Angular momentum of open quantum systems in external magnetic field

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The non-Markovian dynamics of a charged harmonic oscillator linearly coupled to a neutral bosonic heat bath is investigated in external uniform magnetic field. The analytical expression is derived for the asymptotic angular momentum. The orbital diamagnetism of quantum system in a dissipative environment is studied.

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I. INTRODUCTION

In atomic physics, much attention is focused on the hydrogen atom in magnetic field for which the experimental and theoretical studies yield excellent insights into the semiclassical and quantum aspects of nonintegrable systems (see, for example, Refs. [1,2]). In nuclear physics, the study of nuclear properties in the strong field of magnetic trap seems to be interesting. The observation of simultaneous violation of parity and time-reversal invariance is based on the measurement of the linear polarization of gamma transitions produced by the deexcitation of isomeric states of nuclei in the magnetic field at low temperature [3]. The intensive investigations deal with the impact of external magnetic field on such systems as quantum dots, quantum wires, and two-dimensional electronic systems [1–7]. The characteristics of plasma in the homogeneous external field has also to be studied in the physics of gas discharge [8].

The influence of magnetic field on the properties of quantum system was investigated with different approaches. Using the phenomenological Markovian Fokker-Planck equation for the Wigner function, the problem of quantum description of the damped isotropic two-dimensional harmonic oscillator in a uniform magnetic field has been studied in Ref. [9] in the case of arbitrary relations between the proper oscillator frequency, damping coefficients, and temperature. The equations of motion can be obtained by using the quantum Langevin approach or density-matrix formalism which is widely applied to find the effects of fluctuations and dissipation in macroscopic systems [9–34]. Early derivation of the one-dimensional quantum Langevin equation with external force was performed in Ref. [23]. As shown, the particle, coupled to the heat bath and influenced by an arbitrary force to the fixed center, exhibits the Brownian motion. As an application of the quantum Markovian Langevin equation the dynamics of one-dimensional harmonic oscillator coupled to the heat bath was considered. By including the magnetic field in the quantum non-Markovian Langevin equation, the effects of dissipation and magnetic field on localization of a charged particle moving in the confined potential have been investigated in Refs. [23,24,28,30,31]. As found, the weak dissipation delocalizes the oscillation of charged particle when the magnetic field is stronger than a certain critical value [24]. In all cases [20,23–25,28,30], the magnetic field affects neither the memory function nor the random force appearing in the quantum Langevin equation.

The aim of the present work is to derive the analytical expression for the asymptotic angular momentum of a confined charged particle in uniform magnetic field and dissipative environment, and to study the influence of magnetic field on the orbital magnetic moment (orbital diamagnetism). The paper is organized as follows. In Sec. II, we define the Hamiltonian of the system and solve the quantum non-Markovian two-dimensional Langevin equations for a charged particle moving in the plane normal to the field applied. The asymptotic angular momentum is obtained by considering the second moment of the stochastic dissipative equations. The discussions and illustrative numerical results are presented in Sec. III. A summary is given in Sec. IV.

II. NON-MARKOVIAN LANGEVIN EQUATIONS WITH EXTERNAL MAGNETIC FIELD

In order to investigate the influence of external fields on the dynamics of an open quantum system, we consider the motion of a charged particle with effective mass μ and charge $e = |e|$ in the two-dimensional parabolic potential (in xy plane) surrounded by the neutral bosonic heat bath in the presence of perpendicular axisymmetric magnetic field (along z axis). In the case of linear coupling in coordinates between this particle and heat bath the total Hamiltonian of the collective subsystem+heat bath is as follows:

$$H = \frac{1}{2\mu}[\mathbf{p} - e\mathbf{A}(x, y)]^2 + \frac{\mu}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2\right) + \sum_\nu \hbar\omega_\nu b_\nu^+ b_\nu + \sum_\nu (x\alpha_\nu + y g_\nu)(b_\nu^+ + b_\nu), \quad (1)$$

where $\mathbf{A} = (-\frac{1}{2}yB, \frac{1}{2}xB, 0)$ is the vector potential of the magnetic field with the strength $B = |\mathbf{B}|$, $\mathbf{p}$ is the canonically conjugated momentum, ω_x and ω_y are the collective frequencies, b_ν^+ and b_ν are the phonon creation and annihilation operators of the heat bath, and α_ν and g_ν are the coupling parameters [30]. The first term in Eq. (1) includes the magnetic-field energy and the last term describes the interaction between the collective subsystem and heat bath. The bosonic heat bath is modeled by an ensemble of noninteracting harmonic oscillators with frequencies ω_ν. The coupling between the heat bath and collective subsystem is linear in coordinates. The coupling term and external magnetic field do not affect each other.

For convenience, we introduce the following definitions for momenta:

$$\pi_x = p_x + \tfrac{1}{2}\mu\omega_c y, \quad \pi_y = p_y - \tfrac{1}{2}\mu\omega_c x, \tag{2}$$

where $\omega_c = eB/\mu$ is the cyclotron frequency and $[\pi_x, \pi_y] = -[\pi_y, \pi_x] = \imath\hbar\mu\omega_c$. Therefore, the total Hamiltonian (1) is transformed into the form

$$H = \frac{1}{2\mu}\left(\pi_x^2 + \pi_y^2\right) + \frac{\mu}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2\right) + \sum_\nu \hbar\omega_\nu b_\nu^+ b_\nu + \sum_\nu (x\alpha_\nu + y g_\nu)(b_\nu^+ + b_\nu). \tag{3}$$

The system of the Heisenberg equations for the operators x, y, π_x, π_y, and the bath phonon operators b_ν, b_ν^+ is obtained by commuting them with H:

$$\begin{aligned}
\dot{x}(t) &= \frac{i}{\hbar}[H, x] = \frac{\pi_x(t)}{\mu}, \quad \dot{y}(t) = \frac{i}{\hbar}[H, y] = \frac{\pi_y(t)}{\mu},\\
\dot{\pi}_x(t) &= \frac{i}{\hbar}[H, \pi_x] = \pi_y(t)\omega_c - \mu\omega_x^2 x(t) - \sum_\nu \alpha_\nu (b_\nu^+ + b_\nu),\\
\dot{\pi}_y(t) &= \frac{i}{\hbar}[H, \pi_y] = -\pi_x(t)\omega_c - \mu\omega_y^2 y(t) - \sum_\nu g_\nu (b_\nu^+ + b_\nu)
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\dot{b}_\nu^+(t) &= \frac{i}{\hbar}[H, b_\nu^+] = i\omega_\nu b_\nu^+(t) + \frac{i}{\hbar}[\alpha_\nu x(t) + g_\nu y(t)],\\
\dot{b}_\nu(t) &= \frac{i}{\hbar}[H, b_\nu] = -i\omega_\nu b_\nu(t) - \frac{i}{\hbar}[\alpha_\nu x(t) + g_\nu y(t)].
\end{aligned} \tag{5}$$

The solution of Eqs. (5) is

$$\begin{aligned}
b_\nu^+(t) &= f_\nu^+(t) - \frac{\alpha_\nu x(t) + g_\nu y(t)}{\hbar\omega_\nu} + \frac{\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau\, \dot{x}(\tau) e^{i\omega_\nu(t-\tau)} + \frac{g_\nu}{\hbar\omega_\nu}\int_0^t d\tau\, \dot{y}(\tau) e^{i\omega_\nu(t-\tau)},\\
b_\nu(t) &= f_\nu(t) - \frac{\alpha_\nu x(t) + g_\nu y(t)}{\hbar\omega_\nu} + \frac{\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau\, \dot{x}(\tau) e^{-i\omega_\nu(t-\tau)} + \frac{g_\nu}{\hbar\omega_\nu}\int_0^t d\tau\, \dot{y}(\tau) e^{-i\omega_\nu(t-\tau)},
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
f_\nu(t) &= \left[b_\nu(0) + \frac{i}{\hbar\omega_\nu} B_\nu(0)\right] e^{-i\omega_\nu t},\\
B_\nu(t) &= \alpha_\nu x(t) + g_\nu y(t).
\end{aligned}$$

Therefore,

$$\begin{aligned}
b_\nu^+(t) + b_\nu(t) = f_\nu^+(t) + f_\nu(t) - 2\frac{\alpha_\nu x(t) + g_\nu y(t)}{\hbar\omega_\nu} &+ \frac{2\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau\, \dot{x}(\tau)\cos[\omega_\nu(t-\tau)]\\
&+ \frac{2g_\nu}{\hbar\omega_\nu}\int_0^t d\tau\, \dot{y}(\tau)\cos[\omega_\nu(t-\tau)].
\end{aligned} \tag{7}$$

Substituting (7) into (4), we eliminate the bath variables from the equations of motion of the collective subsystem and obtain the nonlinear integrodifferential stochastic dissipative equations

$$\begin{aligned}
\dot{x}(t) &= \frac{\pi_x(t)}{\mu}, \quad \dot{y}(t) = \frac{\pi_y(t)}{\mu},\\
\dot{\pi}_x(t) &= \pi_y(t)\omega_c - x(t)\mu\omega_x^2\left(1 - \frac{1}{\omega_x^2}\sum_\nu \frac{2\alpha_\nu^2}{\mu\hbar\omega_\nu}\right) - \frac{1}{\mu}\int_0^t d\tau\, K_\alpha(t,\tau)\pi_x(\tau) - F_\alpha(t),\\
\dot{\pi}_y(t) &= -\pi_x(t)\omega_c - y(t)\mu\omega_y^2\left(1 - \frac{1}{\omega_y^2}\sum_\nu \frac{2g_\nu^2}{\mu\hbar\omega_\nu}\right) - \frac{1}{\mu}\int_0^t d\tau\, K_g(t,\tau)\pi_y(\tau) - F_g(t).
\end{aligned} \tag{8}$$

The presence of the integral parts in these equations indicates the non-Markovian dynamics. In comparison with Refs. [12,16] we do not introduce the counterterm in the Hamiltonian. So, the stiffnesses of the potentials are renormalized in the equations above. The operators

$$\begin{aligned}
F_\alpha(t) &= \sum_\nu F_\alpha^\nu(t) = \sum_\nu \alpha_\nu (f_\nu^+ + f_\nu),\\
F_g(t) &= \sum_\nu F_g^\nu(t) = \sum_\nu g_\nu (f_\nu^+ + f_\nu)
\end{aligned}$$

play a role of random forces in the coordinates, and Eqs. (8) are the generalized nonlinear quantum Langevin equations. Following the usual procedure of statistical mechanics, we identify these operators as fluctuations because of the uncertainty in the initial conditions for the bath operators. To specify the statistical properties of the fluctuations, we consider an ensemble of initial states in which the fluctuations have the Gaussian distribution with zero average value

$$\langle\langle F_\alpha^\nu(t)\rangle\rangle = \langle\langle F_g^\nu(t)\rangle\rangle = 0. \tag{9}$$

Here, the symbol $\langle\langle \ldots \rangle\rangle$ denotes the average over the bath. We assume that there are no correlations between $F_\alpha^\nu(t)$ and

$F_g^\nu(t)$, so that

$$\sum_\nu \frac{\alpha_\nu g_\nu}{\hbar\omega_\nu} \equiv 0. \tag{10}$$

The dissipative kernels in Eqs. (8) are

$$K_\alpha(t-\tau) = 2\sum_\nu \frac{\alpha_\nu^2}{\hbar\omega_\nu}\cos(\omega_\nu[t-\tau]),$$
$$K_g(t-\tau) = 2\sum_\nu \frac{g_\nu^2}{\hbar\omega_\nu}\cos(\omega_\nu[t-\tau]). \tag{11}$$

Because these kernels do not contain the phonon occupation numbers, they are independent of temperature T (in the energy units) of the heat bath. The temperature enters in the analysis through the distribution of initial conditions. We use the Bose-Einstein statistics for the heat bath:

$$\langle\langle f_\nu^+(t)f_{\nu'}^+(t')\rangle\rangle = \langle\langle f_\nu(t)f_{\nu'}(t')\rangle\rangle = 0,$$
$$\langle\langle f_\nu^+(t)f_{\nu'}(t')\rangle\rangle = \delta_{\nu,\nu'}n_\nu e^{i\omega_\nu(t-t')},$$
$$\langle\langle f_\nu(t)f_{\nu'}^+(t')\rangle\rangle = \delta_{\nu,\nu'}(n_\nu+1)e^{-i\omega_\nu(t-t')}, \tag{12}$$

with occupation numbers for phonons $n_\nu = [\exp(\hbar\omega_\nu/T)-1]^{-1}$ depending on T. Using the properties of random forces, we obtain the quantum fluctuation-dissipation relations

$$\sum_\nu \varphi_{\alpha\alpha}^\nu(t,t')\frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} = K_\alpha(t-t'),$$
$$\sum_\nu \varphi_{gg}^\nu(t,t')\frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} = K_g(t-t'),$$

where

$$\varphi_{\alpha\alpha}^\nu(t,t') = 2\alpha_\nu^2[2n_\nu+1]\cos(\omega_\nu[t-t']),$$
$$\varphi_{gg}^\nu(t,t') = 2g_\nu^2[2n_\nu+1]\cos(\omega_\nu[t-t'])$$

are the symmetrized correlation functions $\varphi_{kk}^\nu(t,t') = \langle\langle F_k^\nu(t)F_k^\nu(t') + F_k^\nu(t')F_k^\nu(t)\rangle\rangle$, $k=\alpha, g$. The quantum fluctuation-dissipation relations differ from the classical ones and are reduced to them in the limit of high temperature T (or $\hbar\to 0$): $\sum_\nu \varphi_{\alpha\alpha}^\nu(t,t') = 2TK_\alpha(t-t')$, $\sum_\nu \varphi_{gg}^\nu(t,t') = 2TK_g(t-t')$.

It is convenient to introduce the spectral density D_ω of the heat bath excitations which allows us to replace the sum over different oscillators, ν, by the integral over frequency: $\sum_\nu \ldots \to \int_0^\infty d\omega\, D_\omega \ldots$. This is accompanied by the following replacements: $\alpha_\nu\to\alpha_\omega$, $g_\nu\to g_\omega$, $\omega_\nu\to\omega$, and $n_\nu\to n_\omega$. Let us consider the following spectral functions [12,21]:

$$D_\omega\frac{\alpha_\omega^2}{\omega} = \frac{\lambda_x^2}{\pi}\frac{\gamma^2}{\gamma^2+\omega^2}, \quad D_\omega\frac{g_\omega^2}{\omega} = \frac{\lambda_y^2}{\pi}\frac{\gamma^2}{\gamma^2+\omega^2}, \tag{13}$$

where the memory time γ^{-1} of dissipation is inverse to the phonon bandwidth of the heat bath excitations which are coupled with the collective oscillator and the coefficients

$$\lambda_x = \frac{1}{\mu}\int_0^\infty d\tau\, K_\alpha(t-\tau)$$

and

$$\lambda_y = \frac{1}{\mu}\int_0^\infty d\tau\, K_g(t-\tau)$$

are the friction coefficients in the Markovian limit. This ohmic dissipation with the Lorenzian cutoff (Drude dissipation) results in the dissipative kernels

$$K_\alpha(t) = \mu\lambda_x\gamma\, e^{-\gamma|t|}, \quad K_g(t) = \mu\lambda_y\gamma\, e^{-\gamma|t|}.$$

The relaxation time of heat bath should be much less than the period of the collective oscillator, i.e., $\gamma\gg\omega_{x,y}$.

As in Ref. [30], the system of Eqs. (8) is solved by applying the Laplace transformations. After the tedious algebra we obtain the solution of this system of equations:

$$\begin{aligned}
x(t) &= A_1(t)x(0) + A_2(t)y(0) + A_3(t)\pi_x(0)\\
&\quad + A_4(t)\pi_y(0) - I_x(t) - I_x'(t),\\
y(t) &= B_1(t)x(0) + B_2(t)y(0) + B_3(t)\pi_x(0)\\
&\quad + B_4(t)\pi_y(0) - I_y(t) - I_y'(t),\\
\pi_x(t) &= C_1(t)x(0) + C_2(t)y(0) + C_3(t)\pi_x(0)\\
&\quad + C_4(t)\pi_y(0) - I_{\pi_x}(t) - I_{\pi_x}'(t),\\
\pi_y(t) &= D_1(t)x(0) + D_2(t)y(0) + D_3(t)\pi_x(0)\\
&\quad + D_4(t)\pi_y(0) - I_{\pi_y}(t) - I_{\pi_y}'(t),
\end{aligned} \tag{14}$$

where $I_x(t) = \int_0^t A_3(\tau)F_\alpha(t-\tau)d\tau$, $I_x'(t) = \int_0^t A_4(\tau)F_g(t-\tau)d\tau$, $I_y(t) = \int_0^t B_3(\tau)F_\alpha(t-\tau)d\tau$, $I_y'(t) = \int_0^t B_4(\tau)F_g(t-\tau)d\tau$, $I_{\pi_x}(t) = \int_0^t C_3(\tau)F_\alpha(t-\tau)d\tau$, $I_{\pi_x}'(t) = \int_0^t C_4(\tau)F_g(t-\tau)d\tau$, $I_{\pi_y}(t) = \int_0^t D_3(\tau)F_\alpha(t-\tau)d\tau$, $I_{\pi_y}'(t) = \int_0^t D_4(\tau)F_g(t-\tau)d\tau$, and the time-dependent coefficients

$$\begin{aligned}
A_1(t) &= \sum_{i=1}^6 \beta_i\{\left[\left(\omega_y^2+s_i^2\right)(s_i+\gamma) - \lambda_y\gamma^2\right]\\
&\quad \times [s_i(s_i+\gamma) + \lambda_x\gamma] + \omega_c^2 s_i(s_i+\gamma)^2\}e^{s_it},\\
A_2(t) &= -\omega_c(\omega_y^2 - \lambda_y\gamma)\sum_{i=1}^6 \beta_i(s_i+\gamma)^2 e^{s_it},\\
A_3(t) &= \frac{1}{\mu}\sum_{i=1}^6 \beta_i(s_i+\gamma)\left[\left(\omega_y^2+s_i^2\right)(s_i+\gamma) - \lambda_y\gamma^2\right]e^{s_it},\\
A_4(t) &= \frac{\omega_c}{\mu}\sum_{i=1}^6 \beta_i s_i(s_i+\gamma)^2 e^{s_it},\\
B_1(t) &= -A_2(t)|_{x\leftrightarrow y}, \quad B_2(t) = A_1(t)|_{x\leftrightarrow y},\\
B_3(t) &= -A_4(t)|_{x\leftrightarrow y}, \quad B_4(t) = A_3(t)|_{x\leftrightarrow y},\\
C_1(t) &= -\mu^2\left(\omega_x^2 - \lambda_x\gamma\right)A_3(t), \quad C_2(t) = \mu\dot{A}_2(t),\\
C_3(t) &= \mu\dot{A}_3(t), \quad C_4(t) = \mu\dot{A}_4(t),\\
D_1(t) &= \mu\dot{B}_1(t), \quad D_2(t) = -\mu^2\left(\omega_y^2 - \lambda_y\gamma\right)B_4(t),\\
D_3(t) &= \mu\dot{B}_3(t), \quad D_4(t) = \mu\dot{B}_4(t).
\end{aligned} \tag{15}$$

Here, s_i are the roots of the following equation:

$$\left[\left(\omega_x^2+s_i^2\right)(s_i+\gamma)-\lambda_x\gamma^2\right]\left[\left(\omega_y^2+s_i^2\right)(s_i+\gamma)-\lambda_y\gamma^2\right] + \omega_c^2 s_i^2 (s_i+\gamma)^2 = 0 \tag{16}$$

and $\beta_i = [\prod_{j\neq i}(s_i - s_j)]^{-1}$ with $i, j = 1$–6. These roots arise when we apply the residue theorem to perform the integration in the inverse Laplace transformation. As seen in Eqs. (14), $A_1(0) = B_2(0) = C_3(0) = D_4(0) = 1$ and $A_{2,3,4}(0) = B_{1,3,4}(0) = C_{1,2,4}(0) = D_{1,2,3}(0) = 0$.

Using Eqs. (14), (15), and the correlations of the random forces at different times, the expressions for the second moments (variances)

$$\Sigma_{q_iq_j}(t) = \tfrac{1}{2}\langle q_i(t)q_j(t) + q_j(t)q_i(t)\rangle - \langle q_i(t)\rangle\langle q_j(t)\rangle,$$

where $q_i = x, y, \pi_x$, or π_y ($i = 1$–4), are derived.

III. ANGULAR MOMENTUM

Using Eqs. (14) and (15), one can find the z component of angular momentum $L_z(t) = \langle x(t)\pi_y(t) - y(t)\pi_x(t)\rangle$ [or magnetic moment per unit volume $M(t) = \frac{neL_z(t)}{2\mu}$, where n is the concentration of charged particles]:

$$L_z(t) = L_z^0(t) + \frac{\mu\hbar\gamma^2}{\pi}\int_0^\infty\int_0^t\int_0^t \frac{d\omega\, d\tau\, d\tilde{\tau}\, \omega \coth\left[\frac{\hbar\omega}{2T}\right]}{\omega^2+\gamma^2}\cos(\omega[\tau-\tilde{\tau}]) \times \{\lambda_x[A_3(\tau)D_3(\tilde{\tau}) - B_3(\tau)C_3(\tilde{\tau})] + \lambda_y[A_4(\tau)D_4(\tilde{\tau}) - B_4(\tau)C_4(\tilde{\tau})]\}, \tag{17}$$

where

$$\begin{aligned} L_z^0(t) = {} & [A_1(t)D_1(t) - B_1(t)C_1(t)]\langle x^2(0)\rangle + [A_2(t)D_2(t) - B_2(t)C_2(t)]\langle y^2(0)\rangle + [A_3(t)D_3(t) - B_3(t)C_3(t)]\langle \pi_x^2(0)\rangle \\ & + [A_4(t)D_4(t) - B_4(t)C_4(t)]\langle \pi_y^2(0)\rangle + [A_1(t)D_3(t) - B_1(t)C_3(t)]\langle x(0)\pi_x(0)\rangle + [A_3(t)D_1(t) - B_3(t)C_1(t)]\langle \pi_x(0)x(0)\rangle \\ & + [A_2(t)D_4(t) - B_2(t)C_4(t)]\langle y(0)\pi_x(0)\rangle + [A_4(t)D_2(t) - B_4(t)C_2(t)]\langle \pi_y(0)y(0)\rangle + [A_3(t)D_4(t) - B_3(t)C_4(t)] \\ & \times \langle \pi_x(0)\pi_y(0)\rangle + [A_4(t)D_3(t) - B_4(t)C_3(t)]\langle \pi_y(0)\pi_x(0)\rangle + [A_1(t)D_2(t) + A_2(t)D_1(t) - B_1(t)C_2(t) \\ & - B_2(t)C_1(t)]\langle x(0)y(0)\rangle + [A_1(t)D_4(t) + A_4(t)D_1(t) - B_1(t)C_4(t) - B_4(t)C_1(t)]\langle x(0)\pi_y(0)\rangle \\ & + [A_2(t)D_3(t) + A_3(t)D_2(t) - B_2(t)C_3(t) - B_3(t)C_2(t)]\langle y(0)\pi_x(0)\rangle. \end{aligned} \tag{18}$$

The angular momentum is equal to $L_z(0) = L_z^0(0) = \langle x(0)\pi_y(0) - y(0)\pi_x(0)\rangle$ at initial time $t = 0$. It changes with time and reaches the asymptotic value at $t\to\infty$. The coupling to the heat bath cause some initial orbital angular momentum $L_z^0(0)$ to dissipate such that the only asymptotic angular momentum remains as time tends to infinity.

Employing $L_z^0(\infty) = 0$ $[A_i(\infty) = B_i(\infty) = C_i(\infty) = D_i(\infty) = 0]$ and the expression for the asymptotic variance $\Sigma_{x\pi_y}(\infty)$, we find the asymptotic z component of angular momentum

$$\begin{aligned} L_z(\infty) &= \langle x(\infty)\pi_y(\infty) - y(\infty)\pi_x(\infty)\rangle = 2\Sigma_{x\pi_y}(\infty) \\ &= -\frac{2\hbar\omega_c\gamma^2}{\pi}\int_0^\infty d\omega\,\omega^3\coth\left[\frac{\hbar\omega}{2T}\right]\frac{\lambda_x\left[(\omega^2+\gamma^2)\left(\omega^2-\omega_y^2\right)+\lambda_y\gamma^3\right]+\lambda_y\left[(\omega^2+\gamma^2)\left(\omega^2-\omega_x^2\right)+\lambda_x\gamma^3\right]}{\left(s_1^2+\omega^2\right)\left(s_2^2+\omega^2\right)\left(s_3^2+\omega^2\right)\left(s_4^2+\omega^2\right)\left(s_5^2+\omega^2\right)\left(s_6^2+\omega^2\right)}. \end{aligned} \tag{19}$$

For positive charge e, the angular momentum is opposite to $\mathbf{B}$. As seen, $L_z(\infty) = 0$ at $\omega_c = 0$ or $\lambda_x = \lambda_y = 0$. So, without external magnetic field and thermostat the angular momentum is zero. So, the combined actions of constant magnetic field (the Lorentz force) and random forces $F_{\alpha,g}$ lead to the emergence of angular momentum. The source of rotational energy is the fluctuations of random forces.

Assuming $\lambda_x = \lambda_y = \lambda$ and $\omega_x = \omega_y = \omega_0$ and employing the roots

$$s_1 = -\tfrac{1}{2}\left[\lambda - i\omega_c + \sqrt{(\lambda-i\omega_c)^2 - 4\omega_0^2}\right], \quad s_2 = -\tfrac{1}{2}\left[\lambda - i\omega_c - \sqrt{(\lambda-i\omega_c)^2 - 4\omega_0^2}\right], \quad s_3 = s_1^*, \quad s_4 = s_2^*, \tag{20}$$

of the Markovian secular equation

$$\left(s[s+\lambda]+\omega_0^2\right)^2 + s^2\omega_c^2 = 0, \tag{21}$$

we obtain the asymptotic angular momentum in the following simple form:

$$\begin{aligned} L_z(\infty) &= -\frac{4\hbar\omega_c\gamma^2\lambda}{\pi}\int_0^\infty \frac{d\omega\,\omega^3\left(\omega^2-\omega_0^2\right)\coth\left[\frac{\hbar\omega}{2T}\right]}{(\omega^2+\gamma^2)\left(s_1^2+\omega^2\right)\left(s_2^2+\omega^2\right)\left(s_3^2+\omega^2\right)\left(s_4^2+\omega^2\right)} \\ &= -\frac{4\hbar\omega_c\gamma^2\lambda}{\pi}\int_0^\infty \frac{d\omega\,\omega^3\left(\omega^2-\omega_0^2\right)\coth\left[\frac{\hbar\omega}{2T}\right]}{(\omega^2+\gamma^2)\left\{\left[\left(\omega^2-\omega_0^2\right)^2+\omega^2\lambda^2\right]^2 - 2\omega^2\omega_c^2\left[\left(\omega^2-\omega_0^2\right)^2-\omega^2\lambda^2\right]+\omega^4\omega_c^4\right\}}. \end{aligned} \tag{22}$$

In the absence of magnetic field ($\omega_c = 0$) we have $L_z = 0$. Setting the frequency of the damped quantum oscillator zero ($\omega_0 \to 0$) in Eq. (22), we find the asymptotic angular momentum for free damped particle:

$$L_z(\infty) = -\frac{4\hbar\omega_c\gamma^2\lambda}{\pi}\int_0^\infty d\omega \frac{\omega \coth\left[\frac{\hbar\omega}{2T}\right]}{\omega^2+\gamma^2}\frac{1}{\omega^4+2\omega^2(\lambda^2-\omega_c^2)+(\lambda^2+\omega_c^2)^2}. \tag{23}$$

At high temperatures and $\gamma \to \infty$, we obtain from Eq. (23)

$$L_z(\infty) = -\frac{2\omega_c T}{\lambda^2+\omega_c^2}. \tag{24}$$

The similar expression was derived in Ref. [20]. As seen, $L_z(\infty)$ approaches zero with increasing friction coefficient. This approach is slower the larger the cyclotron frequency is. Note that the Bohr–Van Leeuwen theorem (there is no diamagnetism in the classical system) is restored in the limit of infinite cyclotron frequency. At low temperature ($T \to 0$) and $\gamma \to \infty$,

$$L_z(\infty) = -\hbar\left(\frac{1}{2} - \frac{\arctan\left[\frac{\lambda^2-\omega_c^2}{\lambda\omega_c}\right]}{\pi}\right) \tag{25}$$

is also nonzero in the presence of dissipation and magnetic field. So, the orbital diamagnetism survives in the dissipative environment. At $\omega_c \gg \lambda$ (strong magnetic field), we obtain

$$L_z(\infty) = -\hbar, \quad M(\infty) = -\frac{ne\hbar}{2\mu}. \tag{26}$$

As seen, for large values of the cyclotron frequency, the asymptotic magnetization equals one (negative) Bohr magneton. So, in the dissipative system we find the quantization conditions ($T \to 0$, $\gamma \to \infty$, $\omega_c \gg \lambda$) for the orbital angular momentum and magnetic moment. The localization of the charged particles with increasing magnetic field was observed in the bosonic system [30]. Because the magnetic field localizes the oscillation of charged particle, the variance $|\Sigma_{x\pi_y}(\infty)|$ reaches its minimum value $\frac{\hbar}{2}$ at high magnetic fields.

We calculate the z component L_z of angular momentum for the system settled in the increasing external magnetic field at different temperatures (Fig. 1). The results indicate the diamagnetism of the system even in the presence of physical heat bath. At large B, the value of L_z (M) approaches $-\hbar$ ($-\frac{ne\hbar}{2\mu}$), which means it tends to the usual quantization of L_z (M) in the dissipative system. It is more pronounced at low temperatures. As seen in Fig. 1, the average value of angular momentum of a free particle ($\omega_0 = 0$) coupled with the heat bath exceeds the average angular momentum of a particle in the harmonic oscillator. For example, in the case of low temperature [$T/(\hbar\lambda) = 0.1$] and $\omega_c/\lambda = 2$, their ratios are about of 2 and 1.1 at $\omega_c/\lambda = 2$ and 8, respectively. In the case of high temperature [$T/(\hbar\lambda) = 2$] and $\omega_c/\lambda = 2$, their ratios are about of 14 and 2 at $\omega_c/\lambda = 2$ and 8, respectively. At high (low) temperature and $\omega_c \approx \lambda$, this ratio is maximal—about 27 (2.5). At low temperature, the absolute value of the magnetization of electric charges confined in the harmonic oscillator decreases with increasing ω_0 (Fig. 1). At high temperature, the dependence of L_z on ω_0 is rather weak.

IV. SUMMARY

The influence of an external magnetic field on the open quantum system was studied beyond the Markov approximation. The explicit expression for the asymptotic angular momentum was obtained for the two-dimensional charged quantum harmonic oscillator in the uniform magnetic field. The linear coupling in coordinates to the neutral bosonic heat bath was treated. In order to average the influence of bosonic heat bath on the charged particle, we applied the spectral function of heat-bath excitations which describes the Drude dissipation with Lorentzian cutoffs. Our formalism is valid at arbitrary coupling strengths, and hence at arbitrary low temperature. At initial time interval, the magnetic field acts on the

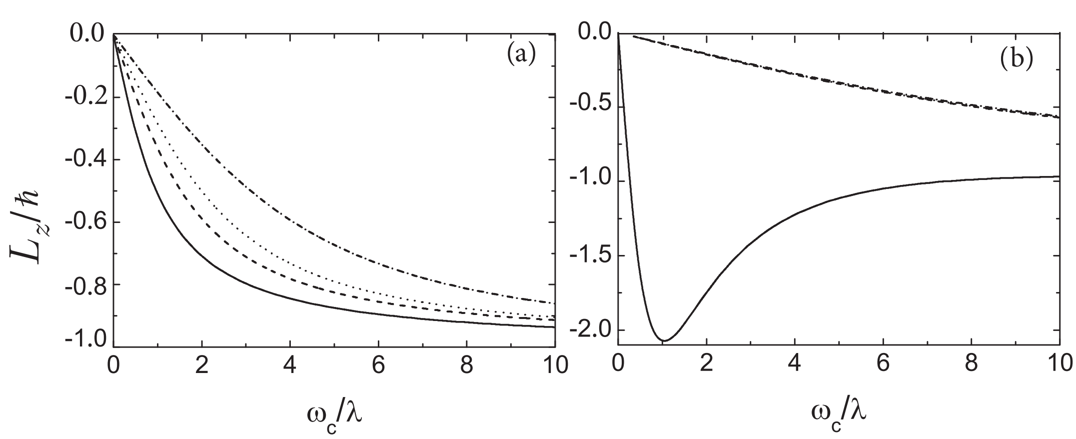

FIG. 1. Calculated magnetic-field dependence of the asymptotic z component L_z of angular momentum at temperature $T/(\hbar\lambda) = 0.1$ (a) or $T/(\hbar\lambda) = 2$ (b) and fixed $\lambda_x = \lambda_y = \lambda$, $\gamma/\lambda = 12$. The solid, dashed, dotted, and dash-dotted lines correspond to the cases with $\omega_0/\lambda = 0$, 0.5, 1, and 2, respectively.

quantum particle through its contribution to the Lorentz force. The dissipation and external magnetic field do affect each other due to the non-Markovian dynamics of the quantum system. The combined action of the constant magnetic field and random forces leads to the emergence of angular momentum. For the two-dimensional charged quantum harmonic oscillator in the uniform magnetic field, we demonstrated the survival of diamagnetism of the system in the presence of realistic heat bath at low and high temperatures. In the dissipative environment and uniform magnetic field with $\omega_c \approx \lambda$, the average angular momentum of a free particle considerably exceeds the average angular momentum of a particle in the harmonic oscillator. In the damped harmonic oscillator, the orbital magnetic moment or angular momentum approaches the quantization limit at $T \rightarrow 0$ and $B \rightarrow \infty$.

For a rotating system confined by a harmonic-oscillator potential, the cyclotron frequency ω_c would have to replaced with the rotating frequency ω_{rot} in Eq. (1). A similar diamagnetic effect as in the uniform magnetic field is expected for the rotating system coupled to the bosonic heat bath. This Hamiltonian with more complicated collective potential can be used for the description of the Bose-Einstein condensation in the rotating frame and in the environment.

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