Comment on "Gain-assisted superluminal propagation and rotary drag of photon and surface plasmon polaritons"

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In their study of superluminal propagation, rotary drag, and surface polaritons [Khan *et al.*, Phys. Rev. A **96**, 013848 (2017); **96**, 049906(E) (2017)], the authors consider a four-level atomic arrangement with transitions in the optical domain. In fact, the values they give to the parameters lead to a probe wavelength lying in the decimeter band and we point out that, in such conditions, all their results are irrelevant.

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In their study of superluminal propagation, rotary drag, and surface polaritons [1,2], Khan *et al.* consider a four-level atomic arrangement with transitions in the optical domain. See Fig. 1(a) in Ref. [1]. On the other hand, they specify in Ref. [2] that all the (angular) frequencies are given in units of $\gamma = (2\pi) \times 1\,\text{MHz}$ and that the probe frequency $\nu_p = 1000\gamma$. The corresponding wavelength is thus $\lambda_p = 30\,\text{cm}$ (in the decimeter band). As shown in the following this invalidates all the results given in Refs. [1,2].

As correctly given in Ref. [1], the electric susceptibility for the probe reads, in SI units,

$$\chi = \frac{2N|\wp_{ac}|^2\rho_{ac}}{\varepsilon_0\hbar\Omega_p}, \tag{1}$$

where N is the atomic number density, a (c) is the upper (lower) level of the probe transition, $\wp_{ac}$ (ρ_{ac}) is the corresponding matrix element of the dipole moment (of the density operator), and Ω_p is the Rabi (angular) frequency of the probe. Expressing the susceptibility as a function of the probe wavelength as made to obtain Eq. (5) in Refs. [1,2] can be achieved by introducing the Einstein's coefficient A_{ac} associated with the transition $a \to c$. From its expression given in Ref. [3], we get

$$|\wp_{ac}|^2 = \left(\frac{3\lambda_p^3}{8\pi^2}\right)\hbar\varepsilon_0 A_{ac}, \tag{2}$$

and finally

$$\chi = \left(\frac{3N\lambda_p^3}{32\pi^3}\right)\left(\frac{8\pi A_{ac}}{\Omega_p}\right)\rho_{ac}. \tag{3}$$

The expression $\chi = (\frac{3N\lambda_p^3}{32\pi^3\Omega_p})\rho_{ac}$ given by Eq. (5) in Ref. [2] thus holds only if Ω_p is expressed in units of $8\pi A_{ac}$. According to the above choice of γ as unit of (angular) frequency, this implies that $8\pi A_{ac} = \gamma$.

It is specified in Ref. [2] that "the susceptibility and group index plotted versus probe detuning have units of $2N|\wp_{ac}|^2/\varepsilon_0\hbar$." As shown in Eq. (1), this quantity has the dimension of an angular frequency and, for consistency, it should also be expressed in units of γ. It then reads $u_\chi = 2N|\wp_{ac}|^2/\varepsilon_0\hbar\gamma$ and, taking into account the above relations,

$$u_\chi = \left(\frac{3N\lambda_p^3}{32\pi^3}\right). \tag{4}$$

For wavelengths λ_p in the visible domain and typical values of the atomic number density N, the susceptibility unit u_χ given by Eq. (4) is in the order of 3×10^{-3}. On the other hand, for $\lambda_p = 30\,\text{cm}$ with $N = 5 \times 10^{12}\,\text{cm}^{-3}$ as considered in Ref. [2], this unit rises to $u_\chi \approx 4 \times 10^{14}$. Figure 2 in Ref. [2] shows that the peak value of the *relative* susceptibility χ/u_χ can exceed 5×10^{-3}. The corresponding *absolute* susceptibility χ is then in the order of 10^{12}. *Such values are meaningless.*

Although this point is less important, we note that, in SI units, the refractive index reads $n = \sqrt{1+\chi}$ and not $n = \sqrt{1+4\pi\chi}$ as used in Ref. [1] to determine the group index. Anyway, the approximation $n \approx 1 + 2\pi\chi$ also made to obtain Eq. (6) in Ref. [1] fails when $|\chi| \gg 1$.

Without examining in detail the parts of [1,2] devoted to rotary drag and surface polaritons, we remark that these phenomena occur when the sample thickness L is large compared to the probe wavelength λ_p. According to [2], $L = 10\,\text{cm}$ and this condition is far from being fulfilled since this thickness is only one-third of the probe wavelength. By the way, we also note the incompatibility of Figs. 3(b) and 4 in Ref. [2] which show rotary drags, respectively, in the order of 10^{-2} and 10^{-7} rad.

Khan *et al.* support their choice of the ratio $\nu_p/\gamma = 1000$ by referring to a paper on the phase control of light velocity [4]. The same ratio ν_p/γ was actually considered in this paper but without specifying the absolute value of the frequencies. We, however, point out that, for a probe frequency in the visible domain, this ratio leads to lifetimes of the excited atomic levels which are fully unrealistic (in the subpicosecond domain).

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Independently of the above criticisms, we remark that, quite generally, large negative group delays are not a sufficient condition to observe significant effects of superluminal propagation. A convincing demonstration of such effects would have required a comparison of the transmitted and incident pulses, which is not made in [1,2].

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