Undulator radiation of premodulated and nonmodulated electron bunches in the negative mass instability regime

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(Received 2 May 2018; published 5 October 2018)

The use of both spatial premodulation and negative mass instability is promising for the energy enhancement and spectrum narrowing of the undulator radiation from dense electron bunches formed in modern laser-driven photoinjectors. Combining these two methods will permit the formation of long-lived trains of short and dense bunches with very large charges, which are able to efficiently emit a much more powerful and narrow-band terahertz radiation than single bunches with the same total charge, or premodulated bunches in the positive mass oscillation regime. The development of radiation instability in the negative mass regime can enable significant quasiperiodic self-modulation of relatively long nonmodulated bunches resulting in their fairly efficient and selective terahertz superradiance.

DOI: 10.1103/PhysRevAccelBeams.21.100701

I. INTRODUCTION

Short electron bunches with a moderately relativistic electron energy of 3 to 6 MeV and a charge of the order of 1 nC, which can be formed in laser-driven photo injectors [1–3], are attractive for the generation of coherent spontaneous terahertz radiation [4–14]. However, the energy and spectral capabilities of the simplest terahertz sources exploiting such bunches are significantly limited due to a strong Coulomb repulsion of particles, which leads to a rapid expansion of bunches and loss of coherency of the radiation emitted from their different parts. A number of authors suggested using preliminary modulation of the electron density in such sources in order to increase drastically the radiation energy and frequency (see, e.g., [10,15,16]). The modulation can be obtained, for example, by means of time modulation in the illuminating laser intensity. The modulation is assumed to allow the use of much longer electron bunches with larger charges, and a significant enhancement of energy and narrowing of the radiation spectra as well.

It was shown in [15] that the Coulomb repulsion of particles from areas with increased density in thin premodulated bunches leads to changes in the initial density modulation and excitation of plasma waves. In this case, the modulation disappears and then reappears at the distance of the quarter and half plasma wavelengths, respectively, and so on. For a sufficient modulation amplitude, the reappearance of modulation is accompanied by the development of sharp density spikes. The "flickering" behavior of such disappearance and reappearance of modulation corresponds to a similar character of coherent radiation that can be generated by the modulated bunches in undulators or other radiation devices.

Another character of the evolution of relatively long modulated bunches and their radiation should be expected when the particles are not repulsed from areas with increased density, but are attracted to them. This "anomalous" situation occurs, in particular, in undulators with a strong (over-resonance) guiding magnetic field due to the development of the so-called undulator negative mass instability (NMI) [17–19] in bunches [20–23]. Due to NMI, an increase/decrease in electron energy under the action of the space charge of a dense bunch can cause approaching to/moving away from the resonance between the cyclotron and undulator oscillations. Correspondingly, the transverse velocity of the particle increases/decreases, which can result in a decrease/increase in the electron's longitudinal velocity. This means that the electrons moving translationally and oscillating in the combined magnetic and bunch space charge fields can behave as relativistic particles that nonisochronously rotate in a uniform magnetic field when their energy changes in the field of a bunch (classical cyclotron NMI) [24,25]. Development of undulator NMI in a short single bunch with a very large charge can produce a dense "core" with a sufficiently small longitudinal size, which results in intense emission of high-energy radiation [22,23]. It is natural to assume that the use of premodulated long bunches in the NMI regime

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may provide sharpening of the modulation peaks and "freezing" of the modulation in the form of a train of short bunches that enables the production of very powerful and efficient THz radiation. This article is devoted to a detailed study of such a situation in comparison with the "normal" evolution of bunches and their undulator radiation under conditions of usual positive masses of the particles.

The paper is organized as follows. In Sec. II, the mechanisms of cyclotron and undulator negative mass instabilities are discussed. The THz source and the numerical code used in the simulations are described in Sec. III. In Secs. IV and V, we compare the radiation of ideally compact pointlike bunches, single bunches with duration of the order of a characteristic half wavelength of radiation, and much longer premodulated and nonmodulated bunches, both under conditions of the "positive" (normal) and "negative" (anomalous) electron masses. A short discussion of the results and the conclusions are given in Sec. VI.

II. NEGATIVE MASS INSTABILITY IN UNDULATORS

Negative mass instability in beams of oscillating charged particles interacting through the Coulomb field is very well known for cyclic accelerators [24,25] (classical cyclotron NMI) and gyrotrons [26]. However, a similar effect [17–19] is much less known for undulators.

In the cyclotron case, electrons rotate along the Larmor circles in a uniform magnetic field B_0 and, being mutually repulsed due to their quasistatic Coulomb interaction, experience effective phase (azimuthal) attraction to the area of increased density. This effect occurs due to the so-called nonisochronity of electron rotation caused by a specific relativistic dependence, $\frac{\partial \Omega_c}{\partial \gamma} < 0$, of the electron cyclotron frequency $\Omega_c = eB_0/mc\gamma$ on the particle energy $E = mc^2\gamma$. The corresponding decrease/increase in the cyclotron frequency with the increase/decrease in the electron energy, caused by an additional acceleration of particles in the Coulomb field of an electron density fluctuation, leads to the attraction of the closest electrons to the fluctuation and growth of the latter.

The NMI in undulators is possible in the presence of a strong uniform guiding field [17–23]. For example, it can be developed in the combined helical undulator and uniform guiding fields. In such a configuration, the electrons can move along stationary helical trajectories with the normalized transverse velocity

$$\beta_\perp = \frac{K}{\gamma|\Delta|}. \tag{1}$$

Here, $K = eB_u\lambda_u/2\pi mc^2$ is the undulator parameter for the helical magnetic field with the amplitude B_u, $\Delta = 1 - \Omega_c/\Omega_u$ is the relative mismatch of the resonance between the forced undulator and free cyclotron particle oscillations, $\Omega_u = 2\pi c\beta_\parallel/\lambda_u$ and λ_u are the undulator frequency and period, and $\beta_\parallel$ is the longitudinal electron velocity normalized to the speed of light c. Within the ultrarelativistic approach, the nonisochronity of longitudinal particle oscillations is described by the following simplified expression for the derivative [18,22]:

$$\frac{\partial \beta_\parallel}{\partial \gamma} = \frac{1}{\gamma^3}\left(1 + \frac{K^2}{\Delta^3}\right). \tag{2}$$

The longitudinal NMI in undulators develops if this value is negative, $\partial\beta_\parallel/\partial\gamma < 0$, which takes place when the following conditions are fulfilled:

$$\Delta < 0, \qquad |\Delta|^3 < K^2. \tag{3}$$

Thus, the cyclotron frequency should be larger than the undulator frequency, $\Omega_c > \Omega_u$ (over-resonance regime), and close to it. The anomalous increase in the longitudinal particle velocity with decreasing energy is caused by the faster resonant decrease in transverse velocity.

III. TERAHERTZ SOURCE AND WB3D NUMERICAL CODE

In this paper, we study the evolution and coherent spontaneous THz radiation of single and modulated dense bunches moving in the combined undulator and guiding magnetic fields inside a cylindrical metal waveguide (Fig. 1). The electron bunches driving such a THz source are assumed to be formed in a laser-driven photoinjector.

The main parameters of the studied system (Table I) are the same as in [23] and are based on the configuration of the Israeli THz source (ITS) developed at the Ariel University [11]. Unlike the original ITS project, a planar undulator is

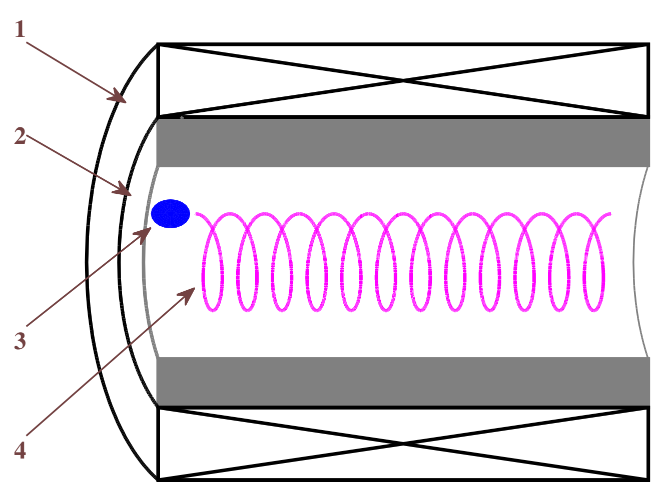


FIG. 1. Scheme of the radiation section: 1—magnetic system creating a transverse helical undulator and strong uniform longitudinal fields; 2—oversized metal circular waveguide; 3—electron bunch; 4—helical stationary electron trajectory.

TABLE I. Basic parameters of the simulated THz source.

Accelerator	
Type	Photoinjector rf linac
Electron kinetic energy	6 MeV ($\gamma \approx 12.7$)
Bunch radius, R_b (rms)	0.5 mm
Waveguide	
Type	Circular cylindrical, metal
Radius	5 mm
Dominant modes at the fundamental undulator harmonic	TE_{11}, TE_{12}, TE_{13}, TM_{11}, TM_{12}
Helical undulator	
Period, λ_u	25 mm
Number of periods, N_u	40–60
Field amplitude, B_u	2 kG
Undulator constant, K_u	0.47
Uniform guiding field, B_0	Up to 75 kG

replaced by a combination of a helical undulator and a focusing solenoid with a strong uniform guiding field, and a circular cylindrical waveguide is used instead of the original rectangular waveguide. A strong over-resonance guiding field of up to $B_0 = 75$ kG enables efficient emission of high-power THz radiation in the NMI regime for given parameters of the electron bunch and undulator field. The operating cylindrical waveguide with a very large (oversized) cross section provides small Brillouin angles for radiated waveguide modes and, correspondingly, relatively high frequencies f. Along with the short and much less charged bunches planned at the first step of the original ITS, relatively long (in comparison with the radiated wavelengths) premodulated and nonmodulated electron bunches with high charges are also considered in this paper.

Injection of a dense bunch into a stationary helical trajectory with small amplitudes of parasitic cyclotron oscillations of electrons can be provided, for example, if an initially converging bunch enters first a solenoid with an operating value of the guiding field and after that propagates along a section with adiabatically increasing undulator field. Methods of the formation and injection of very dense bunches will be discussed in a separate paper.

Numerical simulations of the bunch evolution and THz radiation are carried out within the framework of the space-frequency approach [27] which has already been successfully applied for the simulation of free-electron lasers of various types. In this method, the total rf field in the operating waveguide is considered in the positive-frequency Fourier domain by expansion of the monochromatic field components in terms of transverse eigenmodes of the waveguide. The electron bunch is represented by an ensemble of "macroparticles" interacting with the rf field and each other through a free-space Lorenz-transformed Coulomb field of the rest of the particles. Mutual interaction of the macroparticles simulates space-charge forces in the electron beam. The model neglects the action of the waveguide walls on the space-charge field affecting the particles moving near the waveguide axis. A self-consistent system of equations describes the particle motion and evolution of the radiation along the interaction region. This fairly universal 3D approach realized in the numerical code WB3D is used in the present work for analysis of THz coherent spontaneous undulator radiation (a sort of the so-called undulator superradiance) produced in a metal waveguide by dense electron bunches. The code enables a self-consistent consideration of multimode radiation of particles occurring simultaneously at various undulator harmonics and in a very broad frequency band.

IV. RADIATION OF POINTLIKE CHARGES AND SHORT BUNCHES

Let us discuss the features of the undulator radiation in a waveguide under conditions of both positive (PM) and negative (NM) masses of the oscillating electrons, starting from two simplest cases.

Radiation of a single pointlike charge.—A charge moving along a stationary helical trajectory in the combined helical undulator and uniform guiding magnetic fields inside an oversized waveguide simultaneously radiates at many waveguide modes and at various undulator harmonics. Therefore, its radiation spectrum has a very complex structure. The center frequency of radiation at a definite TE_{mp} or TM_{mp} mode and at the sth undulator harmonic satisfies the usual resonance condition

$$\omega - k_\parallel v_\parallel = s\Omega_u. \tag{4}$$

Here, ω and $k_\parallel$ are the cyclic frequency and the longitudinal wave number. The radiation frequency $f = \omega/2\pi$ coincides with the frequency of the free-space radiation emitted at an angle φ equal to the Brillouin angle of the waveguide mode. The frequency for each radiated waveguide mode is found from an intersection or tangency of the hyperbolic mode dispersion characteristic $k_\parallel(\omega)$ with the resonance line (4). In the first case, there are two intersections, which correspond to low and high frequencies of exact resonance. In the second case, there is only one frequency of the exact resonance and a broad frequency band where the resonance condition is approximately satisfied (grazing regime). The frequency emitted by an ultrarelativistic particle at the sth harmonic in the near-axis forward direction $\varphi \sim \gamma^{-1}$ can be represented in the form

$$f \approx \frac{2s\gamma^2}{1 + (\gamma\beta_\perp)^2 + (\gamma\varphi)^2} \frac{c\beta_\parallel}{\lambda_u}. \tag{5}$$

To provide a high radiation energy of the THz source under consideration, the parameters of the particles and fields (Table I) were chosen in such a way that for the over-resonance guiding field $B_0 = 75$ kG of the positive field direction ($\Omega_c > 0$) the normalized transverse particle velocity $\beta_{\perp 0} = 1.2\gamma_0^{-1}$ is much larger than the velocity $\beta_{\perp 0} = 0.47\gamma_0^{-1}$ in the absence of the guiding field, $B_0 = 0$. Thus, the longitudinal particle velocity and Doppler frequency upshift are essentially smaller in such a regime in comparison with those for a zero guiding field. Due to this, the radiation frequency at the fundamental harmonic $s = 1$ is reduced approximately twofold, down to $f \sim 1.5$ THz.

For the chosen parameters, the radius $r = Kc/\gamma_0(\Omega_c - \Omega_u) = 0.37$ mm of a stationary helical trajectory is small compared with the waveguide radius. In the further analysis, the particles are also supposed to move close to the waveguide axis. It is important to note that due to symmetry, a particle moving along a helical trajectory with the axis coinciding with the axis of a circular waveguide radiates only modes whose azimuthal indices are equal to the number of resonant undulator harmonics, $m = s$. The same strong selection rule significantly rarefying the radiation spectrum is also approximately applicable for the particles whose guiding centers are located close to the waveguide axis.

A particle emits short pulses of electromagnetic waves propagating along the waveguide with different group velocities. The longest efficient interaction between the charge and radiated waves takes place for modes whose group velocities are closest to the longitudinal velocity of the particle. In the dispersion diagram, the parameters of such modes are close to the grazing of the mode dispersion characteristics and the line of the undulator resonance (4), for which $\varphi = \sqrt{\gamma^{-2} + \beta_\perp^{-2}}$ (in the reference frame moving with the longitudinal velocity of the particle, these modes are close to cutoff). The preferential excitation of modes close to the grazing regime was discovered theoretically and confirmed in experiments devoted to the cyclotron superradiance of extended electron bunches, whose lengths are much larger than the characteristic wavelengths of the radiation, but significantly smaller than the undulator length [28].

According to the simulations for a pointlike charge, the dominant high-frequency TE_{11}, TE_{12}, TE_{13}, TM_{11}, and TM_{12} modes and the weaker low-frequency TE_{11}, TE_{12}, and TE_{13} modes are mainly excited at the fundamental ($s = 1$) undulator harmonic in the chosen oversized waveguide. All these modes have azimuthal indices equal to 1, and the TE_{13} mode, which has the lowest group velocity, is excited with the maximum energy. Because of the relatively large transverse velocity of the pointlike particle, a larger energy is simultaneously radiated at modes which are resonant with high harmonics $s > 1$: the corresponding maxima are achieved at the TE_{26} and TE_{39} modes at the second and third harmonics, respectively, and so on. In general, the radiation spectrum of a single pointlike particle is very complicated even in the case of a relatively small charge [Fig. 2(a), the charge 1 pC]. With a high charge, the emitting particle slows down rapidly due to radiation losses, which leads to a notable widening of the spectrum [Fig. 2(b), the charge 10 pC]. This example demonstrates that it is difficult to produce narrow-band radiation by using too compact electron bunches.

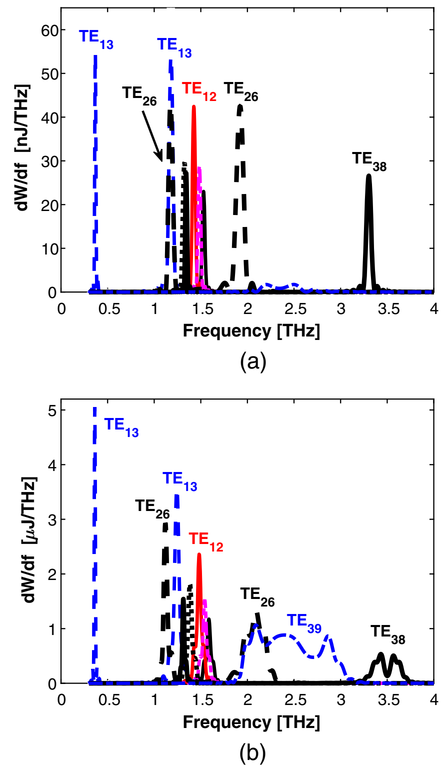


FIG. 2. Radiation spectra of the dominant modes radiated by a pointlike charge of 1 pC (a) and 10 pC (b) in a helical undulator with over-resonance guiding field $B_0 = 75$ kG and oversized waveguide at the first three undulator harmonics after $N_u = 40$ undulator periods.

Due to the anomalous properties of single particle's oscillations in the over-resonance guiding fields (NM regime) loss of the particle energy under the action of the radiative force can be accompanied by a decrease in the transverse and an increase in the axial components of its velocity (compare with the discussed case of space-charge forces in electron bunches). Correspondingly, the radiation frequency will increase during the particle propagation. Conversely, in the positive mass regime, the loss of the particle energy due to radiation leads to a decrease in the average longitudinal particle velocity and, therefore, a decrease in the radiation frequency. In this case, a particle approaches the resonance, and then the amplitudes of its transverse and longitudinal oscillations can be very large. The difference between PM and NM regimes is even more drastic if large charges and long undulators are considered and the particle radiates a considerable part of its initial energy. For example, a charge of the order of 1 nC moving in an under-resonance guiding field of 33 kG (PM) approaches the resonance very rapidly which leads to huge transverse and longitudinal oscillations of the charge resulting in a high radiation energy and almost continuous radiation spectra strongly shifted towards the lower frequencies. However, the same charge moving with the same initial transverse velocity in a symmetric over-resonance guiding field of 75 kG (NM) goes out of resonance; the amplitude of its oscillations decreases, but the radiation energy at high frequencies is much larger than in the previous case.

Radiation of a single short electron bunch.—Due to a partial coherent summation of the waves emitted by various particles composing a distributed bunch, the radiation spectrum is significantly different from that of a pointlike particle. We now consider a 1 nC electron bunch in the form of a cylinder with the radius of $R_b = 0.5$ mm and pulse duration of $T_b = 0.3$ ps. If the particles of the bunch are injected into nearly axial stationary trajectories in the combined magnetic fields with the over-resonance axial component $B_0 = 75$ kG, then the chosen bunch length 0.09 mm is approximately equal to half wavelength at a frequency of 1.54 THz radiated at the fundamental undulator harmonic $s = 1$. Thus, the bunch is initially rather compact for radiation at the fundamental harmonic, but is too long to emit coherently at the higher harmonics.

Because of the large electron density in the bunch the mutual Coulomb repulsion of particles is very strong. In the case of normal positive-mass regime (under-resonance or oppositely directed guiding field), this leads to a fast spreading of the bunch in the longitudinal direction, so that the radiation emitted by different parts of the bunch adds incoherently, resulting in a low radiation energy at high frequencies (Fig. 3). Using simple electrostatic formulas for the uniformly accelerated particle motion we evaluate the characteristic time of the bunch length doubling in the combined undulator and uniform guiding fields:

$$\omega_p' t \sim 1. \tag{6}$$

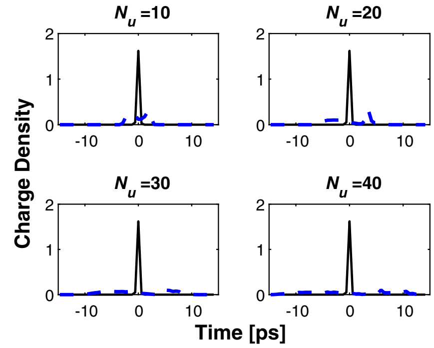
Here,

$$\omega_p' = \sqrt{k_r}\omega_p, \qquad \omega_p = (4\pi e n_0 / m\gamma_\parallel^3)^{1/2} \tag{7}$$

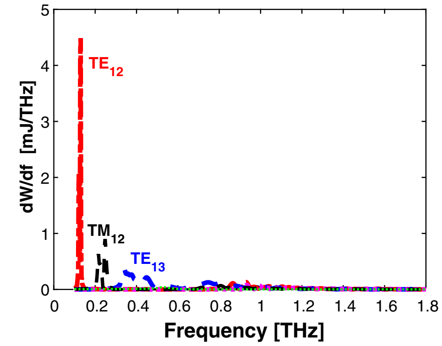
is the longitudinal relativistic plasma frequency with the reduction coefficient k_r for space charge forces; $\gamma_\parallel = 1/\sqrt{\gamma^{-2} + \beta_\perp^{-2}}$ is the longitudinal electron Lorentz factor. The reduction can be estimated by presenting a cylindrical bunch as a set of thin disks with coordinates x_i and surface density σ_i and using a formula for the electric field of a disk at the point x_j on its axis:

$$E_i(x_i, x_j) = 2\pi\sigma_i k_r, \tag{8}$$

where the reduction coefficient



(a)



(b)

FIG. 3. Positive mass regime (under-resonance guiding field of 33 kG) for a single bunch with a charge of 1 nC: (a) initial and current charge density (the time is given relative to the bunch center; the particle motion to the tail of the distribution shifts the peak from the center of mass) and (b) radiation spectra of the dominant waveguide modes after $N_u = 40$ undulator periods.

$$k_r = 1 - \frac{1}{\sqrt{1 + \varrho^2}}, \qquad \varrho = \frac{R_b}{\gamma |x_j - x_i|} \tag{9}$$

describes the transition from a strong mutual repulsion of close quasiplanar electron disks ($\varrho \gg 1$) to a weak repulsion of remote point charges ($\varrho \ll 1$). Taking also into account the bunch density decrease caused by an increase in the effective bunch radius due to the excitation of undulator oscillations, we find that for the given parameters the characteristic time is $t \sim 0.2$ ns. This means fast spreading in the very first undulator periods, in close correspondence to the results of numerical simulations with WB3D code (Fig. 3). In this case, the radiation energy is low even for high-frequency modes radiated at the fundamental harmonic.

Under conditions of the NMI (over-resonance guiding field), effective longitudinal attraction of particles prevents the electron bunch spreading. Contrary to the normal positive-mass regime, a considerable part of the bunch is compressed into a short and dense long-lived core, which can coherently radiate during propagation in a relatively long undulator (Fig. 4). For charges from 0.5 to 1.5 nC, the characteristic duration of a short dense electron bunch is preserved or becomes even shorter. The destructive

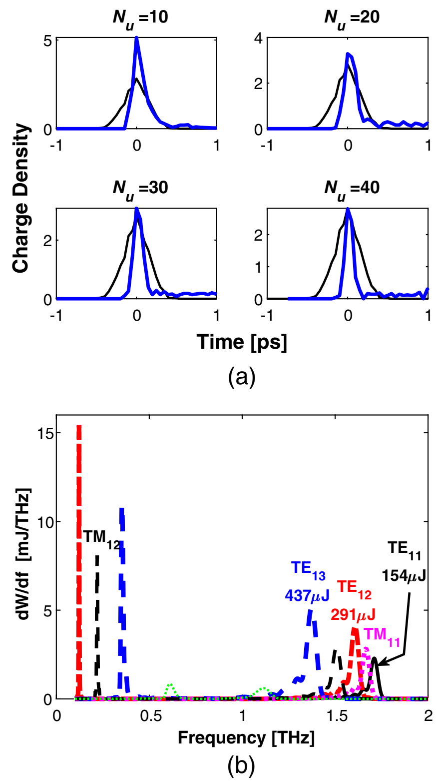


FIG. 4. Negative mass regime (over-resonance guiding field of 75 kG) for a single bunch with a charge of 1 nC: (a) initial and current charge density (the time is given relative to the bunch center; the particle motion to the tail of the distribution shifts the peak from the center of mass) and (b) spectra of the dominant waveguide modes radiated at the fundamental undulator harmonic after $N_u = 40$ undulator periods; the radiation energy is given for high-frequency modes only.

space-charge force increases rapidly with the bunch charge, breaking the balance between the repulsive and attractive forces and reducing the NMI for charges above 2.5 nC. During the bunch propagation, the main peak of the charge density is reduced and broadened, while a considerable part of the particles leaves the central peak and forms a widely extended "tail" after it.

The formation of a long-lived dense core explains a much higher efficiency of high-frequency radiation for the NMI regime with the over-resonance guiding field $B_0 = 75$ kG (Fig. 4) than that for the under-resonance field $B_0 = 33$ kG (Fig. 3). Note that the radiation at high harmonics is negligibly low for the given parameters.

V. EVOLUTION AND RADIATION OF LONG PREMODULATED AND NONMODULATED BUNCHES

A detailed theoretical and experimental study of the properties of the moving thin premodulated bunches of electrons with normal positive masses under the action of the own space charge only (without undulator fields) was done, in particular, in [15] (the evolution of thick quasi-1D modulated bunches was studied theoretically in [16]). Simulations [15] based on the general particle tracer (GPT) code [29] demonstrate that the initial "static" modulation of density in cylindrical bunches with the radius R_b much smaller than the modulation period d' in the own reference frame S' ("threads" in S') leads to the excitation of a plasma wave. In the laboratory reference frame S the corresponding condition for the bunch shape is

$$R_b \ll \gamma d, \tag{10}$$

where γ and d are the Lorentz factor of electrons and the period of modulation in S, respectively. In accordance with Eq. (9), condition (10) means that interaction of remote electron disks is very weak due to the effective cutoff of the longitudinal electric field from the far-away slices even within a single period, $|x_j - x_i| \leq d$. Different periods are almost independent, which leads to the excitation of longitudinal plasma waves. In this process, the density modulation disappears and reappears at a quarter and at a half plasma wavelength, respectively, being converted into velocity modulation and emerging from it after that. We first consider this process qualitatively for an electron bunch of a finite length propagating along the z axis with the harmonic initial density modulation,

$$\rho = \rho_0[1 + m\cos(hz_0)] \tag{11}$$

[Fig. 5(a)]. Due to a strong Coulomb repulsion, each peak of the density modulation splits into two parts moving in opposite directions, which gives rise to the excitation of a plasma wave. In the first quarter of the plasma oscillation, the inner parts of the bunch fill the "valleys" of the initially reduced density, thereby leveling the concentration [Fig. 5(b)]. In the second quarter, they continue

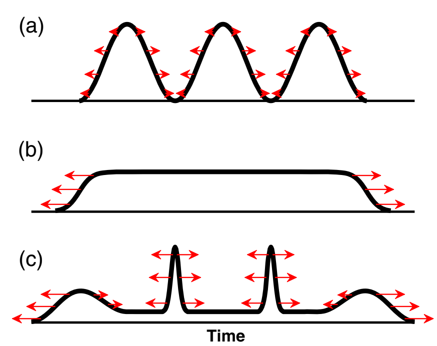


FIG. 5. Schematic illustration of plasma oscillations in a thin bunch with unity modulation: (a) the maxima split into two repulsed parts (the upper picture), (b) in the first quarter of plasma oscillation the inner parts of the maxima fill the valleys of reduced density and equalize concentration, (c) in the second quarter of plasma oscillation, the regions with high density continue to move and create new maxima at the sites of the initial minima.

their movement and create new areas of increased density with maxima at the sites of the initial density minima [Fig. 5(c)]. The edge parts of the bunch moving from the bunch center spread out and move away from the main part of the bunch. It is easy to see that the number of large maxima in the train decreases by one. In the third quarter of plasma oscillations, each new maximum is divided into two parts, which fill the new valleys, and so on. Thus, the modulation flickers in time and degrades because of the escape of the edge particles. Since the magnetic self-field of the bunch does not completely compensate for its radial electric self-field, some guiding magnetic field should be applied to avoid a transverse expansion of the bunch and provide the maintaining of its width. For a sufficiently large amplitude of density modulation, $m \gtrsim 0.5$, the excitation of the plasma wave is an essentially nonlinear process that can lead to the appearance of sharp density spikes [15].

To calculate the characteristic lengths of disappearance and reappearance of the density modulation in a thin cylindrical 3D electron bunch, the authors of [15] proposed, in accordance with GPT simulations, the following formula for the reduced plasma frequency of the bunch moving in free space and a weak guiding magnetic field:

$$\omega_{p_{3\mathrm{D}}} = \frac{2D}{1+2D}\omega_p, \tag{12}$$

where the parameter $D = 2\pi R_b/\gamma d$ is relatively small [compare with Eq. (10)]. Then the first disappearance of the modulation occurs at the distance

$$L_{\mathrm{dis}} = \frac{\pi}{2}\frac{c}{\omega_{p_{3\mathrm{D}}}} \tag{13}$$

and the first reappearance of modulation is observed at a double length.

A natural generalization of these results and expressions for the case of bunches moving along quasistationary trajectories in the combined helical undulator and uniform magnetic fields under conditions of normal positive masses of electron oscillators can be obtained by replacing the Lorentz factor γ by the corresponding longitudinal value: $\gamma_\parallel = 1/\sqrt{\gamma^{-2} + \beta_\perp^{-2}}$. Here, $\beta_\perp$ is the normalized particle velocity determined by Eq. (1). In addition, it is necessary to consider changes in the bunch radius and, correspondingly, changes in the average density due to the excitation of the undulator oscillations of particles. Numerical simulations using WB3D code basically confirm the flickering character of the density modulation, as well as the correctness of the above simplified model for estimations in the absence of radiation. However, strong radiation of particles in the undulator can radically change the bunch evolution.

We now discuss the case with the bunch modulated in the combined magnetic field under conditions of NMI (3). In this case, it is expedient to use relatively thin bunches, whose radii fulfill condition (10), while mutual interaction is basically important for the nearest particles only. Then the density maxima areas accelerate particles before and decelerate the latter behind them. Due to an increase/decrease in transverse velocities and the corresponding decrease/increase in longitudinal velocities, both groups of particles will be attracted to the nearest maxima, making the latter sharper up to the NMI saturation. Interaction between maxima is relatively weak and, as a result, fairly sharp density peaks do not disappear and can propagate almost independently to a large extent. Then, unlike the flickering modulation for positive masses, one will observe “frozen” modulation under conditions of NMI. Such nearly periodic and long-lived trains of dense narrow bunches can obviously be used for the enhancement of radiation energy and a more selective excitation of waveguide modes.

A deterministic distribution of macroparticles corresponding to periodic initial density modulation (11) with an integer number of periods $N_m = f_m T_b$ was used in WB3D simulations for the modulated bunch evolution and radiation. The modulation frequencies f_m chosen in simulations are close to the maxima of radiation energy for various modes emitted by a single charge. For a unity modulation ($m = 1$), a thin cylindrical bunch looks like a train of closely located but weakly dependent pulses. Special tests showed that the correct simulation of space-charge effects in bunches with charges under consideration requires that at least $10^3 N_m$ macroparticles be involved in the simulation. Due to the development of NMI, sharp cores are rapidly formed from the maxima of each pulse [Fig. 6(a)], which hold their shape and quasiperiodic distribution through most of the beam line. The so-called bunching factor

$$b(f_s, z) = |\langle e^{i2\pi f_s t_l(z)}\rangle| \cdot 100\% \tag{14}$$

which reflects the coherent properties of an ensemble of particles at the frequency f_s demonstrates a long valley of highly efficient emission for such a train [Fig. 6(b)]. Here, $t_l(z)$ is the time of arrival of a macroparticle l at the point z.

For 2.5 nC bunches of $N_m = 5$ modulation periods at a modulation frequency of 1.4 THz (Fig. 6), the radiation energy emitted at the strongly dominant TE_{13} mode after 40 undulator periods is as high as 1.6 mJ, which corresponds to the radiation efficiency of 0.11 (in this case, the total radiation energy and efficiency are 2.9 mJ and 0.19, respectively). The radiation efficiency is defined here as a ratio between an energy of the emitted radiation and the initial beam energy. The density distribution keeps a quasiperiodic shape with sharp spikes and high bunching parameter even after 40 undulator periods, both at the modulation frequency and its high harmonics, suggesting the use of a longer undulator for the further beam energy extraction.

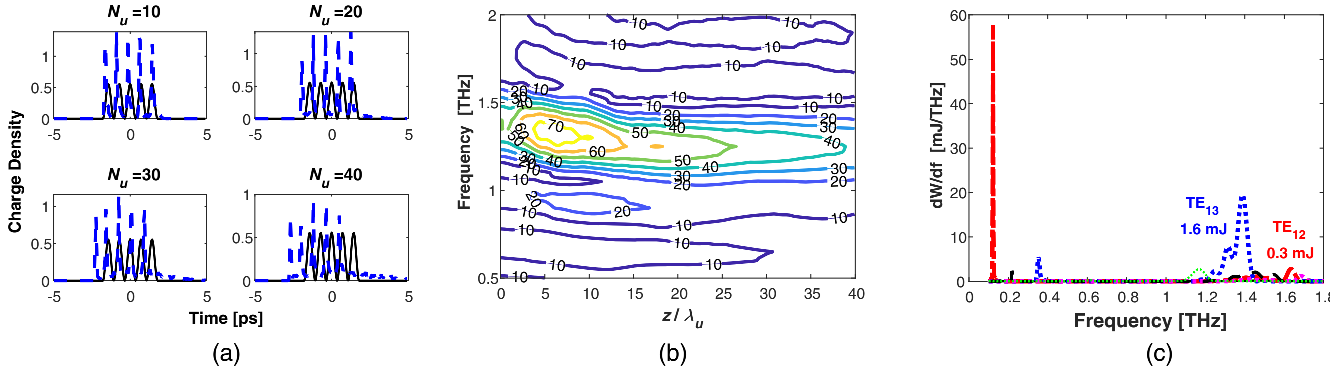


FIG. 6. Bunch with a charge of 2.5 nC and five periods of unity modulation at a frequency of 1.4 THz in NMI regime (over-resonance guiding field of 75 kG): (a) charge density after 10 to 40 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 40 undulator periods.

Fast spreading, a much smaller bunching factor, and a much lower radiation energy were found for the same modulated bunch in the positive mass regime taking place with a symmetric magnetic field of 33 kG (Fig. 7). As was already mentioned, radiation in the undulator changes the character of the bunch evolution in this case: the modulation does not disappear and the particles are bunched at first by a strong radiated wave in sharp dense subpulses, but their number and sharpness quickly decrease.

An increase in the number of modulation periods with the charge per period retained leads to a better mode selectivity and a higher radiation energy efficiency (Figs. 8 and 9). Radiation energy with the dominant TE_{13} mode and 40 undulator periods is 4.1 and 6.2 mJ for 10 and 15 modulation periods, respectively, which corresponds to an efficiency of 0.14. Significant mode selectivity of the dominant radiation and a radiation frequency shift were obtained by varying the bunch modulation frequency f_m.

The radiation efficiency and mode selectivity remain high even with a considerable decrease in the initial modulation factor (Fig. 10). To avoid the effects caused by sharp jumps of the electron density, the density distribution was smoothed out enough at the ends of an electron pulse. The radiation energy and efficiency for the TE_{13} mode and 40 undulator periods are fairly high for the modulation index $m = 0.5$: 2.4 mJ and 0.08, respectively (the total radiation energy and efficiency are 3.8 mJ and 0.13).

As is well known, the radiation is possible and can be fairly efficient even in the case of the complete absence of preliminary modulation, $m = 0$, in relatively long (extended) bunches due to efficient self-modulation and bunching of particles in the combination wave formed by the undulator field and the field of radiated waves. This version of superradiance is successfully used for production of very powerful microwave generation from nanosecond electron beams (see, e.g., [13,28,30] and the literature therein). Such a mechanism of superradiance may be also promising for sources of undulator terahertz radiation utilizing picosecond electron bunches from photoinjectors [13]. According to our simulations, the use of NMI stabilization and related sharpening of the density distribution in an extended bunch leads to an even higher efficiency of superradiance. In this case, almost the same radiation

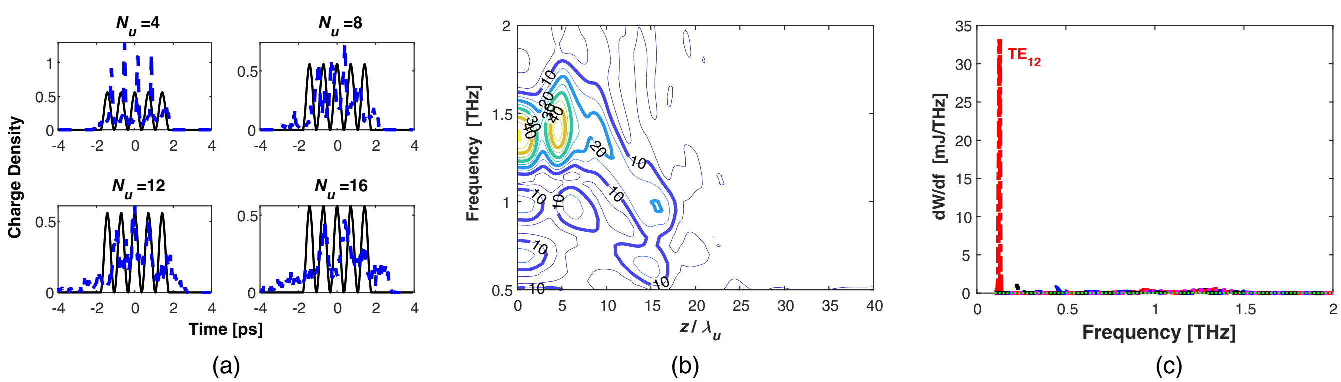


FIG. 7. Bunch with a charge of 2.5 nC and five periods of unity modulation at a frequency of 1.4 THz in the positive mass regime (under-resonance guiding field of 33 kG): (a) charge density after 10 to 40 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 40 undulator periods.

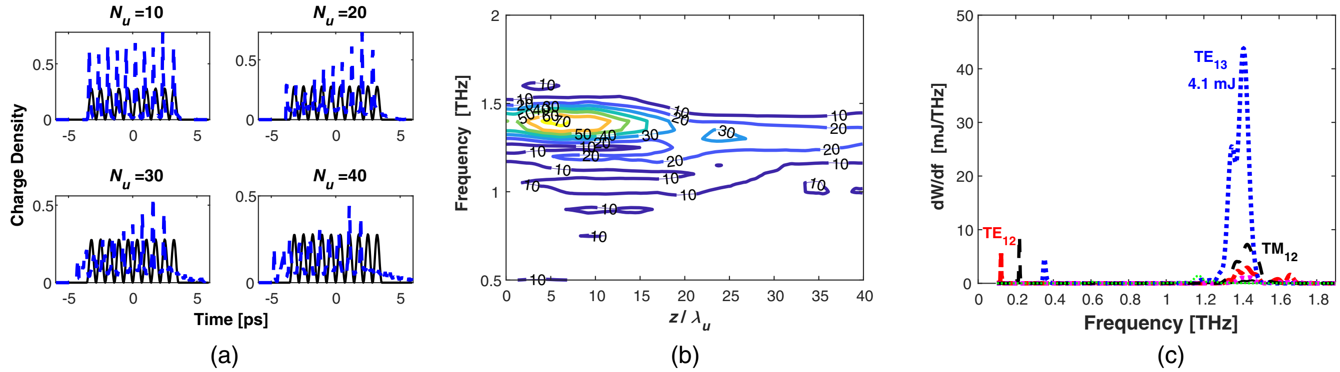


FIG. 8. Bunch with a charge of 5 nC and ten periods of unity modulation at a frequency of 1.4 THz in NMI regime (over-resonance guiding field of 75 kG): (a) charge density after 10 to 40 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 40 undulator periods.

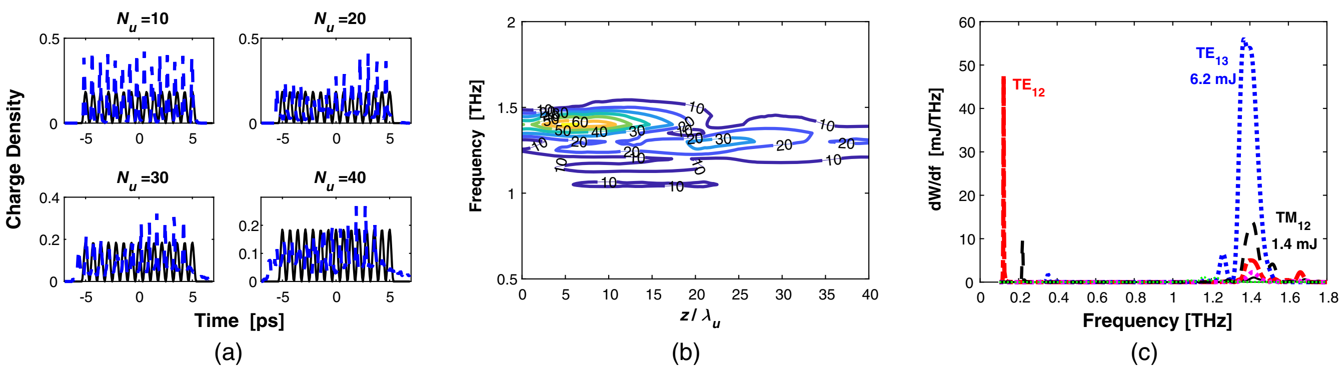


FIG. 9. Bunch with a charge of 7.5 nC and 15 periods of unity modulation at a frequency of 1.4 THz in NMI regime (over-resonance guiding field of 75 kG): (a) charge density after 10 to 40 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 40 undulator periods.

energy and efficiency can be achieved for the dominant TE_{13} mode as in the case of half modulation, $m = 0.5$, but with even higher mode selectivity (Fig. 11); the frequency of the self-modulation is close to 1.4 THz. The efficient electron bunching and radiation are provided due to the closeness of the group velocity of the spontaneously emerged dominant radiated wave to the longitudinal particle velocity. Under such conditions, the particles move for a long time in a strong field of the radiated electromagnetic pulse and are bunched by the combination wave formed by the undulator field and the radiated wave as in free-electron lasers. Comparison of the evolution and superradiance for initially

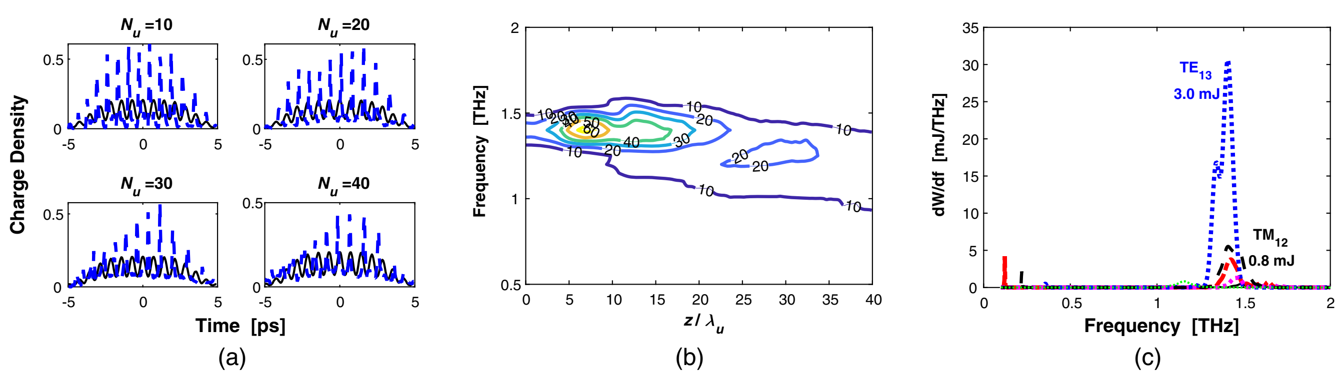


FIG. 10. Bunch with a charge of 5 nC and ten periods of half modulation at a frequency of 1.4 THz and smoothed edges in NMI regime (over-resonance guiding field of 75 kG): (a) charge density after 10 to 40 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 40 undulator periods.

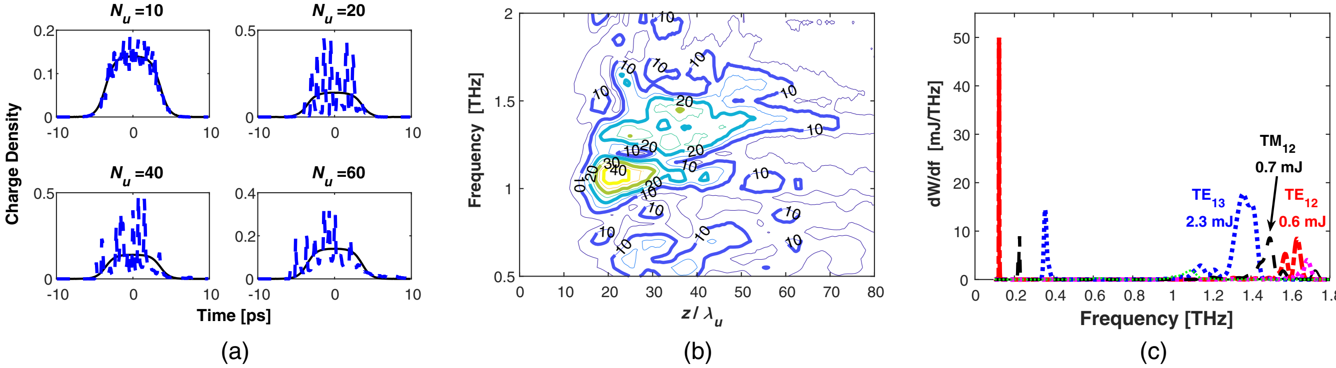


FIG. 11. Nonmodulated bunch with a charge of 5 nC, a duration of 10.6 ps, and smoothed edges in NMI regime (over-resonance guiding field of 75 kG): (a) charge density after 10 to 60 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 60 undulator periods.

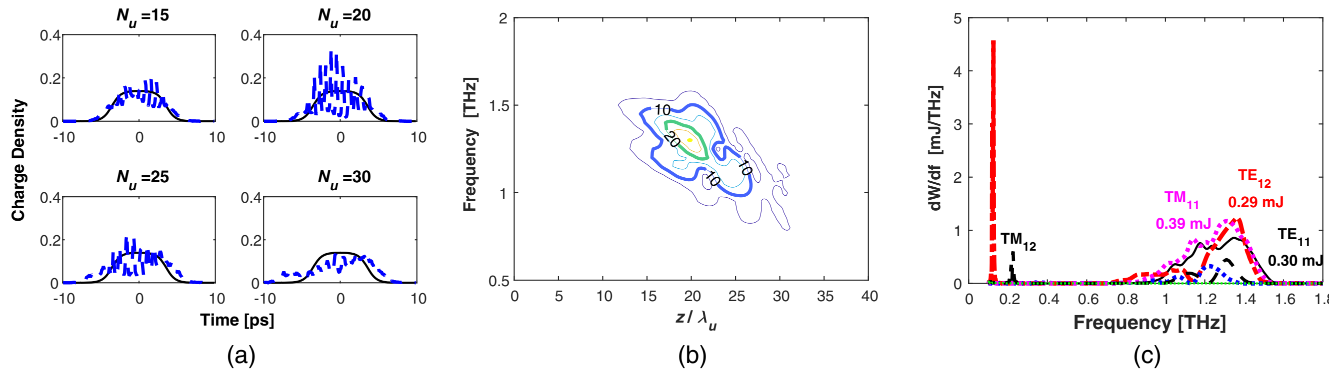


FIG. 12. Nonmodulated bunch with a charge of 5 nC, a duration of 7.1 ps, and smoothed edges in the positive mass regime (under-resonance guiding field of 33 kG): (a) charge density after 15 to 30 undulator periods, (b) contours of the bunching factor, and (c) radiation spectrum after 40 undulator periods.

the same extended nonmodulated bunches moving with the same initial components of the particle velocity at symmetric values of the guiding field 33 kG (Fig. 12) and 75 kG (Fig. 11) demonstrates an appreciable advantage of the negative mass regime.

VI. CONCLUSIONS

The use of the anomalous properties of the electron oscillators propagating in the combined helical undulator and over-resonance uniform guiding magnetic fields under conditions of negative mass instability enables the formation of relatively long-lived quasiperiodic trains of electron bunches. Such trains can have multi-nC charges and efficiently radiate multi-mJ, narrow-band terahertz electromagnetic pulses with duration of a few tens of picoseconds and easy wideband frequency tuning. Efficient and fairly selective terahertz radiation is also possible from bunches with a small modulation amplitude and even from extended nonmodulated bunches with smoothed ends both for positive and negative masses of particles. The latter is possible due to the particle self-modulation and bunching during the development of a superradiant instability [13,28,30], which is especially efficient for the waveguide modes whose group velocities are close to the longitudinal particle velocities [28]. Under such conditions, the particles move for a long time in a strong field of the radiated electromagnetic pulse and are bunched by the combination wave formed by the undulator field and radiated wave. According to our simulations, NMI stabilization and sharpening of the density distribution enable much more efficient terahertz radiation from dense extended electron bunches than in the positive mass regime. The implementation of high-power THz sources utilizing radiation from dense pre-modulated or self-modulated electron bunches can be promising for many applications, including various versions of pump-probe experiments.

ACKNOWLEDGMENTS

This work was supported by the Ministry of Science, Technology and Space, Israel, and by Russian Foundation for Basic Research, Project No. 16-02-00794.

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