Anomalous geometric spin Hall effect of light

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The geometric spin Hall effect of light (GSHEL), similar to the spin Hall effect of light, is also a spin-dependent shift in the centroid of light intensity (energy flux), but it is a purely geometric effect and independent of the light-matter interaction. In this paper, we discuss the GSHEL with respect to momentum flux instead of energy flux. Interestingly, for the symmetric energy-momentum (E-M) tensor, its centroid shift of momentum flux is double that of the energy flux; however, for the canonical E-M tensor, its centroid shift of momentum flux agrees with that of the energy flux. Furthermore, when considering the effect produced by orbital angular momentum, for these two conventional E-M tensors the centroid shifts of the momentum fluxes are twice those of the energy fluxes. To tell which E-M tensor of the electromagnetic field would be more "correct," we propose a possible experimental scheme to test the GSHEL of momentum flux through the mechanical effect of light.

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I. INTRODUCTION AND KEYNOTE

Light can carry energy, momentum, and angular momentum, which play important roles in the light-matter interaction. Nowadays, many techniques have been realized to manipulate microscopic particles by using the linear and angular momentum of light, such as laser cooling, optical tweezers, and optical spanners [1–4]. As reciprocal influences of the matter upon the light, the spin Hall effect of light (SHEL) has attracted a considerable amount of theoretical and experimental investigations. It is a novel phenomenon of spin-dependent centroid displacement of light intensity. In fact, the light reflected or refracted from an optical interface can yield two polarization-dependent shifts in the intensity centroid, namely the longitudinal Goos-Hänchen shift and the transverse Fedorov-Imbert shift [5–8] (for a reference, see Ref. [9]). The latter is regarded as an example of the SHEL, which now is usually interpreted as the spin-orbital interaction of light in terms of the Berry phase [10].

In 2009, another type of SHEL, named geometric SHEL (GSHEL), was proposed [11]. This effect says that a spin-dependent transverse displacement of the centroid of light intensity is observed in a plane not perpendicular to the propagation direction of the light beam. It originates from the nonzero transverse angular momentum observed in the detector frame. Unlike the conventional SHEL that requires the light-matter interaction, the GSHEL is of a purely geometric nature. Later on, the orbital angular momentum of light [12] was shown to cause a similar transverse shift in addition to the shift caused by spin [13]. In 2014, it was reported that the polarized light transmitted across an oblique polarizer [14] yields an intensity centroid displacement larger than that of the conventional SHEL, which was claimed as the observation of the GSHEL [15]. So far, the GSHEL or similar effects have been analyzed for collimated paraxial beams [16], tightly focused vector beams [17], and inhomogeneous polarized beams [18]. For the convenience of reference and comparison, we put here the centroid displacement of the GSHEL with respect to energy flux:

$$\langle y\rangle_P = \int y\,\overline{T}^{z0}_{\rm sym}dxdy \bigg/ \int \overline{T}^{z0}_{\rm sym}dxdy \simeq \frac{\lambda}{4\pi}\sigma\tan\theta. \quad (1)$$

Here $\langle y\rangle_P$ denotes the shift in the barycenter of light intensity and the subscript P indicates that the light intensity is evaluated with the Poynting vector (the energy flux $T^{i0}_{\rm sym}$). The overbar denotes time average. $\sigma = \pm 1$ is the polarization of light and λ the wavelength. θ is the tilted angle between the detector plane and the transverse plane of the light beam. Figure 1 is a schematic diagram of the light beam and the detection system. Strictly speaking, Eq. (1) receives correction for the angular spread of the beam, which we omit in the following discussion.

Light carries energy and its energy transportation is represented by energy flux. Likewise, light carries momentum and its momentum transportation is described by momentum flux. In fact, the energy-momentum (E-M) tensor of the light field, which bands together the energy density, the momentum density, the energy flux density, and the momentum flux density, is a very convenient tool to analyze the properties of light. In this paper, we discuss the GSHEL with respect to the momentum flux density and reach a result different from Eq. (1). To show the difference, for a beam with only spin polarization we get the centroid displacement of momentum flux $T^{zz}_{\rm sym}$:

$$\langle y_{\rm sym}\rangle_T = \int y\,\overline{T}^{zz}_{\rm sym}dxdy \bigg/ \int \overline{T}^{zz}_{\rm sym}dxdy = \frac{\lambda}{2\pi}\sigma\tan\theta. \quad (2)$$

This result differs from the conventional GSHEL with respect to energy flux by a factor of 2, so we call it the anomalous GSHEL. $T^{z0}_{\rm sym}$ in Eq. (1) and $T^{zz}_{\rm sym}$ in Eq. (2) correspond to the

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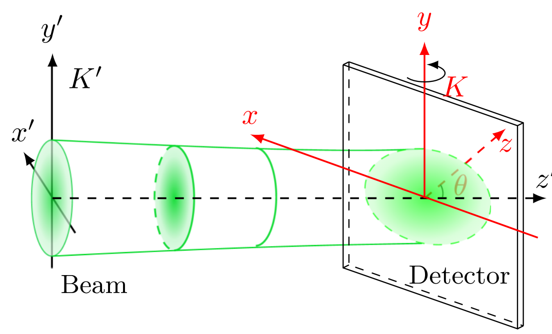

FIG. 1. The light beam propagates along the z' axis, around which the beam is rotationally symmetric. The x-y plane of the laboratory frame K is in the detection plane. The y axis is parallel to the y' axis of the beam frame K'. The z axis is tilted by an angle θ from the z' axis. If the light beam carries angular momentum along the z' direction, a transverse angular momentum J^x can be observed in the K frame.

z-direction energy flux density and the z-direction flux density of the z-direction momentum of the symmetric E-M tensor:

$$T^{\mu\nu}_{\rm sym} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \tfrac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}, \quad (3)$$

where $g^{\mu\nu}$ is the Minkowski metric with signature $(+---)$.

Interestingly, the expression of the E-M tensor is not uniquely determined by the energy and momentum conservation laws. Besides the symmetric E-M tensor, we also have the well-known canonical E-M tensor:

$$T^{\mu\nu}_{\rm can} = -F^{\mu\alpha}\partial^{\nu}A_{\alpha} + \tfrac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}. \quad (4)$$

When calculating the barycenter of momentum flux according to the canonical E-M tensor, we get the result

$$\langle y_{\rm can}\rangle_T = \int y\overline{T}^{zz}_{\rm can}dxdy \Big/ \int \overline{T}^{zz}_{\rm can}dxdy = \frac{\lambda}{4\pi}\sigma\tan\theta. \quad (5)$$

The prediction of the canonical version in Eq. (5) coincides with Eq. (1), but that of the symmetric version in Eq. (2) does not. For this reason, we argue that the GSHEL can be used as a strong criteria to single out a more valid version. In other words, either or both of them must be wrong.

Remarkably, the Poynting vector $(\boldsymbol{E}\times\boldsymbol{B})^i$ is not only the symmetric energy flux density $T^{i0}_{\rm sym}$ (and the symmetric momentum density $T^{0i}_{\rm sym}$) but also the canonical energy flux density $T^{i0}_{\rm can}$ in the radiation gauge. Therefore, Eq. (1) also holds for the canonical E-M tensor in the radiation gauge or the gauge-invariant version of the canonical E-M tensor [19–22]. Certainly, one cannot tell the difference between the two conventional forms of E-M tensor by the GSHEL with respect to energy flux. This is also why the Poynting vector is almost always effective as the energy flux density of a free electromagnetic field.

The E-M tensor and the angular momentum tensor are among the most fundamental quantities of an electromagnetic field. In the field-theory description of the free electromagnetic field, we start with the standard Lagrangian [23]

$$\mathcal{L} = -\tfrac{1}{4}F^{\alpha\beta}F_{\alpha\beta}. \quad (6)$$

From Noether's theorem, the conservation laws are associated with continuous symmetries of the Lagrangian. For the symmetry of space-time translations, applying Noether's theorem yields the canonical E-M tensor $T^{\mu\nu}_{\rm can}$ and the corresponding conservation law:

$$\partial_{\mu}T^{\mu\nu}_{\rm can} = 0, \quad (7)$$

with the conserved charge (the four-momentum P^{ν})

$$P^{\nu} = \int d^3xT^{0\nu}_{\rm can}. \quad (8)$$

The symmetry of space-time rotations generates the conserved canonical angular momentum tensor:

$$M^{\lambda\mu\nu}_{\rm can} = x^{\mu}T^{\lambda\nu}_{\rm can} - x^{\nu}T^{\lambda\mu}_{\rm can} + S^{\lambda\mu\nu} \equiv L^{\lambda\mu\nu} + S^{\lambda\mu\nu}, \quad (9)$$

with the corresponding conservation law

$$\partial_{\lambda}M^{\lambda\mu\nu}_{\rm can} = 0 \quad (10)$$

and the corresponding conserved charge

$$J^{\mu\nu} = \int d^3xM^{0\mu\nu}_{\rm can}. \quad (11)$$

Here $S^{\lambda\mu\nu} = A^{\mu}F^{\lambda\nu} - A^{\nu}F^{\lambda\mu}$ is the spin tensor. Thus, the canonical angular momentum tensor suggests a natural separation of the total angular momentum into the orbital and spin contributions [19–22].

However, the canonical E-M tensor is generally nonsymmetric and is unfavorable to the Einstein's gravitational theory which requires a symmetric E-M tensor as the source of the gravitational field [24]. In addition, the canonical tensors break the gauge symmetry because of their explicit dependence on the vector potential A_{α}, as shown in Eq. (4). In fact, without modifying both the corresponding conservation law and the conserved charges, the expressions of the E-M and angular momentum tensors are not unique. The symmetric E-M tensor can be constructed by adding a suitable total divergence to the canonical one, and such procedure is the so-called Belinfante's method [25]:

$$T^{\mu\nu}_{\rm sym} = T^{\mu\nu}_{\rm can} + \partial_{\alpha}B^{[\alpha\mu]\nu} = T^{\mu\nu}_{\rm can} + \partial_{\alpha}(F^{\mu\alpha}A^{\nu}), \quad (12)$$

where $B^{[\alpha\mu]\nu}$ can be constructed from the spin tensor: $B^{[\alpha\mu]\nu} = (S^{\mu\nu\alpha} + S^{\nu\mu\alpha} - S^{\alpha\nu\mu})/2$. The corresponding symmetric angular momentum tensor can be constructed from the symmetric E-M tensor:

$$M^{\lambda\mu\nu}_{\rm sym} = x^{\mu}T^{\lambda\nu}_{\rm sym} - x^{\nu}T^{\lambda\mu}_{\rm sym}. \quad (13)$$

Quite impressively, the symmetric version of the total angular momentum tensor is expressed in a fully orbital-like form. In other words, the symmetric angular momentum tensor does not discriminate the spin part from the orbital part. However, the spin angular momentum and the orbital angular momentum are largely treated as separated degrees of freedom in classical and quantum optics [19,26–31]: The spin angular momentum is related with the polarization of light, and the orbital angular momentum is associated with the wave-front shape of light. Additionally, optical experiments clearly show that they yield different physical effects [32–35].

The understanding of the E-M tensor determines the understanding of the angular momentum tensor, and vice versa. For the reasons mentioned above, it is not exactly clear how to handle such two different forms of E-M and angular momentum tenors. In this work, the GSHEL, an effect of the angular momentum of light, is therefore used to show the difference between the two conventional E-M tensors. In the next section, our above results can be derived in a simple way. Then, a possible experimental scheme for testing the GSHEL concerning momentum flux is presented to tell us which one of the symmetric and the canonical E-M tensors is more "physical."

II. GSHEL OR ANOMALOUS GSHEL

The GSHEL is related to the nonzero transverse angular momentum of the light beam. In this section, we simply use the sum rule of angular momentum to deduce the above results.

The beam frame K' and the laboratory frame K are connected by a rotation transformation, i.e., $x^\mu = \Lambda^\mu_\nu x'^\nu$ [or $x'^\mu = (\Lambda^{-1})^\mu_\nu x^\nu$], as depicted in Fig. 1. Here the rotation transformation matrix is

$$\Lambda^\mu_\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix}. \tag{14}$$

Then T^{zz} in the K frame can be expressed by the momentum flux in the K' frame:

$$\begin{aligned} T^{zz}(x) = \Lambda^z_\alpha \Lambda^z_\beta T'^{\alpha\beta}(x') &= -(T'^{xz} + T'^{zx})\sin\theta\cos\theta \\ &\quad + (T'^{zz}\cos^2\theta + T'^{xx}\sin^2\theta). \end{aligned} \tag{15}$$

In the K' frame, according to the axial symmetry of the beam around its beam axis, T'^{zz} should show a symmetrical distribution with respect to the x'- and y'-axis. Furthermore, T'^{xx} can be ignored compared to T'^{zz} because the light beam mainly carries the momentum along the propagation direction (z' axis) and the momentum also is mainly transported in the propagation direction. Hence, we obtain

$$\begin{aligned} \langle y\rangle_T &= \int y\overline{T}^{zz}dxdy \Big/ \int \overline{T}^{zz}dxdy \\ &= -\tan\theta \int y(\overline{T}'^{xz} + \overline{T}'^{zx})dxdy \Big/ \int \overline{T}'^{zz}dxdy \\ &= -\frac{\int y'(\overline{T}'^{xz} + \overline{T}'^{zx})dx'dy'}{\int \overline{T}'^{zz}dx'dy'}\tan\theta. \end{aligned} \tag{16}$$

In the last step, expressing the area element from the K frame to the K' frame does not change the final result.

To proceed with the expression in Eq. (16), we have two E-M tensors in hand, the canonical one and the symmetric one:

$$\langle y_{\mathrm{sym}}\rangle_T = -\frac{2\int y'\,\overline{T}'^{zx}_{\mathrm{sym}}dx'dy'}{\int \overline{T}'^{zz}_{\mathrm{sym}}dx'dy'}\tan\theta, \tag{17}$$

$$\langle y_{\mathrm{can}}\rangle_T = -\frac{\int y'(\overline{T}'^{xz}_{\mathrm{can}} + \overline{T}'^{zx}_{\mathrm{can}})dx'dy'}{\int \overline{T}'^{zz}_{\mathrm{can}}dx'dy'}\tan\theta. \tag{18}$$

According to the axial symmetry of the light beam, we have

$$\begin{aligned} \int -y'\,\overline{T}'^{zx}dx'dy' &= \int x'\,\overline{T}'^{zy}dx'dy' \\ &= \frac{1}{2}\int (x'\,\overline{T}'^{zy} - y'\,\overline{T}'^{zx})dx'dy' \end{aligned} \tag{19}$$

and

$$\int \overline{T}'^{zz}_{\mathrm{sym}}dx'dy' \simeq \int \overline{T}'^{zz}_{\mathrm{can}}dx'dy' \simeq P'_z = n\hbar k, \tag{20}$$

where n is the photon number per unit time across the plane x'-y', namely the photon number flux.

For the symmetric E-M tensor, $M'^{zxy}_{\mathrm{sym}} = x'\,T'^{zy}_{\mathrm{sym}} - y'\,T'^{zx}_{\mathrm{sym}}$ represents the flux of the total angular momentum along the propagation direction. Hence, for a beam with only spin polarization we obtain

$$\begin{aligned} \langle y_{\mathrm{sym}}\rangle_T &= \tan\theta \int \left(x'\,\overline{T}'^{zy}_{\mathrm{sym}} - y'\,\overline{T}'^{zx}_{\mathrm{sym}}\right)dx'dy' \Big/ P'_z \\ &= \frac{n\sigma\hbar}{n\hbar k}\tan\theta = \frac{\lambda}{2\pi}\sigma\tan\theta. \end{aligned} \tag{21}$$

For the canonical E-M tensor, $L'^{zxy} = x'\,T'^{zy}_{\mathrm{can}} - y'\,T'^{zx}_{\mathrm{can}}$ gives merely the flux of the orbital angular momentum along the propagation direction. Thus, the integral of the term $y\overline{T}'^{zx}_{\mathrm{can}}$ in Eq. (18) vanishes when the light beam only carries spin angular momentum. For the collimated light beam, the light wave function is approximately in the simultaneous eigenstate of energy and longitudinal momentum. Thus, from the expression of the canonical E-M tensor, we can observe the following relation:

$$\overline{T}'^{xz}_{\mathrm{can}} \simeq \frac{k}{\omega}\overline{T}'^{x0}_{\mathrm{can}} = c\,\overline{T}'^{x0}_{\mathrm{can}}. \tag{22}$$

Then we have

$$\langle y_{\mathrm{can}}\rangle_T \simeq -c\,\tan\theta \int y'\,\overline{T}'^{x0}_{\mathrm{can}}\,dx'dy' \Big/ P'_z. \tag{23}$$

Again, because of the axial symmetry of the light beam, we arrive at

$$\begin{aligned} \int -y'\,\overline{T}'^{x0}_{\mathrm{can}}\,dx'dy' &= \int x'\,\overline{T}'^{y0}_{\mathrm{can}}\,dx'dy' \\ &= \frac{1}{2}\int \left(x'\,\overline{T}'^{y0}_{\mathrm{can}} - y'\,\overline{T}'^{x0}_{\mathrm{can}}\right)dx'dy'. \end{aligned} \tag{24}$$

As mentioned above, the canonical energy flux density T^{i0}_{can} becomes the Poynting vector $(\boldsymbol{E}\times\boldsymbol{B})^i$ in the radiation gauge. Hence, Eq. (24) represents the total time-averaged angular momentum of the beam per unit length and the final result of Eq. (23) is

$$\langle y_{\mathrm{can}}\rangle_T = \frac{cN_s\sigma\hbar}{2n\hbar k}\tan\theta = \frac{\lambda}{4\pi}\sigma\tan\theta. \tag{25}$$

Here N_s is the photon number per unit length along the direction of propagation and we have $n = cN_s$.

So far we have proved the main results presented in the first section. For the light beam carrying the orbital angular momentum $l\hbar$ as well as the spin angular momentum $\sigma\hbar$ per photon along the propagation direction, repeating the above analysis can give the following more general results about the GSHEL:

$$\langle y_{\rm sym}\rangle_P = \langle y_{\rm can}\rangle_P = \frac{\lambda}{4\pi}(l+\sigma)\tan\theta, \tag{26}$$

$$\langle y_{\rm sym}\rangle_T = \frac{\lambda}{4\pi}(2l+2\sigma)\tan\theta, \tag{27}$$

$$\langle y_{\rm can}\rangle_T = \frac{\lambda}{4\pi}(2l+\sigma)\tan\theta. \tag{28}$$

It is valuable to make two remarks on the above results.

(i) As seen in Eq. (16), there are two pieces of momentum flux. One is the transverse flow of the longitudinal momentum T'^{xz}, which is approximately proportional to the corresponding transverse component of energy flux for both the symmetric and the canonical E-M tensors, and its rotation around the direction of the propagation gives the total angular momentum flux. The other is the longitudinal flow of the transverse momentum T'^{zx}, and its rotation refers to the total angular momentum flux for the symmetric E-M tensor, but its rotation only refers to the orbital angular momentum flux for the canonical E-M tensor.

(ii) Therefore, there are two kinds of anomalous GSHEL. One represents the comparison between the GSHEL of momentum flux and that of energy flux, and the difference originates from the two pieces of momentum flux. The other refers to the comparison between the GSHEL of orbital and spin angular momenta, in other words, the different predictions of the symmetric and the canonical E-M tensors for the spin-generated GSHEL concerning momentum flux.

As seen from Eqs. (26)–(28), the GSHEL of orbital angular momentum cannot discriminate these two E-M tensors by both cases of energy and momentum fluxes. However, from Eqs. (27) and (28), the GSHEL concerning momentum flux of the spin-polarized beam can serve as a probe for the possible experimental test of those two E-M tensors. It should be stressed that the above results could also be derived in classical electromagnetism, although these results are obtained in the language of photons.

III. GSHEL MANIFESTED AS THE MOMENT OF FORCE

In the preceding section, we presented the GSHEL in terms of momentum flux and angular momentum flux. As is well known, the momentum and the angular momentum of light can manifest as mechanical effects by interaction with matter; namely, light is capable of exerting force and torque on matter. From the above conclusion, the key difference between the predictions of the two E-M tensors is originated from the spin angular momentum, so the spin-polarized light beam should be used to perform the possible experimental test. Naturally, the torque effect is related to angular momentum and much preferred, but the spin angular momentum of light is equivalent to the orbital angular momentum of light in many ways [36–40]. In Fig. 1, if the spin-polarized light carries orbital angular momentum, the detection plane as a whole undergoes a torque around the x direction:

$$\tau^x = \left\langle\frac{dJ^x}{dt}\right\rangle \simeq -\int \overline{M}^{zyz}dxdy = n(\sigma+l)\hbar\sin\theta. \tag{29}$$

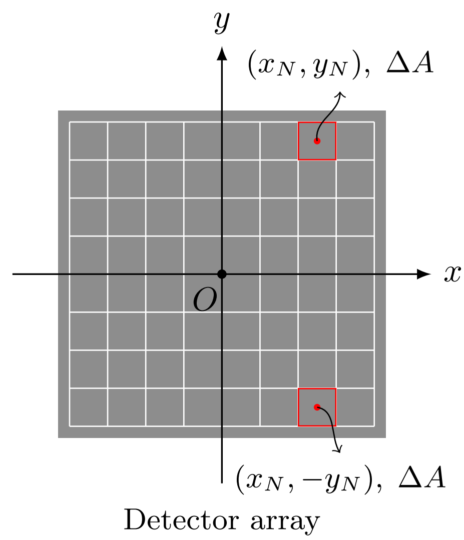

FIG. 2. The detector array consists of individual detection elements.

Here Eq. (29) is valid irrespective of whether the symmetric and the canonical angular momentum tensors are employed, implying that the global torque effect fails to distinguish between the symmetric and the canonical E-M tensors.

Though the spin and orbital angular momenta of light produce the same mechanical effect in many respects, they have different properties which have been verified in numerous experiments [32–35]. Enlightened by such experiments, we propose here a scheme to examine the different predictions of the canonical and the symmetric E-M tensors and determine which E-M tensor is effective. Let the detection plane be served by an array of individual detection elements (or take multiple measurements by placing one detection element at the typical points which also builds an array on the detection plane). Each detection element can measure the time-averaged force $\boldsymbol{f}_N$ exerted by the light beam on the area of the detection element ΔA (see Fig. 2). Each detection element is represented with the coordinate (x_N, y_N) of its center point. The quantity under consideration is the displacement of the barycenter of f_N^z, the longitudinal part of the force $\boldsymbol{f}_N$:

$$\langle y\rangle_f = \frac{\sum_N y_N f_N^z}{\sum_N f_N^z}. \tag{30}$$

Theoretically, the force $\boldsymbol{f}_N$ is the time-averaged rate of change of the field momentum received by the detection element (x_N, y_N). In fact, calculating the optical force and the force density within a material is a complicated issue [41–43], even different electromagnetic tensors lead to different results [44–46]. For simplicity, considering the case that the detector fully absorbs the momentum of light across the detection plane, we have

$$f_N^z = \left\langle\frac{dP_N^z(t)}{dt}\right\rangle \simeq -\overline{T}^{zz}(x_N, y_N)\Delta A, \tag{31}$$

and then the displacement

$$\langle y\rangle_f \simeq \frac{\sum_N y_N\,\overline{T}^{zz}(x_N, y_N)}{\sum_N \overline{T}^{zz}(x_N, y_N)} \simeq \langle y\rangle_T. \tag{32}$$

Our scheme is based on the measurability of the local optical force. Suppose that the power of the laser beam $W = 1$ mW, the wavelength $\lambda = 0.5$ μm, the width of the laser beam $d = 10$ μm, the step $\delta y = y_{N+1} - y_N = 1$ μm, and the ratio of the detection element area ΔA to the cross-sectional area of the beam A_0 is about 1 : 100. Under these conditions, we could rudely estimate the scale of the force exerted on one individual detection element $\Delta f = (W/c)(\Delta A/A_0) \sim 0.1$ pN. Maybe atomic force microscopy (AFM) is an ideal tool for the high-resolution measurement of the optical force required by our scheme. AFMs can be used to measure interaction forces between the probe and the sample and also is applied for highly precise measurement of optical force [47,48]. Nowadays the measurement of force by AFM has made it possible to investigate the optical force in the femtonewton range, even in the attonewton range [49–51]. Therefore, the practical sensitivity of optical force measurement is sufficient to meet the requirement of the above theoretical estimate. Although the optical force in our scheme is tiny, we are confident that the force is detectable. For example, if we set the optical power $W = 10$ W and keep the other conditions unchanged, the force turns out to be about 10^3 pN and could be more easily detected.

To illustrate the different predictions of the two forms of E-M tensor, one can consider a special case: let the laser beam carry spin and orbital angular momenta $s = -l = \hbar$ or $s = -l = -\hbar$ per photon. For this case, as seen from Eqs. (27) and (28), the symmetric E-M tensor predicts the force shift $\langle y_{\rm sym}\rangle_f$ suddenly disappeared and independent of the tilted angle θ, but the canonical one suggests the force shift $\langle y_{\rm can}\rangle_f$ should be as a tangent function of the tilted angle θ. Therefore, for the two predictions of the symmetric and canonical E-M tensors, they show a striking difference as to the dependence of the force shift $\langle y\rangle_f$ on the tilted angle θ. The advantage of this design is that the effects of the spin angular momentum and the orbital angular momentum completely cancel out each other according to the symmetric E-M tensor, but they only partially cancel out each other according to the canonical one. In this case, one can choose to analyze the experimental dependence of the shift $\langle y\rangle_f$ on the tilted angle θ and compare the experimental results with the different predictions. Finally, the experimental results can tell us which prediction is more fit. Thus, this special design can easily test which E-M tensor is the more "physical" one.

In conclusion, we have demonstrated the GSHEL of momentum flux as a natural extension of the GSHEL of energy flux, and we have proposed a possible experimental scheme to test this effect by the mechanical effect of light. For the spin-polarized light beam, the symmetric E-M tensor predicts an anomalous effect, but the canonical E-M tensor does not. On the other hand, for both the symmetric and the canonical E-M tensors, the orbital angular momentum causes the other anomalous GSHEL concerning momentum flux. Therefore, for the spin-polarized light beam, we argue that the GSHEL concerning momentum flux can be regarded as an experimental scheme to test the expressions of E-M tensors. We strongly urge experimentalists to perform the measurement we proposed here, not only because it is an interesting effect but also because it contributes to clarifying our understanding of E-M and angular momentum tensors.

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