instability can be found from the Vlasov analysis about the incoherent tunes. The Comment author has often argued the existence of incoherent (or his so-called "single-particle") resonances with large tune shifts, driven by space charge "pseudo-multipoles" [2]. One of our conclusions is that no such incoherent effect will appear in the core. It is thus useless to make any further discussion assuming the presence of incoherent core resonances. This conclusion holds even at low beam density; the Vlasov formalism is not limited to the high-density regime. We can obtain PIC results analogous to Fig. 20 even when the rms tune depression is much closer to unity.

The second question is not so easy to answer. We agree with the Comment author on the point that the incoherent tune spread has been ignored in past perturbative Vlasov theories on coherent resonances, which kills the Landau damping mechanism. The instabilities of highly nonlinear modes will probably be damped and thus of no serious effect in practice. This point has been made repeatedly in our past papers. Self-consistent simulations or any other approaches are needed to judge which modes are potentially dangerous. In Ref. [1], we deemed it necessary to take care of the coherent modes of second ($m = 2$) and third ($m = 3$) orders at least. The fourth-order ($m = 4$) mode may also demand attention depending on the beam density and on how long the beam stays in the machine at relatively low energy. These requirements are based not only on extensive numerical studies but also on experimental data obtained with the novel ion-trap apparatus "S-POD," the Simulator of Particle Orbit Dynamics.

We disagree with the Comment author's view that "no need has been seen so far to revise the standard picture of nonlinear, incoherent resonances" because "unambiguous experimental signatures have not been reported yet" [3]. First, the lack of unambiguous experimental signatures of coherent modes does not necessarily justify the standard incoherent picture. Everybody is seeking after all for a fuller understanding of space-charge resonances. Second, experimental signatures suggesting the practical importance of the linear and low-order nonlinear coherent instabilities have already been reported in a number of previous publications from the Hiroshima group. Third, we have clear evidence for the nonexistence of incoherent resonances in the beam core.

The Comment author has mostly argued his own personal view about resonances with almost no quantitative discussions that clearly point out any flaw in our theoretical model. The only counterevidence offered in the Comment is the experimental data obtained at the GSI SIS18, but his criticism is obviously based on some misunderstanding or miscalculations. In Sec. II, we first provide additional information about low-order coherent modes and their experimental observations in the S-POD system. Brief discussion is then made in Sec. III about the linearized Vlasov analysis with the Kapchinskij-Vladimirskij (KV) model. Section IV is devoted to a possible interpretation of the SIS18 data from a viewpoint essentially different from the Comment author's. Finally, the present discussion is summarized in Sec. V.

II. OBSERVATION OF LOW-ORDER COHERENT MODES

As remarked by the Comment author, no unambiguous experimental signatures of coherent responses from nonlinear and even linear modes have been observed so far in circular accelerators. The complexity and diverse background sources in real large-scale machines make it challenging to identify clear signals from the coherent modes. This is a primary reason why we developed the S-POD system to explore various space-charge issues. The S-POD is extremely compact in size and much simpler than any accelerator systems, which greatly facilitates the identification of weak coherent signals. Experimental data produced by the S-POD for the last decade have shown the signatures that can be understood most naturally and consistently by accepting the presence of low-order coherent resonances.

We can directly measure the tunes of some coherent modes with the help of the S-POD (Refs. [1; 33] and [1; 34]). If any collective mode of mth order is excited in a plasma bunch, we will detect a pair of frequency components as explained in Ref. [1]; one corresponds to $k_1 + \Omega_m$ and the other to $k_2 - \Omega_m$ where k_1 and k_2 are integers [4]. Figure 1 shows a typical measurement result demonstrating the existence of the coherent quadrupole oscillation. We observe three clear peaks at 298, 702, and 1000 kHz. The large spike at 1 MHz comes from the large envelope modulation driven by the AG focusing force of the linear Paul trap (LPT), which has nothing to do with collective modes. As the operating AG-focusing frequency is 1 MHz here, the first two numbers above correspond to the tunes of $0.298(=\Omega_2)$ and $0.702(=1-\Omega_2)$. The space-charge term $C_2\Delta\bar{\nu}$ in Eq. (2) is responsible for the large shift of Ω_2 from $2\nu_0(=0.320)$. If we keep increasing the operating bare tune, these two peaks eventually merge at which point the bunch becomes unstable. The instability condition is, therefore, $\Omega_2 = 1 - \Omega_2$ that can be rewritten as $\Omega_2 = 1/2$. A similar spectrum can be obtained easily for the dipole mode. This experimental observation is a piece of indisputable evidence for the parametric instability.

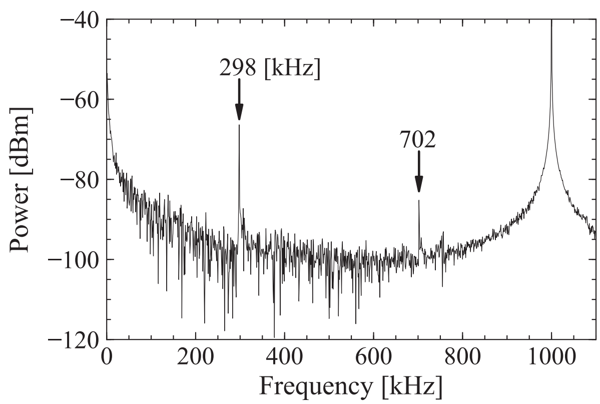

FIG. 1. Frequency spectrum of the quadrupole mode measured in the S-POD trap system at Hiroshima University (Ref. [1; 33]). The operating frequency of the LPT is 1 MHz. The bare betatron tune per unit AG cell is chosen to be $\nu_0 = 0.16$. The number of ions confined in the LPT is about 2×10^6.

A new type of LPT with extra electrodes must be developed for the direct tune measurement of nonlinear coherent oscillations [5]. We have already constructed a prototype and succeeded in exciting the sextupole and octupole modes under the condition in Eq. (13) of Ref. [1]. The result will be published at a later date.

We wish to call general readers' attention to the fact that an ion bunch produced in the LPT always has a "Gaussian-like" profile (see Fig. 9 of Ref. [1; 39]). It must be more like the perfect Gaussian compared with actual beams in circular machines because no complicated technique such as the multiturn injection, painting scheme, etc., is necessary to generate the ion bunch in the LPT.

III. THE KV MODEL

The Comment author has quoted a Vlasov theory [3; 1] by Li and Jameson who nicely generalized the previous KV-based analysis by the Comment author himself (Refs. [1; 12] and [1; 25]). Their theory converts Vlasov's equation into coupled ordinary differential equations that need to be integrated *numerically* to figure out the parameter ranges where the matched KV core becomes unstable. No universal resonance formula has, therefore, been given explicitly in Ref. [3; 1].

We had already developed a 2D Vlasov theory a few years before Ref. [3; 1] was published (Refs. [1; 48] and [6]). Our numerical approach is essentially the same as Li and Jameson's. We can plot a tune diagram under an arbitrary initial condition of the KV beam propagating in an arbitrary AG lattice. Figure 2 is an example where both normal and skew sextupole ($m = 3$) components have been taken simultaneously as resonance driving terms in the space-charge potential. The beam is initially equipartitioned everywhere in the diagram, similarly to the PIC simulations in Subsec. III C of Ref. [1]. Red solid lines indicate the positions of the third-order coherent resonances predicted by Eq. (1), all of which agree remarkably well with the instability bands from the Vlasov analysis. Other instability bands with no red line are practically unimportant or covered by the stronger coherent resonances driven by the lower-order ($m = 2$) terms.

We have confirmed that Eq. (1) can well reproduce the locations of major resonance bands from the Vlasov analysis not only in the equipartitioned case but also under a variety of initial conditions. We here stress the point again that our coherent tune-shift factor is a constant free from any parameters other than the resonance order m. This presents a striking contrast to the previous theoretical

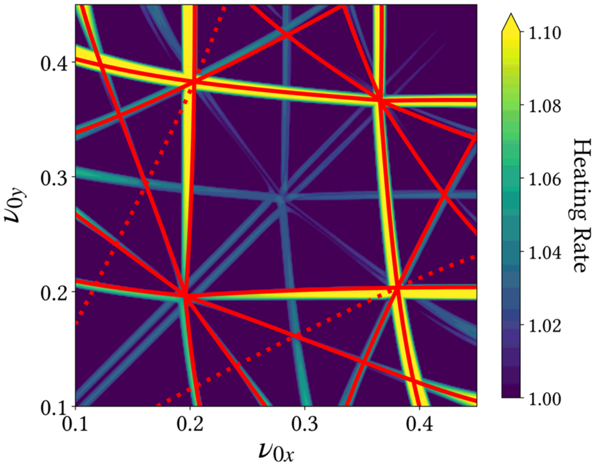

FIG. 2. Stability tune diagram obtained from the 2D Vlasov analysis based on the KV model (Ref. [1; 48] and [6]). All kinds of third-order ($m = 3$) space-charge terms have been taken into account to drive coherent resonances. The contributions from the quadrupole ($m = 2$) and other higher-order ($m \geq 4$) driving self-fields have been ignored. The initial condition is identical to what we assumed in Fig. 6 of Ref. [1] (the equipartitioning case) except that the beam density at the operating point $(\nu_{0x}, \nu_{0y}) = (1/6, 1/6)$ has been increased to $\eta_x = \eta_y = 0.8$ here. Red lines represent the locations of the third-order resonance bands predicted by Eq. (1) with $C_3 = 0.85$. Two difference resonance bands (dotted line) have disappeared because, as pointed out in Ref. [1], the condition $I_{k\ell} = 0$ is fulfilled along them in an equipartitioned beam.

prediction that C_m depends on the integers (k, ℓ), degree of tune split, beam's ellipticity, etc., (Refs. [1; 25], [1; 29], [1; 30]). The proposed semiempirical formula in Eq. (1) enables one to plot resonance lines of practical importance very quickly and easily without solving a set of complicated differential equations numerically. It predicts the locations of instability bands under arbitrary initial beam conditions. All we need to know in advance are the basic lattice design plus only the rms tune depression that can readily be calculated from the rms envelope equations.

IV. EXPERIMENTAL DATA FROM OPERATING MACHINES

Reference [1] is not a review article, so we do not think that it is necessary to make lengthy discussions covering all relevant high-intensity machines around the world. As an important example, we picked the RCS at J-PARC and applied the proposed semi-empirical rule to check if it successfully explains the current operating condition. A preferable operating point determined by the conventional approach in its design stage was $(\nu_{0x}, \nu_{0y}) = (6.68, 6.27)$ [7], but the machine is now operated around $(\nu_{0x}, \nu_{0y}) = (6.45, 6.42)$ after a careful tune survey (Refs. [1; 63] and [1; 70]). As illustrated in Ref. [1], our theoretical prediction is consistent with the current RCS situation.

Following the Comment author's request, we now take a brief look at another two cases, i.e., GSI SIS18 and CERN PS. Let us start from the SIS18. The maximum incoherent tune shift of 0.025 corresponds to the rms tune depression η of about 0.997, assuming a Gaussian core. The core density turns out to be much lower than the RCS case. This number gives an rms tune shift $\Delta\bar{\nu} \approx 0.013$. As the coherent tune shift factor of the sextupole mode is $C_3 \approx 0.8$, the center of the third-order coherent band is expected to be at around $Q_x \approx 4.343$ in the coasting-beam case. According to Fig. 1 in the Comment, the measured peak is located close to this predicted tune. The band width evaluated from the approximate formula in Eq. (16) of Ref. [1] is roughly 0.013 [8]. The tail resonance region extends from 4.333 to the lower boundary of the coherent band.

Even for the bunched-beam case, while it is outside the scope of our 2D resonance model, the position and width of the measured resonance band appear to be well explainable; namely, our theory says that the coherent sextupole band lies between $Q_x \approx 4.339$ and 4.359.

There is a possibility that the observed emittance growth could come mainly from the error-driven tail, considering the low beam density and existence of a sextupole imperfection field in the experiment. It is difficult for us to make a definitive conclusion from limited information, but the point is that no essential discrepancy can be seen between our theoretical predictions and experimental observations. Our theory recommends avoiding the bare-tune range $4.333 \leq Q_x \leq 4.350$ in the case of $\Delta Q_x = -0.025$ and the bare-tune range $4.333 \leq Q_x \leq 4.359$ in the case of $\Delta Q_x = -0.04$. It is informative to notice that the maximum extent of the necktie, i.e., 0.025 for the coasting beam and 0.04 for the bunched beam, overestimates the actual width of the instability region. Finally, we remind the Comment author of the fact that our theory in Ref. [1] was developed for betatron resonances of matched beams. Any longitudinal effects as mentioned by him are of no interest to us here.

The Comment author refers to the CERN PS experiment as if it is a good example supporting the incoherent, frozen space-charge model (FSM). His understanding is, however, questionable. In fact, Bartosik of CERN states in Ref. [1; 46] that "simulations using a frozen adaptive model in PyOrbit for the ideal PS lattice do not explain the observed losses quantitatively". The FSM has also encountered a difficulty in reproducing recent experimental data obtained by Asvesta *et al.* [9]; numerical simulations based on the incoherent model do not appear to be in sufficient agreement with the measurement results. An observed large discrepancy is eventually attributed to unknown octupolar errors exciting a fourth-order sum resonance band along $2Q_x + 2Q_y = 25$. Note that the driving harmonic number has been halved to reduce the resonance order. We suspect that such a strong imperfection might seriously affect the lattice symmetry, giving rise to many additional stop bands. In any case, there seems to be no good agreement at the moment, as opposed to the Comment author's assertion.

V. CONCLUSION

The motions of individual particles forming a dense beam core are correlated because of the long-range nature of Coulomb interaction. Self-consistent PIC simulations clearly show that the core particles do not resonate at their incoherent tunes. Experimental data from the S-POD system supports this view, offering convincing evidence for coherent oscillation modes. The conventional picture relying on the incoherent tune spread is too conservative to make a good estimate of the transverse space-charge limit in a high-intensity hadron ring. The Gaussian necktie gives no useful physical information regarding the stability of the beam's main body.

Once the core becomes unstable, a tail will grow leading to an emittance increase. The core will eventually recover a sort of stability as the density reduction due to the initial instability moves the effective operating point out of the stop band. After that, the emittance growth and beam loss, if any, come mostly from the tail region. Because of weak Coulomb coupling with the beam core, tail particles see the oscillating core space-charge potential as if it were an external driving source. The FSM might then offer a good explanation for measurement data, but that does not necessarily mean the absence of coherent core instability. The situation is similar to why the "particle-core model" works to give a rough picture of mismatch-induced halo formation.

It is true that any ordinary beam has a Gaussian-like profile whose density is peaked around its center, but "Gaussian-like" does not necessarily mean "Gaussian." The actual core configuration is different from the perfect Gaussian in phase space, especially after a complex injection procedure. The exact core potential is no longer spatially symmetric as often assumed in frozen models. Attempts have been even made to intentionally form a non-Gaussian profile for space-charge mitigation. We hardly understand why the Comment author only trust symmetric Gaussian-based predictions and why all possibilities expected in the other types of reasonably realistic distribution functions can be disregarded with confidence. There is no room to doubt that any realistic beam is deviated from a perfect stationary state and thus includes the seeds of various coherent modes that can enhance core resonances as demonstrated in Ref. [1].

In some cases, the incoherent model may work as mentioned above but only for tail dynamics. Self-consistent considerations are vital to establish a reliable picture of core dynamics. It is such an attempt that has been made and reported in Ref. [1]. The work is supported by self-consistent numerical simulations and by *experimental* long-term simulations with the novel tabletop apparatus "S-POD." Not only the linear but also nonlinear coherent modes do exist as mathematically proved by the Vlasov analysis, numerically confirmed with non-Gaussian models, and even experimentally observed in the S-POD system. It sounds unreasonable, at least to us, to insist that only the incoherent mechanism within the scope of frozen models is important in practice.

In the future, the coherent effect studied in Ref. [1] will be more important as the result of growing demand for higher-intensity hadron beams from machine users. The proposed new tune diagram provides information about the stability of the whole matched beam including the tail as well as the core. We believe that our stability map deserves serious consideration as a possible option to search for a good operating region in the betatron tune space.

ACKNOWLEDGMENTS

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[3] I. Hofmann, preceding Comment, Comment on "Empirical condition of betatron resonances with space charge", Phys. Rev. Accel. Beams **23**, 028001 (2020).

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[8] To be on the safe side, we have assumed in Ref. [1] that the band widths of all coherent resonances are equal to $\Delta\bar{\nu}/\eta$. It is possible to derive, from the Vlasov theory in Ref. [1; 27], an approximate m-dependent correction factor to this simplified band-width formula. The factor makes the widths of higher-order stop bands narrower and, as a result, broadens the tail-resonance regions.

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