Quantum model of decoherence in the polarization domain for the fiber channel

Anton Kozubov,* Andrei Gaidash, and George Miroshnichenko
Department of Photonics and Optical Information Technology, ITMO University, 199034 Kadetskaya Line 3b, Saint Petersburg, Russia and Quanttelecom Ltd., 199034 Birzhevaya Line 14, Saint Petersburg, Russia

(Received 1 March 2019; published 28 May 2019)

In this work we consider the Liouville equation; it describes the dynamics of the photon density matrix in the Schrödinger representation based on the Markov approximation in the channel without dispersion. The equation contains a relaxation superoperator dependent on the phenomenological parameters of the optical fiber. These parameters allow one to take into account the phenomena of birefringence and optical activity, isotropic absorption, and dichroism. We also present in our work that these parameters affect not only the polarization of the states but the length of the Stokes vector. Hence the developed technique describes the decoherence process in the polarization domain in the quantum case and allows one to analyze the dynamics of single-photon states in the quantum (depolarizing) channel more properly. We also present a visual illustration of polarization states' evolution in the polarization-coded quantum key distribution BB84 protocol as an example. We estimate quantum bit error rates' dependence on channel length. Also we examine maximal allowed channel length, dependent on various configurations of channel parameters.

DOI: 10.1103/PhysRevA.99.053842

I. INTRODUCTION

Any quantum state is subject to decoherence. According to one of the definitions, decoherence is nonunitary dynamics, which is a consequence of the connection of the quantum system with the external environment [1–3]. External influences on an optical fiber, such as temperature fluctuations, vibrations, bends, torsions, as well as light scattering on random inhomogeneities of the refractive index of the optical fiber and impurities inside it, lead to a change of dielectric permittivity tensor in the optical fiber. The dielectric permittivity tensor in the general case has real and imaginary parts and is anisotropic [4–7]. The real part of the tensor determines the birefringence phenomenon; the imaginary part of the tensor is responsible for dichroism in the optical fiber. Both birefringence and dichroism depend on the frequency of the signal. In optical fibers polarization mode dispersion and polarization dependent losses [8–13] are well studied. We would like to stress that polarization effects can be studied both in linear and nonlinear regimes [14–16]. In this work we focus on the polarization effects in the linear regime for weak (quantum) fields.

It should be noted that methods of dealing with the decoherence of quantum states are constantly being improved. In [17,18], the authors propose to fight decoherence using error correction and error rejection quantum codes. A protocol for suppressing decoherence in the polarization domain is proposed in [19,20]. The authors note that birefringence fluctuates due to the changing environment, temperature, vibrations. In practice, birefringence remains constant for sufficiently long sequences of pulses. In such sequences each pulse is subject to the same polarization perturbation. This channel noise is called collective noise. In these works it was shown that, under the action of collective noise, polarization and time-bin degrees of freedom are not influenced by decoherence and form decoherence-free subspaces [20]. Based on the general theory of relaxation phenomena [21], a photon propagating through an optical fiber should be considered a quantum dynamical system interacting with the external environment. Correlation times of environmental variables can be very short. As a consequence, the dynamics of the photon density matrix in an optical fiber should be described by the Liouville equation containing the relaxation operator in the Markov approximation. A model of decoherence in the polarization domain was proposed in [22]. In this paper, the dynamics of a single photon in a dispersing medium are considered. The length of the Stokes vector may decrease due to nonuniform broadening and dispersion phenomena. In [19], the proposed relaxation operator describes a nonunitary transformation of the photon density matrix in the quantum channel in the Markov approximation.

Nevertheless, for description of quantum channels (especially in the case of quantum key distribution) the trivial model of the depolarizing quantum channel is still commonly used [23–25].

In our work we pursue two main goals. The first goal is to study the dynamics of a single-photon state in a single-mode fiber, whose dielectric constant tensor has anisotropy and dichroism. The Liouville equation is proposed, describing the development of the photon density matrix in the Schrödinger picture considering the Markov approximation. The equation contains a relaxation operator dependent on the phenomenological parameters. These parameters make it possible to take into account the phenomena of birefringence, isotropic absorption, and dichroism. The second goal of our work is to analyze the effect of decoherence in the polarization domain, described by the model Liouville equation, on errors

*avkozubov@itmo.ru

(caused by depolarization) produced by optical fiber quantum channels.

The paper organized as follows. In Sec. II we propose the Liouville equation for the photon density matrix propagating along the optical fiber. The decoherence process in the polarization domain is described using the relaxation superoperator. The exact solution of the equation for the photon density matrix in the Schrödinger representation is also obtained. In Sec. III we consider the polarization-coded BB84 quantum key distribution protocol in terms of the developed approach as an example. Section IV concludes the article.

II. PHENOMENOLOGICAL MODEL

It is assumed that the ideal single-mode optical fiber is isotropic and uniform. But it is obvious that various imperfections, such as torsion, stretching, bending, and temperature and density fluctuations, distort the dielectric permittivity tensor. As a result, the orthogonal quantum states of the photon transform into each other, i.e., "interact." We parametrize the Hamiltonian of interaction $\hat{V}$ with real vector $\boldsymbol{\xi} = \xi \boldsymbol{n}$ [26,27]. We span our space on the three basis vectors $\{|0\rangle, |H\rangle, |V\rangle\}$, where $|0\rangle, |H\rangle, |V\rangle$ are the vacuum state and the horizontally and vertically polarized states of the photon respectively.

In this basis, the interaction operator matrix has the form

$$\hat{V} = \frac{\xi}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & n_z & n_x - in_y \\ 0 & n_x + in_y & -n_z \end{pmatrix}, \quad (1)$$

where $\boldsymbol{n} = \frac{\boldsymbol{\xi}}{|\boldsymbol{\xi}|}$ is the direction vector in the Stokes-Poincaré coordinate system. The Liouville equation for the density matrix of the mixed state (channel density operator) of a photon in a quantum channel affected by anisotropic decoherence in the polarization domain is written as

$$\frac{\partial}{\partial t}\rho(t) = -i[\hat{V}, \rho(t)] + \hat{\Gamma}\rho(t), \quad (2)$$

where $\hat{\Gamma}$ is the superoperator of relaxation in the Markov approximation and it describes the decoherence phenomena. The equations for the matrix elements $\rho_{01}(t)$ and $\rho_{02}(t)$ (here we assume that indexes 0, 1, 2 corresponded to $|0\rangle, |H\rangle, |V\rangle$ accordingly) can be expressed as

$$i\frac{d}{dt}\rho_{01}(t) = -\rho_{01}(t)V_{11} - \rho_{02}(t)V_{21} - i\gamma_{01}\rho_{01}(t), \quad (3)$$

$$i\frac{d}{dt}\rho_{02}(t) = -\rho_{02}(t)V_{22} - \rho_{01}(t)V_{21} - i\gamma_{02}\rho_{02}(t), \quad (4)$$

where γ_{01} and γ_{02} determine the decay rate of off-diagonal elements. Choosing nullified initial conditions, we obtain

$$\rho_{01}(t) = \rho_{02}(t) = 0. \quad (5)$$

The equations for matrix diagonal elements are

$$\frac{d}{dt}\rho_{00}(t) = \gamma\delta(t), \quad (6)$$

$$\frac{d}{dt}\delta(t) = -\gamma\delta(t), \quad (7)$$

where γ denotes the rate of photon absorption in the optical fiber and $\delta(t) = \rho_{11} + \rho_{22}$. The exact solution of the equation has the form

$$\delta(t) = \exp(-\gamma t)\delta(0), \quad (8)$$

where t denotes the time of the photon propagation in the channel. Let us assume $v_{gr} = 1$, where v_{gr} is the group velocity of the signal in the optical fiber. Thus we match the length of the channel with the propagation time as $L = t$. We will represent the photon density matrix in the optical fiber channel using the Schrödinger representation as follows:

$$\rho(t) = [1 - \delta(t)]|0\rangle\langle 0| + \frac{\delta(t)}{2}[|H\rangle\langle H| + |V\rangle\langle V| + (\boldsymbol{P}(t), \hat{\boldsymbol{\sigma}})], \quad (9)$$

where vector operator components of $\hat{\boldsymbol{\sigma}}$ have the form

$$\begin{aligned} \sigma_x &= |H\rangle\langle V| + |V\rangle\langle H|, \\ \sigma_y &= i(|V\rangle\langle H| - |H\rangle\langle V|), \\ \sigma_z &= |H\rangle\langle H| - |V\rangle\langle V|. \end{aligned} \quad (10)$$

The vector operator $\hat{\boldsymbol{\sigma}}$ is the Stokes operator, whose mean values are the Stokes parameters according to [28,29]. So let $\boldsymbol{P}(t)$ be the Stokes vector, and it is denoted as

$$\boldsymbol{P}(t) = \mathrm{Tr}(\hat{\boldsymbol{\sigma}}\rho(t)). \quad (11)$$

Due to the noncommutativity of the projections of the vector $\hat{\boldsymbol{\sigma}}$, there is no quantum state of the photon where the components of the vector $\boldsymbol{P}(t)$ would not have dispersion (with respect to its mean value). Let us define the action of the relaxation operator $\hat{\Gamma}$ by introducing two nonparallel unit vectors $\boldsymbol{\mu}^{(1)}, \boldsymbol{\mu}^{(2)}$ and writing the equation for the operator $(\boldsymbol{P}(t), \hat{\boldsymbol{\sigma}})$ as follows:

$$\frac{d}{dt}(\boldsymbol{P}(t), \hat{\boldsymbol{\sigma}}) = -i\frac{\xi}{2}[(\boldsymbol{n}, \hat{\boldsymbol{\sigma}}), (\boldsymbol{P}(t), \hat{\boldsymbol{\sigma}})] - \varepsilon(\boldsymbol{P}(t), \hat{\boldsymbol{\sigma}}) + \sum_{j=1}^{2}\frac{\beta_j}{4}[(\boldsymbol{\mu}^{(j)}, \hat{\boldsymbol{\sigma}}), [(\boldsymbol{\mu}^{(j)}, \hat{\boldsymbol{\sigma}}), (\boldsymbol{P}(t), \hat{\boldsymbol{\sigma}})]]. \quad (12)$$

We denote this operator the double-axis relaxation operator, referring to the similar term in crystal optics. For the sake of simplicity further we consider the single-axis relaxation operator assuming $\boldsymbol{\mu}^{(1)} = \boldsymbol{\mu}^{(1)} = \boldsymbol{\mu}$. Using properties of the operator $\hat{\boldsymbol{\sigma}}$, we obtain the expression

$$\frac{d}{dt}\boldsymbol{P}(t) = \xi[\boldsymbol{n} \times \boldsymbol{P}(t)] - \varepsilon\boldsymbol{P}(t) + \beta[\boldsymbol{\mu} \times [\boldsymbol{\mu} \times \boldsymbol{P}(t)]]. \quad (13)$$

We will use this phenomenological equation (analogous to the modified Bloch equation [30]) to describe photon state evolution, for instance, in nonideal optical fiber (or other optical media in general). The first term of the equation is parametrized by vector $\boldsymbol{\xi}$ and describes the polarization evolution without changes in length of the Stokes vector $\boldsymbol{P}(t)$. The second term describes isotropic decoherence rates ε; the length of Stokes vector $\boldsymbol{P}(t)$ decreases without changes in direction. The third term is parametrized by unit vector $\boldsymbol{\mu}$ and rates β; it describes anisotropic polarization decoherence. The third term nullifies the Stokes vector's component codirected with $\boldsymbol{\mu}$. The length of the Stokes vector satisfies the inequality $0 \leqslant |\boldsymbol{P}(t)| \leqslant 1$. The photon is fully polarized if $|\boldsymbol{P}(t)| = 1$ and fully depolarized if $|\boldsymbol{P}(t)| = 0$. The latter means that the photon does not contain any polarization-coded information. The contribution of the first term might be compensated with a standard feedback polarization controller unit. However, the decoherence process cannot be compensated in any way. Thus we are more interested in further investigation neglecting the first term (neglecting unitary evolution and assuming one may compensate for corresponding effects, hence focusing only on the relaxation process), i.e., assuming $\xi = 0$, and obtaining the expression

$$\frac{d}{dt}\boldsymbol{P}(t) = M\boldsymbol{P}(t), \tag{14}$$

where the solution is

$$\boldsymbol{P}(t) = \exp{(Mt)}\boldsymbol{P}(0), \tag{15}$$

where matrix M is derived as

$$M = -\beta\begin{pmatrix} \mu_z^2+\mu_y^2+\frac{\varepsilon}{\beta} & -\mu_x\mu_y & -\mu_z\mu_x \\ -\mu_x\mu_y & \mu_z^2+\mu_x^2+\frac{\varepsilon}{\beta} & -\mu_z\mu_y \\ -\mu_z\mu_x & -\mu_z\mu_y & \mu_y^2+\mu_x^2+\frac{\varepsilon}{\beta} \end{pmatrix}. \tag{16}$$

Eigenvalues and eigenvectors of the matrix M are

$$\begin{aligned} &\lambda_1 = -\varepsilon, \quad \boldsymbol{\phi}_1 = \boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \\ \mu_z \end{pmatrix}, \\ &\lambda_2 = -\varepsilon-\beta, \quad \boldsymbol{\phi}_2 = \frac{1}{\sqrt{\mu_z^2+\mu_x^2}}\begin{pmatrix} \mu_z \\ 0 \\ -\mu_x \end{pmatrix}, \\ &\lambda_3 = -\varepsilon-\beta, \quad \boldsymbol{\phi}_3 = \frac{1}{\sqrt{\mu_z^2+\mu_x^2}}\begin{pmatrix} -\mu_y\mu_x \\ \mu_z^2+\mu_x^2 \\ -\mu_z\mu_y \end{pmatrix}. \end{aligned} \tag{17}$$

In order to experimentally observe the polarization transformations induced by the considered relaxation process, one may implement standard quantum measurements of Stoke's operators for single photons, e.g., as in [31,32], implying that all the polarization transformation related to unitary evolution is to be compensated.

III. EXAMPLE OF PRACTICAL IMPLEMENTATION

Let us consider the case of the famous BB84 quantum key distribution protocol, where one uses four polarization states $|\chi_n\rangle$ as follows:

$$\begin{aligned} &|\chi_1\rangle = |H\rangle, \quad |\chi_2\rangle = |V\rangle, \\ &|\chi_3\rangle = |S\rangle = \frac{|H\rangle+|V\rangle}{\sqrt{2}}, \\ &|\chi_4\rangle = |F\rangle = \frac{|H\rangle-|V\rangle}{\sqrt{2}}, \end{aligned} \tag{18}$$

hence the unconditioned channel density operator is

$$\rho = \frac{1}{4}\sum_{n=1}^{4}\rho^{(n)}, \tag{19}$$

$$\rho^{(n)} = [1-\delta(0)]|0\rangle\langle 0| + \delta(0)|\chi_n\rangle\langle\chi_n|, \tag{20}$$

where $\delta(0)$ defines the contribution of the vacuum states. Initial Stokes vectors $\boldsymbol{P}^{(n)}(0)$ have the following form:

$$\boldsymbol{P}^{(1)}(0) = -\boldsymbol{P}^{(3)}(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{21}$$

$$\boldsymbol{P}^{(2)}(0) = -\boldsymbol{P}^{(4)}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{22}$$

Let us express initial states in terms of basis (17) and utilize solutions (8) and (15) in order to derive vectors $\boldsymbol{P}^{(n)}(t)$ as follows:

$$\boldsymbol{P}^{(1)}(t) = -\boldsymbol{P}^{(3)}(t) = e^{(-\varepsilon t)}\begin{pmatrix} \mu_x\mu_z[1-\exp(-\beta t)] \\ \mu_y\mu_z[1-\exp(-\beta t)] \\ \mu_z^2+\exp(-\beta t)\left(1-\mu_z^2\right) \end{pmatrix}, \tag{23}$$

$$\boldsymbol{P}^{(2)}(t) = -\boldsymbol{P}^{(4)}(t) = e^{(-\varepsilon t)}\begin{pmatrix} \mu_x^2+\exp(-\beta t)\left(1-\mu_x^2\right) \\ \mu_y\mu_x[1-\exp(-\beta t)] \\ \mu_x\mu_z[1-\exp(-\beta t)] \end{pmatrix}. \tag{24}$$

Vectors $\boldsymbol{P}^{(n)}(t)$ are conveniently described in a spherical Stokes-Poincaré coordinate system with an axis directed along the y axis with a zenith angle 2Θ and azimuth angle 2Φ:

$$\begin{aligned} P_z(t) &= |\boldsymbol{P}(t)|\cos[2\Phi(t)]\cos[2\Theta(t)], \\ P_x(t) &= |\boldsymbol{P}(t)|\cos[2\Phi(t)]\sin[2\Theta(t)], \\ P_y(t) &= |\boldsymbol{P}(t)|\sin[2\Theta(t)]. \end{aligned} \tag{25}$$

We consider the features of anisotropic decoherence in the particular case where

$$\mu_x = -\mu_z = \frac{1}{\sqrt{2}}. \tag{26}$$

In this case $\Theta(t)$ and the vectors $\boldsymbol{P}^{(n)}(t)$ are located on the plane xz. By analogy with classical optics, such photon states should be called linearly polarized:

$$|\chi\rangle = |V\rangle\sin{(\Phi(t))} + |H\rangle\cos{(\Phi(t))}. \tag{27}$$

Lengths (absolute values) of the $\boldsymbol{P}^{(n)}(t)$ vectors (degree of the polarization) are equal and are expressed as

$$|\boldsymbol{P}^{(n)}(t)| = \frac{1}{\sqrt{2}}\exp{(-\varepsilon t)}\sqrt{1+\exp{(-2\beta t)}}. \tag{28}$$

Angle $\Phi(t)$ does not depend on ε. The latter defines the isotropic contribution to the polarization decoherence process, and β defines the rotation of the polarization plane (xz plane). For instance, two regimes of polarization decoherence in

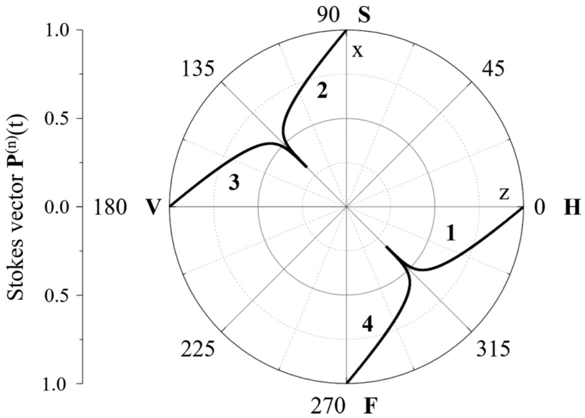

FIG. 1. Vector trajectories $\boldsymbol{P}^{(n)}(t)$ presented on the xz Stokes-Poincaré plane with solid black lines. Numbers $n = 1, 2, 3, 4$ mark vector numbers. These dependences were obtained using Eqs. (23), (24), (25) conditioned by Eq. (26). Here the channel is parametrized as $\frac{\beta}{\varepsilon} = 10$, and $\tau = \varepsilon t$ is the dimensionless channel length ($\tau \leqslant 1$). H, V, S, F denote the starting points of the trajectories and correspond to the pure states from Eq. (18).

the xz polarization plane of the Stokes-Poincaré coordinate system are shown in Figs. 1 and 2. Trajectories of $\boldsymbol{P}^{(n)}(t)$ in the case of $\varepsilon < \beta$ are shown in Fig. 1, where rotation of the polarization plane is the prevailing process. Otherwise, decrease of $\boldsymbol{P}^{(n)}(t)$ vectors lengths is the more prevalent process compare to the rotation of the polarization plane.

The probabilities of detecting the correct quantum bit and the one with bitflip are derived correspondingly as follows:

$$\mathcal{P}_{0,0} = \mathcal{P}_{1,1} = \frac{1+B}{4}, \quad \mathcal{P}_{0,1} = \mathcal{P}_{1,0} = \frac{1-B}{4}, \tag{29}$$

where

$$B = \tfrac{1}{2}[1 + \exp(-\beta t)] \exp(-\varepsilon t). \tag{30}$$

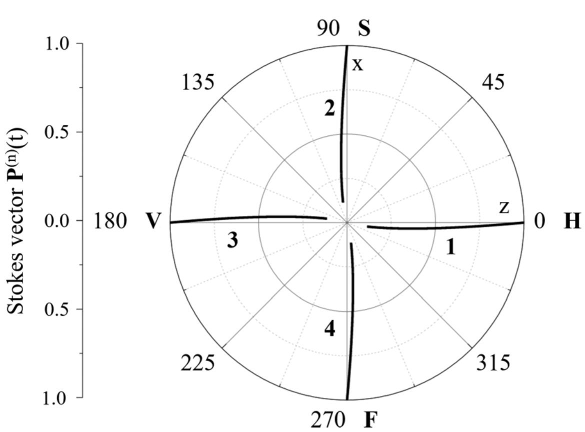

FIG. 2. Vector trajectories $\boldsymbol{P}^{(n)}(t)$ presented on the xz Stokes-Poincaré plane with solid black lines. Numbers $n = 1, 2, 3, 4$ mark vector numbers. These dependences were obtained using Eqs. (23), (24), (25) conditioned by Eq. (26). Here the channel is parametrized as $\frac{\beta}{\varepsilon} = 0.2$, and $\tau = \varepsilon t$ is the dimensionless channel length ($\tau \leqslant 1$). H, V, S, F denote the starting points of the trajectories and correspond to the pure states from Eq. (18)

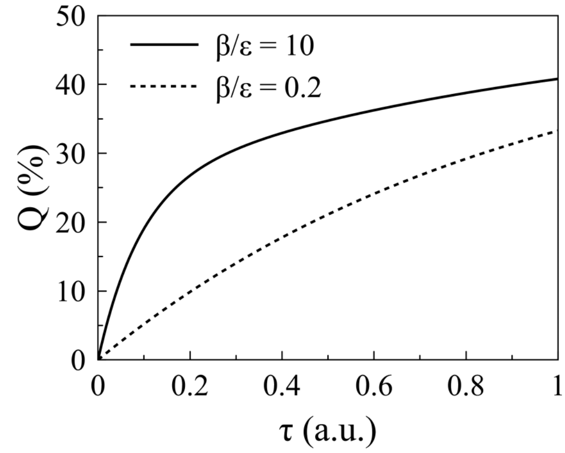

FIG. 3. Quantum bit error rate $\mathcal{Q}$ values dependent on dimensionless channel length $\tau = \varepsilon t$ ($\tau \leqslant 1$), with its properties equal to cases of depolarization in Figs. 1 and 2, i.e., $\beta/\varepsilon = 10$ and $\beta/\varepsilon = 0.2$ respectively.

It should be noted that we assume an ideal single photon source and detector in order to investigate the impact of only depolarization on the quantum key generation process.

Hence, the quantum bit error rate (QBER) can be estimated as

$$\mathcal{Q} = \frac{\mathcal{P}_{0,1} + \mathcal{P}_{1,0}}{\mathcal{P}_{0,1} + \mathcal{P}_{1,0} + \mathcal{P}_{0,0} + \mathcal{P}_{1,1}} = \frac{1-B}{2}. \tag{31}$$

One may examine possible QBER values in Fig. 3 with channel properties equal to cases of depolarization in Figs. 1 and 2, i.e., $\beta/\varepsilon = 10$ and $\beta/\varepsilon = 0.2$ respectively. Further, we demonstrate the critical channel length τ_{crit} [where $\tau = \varepsilon t$ ($\tau \leqslant 1$) is dimensionless channel length], i.e., where $1 - 2h(Q(\tau_{crit})) = 0$ [33], dependent on different channel configurations (different β/ε) in Fig. 4, where $h(x)$ is the binary entropy function.

It was found that the critical channel length $\tau_{\rm crit}$ corresponds to an approximately 20% decrease of the Stokes vector

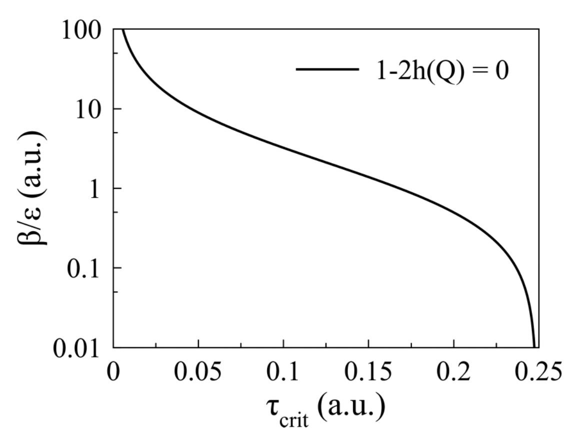

FIG. 4. Critical channel length $\tau_{\rm crit}$, dependent on different values of the channel parameters' relation β/ε. Here $\tau = \varepsilon t$ ($\tau \leqslant 1$) is the dimensionless channel length and $\tau_{\rm crit}$ is the critical channel length where the secure key rate drops to zero, i.e., where $1 - 2h(Q(\tau_{\rm crit})) = 0$ [33], where $h(x)$ is the binary entropy function.

length; it should be less than this value in order to maintain a positive secret key rate.

IV. CONCLUSION

In this paper we propose and investigate the model Liouville equation, see Eq. (2), for the density matrix of the mixed state of a photon in a quantum channel, which is subject to anisotropic decoherence in the polarization domain. The equation takes into consideration the anisotropy of the real part of the dielectric constant (vector $\xi \boldsymbol{n}$) and relaxation superoperator $\hat{\Gamma}$. The latter is parameterized by the decay rates of off-diagonal elements γ_{01}, γ_{02}, photon absorption rate γ, isotropic decoherence rate ε, and vector $\beta \boldsymbol{\mu}$ considering decoherence anisotropy in the polarization domain. These parameters can be interpreted as the characteristics of a complex process of photon states' decoherence in an optical fiber. As shown in our work, these parameters affect not only the polarization of the states but the length of the Stokes vector (see Figs. 1 and 2). The constructed model allows one to take into consideration a more realistic and specific description of the depolarizing fiber channel. As an example, we estimate the impact of quantum states decoherence on the performance of the polarization-coded quantum key distribution protocol BB84. Quantum bit error rates' dependence on channel length (see Fig. 3) and critical (maximal allowed) channel length dependence on channel configuration (see Fig. 4) are presented.

ACKNOWLEDGMENTS

This work was financially supported by the Government of Russian Federation (Grant 08-08). All authors contributed equally to the work.

[1] D. A. Lidar and K. B. Whaley, in *Irreversible Quantum Dynamics* (Springer, Berlin, 2003), pp. 83–120.
[2] R. Omnes, Phys. Rev. A **56**, 3383 (1997).
[3] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[4] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
[5] M. O. Scully and M. Suhail Zubiary, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
[6] M. Born and E. Wolf, *Principles of Optics*, 4th ed. (Pergamon, Oxford, 1968).
[7] V. M. Agranovič and V. L. Ginzburg, *Kristallooptika s učetom prostranstvennoj dispersii i teorija eksitonov* (Nauka, Moscow, 1965).
[8] N. Gisin and B. Huttner, Opt. Commun. **142**, 119 (1997).
[9] H. Dong, P. Shum, M. Yan, J. Zhou, G. Ning, Y. Gong, and C. Wu, Opt. Commun. **274**, 116 (2007).
[10] L. Palmieri, J. Lightwave Technol. **24**, 4075 (2006).
[11] S. Rashleigh and R. Ulrich, Opt. Lett. **3**, 60 (1978).
[12] C. D. Poole, J. H. Winters, and J. A. Nagel, Opt. Lett. **16**, 372 (1991).
[13] D. A. Nolan, X. Chen, and M.-J. Li, J. Lightwave Technol. **22**, 1066 (2004).
[14] H. Zhang, D. Y. Tang, L. M. Zhao, and X. Wu, Phys. Rev. B **80**, 052302 (2009).
[15] H. Zhang, D. Tang, L. Zhao, and R. Knize, Opt. Express **18**, 4428 (2010).
[16] H. Zhang, D. Tang, L. Zhao, and X. Wu, Opt. Express **19**, 3525 (2011).
[17] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997).
[18] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[19] C. Han, Z.-W. Zhou, and G.-C. Guo, J. Phys. B **39**, 1677 (2006).
[20] Z. D. Walton, A. F. Abouraddy, A. V. Sergienko, B. E. A. Saleh, and M. C. Teich, Phys. Rev. Lett. **91**, 087901 (2003).
[21] U. Fano, Rev. Mod. Phys. **29**, 74 (1957).
[22] N. Gisin, J. Brendel, J.-D. Gautier, B. Gisin, B. Huttner, G. Ribordy, W. Tittel, and H. Zbinden, *Decoherence: Theoretical, Experimental, and Conceptual Problems* (Springer, Berlin, 2000), pp. 191–200.
[23] K. Tamaki, M. Koashi, and N. Imoto, Phys. Rev. Lett. **90**, 167904 (2003).
[24] M. Christandl, R. Renner, and A. Ekert, arXiv:quant-ph/0402131.
[25] D. Sutter, V. B. Scholz, A. Winter, and R. Renner, IEEE Trans. Inf. Theory **63**, 7832 (2017).
[26] G. Miroshnichenko, Opt. Spectrosc. **112**, 777 (2012).
[27] G. Miroshnichenko and A. Sotnikova, Opt. Spectrosc. **112**, 327 (2012).
[28] N. Korolkova, G. Leuchs, R. Loudon, T. C. Ralph, and C. Silberhorn, Phys. Rev. A **65**, 052306 (2002).
[29] J. Lehner, U. Leonhardt, and H. Paul, Phys. Rev. A **53**, 2727 (1996).
[30] R. M. White, R. M. White, and B. Bayne, *Quantum Theory of Magnetism* (Springer, Berlin, 1983), Vol. 1.
[31] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A **64**, 052312 (2001).
[32] D. Klyshko, J. Exp. Theor. Phys. **84**, 1065 (1997).
[33] P. W. Shor and J. Preskill, Phys. Rev. Lett. **85**, 441 (2000).