Vortex nucleation in nonlocal nonlinear media

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Spontaneous vortex nucleation is a universal feature of open and nonlinear physical systems. We investigate theoretically vortex rings and vortex lines emerging during propagation of self-trapped wave beams in nonlocal nonlinear media. We demonstrate how radially perturbed fundamental solitons exhibit extremely robust and long-lived oscillations with the spontaneous generation of a regular set of vortex rings at the wave beam periphery. We find numerically a class of cylindrically symmetric higher-order spatial solitons and investigate their stability and nonlinear dynamics. The formation of external vortex rings, similar to fundamental soliton, is accompanied by emergence of additional internal vortex-antivortex pairs nucleating from the edge-ring phase dislocation of perturbed higher-order soliton.

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I. INTRODUCTION

Vortex rings are topological structures with a closed-loop core which play a crucial role in the decay of superflow and in quantum turbulence in condensed-matter physics. The spontaneous generation of optical vortex rings was predicted theoretically in self-focusing saturable optical media [1] as a regular sequence of vortex loops perpendicular to the propagation direction of radially perturbed solitons. This spontaneous vortex nucleation is a consequence of the nonlinear phase accumulation between the soliton's peak and its tail: phase singularities nucleate if this phase difference reaches the value of π during evolution along the optical axis z. Similar conclusions regarding the nature of spontaneous vortex nucleation were reached in recent experimental observations of the spatiotemporal optical vortex rings [2]. In contrast to spatiotemporal vortices or vortex rings in fluids, the nonlinear phase of the continuous-wave self-trapped light beam breaks the wave front into a sequence of optical vortex loops *static in time*. In this paper we confirm the generic nature of this phenomenon by demonstrating theoretically that vortex rings can be generated at the periphery of a fundamental soliton propagating in nonlocal nonlinear media.

In contrast to local nonlinear media, where higher-order solitons suffer from symmetry-breaking instabilities [3], nonlocality can suppress such instabilities [4,5] and even support exotic states, e.g., generalizing the well-known Laguerre- and Hermite-Gaussian linear modes [6]. The Laguerre-Gaussian radially symmetric solitons, with a central bright spot surrounded by alternating dark and bright rings of varying size, were first discovered in Ref. [7] for the local Kerr-type nonlinear media. Despite carrying zero angular momentum, they develop azimuthal instability [8,9], with the bright rings decaying into several fundamental solitons, similar to the instability of vortex solitons [3].

The dark intensity rings of the Laguerre-type solitons represent cylindrically shaped edge phase dislocations. We are interested to learn here how the perturbations and nonlinear dynamics can affect the topological structure of these "dark cylinders" and their possible relation to the vortex rings discussed above. We recall that stationary nonspinning higher-order solitons were investigated in Ref. [10] by approximate variational method in a nonlocal medium with Gaussian-type response function. The specific feature of the media with a Gaussian-type nonlocal response function is that higher-order solitons may exhibit dynamics with revivals and periodic robust oscillations between two or more spatially localized states with distinctly different symmetries [6,10]. This is different from nonlinear media with thermal nonlocal response functions [4], but to the best of our knowledge, no such solitons have been obtained so far in the later model.

In this paper, we find higher-order solitons by numerical solution of the stationary nonlinear Schrödinger equation (NLSE) with thermal optical nonlinearity. We investigate the stability of the higher-order solitons by numerical simulations of the dynamic NLSE in the nonlocal regime, in particular, showing the spontaneous formation of external vortex rings. We demonstrate that as the cylindrically shaped edge dislocation undergoes topological transformations, the radially symmetric perturbation drives nucleation of an additional internal sequence of vortex-antivortex ring pairs.

II. STEADY STATES IN NONLOCAL NONLINEAR MEDIA

A bright spatial soliton is a wave beam of finite cross section which propagates in a nonlinear medium without changing its structure. The dynamics and stability of spatial solitons with respect to collapse [11] were extensively investigated in various nonlinear media, including dissipative systems [12,13]; see Ref. [14] for a review and detailed discussion on the thresholds for self-focusing and collapse instability.

In the spatially nonlocal media the nonlinear response depends on the wave-packet intensity at some extensive spatial domain. Nonlocality naturally arises in many nonlinear media. In particular, a nonlocal response is induced by heating and ionization, and it is known to be important in media with thermal nonlinearities such as thermal glass [15] and plasmas [16]. Nonlocal response is a key feature of the orientational nonlinearities due to long-range molecular interactions in nematic liquid crystals [17]. An interatomic interaction potential in Bose-Einstein condensates (BECs) with dipole-dipole interactions is also known to be substantially nonlocal [18–20]. The feedback of the BEC on propagation of electromagnetic waves induces substantially nonlocal effective interactions [21] (the local-field effect). In all such systems, nonlocal nonlinearity can be responsible for many features, such as the familiar effect of the collapse arrest [22,23] and stabilization of various coherent structures.

The basic dimensionless equations describing the propagation of the electric field envelope $\Psi(x, y, z)$ coupled to the temperature perturbation $\theta(x, y, z)$ has the following form [4]:

$$i\frac{\partial \Psi}{\partial z} + \Delta_{\perp}\Psi + \theta\Psi = 0,$$
$$\alpha^2\theta - \Delta_{\perp}\theta = |\Psi|^2, \tag{1}$$

where $\Delta_{\perp} = d^2/dx^2 + d^2/dy^2$. They describe the light propagation in bulk medium with thermal nonlinearities, and it appears also in the study of two-dimensional bright solitons in nematic liquid crystals [17] and in partially ionized plasmas [15].

In the limit $\alpha^2 \gg 1$, we can neglect the second term in the equation for the field θ of Eqs. (1) and reduce this system to the standard local nonlinear Schrödinger (NLS) equation with cubic nonlinearity. The opposite case, i.e., $\alpha^2 \ll 1$, will be referred to as a strongly nonlocal regime of the beam propagation. The second equation of the system (1) can be readily solved using Green functions presented in [4] for $\alpha \neq 0$ and in [21] for the limiting case $\alpha = 0$.

We are interested in stationary nonspinning solutions of (1) in the form

$$\Psi(x, y, z) = \psi_n(r)\exp(i\Lambda z), \tag{2}$$

where $r = \sqrt{x^2 + y^2}$ is the radial coordinate, and Λ is the beam propagation constant; n is the number of zeros (nodes) of the profile $\psi_n(r)$. Such solutions describe either the fundamental optical soliton, when $n = 0$, or the higher-order soliton with n nodes when $n > 0$. Substituting (2) into (1) yields the following system:

$$-\lambda\psi_n + \Delta_r\psi_n + \theta\psi_n = 0,$$
$$\theta - \Delta_r\theta = |\psi_n|^2. \tag{3}$$

Here $\lambda = \Lambda/\alpha^2$ is the rescaled propagation constant, $\Delta_r = d^2/dr^2 + (1/r)d/dr$. We consider the case $\alpha \neq 0$, which implies that both field $\psi(r)$ and temperature $\theta(r)$ distributions rapidly decay at $r \to \infty$.

The system of Eqs. (3) was solved by the Petviashvili method [24] in the case of a fundamental soliton and by the shooting method to find higher-order solutions. Examples of numerical solutions in the form of higher-order solitons with

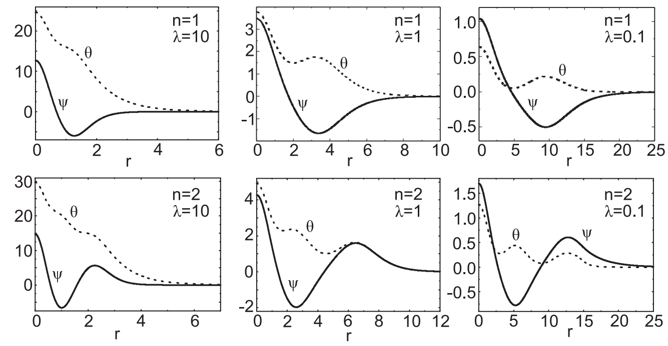

FIG. 1. Typical examples of numerical stationary solutions in the form of single-ring ($n = 1$) and double-ring ($n = 2$) optical solitons.

$n = 1$ and $n = 2$ are shown in Fig. 1. Figure 2(a) shows the beam power $P = \int |\psi_n|^2 d^2\mathbf{r}$ as a function of the rescaled propagation constant λ. Note that in a strongly nonlocal regime ($\alpha^2 \ll 1$, i.e., $\lambda \gg 1$) the profile of the temperature distribution is wider than the effective radius of the central bright core of the solitonic wave beam. Let us define the effective radii r_ψ and r_θ of the intensity distribution $|\psi|^2$ and the temperature distribution θ, respectively, as follows:

$$r_\psi^2 = \frac{1}{P}\int r^2|\psi_n(r)|^2 d^2\mathbf{r}, \quad r_\theta^2 = \frac{\int r^2\theta(r)d^2\mathbf{r}}{\int \theta(r)d^2\mathbf{r}}.$$

Figure 2(b) shows the radii r_ψ and r_θ as functions of λ. Both r_ψ and r_θ decrease monotonically when λ grows.

Propagation of these solutions and the fundamental soliton solution ($n = 0$) under the perturbation was simulated by employing the split-step Fourier method along the z axis with monitoring of conservation of integrals of motion: (i) beam power P, (ii) momentum $\mathbf{I}_{\perp} = \int \mathbf{j}d^2\mathbf{r}$, where $\mathbf{j} = -\frac{i}{2}\{\Psi^*\nabla_{\perp}\Psi - \Psi\nabla_{\perp}\Psi^*\}$, (iii) angular momentum $\mathbf{M}_z = \int \mathbf{r} \times \mathbf{j}d^2\mathbf{r}$, and (iv) Hamiltonian $H = \int\{|\nabla_{\perp}\Psi|^2 - \frac{1}{2}\theta\Psi\}d^2\mathbf{r}$. Note that for the axially symmetric perturbations, all considered structures have both the transverse momentum and the angular momentum equal to zero.

III. VORTEX RINGS ON FUNDAMENTAL SOLITON

Here we simulate evolution of the fundamental soliton along the z axis with initial conditions of the form $\Psi(x, y, z = 0) = a^{-1}\psi(r/a)$. The initial stretching with $a \neq 1$ leads to radial oscillations and additional nonlinear accumulation of phase. Figure 3 demonstrates this process for a perturbed

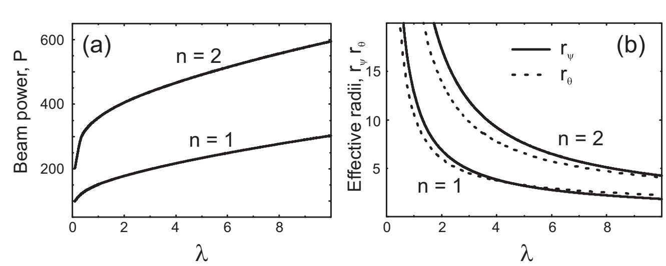

FIG. 2. Parameters of metastable single- and double-ring solitons ($n = 1, 2$): (a) beam power vs rescaled propagation constant, and (b) effective radii r_ψ (solid curves) and r_θ (dashed curves).

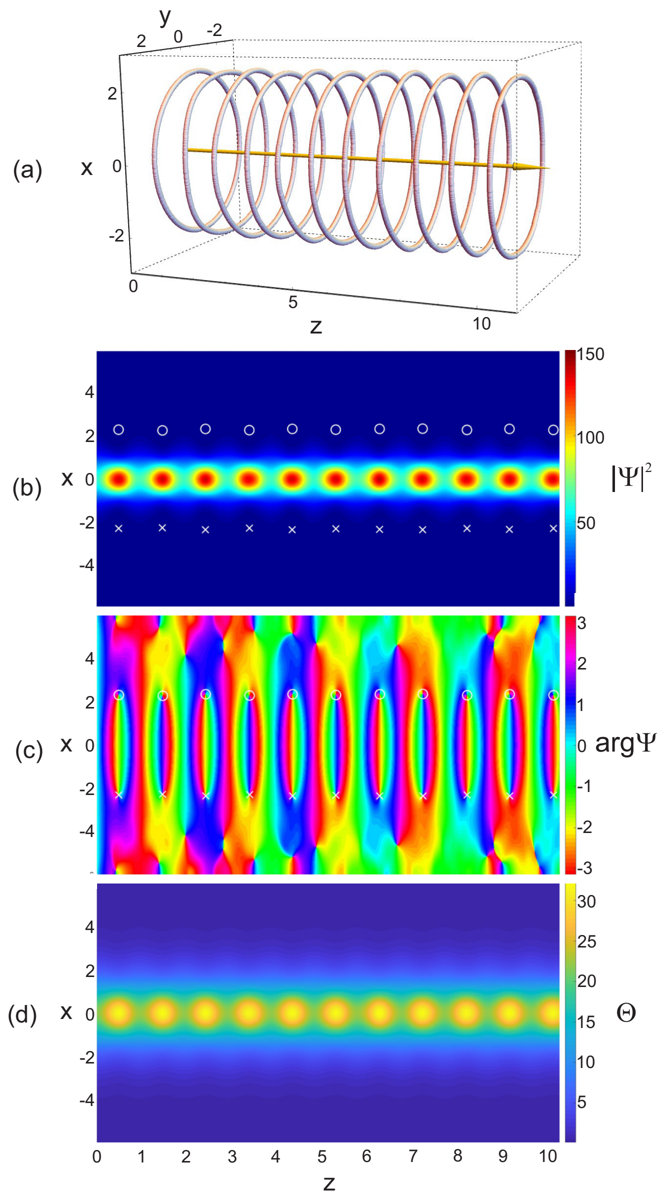

FIG. 3. Spontaneous generation of a regular set of vortex rings appears as the result of a strong perturbation level of $a = 1.3$: (a) position of the vortex cores; cross section in the plane $y = 0$ for (b) intensity distribution $|\Psi(x, z)|^2$, (c) phase $\arg\Psi(x, z)$, and (d) temperature perturbation $\Theta(x, z)$.

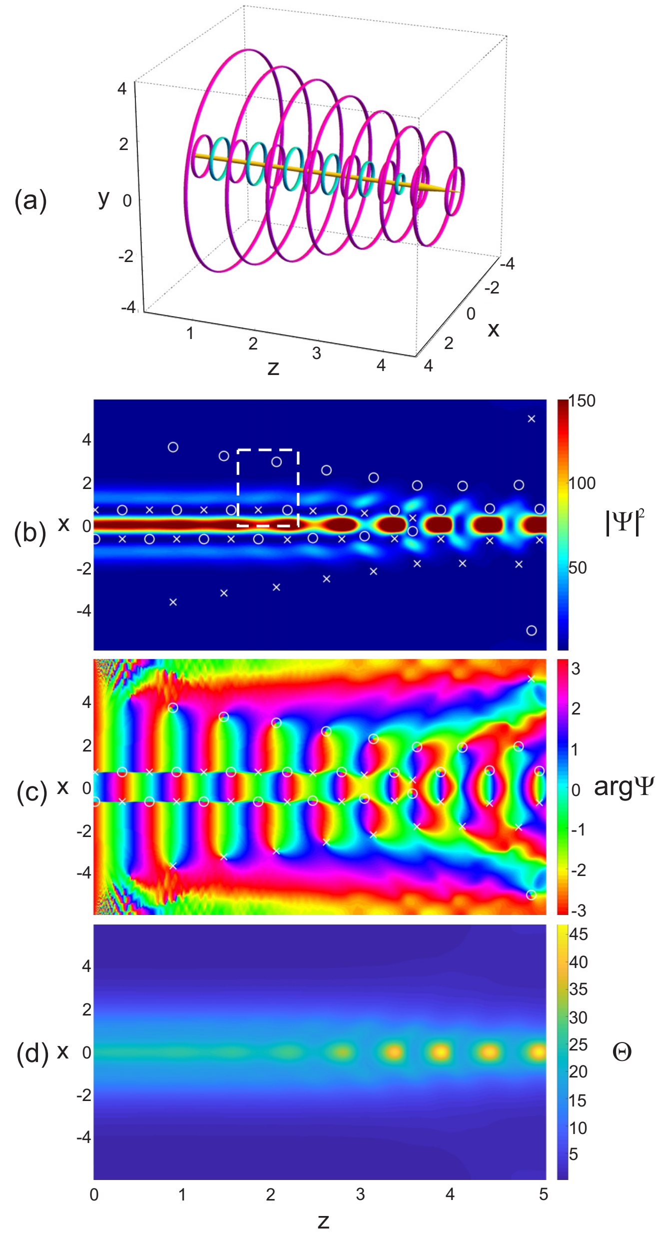

FIG. 4. Spontaneous generation of a series of pairs of vortex and antivortex rings which appear as the result of a weak perturbation level, $a = 1.01$: (a) position of the vortex cores; cross section in the plane $y = 0$ for (b) intensity distribution $|\Psi(x, z)|^2$, (c) phase $\arg\Psi(x, z)$, and (d) temperature perturbation $\Theta(x, z)$.

fundamental soliton with the field's topological structure, namely, the appearance of a regular set of vortex rings similar to vortex rings revealed in [1] for media with local saturating nonlinearity. Taking a very strong perturbation level of $a = 1.3$ and $\lambda = 10$ (the case of strong nonlocality), we observe extremely robust and long-lived oscillations with the spontaneous generation of a regular set of vortex rings.

IV. VORTEX RINGS AND LINES ON HIGHER-ORDER SOLITONS

While the fundamental solitons are known to be stable in nonlocal nonlinear media [22], stability of the higher-order structures (such as vortex solitons, bound states of solitons, and other higher-order structures) crucially depends on the specific form of the nonlocality. For the model with Gaussian-type response function, an example of robust propagation of a single-charge one-node ($m = 1$, $n = 1$) Laguerre-Gaussian LG_1^1 wave beam has been demonstrated in Ref. [5]. In this section we consider the dynamics of the numerically found higher-order solitons in nonlocal media with a thermal response function for different types of perturbation.

For reasons of simplicity, we analyze evolution of the higher-order soliton with the $n = 1$ node, which has an edge-ring dislocation. First, to have an ability to compare results

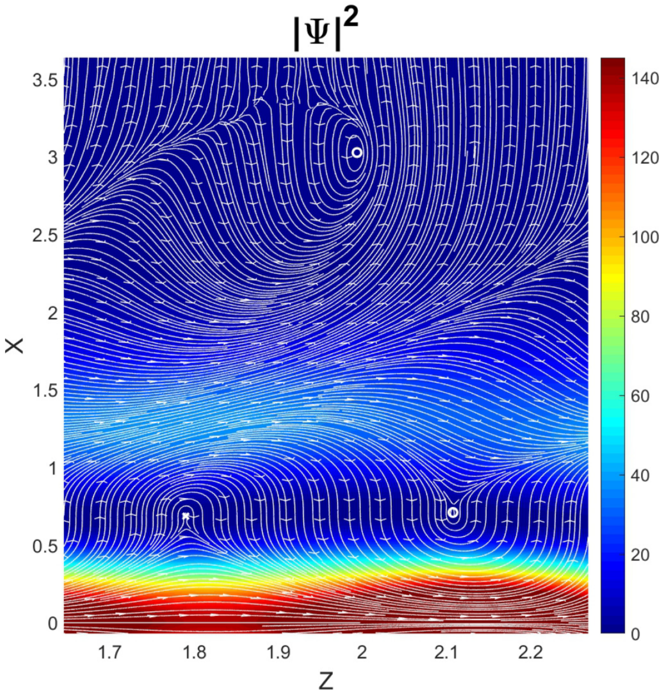

FIG. 5. Enlarged fragment of intensity plot [indicated as dashed rectangle in Fig. 4(b)] with streamlines of the Poynting vector shown. It corresponds to a single beam oscillation. Vortex and antivortex cores, which come from cut vortex rings, are illustrated by white zeros and crosses, respectively.

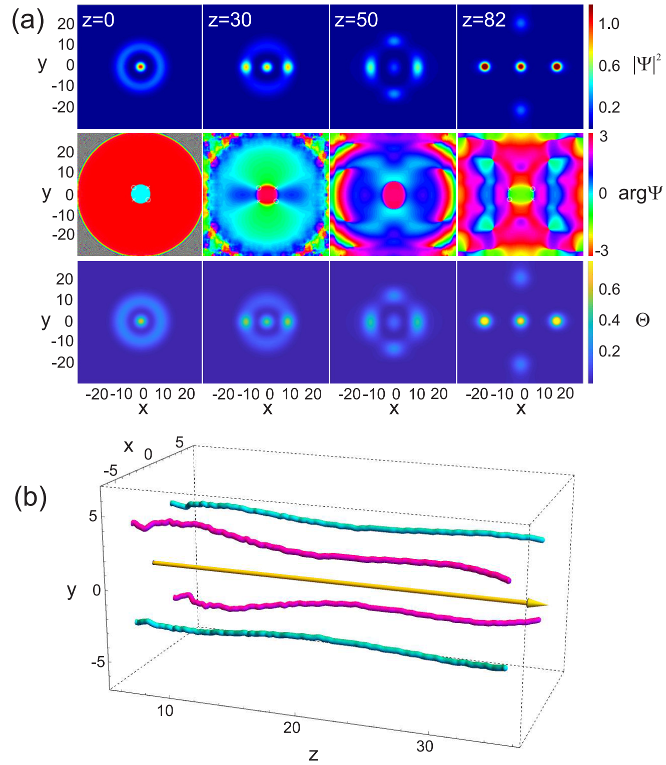

FIG. 6. (a) Snapshots of the evolution in z direction single-ring ($n = 1$) optical solitons for $\lambda = 0.1$. Shown are intensity distribution $|\Psi(x, y)|^2$, phase $\arg\Psi(x, y)$, and $\Theta(x, y)$. (b) Trajectories of vortex (green) and antivortex (red) cores propagating in z direction.

with fundamental soliton behavior, we take also $\lambda = 10$. The solution is initially slightly perturbed radially with $a = 1.01$, like in the case of the previous section. Topological instability of the edge dislocation leads to its splitting into a series of pairs of vortex and antivortex rings, one pair per oscillation (see Fig. 4). Emergence of external vortex rings is connected with radial constriction of the beam and results in energy transfer from the bright ring to the bright core of the beam; therefore an energy flow forms the saddle points in a low-intensity region where the internal vortex rings are situated (see Fig. 5).

This phenomenon can be explained by the topology of the system. For the unperturbed soliton with nodes propagating along the z axis, surfaces of zero real and imaginary parts of the field envelope Ψ are ideal circular cylinders which coincide. Being perturbed radially, these cylinders begin to oscillate with the same frequency but generally with a different phase, so they should intersect each other twice per period of oscillations in this case. Therefore the external vortex rings and internal pairs of vortex-antivortex rings have a quite different topological origin, and the existence of internal rings is a specific topological feature of the solitons with nodes.

Second, we perturb the single-node soliton by a quadrupole mode with initial conditions in the form $\Psi(x, y, z = 0) = \psi(r)[1 + \epsilon\cos(L\varphi)]$, where $\epsilon = 0.025$ and $L = 2$. Results of soliton evolution are shown in Figs. 6 and 7 in cases of local and strongly nonlocal regimes.

The higher-order solitons appear to be only *metastable* in the nonlocal media with thermal response function. These structures, being even strongly perturbed, do not decay immediately but propagate for a considerable distance. However, symmetry-breaking modulational instability developing after several oscillations of the wave-beam envelope leads to strong transformations of spatial profile of the intensity distribution. Finally, azimuthal instability leads to decay of the solitons with nodes in the weakly nonlocal regime, as is seen from Fig. 6.

In our numerical simulations, in the strongly nonlocal regime we observed dynamics with partial revival of the nonspinning higher-order soliton similar to that observed in Ref. [10] for the model based on a Gaussian-type kernel of the nonlocal medium response function. However, in sharp contrast to the model with the Gaussian-type response function, the higher-order soliton does not exhibit robust oscillations between eigenstates of different symmetry in a medium with thermal nonlocal nonlinearity. As is seen from Fig. 7, the higher-order soliton first transforms into Hermit-Gauss mode but than rapidly decays. Solitons with more rings($n \geqslant 2$) decay in a similar way, even faster than a single-node soliton.

A common feature of perturbed soliton dynamics in both regimes is the spatially robust quadrupole set of vortex lines. It emerges in a low-intensity region inside the solitonic beam independently of the range of nonlocality. The existence of the four vortex lines also has a topological nature. When the beam is perturbed by quadrupole mode ($L = 2$), the surfaces of real and imaginary parts of Ψ deform into two elliptic cylinders which intersect each other along four parallel lines in the direction of the z axis.

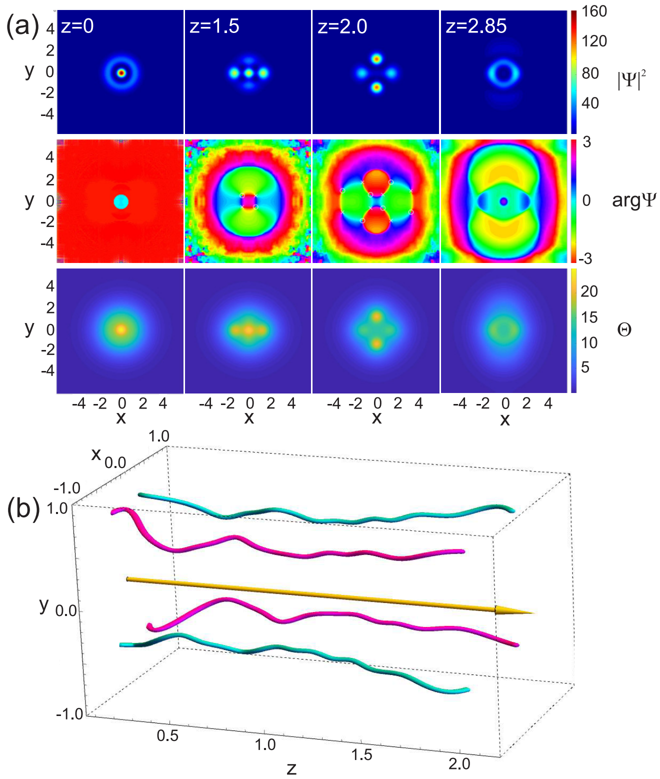

FIG. 7. (a) Snapshots of the evolution in z direction single-ring ($n = 1$) optical solitons for $\lambda = 10$. Shown are intensity distribution $|\Psi(x, y)|^2$, phase $\arg\Psi(x, y)$, and $\Theta(x, y)$. (b) Trajectories of vortex (green) and antivortex (red) cores propagating in z direction.

The main problem in experimental observation of external vortex rings is that they are located at the periphery of the wave beam in the low-intensity region. The internal vortex rings and vortex lines live sufficiently long in the strong nonlocal regime, and they can be readily detected in central region of the wave beam.

V. SUMMARY AND CONCLUSIONS

We investigated different vortex complexes which spontaneously emerge on perturbed solitonic structures in nonlocal nonlinear media with thermal response function. Using direct numerical simulations, we found the nth bound solitonic state which has a central bright spot surrounded by n rings of varying size. These structures rapidly decay into several fundamental solitons due to symmetry-breaking azimuthal instability in a weakly nonlocal regime, but they appear to be metastable and exhibit dynamics with partial revivals for highly nonlocal regime.

We show that the vortex rings appear spontaneously at the periphery of the radially perturbed fundamental soliton. A remarkable topological feature of radially perturbed higher-order solitons is the emergence of additional internal vortex-antivortex ring pairs perpendicular to the optical axis. In contrast, the perturbation mode with quadrupole symmetry leads to the emergence of spatially robust internal pairs of vortex and antivortex lines codirected with the optical axis. These vortex-antivortex pairs nucleate from the edge phase dislocation, or the dark intensity ring, and thus preserve conservation of topological charge and orbital angular momentum.

Emerging vortex rings and lines investigated in this work can be seen as building blocks for spontaneous or even engineered creation of more complex topological structures peculiar to structured light, such as vortex links and knots [1].

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