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Cavity-QED simulator of slow and fast scrambling

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We study information scrambling, as diagnosed by the out-of-time order correlations (OTOCs), in a system of large spins collectively interacting via spatially inhomogeneous and incommensurate exchange couplings. The model is realizable in a cavity QED system in the dispersive regime. Fast scrambling, signaled by an exponential growth of the OTOCs, is observed when the couplings do not factoriZe into the product of a pair of local interaction terms, and at the same time the state of the spins points initially coplanar to the equator of the Bloch sphere. When one of these conditions is not realized, OTOCs grow algebraically with an exponent sensitive to the orientation of the spins in the initial state. The impact of initial conditions on the scrambling dynamics is attributed to the presence of a global conserved quantity, which critically slows down the evolution for initial states close to the poles of the Bloch sphere.

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Introduction. Information scrambling, and its intimate relation to quantum chaos and holographic duality [1–3], represents one of the frontiers of research in condensed matter and many-body physics. Out-of-time order correlations (OTOCs) have been raised to the rank of prime quantifiers of information scrambling in this field: for two commuting unitary operators $\hat{W}$ and $\hat{V}$, OTOCs are defined as

$$\mathcal{C}(t) = \langle [\hat{W}(t), \hat{V}(0)]^2 \rangle, \tag{1}$$

with $\hat{W}$ evolving with the Hamiltonian of the system, H. This quantity is currently considered an indicator of the loss of memory of initial conditions in a quantum system. Specifically, $\mathcal{C}(t)$ measures the overlap between two states: one state is prepared through the subsequent application of $\hat{V}$ at time $t = 0$ and $\hat{W}$ at a later time t, while the second reverses this procedure in time.

A system which deteriorates information exponentially in time is called a fast scrambler, and the associated scrambling rate, λ, has been regarded as a quantum analog of the Lyapunov exponent, which in classical chaotic systems dictates the rate at which two initially closed trajectories diverge in phase space [4]. In quantum systems, λ, as measured by the OTOC, is bounded, $\lambda \leqslant 2\pi T$ (with T the temperature of the system), as first derived in the context of holographic theory and black holes [1,2,5]. However, in any system with finite Hilbert space dimension, the growth in time of the OTOC is a transient phenomenon before saturation to a constant value occurs, since, using triangular inequalities, the norm of $\mathcal{C}(t)$ in Eq. (1) can be always bounded by the norm of the operators therein involved. Few condensed matter systems scramble fast [6–12], and, to the best of our current knowledge, none saturates the bound on the quantum Lyapunov exponent, λ, with the exception of the so called SYK model, a recent extension by Kitaev of an original model introduced by Sachdev and Ye [13–26], where information is scrambled swiftly as in a black hole. As counterpart, slow scramblers are identified with those systems where out-of-time order correlations grow polynomially in time, with examples ranging from interacting fermions in infinite dimensions [27], to Luttinger liquids [28], to many-body localized systems [29–31], encompassing periodically kicked quantum Ising chains [32] and spin chains driven by noise [33] (for a connection between scrambling and quantum critical points, see also [34,35]).

More recent developments include operator spreading and entanglement growth in quantum random circuits [36–43], the search for velocity-dependent Lyapunov exponents in spatially extended quantum systems [44–46], and a general resurgence of interest in the concept of quantum chaos [47–50]. Despite the proliferation of theoretical studies in the last few years, only few experimental proposals and realizations for the measurement of scrambling are currently available [51], encompassing ion traps [52], Ramsey interferometry [53], cavity QED systems [54], and nuclear magnetic resonance simulators [55–57].

In this work, we consider a quantum many-body simulator of pairwise collectively interacting spins through position dependent couplings, a model realizable with atoms coupled to cavity photons, in the dispersive limit of the large cavity detuning and therefore capable of mediating interactions among the atoms. We find that in our system the scrambling of spin operators grows in a power law fashion, with the noticeable exception of exponential dynamics of the OTOCs starting from spin states pointing close to the $\hat{x}$ direction (or equivalently any one coplanar with the equator of the Bloch sphere), and ruled by many-body interactions which do not factorize into pairs of local interaction couplings (see Fig. 1 for a summary of our results). We use a semiclassical treatment to study the scrambling dynamics, as done in several other models ranging from kicked rotors [58] to classical interacting spin chains [7,59,60]. This is motivated by the observation that the chaotic behavior in the SYK model is essentially of semiclassical nature, as confirmed in Refs. [22,61,62] which

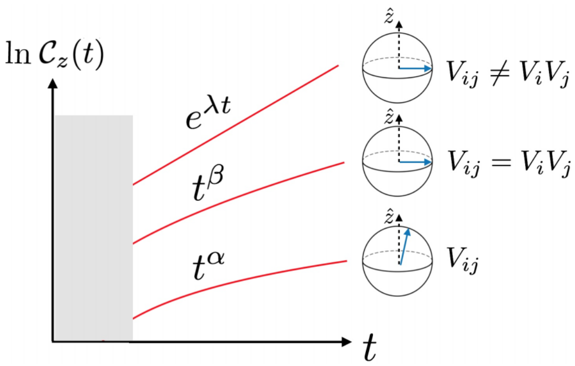

FIG. 1. Sensitivity of the scrambling dynamics to different initial conditions and to the nature of many-body coupling V_{ij}. For nonseparable interactions $V_{ij} \neq V_i V_j$, and for initial spin states pointing close to the equator of the Bloch sphere, information is scrambled fast in $\mathcal{C}_z(t)$, an OTOC computed from J_i^z spin operators [see Eq. (7)], growing exponentially before saturation. In all other instances, the growth of $\mathcal{C}_z(t)$ is algebraic in time ($\alpha \simeq 5/2$ and $\beta \simeq 9/2$).

showed that quantum interference effects renormalize the Lyapunov exponent to values consistent with the bound in Ref. [2].

The model. We consider interacting particles of spin L on the lattice, governed by the Hamiltonian

$$H = \frac{1}{NL} \sum_{i,j=1}^{N} V_{ij} J_i^+ J_j^-, \tag{2}$$

with $i = 1, \dots, N$, and where $J_i^{\pm}$ are the raising and lowering operators of the SU(2) algebra. The prefactors $1/N$ and $1/L$ are introduced to render the Hamiltonian extensive in the thermodynamic limit, $H \propto N$, and scaling as $H \propto L$. Rescaling the spin operators as $\tilde{J}_i^{(\alpha)} \equiv J_i^{(\alpha)}/L$, the commutation relations read ($\alpha, \beta, \gamma = x, y, z$; with i the imaginary unit)

$$\left[\tilde{J}_i^{(\alpha)}, \tilde{J}_j^{(\beta)}\right] = \frac{i}{L} \epsilon^{\alpha\beta\gamma} \delta_{ij} \tilde{J}_j^{(\gamma)}. \tag{3}$$

Since $\tilde{J}_i^{(\alpha)}$ are bounded, Eq. (3) implies that for large L, the Hamiltonian (2) describes the interaction among large, classical spins. Note that the magnitude of the spins is not controlled by the system size, rather by the independent parameter L.

The Hamiltonian (2) can be realized for example by considering a system of alkaline earth atoms exhibiting a long-lived optical transition, loaded in a cavity and tightly trapped in the ground vibrational level of a one-dimensional deep optical lattice, with lattice spacing $\lambda_l/2$, and with N_i atoms per site [63–65]. In the dispersive regime of large cavity detuning (see Supplemental Material (SM) [66]), we can adiabatically eliminate the photons, and derive an effective Hamiltonian in the form (2). We will consider spatially dependent interactions which factorize, $V_{ij} = V_i V_j$ ("separable" interactions), with

$$V_i = v \cos(2\pi i \sigma), \tag{4}$$

and $\sigma = F_{M-1}/F_M$, where $F_M \equiv N$ is the Mth term of the Fibonacci sequence; or "nonseparable" interactions ($V_{ij} \neq V_i V_j$), of the form

$$V_{i,j} = v^2 \cos[2\pi(i-j)\sigma]. \tag{5}$$

This can be realized by considering, in the former case, a cavity which admits a single resonant mode propagating along the one-dimensional lattice with wave vector $k = 2\pi/\lambda_c$, and, in the latter case, a ring cavity, which supports two degenerate running modes [67–69]. In both cases, we assume that the ratio between λ_c and λ_l is incommensurate, and given by $\sigma = \lambda_l/\lambda_c = 2F_M/F_{M+1}$ (see SM for further details).

Out of time order correlations (OTOCs). As customary in spin systems with emerging classical behavior, we introduce N pairs of canonical coordinates (q_i, p_i) with $i = 1, \dots, N$, representing, respectively, on the Bloch sphere, the azimuthal angle, $0 \leqslant q_i < 2\pi$, and the polar angle, $0 < \theta_i \leqslant \pi$, via the relation $p_i = \cos\theta_i$. As anticipated, this constitutes an appropriate effective description of the degrees of freedom of the model, in the limit of a large number of atoms per lattice site ($N_i \simeq L \gg 1$).

In turn, this allows one to rewrite the normalized angular momentum components as

$$\tilde{J}_i^x = \cos q_i \sqrt{1-p_i^2}, \quad \tilde{J}_i^y = \sin q_i \sqrt{1-p_i^2}, \quad \tilde{J}_i^z = p_i, \tag{6}$$

and accordingly one can write the classicalized version of the OTOC for the $\hat{z}$-spin component,

$$\begin{aligned}\mathcal{C}_z &\equiv \left\langle \left[\tilde{J}_i^z(t), \tilde{J}_i^z(0)\right]^2 \right\rangle \\ &= \langle \{p_i(t), p_i(0)\}^2 \rangle = \left\langle \left(\frac{\partial p_i(t)}{\partial q_i(0)} \right)^2 \right\rangle.\end{aligned} \tag{7}$$

In Eq. (7), the quantum mechanical commutator has been converted into a Poisson bracket, given the emerging classical dynamics of (2). Since the main focus of this work is on the slow and fast scrambling properties of the many body simulator in (2), we shall restrict our calculations for the rest of the paper to equal-site correlators for simplicity.

The classical equations of motions for (q_i, p_i) read

$$\begin{aligned}\dot{p}_i &= -\frac{2}{N}\sqrt{1-p_i^2} \sum_l V_{il} \sin(q_i - q_l)\sqrt{1-p_l^2}, \\ \dot{q}_i &= \frac{2}{N} \frac{p_i}{\sqrt{1-p_i^2}} \sum_l V_{il} \cos(q_i - q_l)\sqrt{1-p_l^2},\end{aligned} \tag{8}$$

as they can be straightforwardly derived from the Hamilton-Jacobi equations

$$\dot{p}_i = -\partial\mathcal{H}/\partial q_i, \quad \dot{q}_i = \partial\mathcal{H}/\partial p_i, \tag{9}$$

with

$$\mathcal{H} = \frac{1}{N} \sum_{ij} V_{ij} \exp\left[i(q_i - q_j)\right] \sqrt{\left(1-p_i^2\right)\left(1-p_j^2\right)} \tag{10}$$

the classical limit of H.

The instances of OTOC dynamics discussed in this work are realized starting from a product state of identical coherent states at every lattice site, $|\psi\rangle_{(t=0)} = \prod_{i=1}^N \otimes |q_i, \theta_i\rangle$ (see also SM). The state $|q_i, \theta_i\rangle$ is obtained by rotating on the Bloch sphere the state $|L, -L\rangle_i$ on site i by the pair of angles (q_i, θ_i) (see the SM for the related expression), and, in the large L limit, this corresponds to evolving the effectively classical dynamics of (2) from a set of initial random conditions drawn

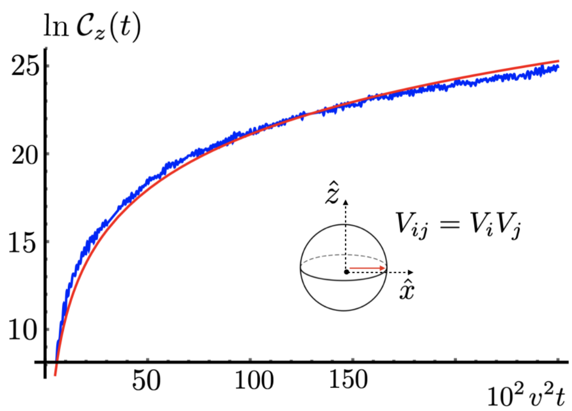

FIG. 2. Slow scrambling is signaled by the algebraic growth of $\mathcal{C}_z(t)$ (blue curve) as a function of time for separable interactions, $V_{ij} = V_i V_j$, and initial state pointing along the $\hat{x}$ direction. Here $N = 55$ sites ($N = F_{10}$) and $R = 500$. The red curve is the semilogarithmic fit, $\ln \mathcal{C}_z(t) \simeq \beta \ln(v^2 t) + C$, with $\beta = 4.55$ and $C = 0.15$ (in the figure we plot the actual evolution over four decades; the logarithm is in natural basis). At longer times (not shown in the plot), $\mathcal{C}_z(t)$ saturates to a constant value.

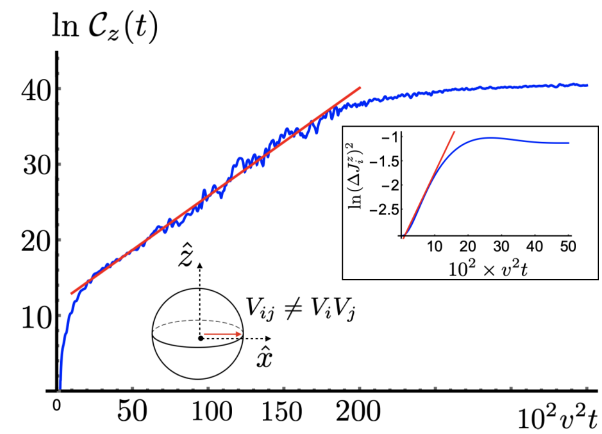

FIG. 3. Fast scrambling is signaled by the exponential growth of $\mathcal{C}_z(t)$ (blue curve) as a function of time for nonseparable couplings, $V_{ij} \neq V_i V_j$, and initial state pointing along the $\hat{x}$ direction. Here $N = 55$ sites ($N = F_{10}$) and $R = 500$. The red curve is the semilogarithmic fit, $\ln \mathcal{C}_z(t) \simeq \lambda v^2 t + C$, with $\lambda = 0.14$ and $C = 11.5$ (in the figure we plot the actual evolution over three decades; the logarithm is in natural basis). At longer times $\mathcal{C}_z(t)$ saturates to a constant value. *Inset*: Semilogarithmic plot of the variance $\ln[(\Delta J_i^z)^2](t) \simeq \lambda' v^2 t + C'$, with $\lambda' = 0.15$ and $C' = -3.3$.

from a certain distribution, as done in semiclassical phase-space methods [70].

For instance, for initial states close to the north pole, one has (see Ref. [70]) that $\langle J_i^z \rangle \simeq L$, $\langle J_i^{x,y} \rangle \simeq 0$ while $\langle (J_i^{x,y})^2 \rangle = L/2$. Following the semiclassical approach, we average over an ensemble of R different trajectories, each one resulting from a different realisation of the random initial condition. Specifically, we calculate $\mathcal{C}_z(t)$ averaged over these R classical trajectories, $\mathcal{C}_z^a(t)$, with $a = 1, \dots, R$, as

$$\mathcal{C}_z(t) = \frac{1}{R} \sum_{a=1}^{R} \mathcal{C}_z^a(t). \tag{11}$$

We have compared the classical dynamics sampled over this initial state distribution with a perturbative quantum mechanical calculation valid at short times, and found quantitative agreement (see SM for details).

Slow scrambling. The OTOCs of the model show at short times a growth $\propto t^2$, as can be shown with perturbation theory (see SM). However, the nonlinear character of interactions changes the time-dependent behavior of the OTOC at later times. In particular, we find that the choice of initial state and the separability of the interaction (or lack of it) influences the temporal dependence of the OTOC. For initial states pointing close the poles of the Bloch sphere, the growth of $C_z(t)$ is power law with $C_z(t) \propto t^{\alpha}$ for $\alpha \simeq 2.5$ irrespectively of the separability of the interaction couplings, V_{ij}. Figure 2 shows a second, qualitatively different instance of slow scrambling in the model: $\mathcal{C}_z(t) \propto t^{\beta}$ with $\beta \simeq 4.5$, for separable interactions and for an initial state pointing along the $\hat{x}$ direction on the Bloch sphere.

Fast scrambling. The nonseparability of the exchange couplings, $V_{ij} \neq V_i V_j$, has important consequences for initial states pointing along directions coplanar with the equator of the Bloch sphere. In this case, the scaling of the OTOCs at intermediate times is exponential, see Fig. 3, indicating that, in order to realize fast scrambling, the spatial structure of the interactions is crucial. Concerning the late-time saturation of the OTOCs, they both reach the same asymptotic value in Figs. 2 and 3: different scrambling properties describe different ways for the OTOCs to relax, but the final steady state value is identical, provided one uses the same initial conditions and operators for the OTOCs.

We now consider initial conditions in the form of tensor products of coherent states with every spin pointing in a direction tilted by a certain polar angle $0 < \vartheta < \pi/2$ with respect to the north pole of the Bloch sphere. Specifically, we want to study how the onset of fast scrambling in $\mathcal{C}_z(t)$ depends on ϑ. In Fig. 4 we report a specific instance for a system of $N = 55$ sites and we find that above a certain critical angle, $\vartheta_c \simeq 0.44\pi$, the out-of-time order correlator $\mathcal{C}_z(t)$ still exhibits exponential growth before saturation, with a

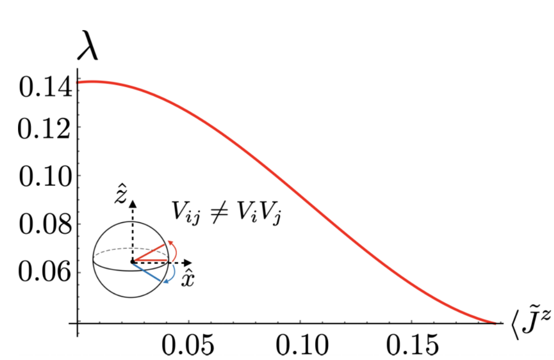

FIG. 4. Dependence of the scrambling exponent, λ, as a function of the initial value of the rescaled magnetization $\langle \tilde{J}_i^z \rangle$ for nonseparable interactions. The red arrow on the Bloch sphere delimits the critical angle ϑ_c, below which scrambling is slow (the blue line and arrow indicate that identical phenomenology holds in the southern hemisphere). λ is extracted from $\mathcal{C}_z(t)$ evaluated on a system of $N = 55$ sites, with $R = 300$.

Lyapunov exponent which vanishes continuously as $\vartheta \to \vartheta_c^+$. Approximatively for $\vartheta \lesssim 0.3\pi$ slow scrambling $\propto t^{5/2}$ takes over again, while, in between these two critical angles, $\mathcal{C}_z(t)$ still grows and saturates, although a neat fit is harder to find. The presence of fast scrambling only for initial states pointing in a neighborhood of the $\hat{x}$ direction can be rationalized by recalling that $J^z \equiv \sum_{i=1}^N \hat{J}_i^z$, the total magnetization along the $\hat{z}$ direction, is a conserved quantity in our model, and for states closer to the north pole dynamics becomes slower; in the limit case of maximal absolute value of $\langle J^z \rangle$ (all spins pointing initially close to the north or south pole) no evolution occurs. In Fig. 4, we plot the dependence of the exponent λ, extracted from $\mathcal{C}_z(t)$, as a function of the expectation value of $\tilde{J}_i^z$ (identical at each site, for the initial conditions we have chosen). We have checked that for the next Fibonacci number (i.e., changing the system's size), $N = F_{11} = 89$, this scenario remains qualitatively unaltered.

Fidelity OTOCs. As recently pointed out in Ref. [46], the *fidelity* OTOC, $\mathcal{C}_G(t)$, obtained setting in Eq. (1) the operator $\hat{V} = |\psi_0\rangle\langle\psi_0|$ equal to the projector on the state $|\psi_0\rangle$, and $\hat{W} = e^{i\delta\phi\hat{G}}$, with a Hermitian operator $\hat{G} = J_i^z$ (in the case we analyze here), reduces for small perturbations, $\delta\phi \ll 1$, to a measure of the variance, $\Delta G^2(t)$, of the observable $\hat{G}$. Specifically, up to second order in $\delta\phi$, we have [46]

$$1 - \mathcal{C}_G(t) = 1 - |\langle\psi_0|e^{i\delta\phi\hat{G}}|\psi_0\rangle|^2 \simeq \delta\phi^2(\langle G^2(t)\rangle - \langle G(t)\rangle^2) \equiv \delta\phi^2\Delta G^2(t). \quad (12)$$

The *fidelity* OTOC thus links fast scrambling with the exponential grow of $\Delta G^2(t)$, and represents the most direct diagnostic of chaotic behavior [4]. The importance of $\mathcal{C}_G(t)$ resides in the fact that variance is an easily accessible quantity, as demonstrated in trapped ion magnets [71] or cavity QED platforms [25,65,72–76].

We have calculated the variance $\Delta G^2(t)$ of initial states pointing along the $\hat{x}$ direction, in the scrambler given by (2) with nonseparable interactions, and we have found that it grows as $\Delta G^2(t) \propto e^{\lambda' t}$, with $\lambda' \simeq 0.15$, and saturates quickly, in agreement with the behavior of $\mathcal{C}_z(t)$ (see also inset of Fig. 3). The fact that the Lyapunov exponents extracted from the OTOC and the variance match is consistent with the picture that the onset of fast scrambling in our model is probably rooted in the presence of an underlying classical chaotic regime for states pointing along the equator of the Bloch sphere.

Perspectives. In summary, our study presents an experimentally viable route for a system where the interplay of initial conditions and the structure of the many-body interactions discriminate between the emergence of slow (algebraic in time) or fast (exponential) information scrambling in the OTOCs dynamics. Notice that upon reducing the number of atoms per lattice site, L, the system enters a regime dominated by quantum fluctuations. Although computations become more challenging, we donot expect that our main conclusions on how separability of interactions gives rise to different scrambling properties will be modified; indeed, in the case of systems with reduced L, the semiclassical dynamics of the OTOCs would most likely represent a saddle point solution which will be dressed by quantum corrections. One of the exciting aspects of our work is the possibility to test these theoretical predictions, and resolve the "quantum scrambling" properties of this model directly in the laboratory.

The results discussed here open the way to the exciting perspective of studying the scrambling properties of cavity QED simulators in the large cooperativity regime [77–79], upon tuning the "level" of randomness [80] in the spin-spin collective interactions, V_{ij}. For instance, it would be interesting to explore the impact of truly disordered interactions on slow scrambling, or the robustness of fast scrambling when perfect periodicity is restored in the spatial structure of the nonseparable many-body interactions [we do not expect that truly disordered couplings may compromise the fast scrambling behavior, since this already occurs with the clean, yet irregular, distribution given by (5)]. Furthermore, it would be worthwhile to explore whether slow and fast scrambling leave signatures on easily accessible observables through measurements of the output cavity field and collective spin distribution.

On the theory side, we believe it would be interesting to study, in the future, the scrambling of extended quantum systems with quasiperiodic (or quasirandom) short range many-body interactions, in order to explore how they would impact on the butterfly velocities and front spreading properties of out-of-time order correlators.

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