Directional steering as a sufficient and necessary condition for Gaussian entanglement swapping: Application to distant optomechanical oscillators

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Einstein-Podolsky-Rosen (EPR) steering is a quantum nonlocal effect that characterizes the ability to remotely control quantum states of one system via local measurements on another distant system entangled with the controlled one. Entanglement swapping is an effective way to realize distant entanglement, which is useful for building quantum networks. Previous studies show that the presence of entanglement of subsystems in mixed states is merely necessary for achieving entanglement swapping. In this paper, we find that for Gaussian entanglement swapping with two pairs of entangled modes $\hat{a}_j$ and $\hat{b}_j$ ($j = 1, 2$), the existence of directional EPR steering from the modes $\hat{b}_j$ under homodyne detection to the other modes $\hat{a}_j$ is a sufficient and necessary condition for achieving swapped entanglement between modes $\hat{a}_1$ and $\hat{a}_2$, and moreover this achievement is independent of the reverse steering from modes $\hat{a}_j$ to $\hat{b}_j$. We further reveal that this is because the steering in that direction enables the amplitude and phase squeezing of the two composite modes of modes $\hat{a}_1$ and $\hat{a}_2$ via homodyne detection, which is necessary and sufficient for swapped entanglement. As a concrete example, we next investigate light-mechanical steering in a dispersively or dissipatively coupled optomechanical system, and we consider the generation of entanglement between two distant optomechanical oscillators via entanglement swapping by utilizing steering. It is interesting to find that even without the constraint that the cavity linewidth should be smaller than the mechanical frequency, robust mechanical entanglement can be obtained in the steady-state regime.

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I. INTRODUCTION

The famous Einstein-Podolsky-Rosen (EPR) paradox describes the situation in which two distant observers—Alice and Bob—share entangled particles, and one observer, say Alice, is able to control the states of Bob's particle by performing local measurements on her particle [1]. Such an ability to nonlocally control states of remote particles was termed "steering" by Schrödinger [2]. It is an intrinsic quantum nonlocal effect. It has recently been shown by Wiseman, Jones, and Doherty that EPR steering can be regarded as verifiable entanglement distribution by an untrusted party [3,4], while Bell nonlocality and quantum inseparability can be defined as entanglement distribution among distrust parties and between trust parties, respectively. Therefore, EPR steering is intermediate between plain entanglement (nonseparability) [5] and Bell nonlocality (the violation of Bell inequality) [6,7]. Concretely, the states exhibiting Bell nonlocality are a subset of steerable states, which are in turn a subset of entangled (inseparable) states. Furthermore, in contrast with plain entanglement and Bell nonlocality, steering is intrinsically asymmetric with respect to the two observers and is thus directional. This means that the steering from Alice to Bob or the reverse steering from Bob to Alice may be different, although the two particles of the observers are entangled with each other, and there is even one-way steering, which allows, for example, Alice to steer the states of Bob's particle but not vice versa [8–13].

In addition to being of fundamental interest, EPR steering has recently attracted increasing interest due to its potential applications, e.g., one-sided device-independent quantum cryptography [14,15], subchannel discrimination [16], and secure quantum teleportation [17]. Quantum steering has been experimentally realized in a variety of physical systems [10–13,18–27]. In addition, by utilizing steerable correlations, desirable quantum states can be achieved via local measurements [28–30].

On the other hand, entanglement swapping is now considered an effective approach to entangling two distant objects that do not directly interact with each other [31–33]. By using entanglement swapping, the entanglement among different nodes can be realized, which is of importance for building up a large-scale quantum-information network and transferring quantum states. Nowadays, entanglement swapping has been experimentally demonstrated in both discrete and continuous variable systems [34,35]. For Gaussian states, it has been shown that for pure entangled input states, swapped

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entanglement can be achieved [36]. In reality, however, due to losses in channels, such as in the context of quantum communication, one typically deals with mixed states, whereas for mixed input states, the input entanglement is merely necessary for achieving entanglement swapping. We therefore naturally ask, what is the sufficient and necessary condition for entanglement swapping of continuous variable Gaussian states with mixed input states?

Cavity optomechanics involves coupling between a nano- or micromechanical oscillator and the radiation field inside a cavity [37]. Recent experiments have achieved quantum squeezed states of light and mechanical modes [38,39], light-mechanical Gaussian entangled states [40], nonclassical correlations between single photons and phonons from a mechanical oscillator [41], and Gaussian entanglement between two mechanical oscillators inside a cavity [42]. More interestingly, the first observation of remote entanglement of discrete variables of two micromechanical oscillators with the DLCZ protocol [43] was reported very recently [44]. These achievements make cavity optomechanics not only an intriguing platform for the study of fundamental physics, such as EPR steering, but also a potential candidate for the development of novel quantum devices. For instance, remote mechanical entanglement can offer a new compelling route toward scalable quantum networks [44].

In this paper, we first study the connection between EPR steering and entanglement swapping of Gaussian states. We find that for Gaussian entanglement swapping, the steering from the modes mixed at the beam splitter to the other modes of two identical input states is sufficient and necessary for achieving swapped entanglement. The achievement of entanglement is independent of the reverse steering. We show that this is because the steering in that direction enables the squeezing of the composite modes of the two distant modes to be entangled via homodyne detection. Then, as a realistic example, we investigate light-mechanical steering in a dispersively or dissipatively coupled optomechanical system, and we consider the generation of entanglement between two distant mechanical oscillators via entanglement swapping with steering. It is found that for both types of coupling, by filtering the output field from the cavities, robust mechanical entanglement can be obtained in the regime of steady states without the constraint that the cavity linewidth should be smaller than the mechanical frequency.

The remainder of this paper is organized as follows. In Sec. II, Gaussian entanglement, steering, and entanglement swapping are reviewed. In Sec. III, we investigate in detail the connection between the steering of subsystems and Gaussian entanglement swapping. In Sec. IV, we consider the establishment of entanglement between two distant optomechanical oscillators by utilizing the steering of the subsystems. In Sec. V, we give the main summary.

II. GAUSSIAN ENTANGLEMENT, STEERING, AND SWAPPING

We consider entanglement swapping of two pairs of bosonic modes [described by the annihilation operators $\hat{a}_j$ and $\hat{b}_j$ $(j=1,2)$] in the same Gaussian states, which are characterized by the correlation matrix (CM) $(\sigma_{ab})_{ll'} = \langle\lambda_l\lambda_{l'} + \lambda_{l'}\lambda_l\rangle/2$, expressed in the form

$$\sigma_{ab} = \begin{pmatrix} A_j & C_j \\ C_j^T & B_j \end{pmatrix} \ (j=1,2). \tag{1}$$

Here $\lambda = (\hat{X}_{a_j}, \hat{P}_{a_j}, \hat{X}_{b_j}, \hat{P}_{b_j})$, the quadrature operators $\hat{X}_O = (\hat{O}+\hat{O}^\dagger)/\sqrt{2}$ and $\hat{P}_O = -i(\hat{O}-\hat{O}^\dagger)/\sqrt{2}$ $(\hat{O} = \{\hat{a}_j, \hat{b}_j\})$, and the entries A_j, B_j, and C_j are 2×2 matrices. In the following discussion, we will omit the subscript "j" for simplicity. The entanglement between the modes $\hat{a}$ and $\hat{b}$ can be quantified by the logarithmic negativity [45],

$$E_N = \max[0, -\ln(2\lambda)], \tag{2}$$

where $\lambda = 2^{-1/2}\sqrt{\Sigma - \sqrt{\Sigma^2 - 4\det\sigma_{ab}}}$ and $\Sigma = \det A + \det B - 2\det C$. The steering from mode $\hat{b}$ to mode $\hat{a}$ (i.e., b-to-a steering) can be quantified by the measure [46]

$$\mathcal{S}^{b\to a} = \max\left\{0, \frac{1}{2}\ln\frac{\det B}{4\det\sigma_{ab}}\right\}, \tag{3}$$

and similarly for the reverse steering from mode $\hat{a}$ to mode $\hat{b}$ (a-to-b steering).

By local Gaussian operations, the CM σ_{ab} can be transformed into the following standard form [47]:

$$\sigma_{ab} = \begin{pmatrix} f_a & 0 & c_x & 0 \\ 0 & f_a & 0 & c_y \\ c_x & 0 & f_b & 0 \\ 0 & c_y & 0 & f_b \end{pmatrix}, \tag{4}$$

for which the condition for b-to-a steering becomes

$$\left(f_a - \frac{c_x^2}{f_b}\right)\left(f_a - \frac{c_y^2}{f_b}\right) - \frac{1}{4} < 0, \tag{5}$$

while the reverse steering requires the condition

$$\left(f_b - \frac{c_x^2}{f_a}\right)\left(f_b - \frac{c_y^2}{f_a}\right) - \frac{1}{4} < 0. \tag{6}$$

One-way steering is achieved when either Eq. (5) or Eq. (6) holds.

We consider entanglement swapping via mixing modes $\hat{b}_1$ and $\hat{b}_2$ at a balanced beam splitter (BS). With the CM in Eq. (4), the corresponding Wigner functions can be found to be

$$W_j(\vec{R}_j) = \mathcal{N}_j^{-1}\exp\left(-\frac{1}{2}\vec{R}_j^T M \vec{R}_j\right), \tag{7}$$

where the vectors of phase-space variables $\vec{R}_j^T = (x_{a_j}, p_{a_j}, x_{b_j}, p_{b_j})$, the matrices $M = \sigma^{-1}$, and the normalization factors $\mathcal{N}_j = \int d^4\vec{R}_j \exp(-\frac{1}{2}\vec{R}_j^T\sigma_j^{-1}\vec{R}_j)$. Thus, the whole Wigner function $W_{\text{in}}(\vec{R}_{ab})$ of the two input states at the beam splitter is given by

$$\begin{aligned} W_{\text{in}}(\vec{R}_{ab}) &= W_1(\vec{R}_1)W_2(\vec{R}_2) \\ &= \mathcal{N}^{-1}\exp\left(-\frac{1}{2}\vec{R}_{ab}^T M_{ab}\vec{R}_{ab}\right), \end{aligned} \tag{8}$$

where $\vec{R}_{ab}^T = (x_{a_1}, p_{a_1}, x_{a_2}, p_{a_2}, x_{b_1}, p_{b_1}, x_{b_2}, p_{b_2})$, $\mathcal{N} = \mathcal{N}_1\mathcal{N}_2$, and the matrix M_{ab} can be easily obtained; its explicit expression is not presented here.

The two output modes $\hat{c}_1$ and $\hat{c}_2$ from the beam splitter can be expressed as

$$\hat{c}_\pm = \frac{1}{\sqrt{2}}(\hat{b}_1 \pm \hat{b}_2). \tag{9}$$

In terms of phase-space variables (x_{a_j}, p_{a_j}) and $(x_{c_\pm}, p_{c_\pm})$ corresponding to modes $\hat{a}_j$ and $\hat{c}_\pm$, the Wigner function $W_{\text{out}}(\vec{R}_{ac})$ of the state of these four modes can be obtained as

$$W_{\text{out}}(\vec{R}_{ac}) = \mathcal{N}^{-1} \exp\left(-\frac{1}{2}\vec{R}_{ac}^T M_{ac} \vec{R}_{ac}\right), \tag{10}$$

where the vector $\vec{R}_{ac}^T = (x_{a_1}, p_{a_1}, x_{a_2}, p_{a_2}, x_{c_+}, p_{c_-}, x_{c_-}, p_{c_+})$, the matrix $M_{ac} = D^T M_{ab} D$, where $D = I_2 \oplus d_0$, I_2 is a 2×2 identity matrix, and $d_0 = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} I_2 & I_2 \\ \sigma_z & -\sigma_z \end{smallmatrix}\right)$ for the Pauli matrix σ_z.

Consider that the quadratures $\hat{P}_{c_+}$ and $\hat{X}_{c_-}$ of the output modes from the beam splitter are simultaneously measured under homodyne detection. Depending on the detection results $\tilde{p}_{c_+}$ and $\tilde{x}_{c_-}$, the unnormalized Wigner function of modes $\hat{a}_1$ and $\hat{a}_2$ is then obtained as

$$W_{\text{con}}(\vec{R}_a) = \int dx_{c_1} dp_{c_2} W_{\text{out}}(\vec{R}_{ac})\Big|_{x_{c_-}=\tilde{x}_{c_-}, p_{c_+}=\tilde{p}_{c_+}}, \tag{11}$$

where $\vec{R}_a^T = (x_{a_1}, p_{a_1}, x_{a_2}, p_{a_2})$, and integration leads to

$$W_{\text{con}}(\vec{R}_a) = \tilde{\mathcal{N}}^{-1} \exp\left(-\frac{1}{2}\vec{R}_a^T M_a \vec{R}_a - \vec{R}_a^T \vec{d}_a\right), \tag{12}$$

where the vector $\vec{d}_a^T = (d_1\tilde{x}_{c_-}, d_2\tilde{p}_{c_+}, d_3\tilde{x}_{c_-}, d_4\tilde{p}_{c_+})$ and the expressions of the 4×4 matrix M_a and $d_{1,2,3,4}$ can be readily obtained and are not given here. One can see that the first-order moments of the conditional state of Eq. (12) are dependent on the measurement results and they can be displaced via classical communication. Considering the displacement $\vec{R}_a \to \vec{R}_a + \mathcal{K}\vec{\mathcal{G}}$ on the modes $\hat{a}_1$ and $\hat{a}_2$, where $\mathcal{K} = \text{diag}\,[\tilde{x}_{c_-}, \tilde{p}_{c_+}, \tilde{x}_{c_-}, \tilde{p}_{c_+}]$ and the phase-dependent gains $\vec{\mathcal{G}}^T = (g_{a_1}^x, g_{a_1}^y, g_{a_2}^x, g_{a_2}^y)$ [48], one can find that the first-order moments of the state (12) vanish when choosing the phase-dependent gains

$$\vec{\mathcal{G}} = -\mathcal{K}^{-1} M_a^{-1} \vec{d}_a. \tag{13}$$

Therefore, for this choice the final ensemble-average state of the modes $\hat{a}_1$ and $\hat{a}_2$ over all measurement results is equal to this one-shot conditional state of Eq. (12), and thus its CM V_a^{ens} is given by

$$\sigma_a^{\text{ens}} = M_a^{-1} = \begin{pmatrix} \sigma_{a,11}^{\text{ens}} & 0 & \sigma_{a,13}^{\text{ens}} & 0 \\ 0 & \sigma_{a,22}^{\text{ens}} & 0 & \sigma_{a,24}^{\text{ens}} \\ \sigma_{a,13}^{\text{ens}} & 0 & \sigma_{a,33}^{\text{ens}} & 0 \\ 0 & \sigma_{a,24}^{\text{ens}} & 0 & \sigma_{a,44}^{\text{ens}} \end{pmatrix}, \tag{14}$$

where

$$\sigma_{a,11}^{\text{en}} = \sigma_{a,33}^{\text{en}} = f_a - \frac{c_x^2}{2f_b}, \tag{15a}$$

$$\sigma_{a,22}^{\text{en}} = \sigma_{a,44}^{\text{en}} = f_a - \frac{c_y^2}{2f_b}, \tag{15b}$$

$$\sigma_{a,13}^{\text{en}} = -\frac{c_x^2}{2f_b}, \quad \sigma_{a,24}^{\text{en}} = \frac{c_y^2}{2f_b}. \tag{15c}$$

III. ONE-WAY STEERING: A SUFFICIENT AND NECESSARY CONDITION FOR GAUSSIAN ENTANGLEMENT SWAPPING

According to Eq. (2), it can be readily found that for the two-mode Gaussian swapped state (14), the entanglement parameter

$$\lambda = \frac{\sqrt{\left(f_a f_b - c_x^2\right)\left(f_a f_b - c_y^2\right)}}{f_b}, \tag{16}$$

and thus from Eq. (2) the entanglement condition of $\lambda < \frac{1}{2}$ reduces to

$$\left(f_a - \frac{c_x^2}{f_b}\right)\left(f_a - \frac{c_y^2}{f_b}\right) - \frac{1}{4} < 0, \tag{17}$$

which is the exact condition Eq. (5) of the steering from mode $\hat{b}_j$ to mode $\hat{a}_j$ of the subsystems. Therefore, it is revealed that the b_j-to-a_j steering of the input states is sufficient and necessary for realizing the entanglement between the indirect coupling modes $\hat{a}_1$ and $\hat{a}_2$. Moreover, the entanglement condition is independent of the reverse steering from modes $\hat{a}_j$ to $\hat{b}_j$, although the reverse steering is also present when $f_b \leqslant f_a$, because with it the inequality (17) can lead to the inequality

$$\left(f_b - \frac{c_x^2}{f_a}\right)\left(f_b - \frac{c_y^2}{f_a}\right) < \frac{f_b^2}{4f_a^2} < \frac{1}{4}. \tag{18}$$

It should be noted that for the standard form of the CM in Eq. (4), the entanglement condition of $\lambda < \frac{1}{2}$ reduces to

$$4\left(f_a f_b - c_x^2\right)\left(f_a f_b - c_y^2\right) - \left(f_a^2 + f_b^2 + 2|c_x c_y|\right) + \frac{1}{4} < 0. \tag{19}$$

It can be immediately found that the above inequality (19) holds when the inequality (17) is satisfied, but not vice versa, since we have the relation $f_{a,b} > \frac{1}{2}$. This means that for Gaussian entanglement swapping, the existence of entanglement of two subsystems in mixed states is merely necessary for the presence of swapped entanglement. Note that in deriving the inequality (19), the necessary condition that $c_x c_y < 0$ for the entanglement is utilized.

To understand the above result, let us introduce the new operators

$$\hat{d}_+ = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2), \tag{20a}$$

$$\hat{d}_- = \frac{1}{\sqrt{2}}(\hat{a}_1 - \hat{a}_2). \tag{20b}$$

Due to the symmetry between the two input states, the state $W_{\text{out}}(\vec{R}_{ac})$ of modes $\hat{a}_j$ and the two output modes $\hat{c}_\pm$ can be written as the product of the Wigner function of modes $\hat{c}_+$ and $\hat{d}_+$ and the Wigner function of modes $\hat{c}_-$ and $\hat{d}_-$, i.e.,

$$W_{\text{out}}(\vec{R}_{ac}) = W_{\text{out}}(\vec{R}_+) W_{\text{out}}(\vec{R}_-), \tag{21}$$

where

$$W_{\text{out}}(\vec{R}_+) = \sqrt{\mathcal{N}^{-1}} \exp\left(-\frac{1}{2}\vec{R}_+^T\sigma^{-1}\vec{R}_+\right), \tag{22a}$$

$$W_{\text{out}}(\vec{R}_-) = \sqrt{\mathcal{N}^{-1}} \exp\left(-\frac{1}{2}\vec{R}_-^T\sigma^{-1}\vec{R}_-\right), \tag{22b}$$

with $\vec{R}_+ = (x_{d_+}, p_{d_+}, x_{c_+}, p_{c_+})$ and $\vec{R}_- = (x_{d_-}, p_{d_-}, x_{c_-}, p_{c_-})$. Thus, in terms of modes $\hat{c}_\pm$ and $\hat{d}_\pm$, the output state is decoupled and moreover the CMs of the states $W_{\text{out}}(\vec{R}_+)$ and $W_{\text{out}}(\vec{R}_-)$ have the same form as those of the input modes $\hat{a}_j$ and $\hat{b}_j$, showing that the steering property of modes $\hat{c}_+$ and $\hat{d}_+$ ($\hat{c}_-$ and $\hat{d}_-$) is the same as that of the couple of modes $\hat{a}_j$ and $\hat{b}_j$. When homodyne detecting on the quadrature $\hat{P}_{c_+}$ and $\hat{X}_{c_-}$, the CMs of the conditional states of modes $\hat{d}_+$ and $\hat{d}_-$ take the form

$$\sigma_{d_+|\hat{P}_{c_+}} = \begin{pmatrix} f_a & 0 \\ 0 & f_a - \frac{c_y^2}{f_b} \end{pmatrix}, \tag{23a}$$

$$\sigma_{d_-|\hat{X}_{c_-}} = \begin{pmatrix} f_a - \frac{c_x^2}{f_b} & 0 \\ 0 & f_a \end{pmatrix}. \tag{23b}$$

Then, the variances $V(\hat{P}_{d_\pm})$ of the quadratures $\hat{X}_{d_+}$ and $\hat{X}_{d_-}$ of the two conditional states can be found to be

$$V(\hat{P}_{d_+}) = f_a - \frac{c_y^2}{f_b}, \tag{24a}$$

$$V(\hat{X}_{d_-}) = f_a - \frac{c_x^2}{f_b}. \tag{24b}$$

Similarly, when detecting the quadrature $\hat{X}_{c_+}$ and $\hat{P}_{c_-}$, we have the variances

$$V(\hat{X}_{d_+}) = V(\hat{X}_{d_-}), \quad V(\hat{P}_{d_-}) = V(\hat{P}_{d_+}). \tag{25}$$

When the b_j-to-a_j steering of the input states is present, it means that the c_+-to-d_+ and c_--to-d_- steering also exists. The presence of steering implies that the product of the variances (inferred variances via measurements)

$$V(\hat{X}_{d_+})V(\hat{P}_{d_+}) = V(\hat{X}_{d_-})V(\hat{P}_{d_-}) < \frac{1}{4}, \tag{26}$$

i.e., the predictions of the quadratures of the modes $\hat{d}_\pm$ via the measurement strategies have an error product that is lower than the Heisenberg uncertainty. This is the essence of the EPR paradox [49]. The inequality (26) is identical to the steering condition of Eq. (5), indicating that the variances $V(\hat{X}_{d_-}) < \frac{1}{2}$ or $V(\hat{P}_{d_+}) < \frac{1}{2}$, i.e., quadrature quantum squeezing of the modes of $\hat{d}_\pm$. According to Eq. (20), modes $\hat{a}_1$ and $\hat{a}_2$ can be considered as the output of the balanced beam splitter with the input modes being $\hat{d}_+$ and $\hat{d}_-$. The quadrature squeezing of the input modes is sufficient and necessary for the Gaussian entanglement between symmetric output modes. It can be easily shown that the two input states in Eq. (23) with the constraint of Eq. (26) can lead to output entanglement. We therefore reveal that the steering from mode $\hat{b}_j$ to mode $\hat{a}_j$ is sufficient and necessary for the entanglement between modes $\hat{a}_1$ and $\hat{a}_2$ by swapping.

IV. APPLICATION TO DISTANT MECHANICAL ENTANGLEMENT WITH OPTOMECHANICAL STEERING

As a realistic example, we consider in this section the generation of Gaussian entanglement between two distant optomechanical resonators via entanglement swapping. Existing schemes have already been proposed for realizing the entanglement between two mechanical oscillators that are coupled directly to each other or mediated by other auxiliary systems [50–54]. Experimentally, the Gaussian entanglement between two mechanical resonators coupled to a cavity field has been realized [42]. Moreover, the entanglement of discrete variables between two distant mechanical oscillators has also been achieved [44]. Here, to achieve distant Gaussian mechanical entanglement via swapping, as verified in the previous section, steering between the mechanical oscillators and the optical fields is required. As depicted in Fig. 1, we consider two identical subsystems of a cavity optomechanical system. For generical consideration, we further assume that for each optomechanical subsystem, the resonant frequency $\tilde{\omega}_c$ or the dissipation rate $\tilde{\kappa}_c$ of each cavity is dependent on the displacement x_m of the mechanical oscillators of frequency ω_m and mass m. The former gives rise to purely dispersive optomechanical coupling, while the latter leads to purely dissipative coupling between the cavity field and the mechanical oscillator. Dispersive optomechanical coupling has been realized in a variety of optomechanical systems ranging from microwave to optical regimes [37]. Very recently, a proposal for achieving purely dissipative optomechanical coupling has been put forward [55,56] and experimentally realized with a cavity-enhanced Michelson-type interferometer [57]. In addition, the simultaneous achievement of dispersive and dissipative optomechanical coupling has also been realized, with a single-crystal diamond nanobeam waveguide [58].

For the jth subsystem, we consider that the cavity frequency $\tilde{\omega}_c(\hat{x}_{m,j})$ is expanded to first order in the mechanical displacement $\hat{x}_{m,j} \equiv \sqrt{\frac{\hbar}{2m\omega_m}}(\hat{a}_{m,j} + \hat{a}_{m,j}^\dagger)$, i.e., $\tilde{\omega}_c(\hat{x}_{m,j}) \approx \omega_c - g_\omega(\hat{a}_{m,j} + \hat{a}_{m,j}^\dagger)$, where the dispersive coupling $g_\omega = -\sqrt{\frac{\hbar}{2m\omega_m}}\frac{\partial\tilde{\omega}_c}{\partial x_m}$ and the annihilation operator $\hat{a}_{m,j}$ denotes the jth mechanical mode. Then, the dispersive optomechanical

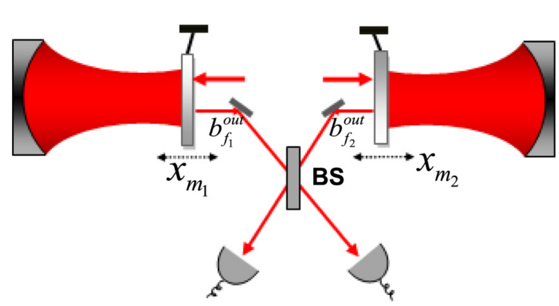

FIG. 1. The schematic plot of entanglement swapping with two dispersively or dissipatively coupled optomechanical subsystems. The filtered output modes $b_{f_1}^{\text{out}}$ and $b_{f_2}^{\text{out}}$ from the cavities are under Bell-type detection to establish the entanglement between the two mechanical oscillators a_{m_1} and a_{m_2}.

coupling can be described by the Hamiltonian

$$\begin{aligned}\hat{H}_{\omega,j} &= \hbar\omega_m\hat{a}^\dagger_{m,j}\hat{a}_{m,j} + \hbar\tilde{\omega}_c(\hat{x}_{m,j})\hat{b}^\dagger_{c,j}\hat{b}_{c,j}\\ &= \hbar\omega_m\hat{a}^\dagger_{m,j}\hat{a}_{m,j} + \hbar\omega_c\hat{b}^\dagger_{c,j}\hat{b}_{c,j}\\ &\quad - \hbar g_\omega(\hat{a}_{m,j} + \hat{a}^\dagger_{m,j})\hat{b}^\dagger_{c,j}\hat{b}_{c,j},\end{aligned} \tag{27}$$

where the annihilation operator $\hat{b}_{c,j}$ represents the jth cavity field.

The dissipative optomechanical coupling describes the coupling of the cavity field to the external bath, which is dependent on the motion of the mechanical oscillator. Hence, the cavity-bath coupling of the jth subsystem, which leads to the damping of the cavity, can be effectively described by [55,56,59]

$$\hat{H}_{\kappa,j} = i\hbar\sqrt{\tilde{\kappa}_c(\hat{x}_{m,j})}(\hat{b}^\dagger_{\text{in},j}\hat{b}_{c,j} - \hat{b}^\dagger_{c,j}\hat{b}_{\text{in},j}), \tag{28}$$

where the bath's collective operator $\hat{b}_{\text{in},j}(t) = \frac{1}{\sqrt{2\pi}}\int d\omega\,\hat{b}_{\omega,j}e^{-i\omega t}$, with the nonzero correlations $\langle\hat{b}_{\text{in},j}(t)\hat{b}^\dagger_{\text{in},j'}(t')\rangle = \delta(t-t')\delta_{jj'}$ for the bath modes $\hat{b}_{\omega,j}$ in vacuum and satisfying the commutation relation $[\hat{b}_{\omega,j}, \hat{b}^\dagger_{\omega',j'}] = \delta(\omega-\omega')\delta_{jj'}$. Likewise, by expanding $\tilde{\kappa}_c(\hat{x}_{m,j}) \approx \kappa_c + g_\kappa(\hat{a}_{m,j} + \hat{a}^\dagger_{m,j})$, with the dissipative coupling $g_\kappa = \sqrt{\frac{\hbar}{2m\omega_m}}\frac{\partial\tilde{\kappa}_c}{\partial x_m}$, the Hamiltonian of Eq. (28) is reduced approximately to

$$\hat{H}_{\kappa,j} = i\hbar\sqrt{\kappa_c}\left[1 + \frac{g_\kappa(\hat{a}_{m,j} + \hat{a}^\dagger_{m,j})}{2\kappa_c}\right](\hat{b}^\dagger_{\text{in},j}\hat{b}_{c,j} - \hat{b}^\dagger_{c,j}\hat{b}_{\text{in},j}), \tag{29}$$

which explicitly shows that the dissipative coupling between the cavity field and the mechanical oscillator is mediated by input noise.

In addition to the above input loss of cavities, we also take into account intrinsic losses of the cavities, which can be effectively described by the coupling

$$\hat{H}^{\text{int}}_{\kappa,j} = i\hbar\sqrt{\kappa^{\text{int}}_c}(\hat{b}^\dagger_{\text{int},j}\hat{b}_{c,j} - \hat{b}^\dagger_{c,j}\hat{b}_{\text{int},j}), \tag{30}$$

with the loss rate κ^{int}_c of the jth cavity, where $\hat{b}_{\text{int},j}$ accounts for the environment that induces intrinsic loss and also satisfying the nonzero correlations $\langle\hat{b}_{\text{int},j}(t)\hat{b}^\dagger_{\text{int},j'}(t')\rangle = \delta(t-t')\delta_{jj'}$. Finally, we consider that each optomechanical cavity is driven by a laser with a frequency of $\omega_{d,j}$ and amplitudes $\bar{b}_{\text{in},j}$, described by

$$\hat{H}_{d,j} = i\hbar\sqrt{\tilde{\kappa}_c(\hat{x}_{m,j})}(\bar{b}^*_{\text{in},j}\hat{b}_{c,j}e^{i\omega_{d,j}t} - \bar{b}_{\text{in},j}\hat{b}^\dagger_{c,j}e^{-i\omega_{d,j}t}). \tag{31}$$

By taking into account the mechanical damping with the rate γ_m, the Langevin equations of the subsystem's operators $\hat{a}_{m,j}$ and $\hat{b}_{c,j}$ in the rotating frame at the driving frequency $\omega_{d,j}$ (i.e., moving into the interaction picture with respect to $\hbar\omega_{d,j}b^\dagger_{c,j}\hat{b}_{c,j}$ to cancel the time dependence of $\hat{H}_{d,j}$) can be derived as

$$\begin{aligned}\frac{d}{dt}\hat{a}_m &= -\left[\frac{\gamma_m}{2} + i\omega_m\right]\hat{a}_m + ig_\omega\hat{b}^\dagger_c\hat{b}_c\\ &\quad + \frac{g_\kappa}{2\sqrt{\kappa_c}}(\hat{\bar{b}}^\dagger_{\text{in}}\hat{b}_c - \hat{b}^\dagger_c\hat{\bar{b}}_{\text{in}}) - \sqrt{\gamma_m}\hat{a}_{\text{in}}(t),\end{aligned} \tag{32a}$$

$$\begin{aligned}\frac{d}{dt}\hat{b}_c &= -\left\{\frac{\kappa_c}{2}\left[1 + \frac{g_\kappa}{\kappa_c}(\hat{a}_m + \hat{a}^\dagger_m)\right] + \frac{\kappa_{\text{int}}}{2} + i\delta_c\right.\\ &\quad \left. - ig_\omega(\hat{a}_m + \hat{a}^\dagger_m)\right\}\hat{b}_c - \sqrt{\kappa_{\text{int}}}\hat{b}_{\text{int}}(t)\\ &\quad - \sqrt{\kappa_c}\left[1 + \frac{g_\kappa}{2\kappa_c}(\hat{a}_m + \hat{a}^\dagger_m)\right]\hat{\bar{b}}_{\text{in}}(t),\end{aligned} \tag{32b}$$

where the subscript "j" is omitted for simplicity, the detuning $\delta = \omega_c - \omega_d$, and $\hat{\bar{b}}_{\text{in}} = \hat{b}_{\text{in}} + \bar{b}_{\text{in}}$. Here, $\hat{a}_{\text{in}}(t)$ denotes a thermal mechanical noise operator and it satisfies the nonzero correlations $\langle\hat{a}^\dagger_{\text{in}}(t)\hat{a}_{\text{in}}(t')\rangle = n_{\text{th}}\delta(t-t')$ and $\langle\hat{a}_{\text{in}}(t)\hat{a}^\dagger_{\text{in}}(t')\rangle = (n_{\text{th}}+1)\delta(t-t')$ [37,60], with the mean thermal phonon number $n_{\text{th}} = (e^{\hbar\omega_m/k_BT} - 1)^{-1}$ of the mechanical environment at temperature T, and k_B is the Boltzmann constant.

For the strong driving fields, the above nonlinear equation can be linearized around the steady-state amplitudes of the operators $\hat{a}_m$ and $\hat{b}_c$ as $\hat{a}_m = \bar{a}^{\text{ss}}_m + \delta\hat{a}_m$ and $\hat{b}_c = \bar{b}^{\text{ss}}_c + \delta\hat{b}_c$, where the steady-state amplitudes $\bar{b}^{\text{ss}}_c \approx -\frac{\sqrt{\kappa_c}}{\frac{\kappa_c}{2}+i\Delta}\bar{b}_{\text{in}}$ with the detuning $\Delta = \delta_c + \sqrt{2}g_\omega\text{Re}[\bar{a}^{\text{ss}}_m]$ and $\bar{a}^{\text{ss}}_m \approx \frac{g_\omega(\bar{b}^{\text{ss}}_c)^2}{\omega_m + i\frac{\gamma_m}{2}}$, and the operators $\delta\hat{O}$ describe the corresponding quantum fluctuations of the system, and in the following "δ" is omitted for simplicity. The linearized Langevin equations of the fluctuating operators can be derived as

$$\begin{aligned}\frac{d}{dt}\hat{a}_m &= -\left(\frac{\gamma_m}{2} + i\omega_m\right)\hat{a}_m - \left(\frac{G_\kappa}{4} - iG\right)\hat{b}_c\\ &\quad + \left(\frac{G_\kappa}{4} + iG\right)\hat{b}^\dagger_c - \frac{G_k}{2\sqrt{\kappa_c}}\hat{b}_{\text{in}}(t)\\ &\quad + \frac{G_k}{2\sqrt{\kappa_c}}\hat{b}^\dagger_{\text{in}}(t) - \sqrt{\gamma_m}\hat{a}_{\text{in}}(t),\end{aligned} \tag{33a}$$

$$\begin{aligned}\frac{d}{dt}\hat{b}_c &= -\left(\frac{\kappa_c + \kappa_{\text{int}}}{2} + i\Delta\right)\hat{b}_c - \left(\frac{G_\kappa}{4} - iG\right)(\hat{a}_m + \hat{a}^\dagger_m)\\ &\quad - \sqrt{\kappa_c}\hat{b}_{\text{in}}(t) - \sqrt{\kappa_{\text{int}}}\hat{b}_{\text{int}}(t),\end{aligned} \tag{33b}$$

where $G_\omega \equiv g_\omega\bar{b}^{\text{ss}}_c$ and $G_\kappa \equiv g_\kappa\bar{b}^{\text{ss}}_c$ are, respectively, collective dispersive and dissipative optomechanical coupling strengths, and $G = G_\omega + \frac{G_\kappa\Delta}{2\kappa_c}$.

With Eq. (33), we can study quantum steerable correlations between the mechanical oscillator and the cavity field. But in reality, it is the output field leaking from the cavity rather than the intracavity field under homodyne detection that achieves entanglement swapping. The output field $\hat{b}^{\text{out}}_c(t)$ is related to the intracavity field $\hat{b}_c(t)$ via the relation [60,61]

$$\hat{b}^{\text{out}}_c(t) = \sqrt{\tilde{\kappa}_c}\hat{b}_c(t) + \hat{b}_{\text{in}}(t), \tag{34a}$$

$$\simeq \sqrt{\kappa_c}\hat{b}_c(t) + \frac{G_\kappa}{2\sqrt{\kappa_c}}(\hat{a}_m + \hat{a}^\dagger_m) + \hat{b}_{\text{in}}(t), \tag{34b}$$

where the equation in the second line is the result of expanding $\tilde{\kappa}_c$ to first order in the mechanical displacement as well as linearization. The output field satisfies the commutation relation $[\hat{b}^{\text{out}}_c(t), \hat{b}^{\text{out}\dagger}_c(t')] = \delta(t-t')$, and thus to discuss the steering between the output field and the mechanical oscillator, we should define a temporal mode $\hat{b}_f^{\rm out}$ of the continuous output field $\hat{b}_c^{\rm out}$ [62], which should possess a definite central frequency and bandwidth and satisfy $[\hat{b}_f^{\rm out}, \hat{b}_f^{\dagger\rm out}] = 1$, given by

$$\hat{b}_f^{\rm out}(t) = \int_{-\infty}^{t} f(t-s)\hat{b}_c^{\rm out}(s)ds, \tag{35}$$

where the function $f(t)$ satisfies $\int_0^\infty |f(t)|^2 dt = 1$ due to the commutation relation of $\hat{b}_f^{\rm out}$ and it can be chosen as

$$f(t) = \sqrt{\frac{2}{\tau}}\theta(t)e^{-(\frac{1}{\tau}+i\Omega_f)^{-1}t}, \tag{36}$$

with the Heaviside step function θ. The function $f(t)$ defines a filtered mode with a bandwidth of τ^{-1} and a central frequency of Ω_f from the continuous output field $\hat{b}_c^{\rm out}(t)$.

For the given filtered mode $\hat{b}_f^{\rm out}$, the stationary bipartite correlation matrix of the mechanical and this temporal mode can be obtained by first introducing $\chi^{\rm out}(\omega) = (\hat{a}_m(\omega), \hat{a}_m^\dagger(-\omega), \hat{b}_f^{\rm out}(\omega), \hat{b}_f^{\rm out\dagger}(-\omega))^T$, where $\hat{a}_m(\omega)$ and $\hat{b}_f^{\rm out}(\omega)$ are, respectively, the Fourier transformation of $\hat{a}_m(t)$ and $\hat{b}_f^{\rm out}(t)$. Rearrange Eq. (33) in the form $\frac{d}{dt}\psi(t) = \mathcal{A}\psi(t) - \mathcal{B}_1\psi_{\rm in}(t) - \mathcal{B}_2\psi_{\rm int}(t)$, where $\psi = (\hat{a}_m, \hat{a}_m^\dagger, \hat{b}_c, \hat{b}_c^\dagger)^T$, $\psi_{\rm in} = (\hat{a}_{\rm in}, \hat{a}_{\rm in}^\dagger, \hat{b}_{\rm in}, \hat{b}_{\rm in}^\dagger)^T$, and $\psi_{\rm int} = (0, 0, \hat{b}_{\rm int}, \hat{b}_{\rm int}^\dagger)^T$. The matrices $\mathcal{A}$, $\mathcal{B}_1$, and $\mathcal{B}_2$ can be easily obtained and are not presented here. By using Eqs. (34) and (35), we can obtain

$$\chi^{\rm out}(\omega) = \mathcal{F}_1(\omega)\chi_{\rm in}(\omega) + \mathcal{F}_2(\omega)\chi_{\rm int}(\omega), \tag{37}$$

where $\chi_{\rm in}(\omega)$ and $\chi_{\rm int}(\omega)$ are, respectively, the corresponding Fourier transform of $\psi_{\rm in}(t)$ and $\psi_{\rm int}(t)$, $\mathcal{F}_{1\omega} = \mathcal{T}_\omega\mathcal{A}_\omega\mathcal{B}_1 + \mathcal{D}_\omega$, $\mathcal{F}_{2\omega} = \mathcal{T}_\omega\mathcal{A}_\omega\mathcal{B}_2$,

$$\mathcal{T}_\omega = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{G_\kappa f(\omega)}{\sqrt{2\kappa_c}} & \frac{G_\kappa f(\omega)}{\sqrt{2\kappa_c}} & \sqrt{\kappa_c}f(\omega) & 0 \\ \frac{G_\kappa f^*(-\omega)}{\sqrt{2\kappa_c}} & \frac{G_\kappa f^*(-\omega)}{\sqrt{2\kappa_c}} & 0 & \sqrt{\kappa_c}f^*(-\omega) \end{pmatrix}, \tag{38}$$

$$\mathcal{D}_\omega = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & f(\omega) & 0 \\ 0 & 0 & 0 & f^*(-\omega) \end{pmatrix}, \tag{39}$$

and $\mathcal{A}_\omega = (\mathcal{A} + i\omega I)^{-1}$. In the above, $f(\omega)$ is the Fourier transform of the function $f(t)$. With Eq. (37), the bipartite CM σ_{ab} [defined in Eq. (1)] with respect to the mechanical mode $\hat{a}_m$ and the filtered output field $\hat{b}_f^{\rm out}$ in the steady-state regime can be found to be

$$\sigma_{ab} = \int_{-\infty}^{\infty} d\omega \sum_{j=1}^{2} \mathcal{H}\mathcal{F}_j(\omega)\mathcal{C}_j\mathcal{F}_j^T(-\omega)\mathcal{H}^T, \tag{40}$$

where $\mathcal{H} = {\rm diag}(\mathcal{H}_1, \mathcal{H}_2)$, $\mathcal{H}_j = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$, $\mathcal{C}_1 = \frac{1}{2}(2n_{\rm th} + 1)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, and $\mathcal{C}_2 = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

It should be noted that the Langevin equation (33) is stable only if all the eigenvalues of the matrix $\mathcal{A}$ have negative real parts. With the Routh-Hurwitz criterion [63], the stability condition can be found to be that the following three inequalities are simultaneously held, i.e.,

$$S_1 = \kappa_c\left(\frac{\kappa_c^2}{4} + \kappa_c\gamma_m + \Delta^2\right) + \gamma_m\left(\frac{\gamma_m^2}{4} + \kappa_c\gamma_m + \omega_m^2\right) - 2\omega_m G_\kappa G > 0, \tag{41a}$$

$$S_2 = \left(\frac{\gamma_m^2}{4} + \omega_m^2\right)\left(\frac{\kappa_c^2}{4} + \Delta^2\right) + \kappa_c\omega_m G_\kappa G + 2\omega_m\Delta\left(\frac{G_\kappa^2}{8} - 2G^2\right) > 0, \tag{41b}$$

$$\begin{aligned} S_3 = {}& \left[\kappa_c\left(\frac{\kappa_c\gamma_m}{4} + \omega_m^2\right) + \gamma_m\left(\frac{\kappa_c\gamma_m}{4} + \Delta^2\right) + 2\omega_m G_\kappa G\right]\left[\kappa_c\left(\frac{\kappa_c^2}{4} + \kappa_c\gamma_m + \Delta^2\right) + \gamma_m\left(\frac{\gamma_m^2}{4} + \kappa_c\gamma_m + \omega_m^2\right)\right. \\ & \left. -2\omega_m G_\kappa G\right] - (\kappa_c + \gamma_m)^2\left[\left(\frac{\gamma_m^2}{4} + \omega_m^2\right)\left(\frac{\kappa_c^2}{4} + \Delta^2\right) + \kappa_c\omega_m G_\kappa G + 2\omega_m\Delta\left(\frac{G_\kappa^2}{8} - 2G^2\right)\right] > 0. \end{aligned} \tag{41c}$$

Thus, with the steady-state CM σ_{ab}, the entanglement and steering between the mechanical oscillator and the filtered output field can be discussed. The entanglement swapping is accomplished by mixing the two filtered output fields $\hat{b}_{f,1}^{\rm out}$ and $\hat{b}_{f,2}^{\rm out}$ on a balanced beam splitter and performing homodyne detection on the two beam-splitter outputs, as shown in Fig. 1. The entanglement between the two mechanical oscillators can be analyzed by obtaining the corresponding CM in Eq. (14).

In Figs. 2–5, the dependences of the steady-state optomechanical entanglement E_{ab}, light-mechanical steering $\mathcal{S}_{b|a}$ and $\mathcal{S}_{a|b}$ of the subsystems, and entanglement $E_{m_{12}}$ between the two mechanical oscillators on the detuning Δ, the coupling $G_{\omega/\kappa}$, and the cavity dissipation rate κ_c are presented, respectively, for dispersive and dissipative coupling. It is shown from them that for the optomechanical system, the steering between the output filtered field and the mechanical oscillator in two directions can be achieved. Moreover, there exist asymmetric and even one-way steering. We can see that the presence of optomechanical entanglement E_{ab} does not mean the entanglement between the two mechanical oscillators via entanglement swapping, since the subsystem's entanglement is merely necessary for swapped entanglement. However, it can be seen that the values of the optomechanical steering $\mathcal{S}_{a|b}$ from the output field and the mechanical oscillator are equal to that of the mechanical entanglement $E_{m_{12}}$, showing that the existence of light-to-mechanical steering is

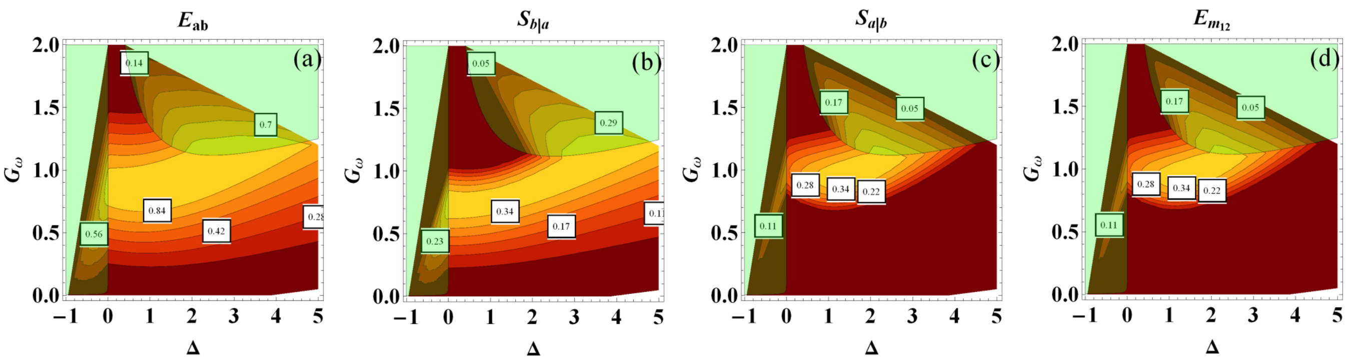

FIG. 2. The light-mechanical entanglement E_{ab}, steering $S_{b|a}$ and $S_{a|b}$ of the subsystems, and distant mechanical entanglement $E_{m_{12}}$ via entanglement swapping for dispersive optomechanical coupling, with the parameters $\omega_m = 1$, $G_\kappa = 0$, $\kappa_c = 5$, $\Omega_f = 1$, $\tau^{-1} = 1$, $\gamma_m = 10^{-5}$, $\kappa_{\rm int} = 0$, and $n_{\rm th} = 0$. The green shaded areas correspond to the regions of instability.

the sufficient and necessary condition for achieving mechanical entanglement via swapping.

Specifically, as shown in Figs. 2 and 3, we see that for dispersive optomechanical coupling, the optomechanical entanglement E_{ab}, the light-mechanical steering $\mathcal{S}_{b|a}$ and $\mathcal{S}_{a|b}$, and the mechanical entanglement $E_{m_{12}}$ are mainly presented in the red-detuned regime ($\Delta > 0$), while for dissipative coupling, stronger light-mechanical entanglement and steering appears not only in the red-detuned regime but also in the region of blue detuning ($\Delta < 0$). This is because for the dispersive coupling case, the optomechanical coupling strength G_ω is considerably limited by the stability condition in the blue-detuned regime, as depicted by the shaded areas in the figures, which therefore renders the entanglement and steering weak and fragile, whereas for the dissipative coupling case, the larger value of the coupling strength G_κ can be allowed in both regimes of red and blue detuning. Thus, for the latter case, significant entanglement E_{ab} and $E_{m_{12}}$ and steering in two directions can be generated in the blue-detuned regime via dissipative coupling. Nevertheless, even in the red-detuned regime, it is shown that the achievable entanglement and steering via dissipative coupling are stronger than those by dispersive coupling. In addition, from Figs. 2 and 3 one can also see that the optimal entanglement and steering are achieved around the detuning $\Delta \approx 1.5\omega_m$ for dispersive coupling, but for dissipative coupling it occurs near the larger detuning $\Delta \approx 8\omega_m$, both with the coupling $G_{\omega,\kappa} \geqslant \omega_m$. It should be noted that in reality the coupling $G_{\omega,\kappa}$ and the detuning Δ are not independent of each other, but for the fixed value of the detuning, the coupling strengths can be controlled by modulating the driving strength.

In Figs. 4 and 5, the dependence of the optomechanical entanglement E_{ab}, steering $\mathcal{S}_{b|a}$ and $\mathcal{S}_{a|b}$, and the mechanical entanglement $E_{m_{12}}$ on the cavity dissipation rate κ_c are presented. It is very interesting to see that for both kinds of optomechanical coupling, the larger dissipation rate is more beneficial for the generation of entanglement and steering, which are optimized for the bad cavity limit $\kappa_c \gg \omega_m$, as shown in the figures. This means that to achieve significant entanglement and steering, the constraint that the cavity dissipation rate κ_c should be smaller than the mechanical frequency ω_m can be discarded. This makes our scheme quite different from that in Ref. [48] in which the mechanical oscillator is coupled to two cavity modes that are, respectively, driven by a blue-detuned and a red-detuned laser, and moreover the swapped entanglement is achieved only in the sideband-resolved regime. As a matter of fact, except for some special optomechanical systems, such a condition is hard to fulfill in many experimental optomechanical systems, since it typically requires optical cavities with very high finesse,

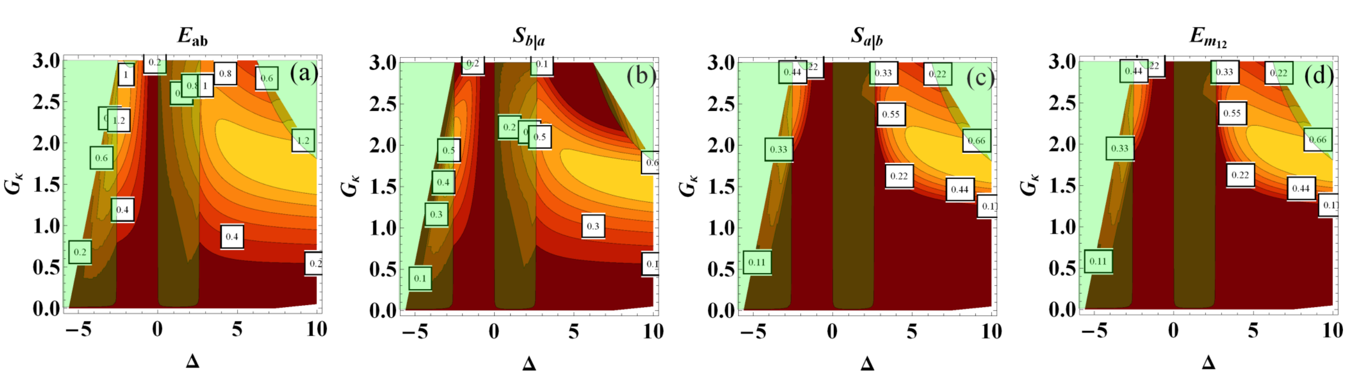

FIG. 3. The light-mechanical entanglement E_{ab}, steering $S_{b|a}$ and $S_{a|b}$ of the subsystems, and distant mechanical entanglement $E_{m_{12}}$ via entanglement swapping for dissipative optomechanical coupling, with the parameters $\omega_m = 1$, $G_\omega = 0$, $\kappa_c = 5$, $\Omega_f = 1$, $\tau^{-1} = 1$, $\gamma_m = 10^{-5}$, $\kappa_{\rm int} = 0$, and $n_{\rm th} = 0$. The green shaded areas correspond to the regions of instability.

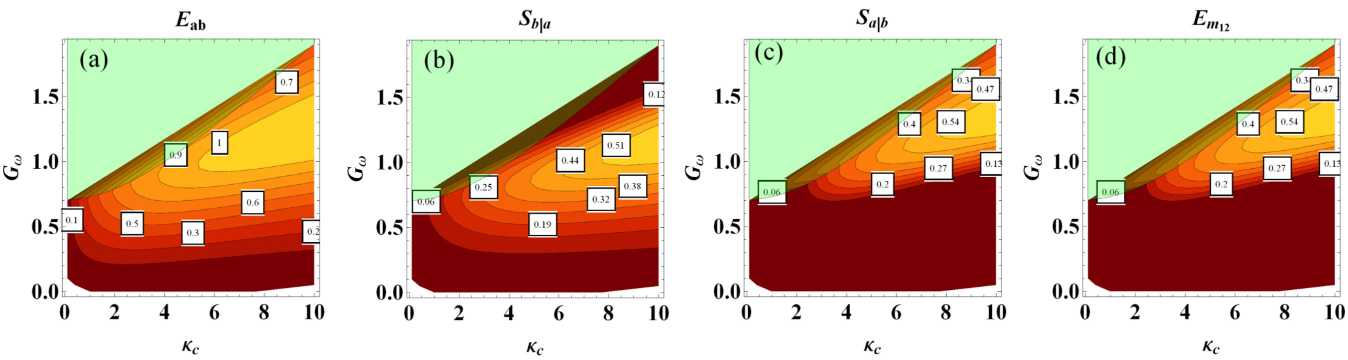

FIG. 4. The light-mechanical entanglement E_{ab}, steering $S_{b|a}$ and $S_{a|b}$ of the subsystems, and distant mechanical entanglement $E_{m_{12}}$ via entanglement swapping for dispersive optomechanical coupling, with the parameters $\omega_m = 1,\ G_\kappa = 0,\ \Delta = 2,\ \Omega_f = 1,\ \tau^{-1} = 1,\ \gamma_m = 10^{-5},\ \kappa_{\rm int} = 0$, and $n_{\rm th} = 0$. The green shaded areas correspond to the regions of instability.

and considerably limits the size and mass of the mechanical oscillator. Thus, with our scheme, the distant entanglement between two low-frequency mechanical oscillators can be generated.

Finally, we study the effect of cavity intrinsic loss and thermal mechanical fluctuations on entanglement and steering for the present system, which are, respectively, depicted in Figs. 6 and 7. We can see from Fig. 6 that the intrinsic loss decreases the entanglement and steering. Moreover, as the intrinsic loss increases, the steering from the output field to the mechanical oscillator decreases much faster than the reverse steering. This is because as the vacuum intrinsic loss increases, the intracavity field tends to a vacuum and thus can be more easily steered by the mechanical oscillator, which has larger quantum fluctuations than the former and hence we have stronger steering from the mechanics to the output field. From Fig. 7 it is shown that the distant mechanical entanglement can still exist for $n_{\rm th} \approx 2 \times 10^4$ and $n_{\rm th} \approx 3.0 \times 10^4$, respectively, by dispersive and dissipative coupling with $\kappa_{\rm int} = 0.1\kappa_c$. This means that the robust entanglement between the two mechanical oscillators can be achieved. Consider realistic parameters close to the recent experiment in Ref. [57], with the mechanical frequency $\omega_m \approx 2\pi \times 140$ kHz, the mechanical quality factor $Q = \omega_m/\gamma_m = 10^5$, the cavity linewidth $\kappa_c = 8\omega_\omega \approx 2\pi \times 1.12$ MHz, the single-photon coupling rates $g_\omega \approx 2\pi \times 1.5$ Hz, and $g_\kappa \approx 2\pi \times 0.3$ Hz. For the powers $\mathcal{P}_\omega \approx 4.5$ mW and $\mathcal{P}_\kappa \approx 2.2$ W of the driving lasers with the wavelength $\lambda = 1064$ nm, the coupling strengths $G_\omega \approx 1.25\omega_m$ and $G_\kappa \approx 2.5\omega_m$, as accepted in Fig. 6, can be achieved under the detuning $\Delta = 2\omega_m$ and $8\omega_m$, respectively. With these parameters, it can be found that the mechanical entanglement can still be achieved at the environment temperature $T \approx 0.12$ K and $T \approx 0.2$ K, respectively, for dispersive and dissipative coupling.

V. CONCLUSION

To summarize, in the paper we first study the connection between EPR steering and entanglement swapping of Gaussian states. It is found that for Gaussian entanglement swapping, the steering from the modes subject to detection to the other modes of two input states is sufficient and necessary for achieving swapped entanglement, which is moreover independence of reverse steering. We further reveal that this is due to the steering in such a direction, with homodyne detection enabling the quadrature quantum squeezing of the composite modes of the two distant modes to be entangled. Then, as a realistic example, we investigate light-mechanical steering in dispersively and dissipatively coupled optomechanical systems, and we consider the generation of entanglement between two distant mechanical oscillators via entanglement swapping with the steering. It is found that for both cases

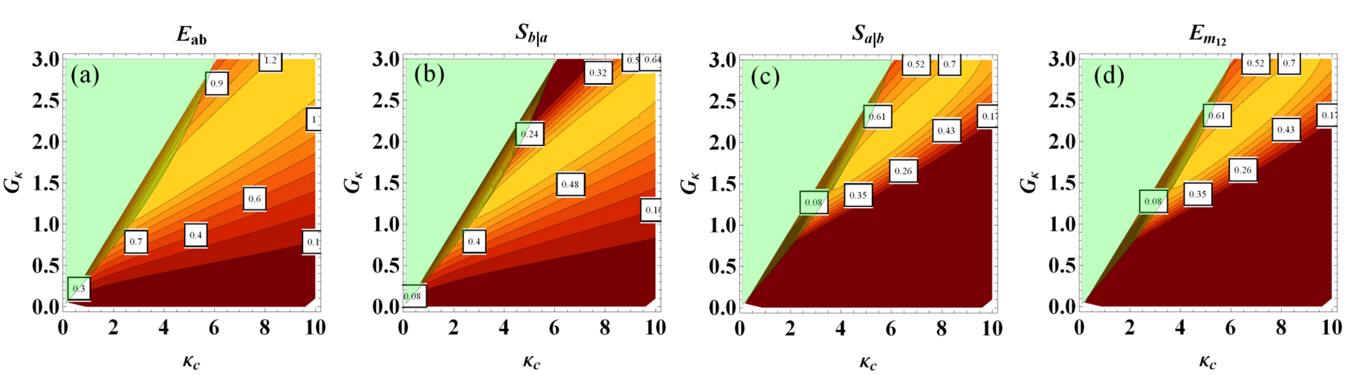

FIG. 5. The light-mechanical entanglement E_{ab}, steering $S_{b|a}$ and $S_{a|b}$ of the subsystems, and distant mechanical entanglement $E_{m_{12}}$ via entanglement swapping for dissipative optomechanical coupling, with the parameters $\omega_m = 1,\ G_\kappa = 0,\ \Delta = 8,\ \Omega_f = 1,\ \tau^{-1} = 1,\ \gamma_m = 10^{-5},\ \kappa_{\rm int} = 0$, and $n_{\rm th} = 0$. The green shaded areas correspond to the regions of instability.

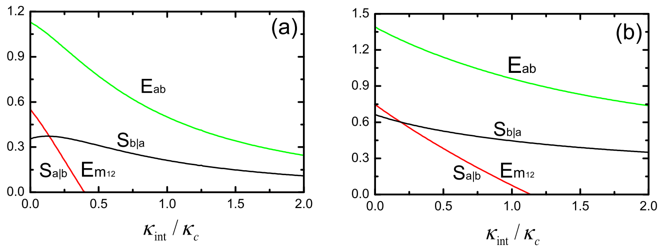

FIG. 6. The effect of the intrinsic loss of the cavity on the light-mechanical entanglement E_{ab}, steering $S_{b|a}$ and $S_{a|b}$ of the subsystems, and distant mechanical entanglement $E_{m_{12}}$ for dispersive coupling in (a) and dissipative optomechanical coupling in (b), with $\Delta = 2$ and $G_\omega = 1.25$ in (a), and $\Delta = 8$ and $G_\kappa = 2.5$ in (b). The other parameters are $\omega_m = 1$, $\kappa = 8$, $\Omega_f = 1$, $\tau^{-1} = 1$, $\gamma_m = 10^{-5}$, and $n_{\rm th} = 0$.

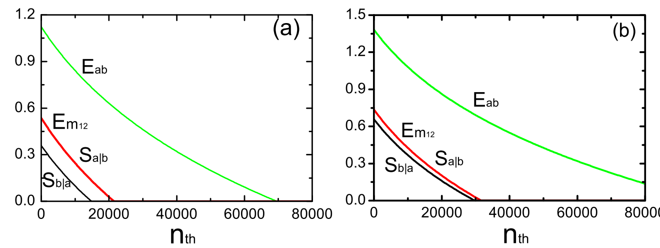

FIG. 7. The effect of thermal mechanical noise $n_{\rm th}$ on the light-mechanical entanglement E_{ab}, steering $S_{b|a}$ and $S_{a|b}$ of the subsystems, and distant mechanical entanglement $E_{m_{12}}$ for dispersive coupling in (a) and dissipative optomechanical coupling in (b). The parameters are the same as in Fig. 6, expect for the intrinsic loss $\kappa_{\rm int} = 0.1\kappa_c$.

of coupling, robust mechanical entanglement can be obtained in the regime of steady states without the constraint that the cavity linewidth should be smaller than the mechanical frequency. Thus, with the present scheme, distant entanglement between two low-frequency mechanical oscillators can be generated.

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