

Particle-number scaling of the quantum work statistics and Loschmidt echo in Fermi gases with time-dependent traps

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We investigate the particle-number dependence of some features of the out-of-equilibrium dynamics of d -dimensional Fermi gases in the dilute regime. We consider protocols entailing the variation of the external potential which confines the particles within a limited spatial region, in particular, sudden changes of the trap size. To characterize the dynamic behavior of the Fermi gas, we consider various global quantities such as the ground-state fidelity for different trap sizes, the quantum work statistics associated with the protocol considered, and the Loschmidt echo measuring the overlap of the out-of-equilibrium quantum states with the initial ground state. Their asymptotic particle-number dependencies show power laws for noninteracting Fermi gases. We also discuss the effects of short-ranged interactions to the power laws of the average work and its square fluctuations, within the d -dimensional Hubbard model and its continuum limit, arguing that they do not generally change the particle-number power laws of the free Fermi gases.

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I. INTRODUCTION

The recent progress of experiments in atomic physics has provided a great opportunity for a thorough investigation of the thermodynamics of quantum systems, and the interplay between quantum and statistical behaviors. Atomic systems are realized with a great control, thanks to the impressive progress in the manipulation of cold atoms [1]. The realization of physical systems which are described by theoretical models, such as dilute Fermi and Bose gases, Hubbard and Bose-Hubbard models, with spatial dimensions from one to three, provides thorough experimental checks of the fundamental paradigms of statistical and quantum physics. In particular, they allow us to investigate the unitary quantum evolution of closed many-body systems, exploiting their low dissipation rate which maintains phase coherence for a long time [1,2]. Therefore the theoretical investigation of the out-of-equilibrium unitary dynamics of many-body systems is of great importance for a deep understanding of the fundamental issues of quantum dynamics, their possible applications, and new developments.

In this paper we study some features of the out-of-equilibrium quantum dynamics of Fermi gases, arising from variations of the external potential which confines them within a limited spatial region. We consider generic d -dimensional traps arising from external power-law potentials, and, in particular, the cases of harmonic traps and hard-wall traps. Some aspects related to this issue have been discussed in the literature, such as the time dependence of the particle density and fixed-time correlation functions, spatial entanglement, and so on, in particular for one-dimensional systems, see, e.g., Refs. [3–16].

We focus on the particle-number dependence of the out-of-equilibrium dynamics of N -particle Fermi gases in the dilute regime, when the external potential is changed in such a way as to give rise to sudden variations of the trap size, or shifts of the trap. To characterize the evolution of the quantum states,

we consider various global quantities, such as the ground-state fidelity associated with changes of the trap size, the quantum work associated with a sudden change of the trap size, the overlap between the quantum state at a given time t and the initial ground state, as measured by the so-called Loschmidt echo. We show that large- N power laws characterize their dependence on the particle number.

We mostly consider lattice gas models of spinless noninteracting Fermi particles in the dilute regime, realized in the limit of large trap size keeping the particle number fixed. This corresponds to the trap-size scaling limit, or continuum limit, whose scaling functions are related to the correlation functions of a continuum many-body theory of free Fermi particles in an external confining potential [17,18]. In the case of the quantum work and its fluctuations, we also discuss the effects of particle interactions, in the framework of the Hubbard model and its continuum limit in the dilute regime.

The paper is organized as follows. In Sec. II we present the general setting of the problem for free Fermi lattice gases in the dilute regime, and their continuum limit. In Sec. III we study the particle-number dependence of the ground-state fidelity associated with variations of the trap size; the corresponding equilibrium condition is realized in the limit of adiabatic changes of the trap features. Section IV is devoted to the computation of the first few moments of the quantum work distribution associated with sudden changes of the trap size, starting for an equilibrium (ground-state) condition. In Sec. V we study the particle-number dependence of the overlap between the quantum states along the out-of-equilibrium evolution and the initial states, as measured by the so-called Loschmidt echo. In Sec. VI we discuss the effects of short-ranged particle interactions within the Hubbard model and its continuum limit, arguing that the power laws of the asymptotic particle-number dependence of the quantum work, and its fluctuations, do not generally change with respect to the case of free Fermi gases. Finally, in Sec. VII we

summarize our main results, and draw our conclusions. Some useful formulas for the ground state of noninteracting Fermi gases are reported in the Appendix.

II. GENERAL SETTING OF THE PROBLEM

We consider d -dimensional lattice gases of N noninteracting spinless Fermi particles constrained within a limited spatial region by an external force. The corresponding lattice many-body Hamiltonian reads

$$H(\ell) = -t \sum_{\langle xy \rangle} [c_x^\dagger c_y + \text{H.c.}] + \sum_{\mathbf{x}} V(\mathbf{x}, \ell) n_{\mathbf{x}}, \quad (1)$$

where \mathbf{x} are the sites of a d -dimensional cubic-like lattice, $\langle xy \rangle$ indicates nearest-neighbor sites, $c_{\mathbf{x}}$ is a spinless fermionic operator, $n_{\mathbf{x}} = c_{\mathbf{x}}^\dagger c_{\mathbf{x}}$ is the particle-density operator. In the rest of the paper we set the lattice spacing $a = 1$, the kinetic constant $t = 1$, and $\hbar = 1$; their dependence can be easily inferred by dimensional analysis. The confining potential $V(\mathbf{x}, \ell)$ is coupled to the particle density operator; it is such that $V(\mathbf{x}, \ell) \rightarrow \infty$ for $|\mathbf{x}| \rightarrow \infty$, so that $\langle n_{\mathbf{x}} \rangle \rightarrow 0$ for $\mathbf{x} \rightarrow \infty$. We assume it is isotropic, and characterized by a generic power law, i.e.,

$$V(\mathbf{x}, \ell) = \frac{1}{p} v^p |\mathbf{x}|^p, \quad \ell = v^{-1}, \quad (2)$$

where ℓ should be considered as the trap size [7,17]. The potential with power law $p = 2$ gives rise to harmonic traps, where $\omega = v$ is the corresponding frequency. In the limit $p \rightarrow \infty$ we recover hard-wall traps, so that $V = 0$ for $|\mathbf{x}| < \ell$ and $V = \infty$ for $|\mathbf{x}| > \ell$. The particle number operator $\hat{N} = \sum_{\mathbf{x}} n_{\mathbf{x}}$ is conserved, i.e., $[\hat{N}, H(\ell)] = 0$. We consider the lattice model (1) at a fixed number N of particles, $N \equiv \langle \hat{N} \rangle$.

We consider the dilute regime, when the particles are sufficiently diluted, i.e., $N/\ell^d \ll 1$. This is effectively defined as the asymptotic behavior in the large trap-size limit, keeping the particle number N fixed. This limit can be studied in the trap-size scaling framework [10,17], which relates the asymptotic trap-size dependence of lattice gases in the dilute regime with the corresponding vacuum-to-metal quantum transition of the many-body Hamiltonian (1) in the presence of a chemical potential [adding a term $-\mu \sum_{\mathbf{x}} n_{\mathbf{x}}$ to the Hamiltonian (1), releasing the constraint on the number of particles]. We recall that the large trap-size limit in the presence of a chemical potential μ corresponds to taking the large- ℓ limit keeping the ratio N/ℓ^d fixed. A detailed discussion of the connection between the dilute regime and the one keeping N/ℓ^d fixed can be found in Ref. [17]. In particular, when increasing the chemical potential across the value $\mu = \mu_c = -2d$, the Fermi gas experiences a quantum phase transition, from a trivial phase where the ground state is given by the vacuum (absence of particles, for $\mu < \mu_c$), to the metallic phase characterized by algebraically decaying correlations ($\mu > \mu_c$), see, e.g., Ref. [20].

The critical behavior at the vacuum-to-metal transitions (located at $\mu = \mu_c = -2d$) is characterized by the trap-size exponent [17,19]

$$\theta = \frac{p}{p+2}, \quad (3)$$

depending on the power of the confining potential (2). Its meaning is related to the fact the presence of an external inhomogeneous potential induces a nontrivial length scale $\xi \sim \ell^\theta$ in the correlation functions of the system (we recall that we are using the particular physical units $\hbar = 1$, $a = 1$, $t = 1$). Thus, the critical length scale does not scale as the trap size, but as a nontrivial power with exponent θ . Only in the limit $p \rightarrow \infty$ we have that $\xi \sim \ell$ as expected from standard finite-size scaling arguments [21]. For example, the trap-size dependence of the gap $\Delta(\ell)$ of the Fermi gas (i.e., the difference of the lowest energy levels) behaves asymptotically as

$$\Delta(\ell) \sim \xi^{-z} \sim \ell^{-z\theta}, \quad (4)$$

where $z = 2$ is the dynamic exponent associated with the vacuum-to-metal transition of Fermi gases. Moreover, correlation functions of generic local operators $\mathcal{O}(\mathbf{x})$ develop a trap-size scaling behavior [10,17], such as

$$F(\mathbf{x}_1, \dots, \mathbf{x}_n; \ell, N) \equiv \langle \mathcal{O}(\mathbf{x}_1) \dots \mathcal{O}(\mathbf{x}_n) \rangle \approx \ell^{-\varepsilon} \mathcal{F}(\mathbf{X}_1, \dots, \mathbf{X}_n; N), \quad (5)$$

where

$$\mathbf{X}_i = \mathbf{x}_i / \ell^\theta, \quad \varepsilon = n\theta y_o, \quad (6)$$

and y_o is the renormalization-group dimension of the operator $\mathcal{O}(\mathbf{x})$ at the fixed point associated with the vacuum-to-metal transition [20,21]. Of course, corrections to this asymptotic behavior arise in lattice models due to the space discretization. They are generally suppressed by powers of ℓ , more precisely they are expected to vanish as $\ell^{-2\theta}$ for lattice free-fermion gases.

In the *continuum* limit $a \rightarrow 0$, where a is the lattice spacing, or equivalently in the limit $\ell/a \rightarrow \infty$ keeping fixed a , we recover a continuum model for a Fermi gas of N particles in a trap of size ℓ , corresponding to the many-body problem with one-particle Hamiltonian

$$\mathcal{H}(\ell) = \frac{p^2}{2m} + V(\mathbf{x}, \ell). \quad (7)$$

We set $m = 1$, so that the trap size ℓ corresponds to that of the lattice model (1), using the same unit ($\hbar = 1$ and $t = 1$). Such a continuum limit corresponds to the trap-size scaling limit of the lattice model [10,19]. This implies that the scaling functions $\mathcal{F}(\mathbf{X}_1, \dots, \mathbf{X}_n; N)$ entering the trap-size scaling relation (5) are exactly given by the continuum many-body problem associated with the one-particle Hamiltonian (7). Some useful formulas for the ground state of Fermi gases with the one-particle Hamiltonian (7) are reported in the Appendix.

In this paper we mostly focus on the evolution of the Fermi gas arising from variations of the trap size, starting from the ground state associated with an initial trap size ℓ_0 . We study the relations between the initial and evolving states, as they are quantified by a number of quantum information concepts, such as ground-state fidelity, quantum work statistics, and Loschmidt echo.

In the protocol that we consider the initial condition of the Fermi gas is the ground state associated with the initial Hamiltonian parameters. Therefore, in the continuum limit,

the $t = 0$ state is represented by the many-body wave function

$$\Psi(\mathbf{x}_1, \dots, \mathbf{x}_N, t = 0; \ell_0) = \frac{1}{\sqrt{N!}} \det[\psi_i(\mathbf{x}_j; \ell_0)], \quad (8)$$

where $\psi_k(\mathbf{x}; \ell_0)$ are the lowest N eigenstates of the one-particle Hamiltonian $\mathcal{H}(\ell_0)$, cf. Eq. (7). Then the trapping potential generally changes as

$$V(\mathbf{x}, t) = \frac{1}{p} \kappa(t) |\mathbf{x}|^p. \quad (9)$$

The time dependence of the function $\kappa(t)$ has a time scale t_s . In the limit $t_s \rightarrow 0$ we may consider it as a sudden change of the confining potential, while for $t_s \rightarrow \infty$ we recover the adiabatic limit [more precisely when $t_s \gg \ell(t)^{\frac{d}{p}}$ with $\ell(t) \equiv \kappa(t)^{-1/p}$], when the quantum evolution passes through equilibrium ground states associated with the varying trap sizes. The time variation of the external potential gives generally rise to a nontrivial quantum evolution of the Fermi gas, whose many-body wave function in the continuum limit can be written as

$$\Psi(\mathbf{x}_1, \dots, \mathbf{x}_N, t; \ell_0) = \frac{1}{\sqrt{N!}} \det[\psi_i(\mathbf{x}_j, t; \ell_0)], \quad (10)$$

where the one-particle wave functions $\psi_i(\mathbf{x}_j, t; \ell_0)$ are solutions of the one-particle Schrödinger problem

$$i \frac{d\psi_i(\mathbf{x}_j, t; \ell_0)}{dt} = \left[\frac{\mathbf{p}^2}{2} + V(\mathbf{x}, t) \right] \psi_i(\mathbf{x}_j, t; \ell_0), \quad (11)$$

$$\psi_i(\mathbf{x}_j, t = 0; \ell_0) = \psi_i(\mathbf{x}, \ell_0). \quad (12)$$

In particular, we will consider the out-of-equilibrium dynamics arising from sudden changes of the trap size.

III. GROUND-STATE FIDELITY RELATED TO VARIATIONS OF THE TRAP SIZE

Before discussing the out-of-equilibrium dynamics arising from sudden variations of the trap size of the system, we investigate the adiabatic limit of our dynamic problem, which corresponds to slow variations of the trap size $\ell(t) \equiv \kappa(t)^{-1/p}$, when the time scale of the time-dependent external potential gets large, so that the system is always in the ground state associated with the actual value $\ell(t)$. Thus the global changes of the system properties are related to the variation of the ground-state many-body wave function, and in particular, to the quantum overlap between the ground states for different trap sizes. This is quantified by the equilibrium ground-state fidelity associated with variations of the trap size.

The concept of ground-state fidelity has been introduced to quantify the overlap between ground states associated with different parameters of the model [22,23]. The usefulness of the fidelity as a tool to distinguish quantum states can be traced back to Anderson's orthogonality catastrophe [24]: the overlap of two many-body ground states corresponding to Hamiltonians differing by a small perturbation vanishes in the thermodynamic limit.

The ground-state fidelity monitors the changes of the ground-state wave function $|0_{\ell,N}\rangle$ of the N -particle Fermi gas trapped by the potential with length scale ℓ , when varying the

control parameter $v = \ell^{-1}$. We define it as [22]

$$F(\ell_0, \ell_1, N) \equiv \left| \langle 0_{\ell_1,N} | 0_{\ell_0,N} \rangle \right|. \quad (13)$$

Defining

$$\delta_\ell \equiv \ell_1/\ell_0 - 1, \quad (14)$$

and assuming δ_ℓ sufficiently small, we can expand the ground-state fidelity in powers of δ_ℓ : [22]

$$F = 1 - \frac{1}{2} \delta_\ell^2 \chi_F(\ell_0, N) + O(\delta_\ell^3), \quad (15)$$

where χ_F may be considered as the corresponding susceptibility. The cancellation of the linear term in the expansion (15) is essentially related to the fact that the fidelity is bounded, i.e., $0 \leq F \leq 1$. The fidelity susceptibility gives a quantitative idea of the *speed* of the flow of ground states within the global Hilbert space of the quantum states, when varying the trap size. The behavior of the ground-state fidelity, and in particular, its susceptibility, at quantum transitions has been discussed in the literature, see, e.g., Refs. [25–28], finding a sizable enhancement of the fidelity susceptibility with respect to the behavior of systems in normal conditions.

We compute the ground-state fidelity in the trap-size scaling limit, or equivalently in the continuum limit. As we shall see, the fidelity susceptibility turns out to be independent of ℓ_0 in this limit, i.e.,

$$\chi_F(\ell_0, N) \equiv \chi_F(N). \quad (16)$$

We then determine the large- N asymptotic behaviors. It is important to note that such large- N asymptotic behaviors should be always meant within the dilute regime of the lattice gas model, i.e., when the condition $N/\ell^d \ll 1$ is satisfied.

To begin with, we consider N -particle Fermi gases constrained within one-dimensional harmonic traps, whose ground-state wave function can be written as [29]

$$\Psi(x_1, \dots, x_N; \ell) = \ell^{-N/4} c_N A(X_1, \dots, X_N) e^{-\sum_i X_i^2/2},$$

$$A(x_1, \dots, x_N) = \prod_{1 \leq i < j \leq N} (X_i - X_j),$$

where $X_i = x_i/\sqrt{\ell}$, and c_N is the appropriate normalization constant so that $\int \prod_{i=1}^N dx_i |\Psi|^2 = 1$. The fidelity between one-dimensional ground states associated with the trap sizes ℓ_0 and ℓ_1 can be analytically computed, obtaining

$$F(\ell_0, \ell_1, N) = \int \prod_{i=1}^N dx_i \Psi(x_1, \dots, x_N; \ell_1)^* \Psi(x_1, \dots, x_N; \ell_0) = \left[\frac{4\ell_0\ell_1}{(\ell_0 + \ell_1)^2} \right]^{N^2/4}. \quad (17)$$

By expanding it as in Eq. (15), we obtain the corresponding susceptibility, which is given by

$$\chi_F(N) = \frac{1}{8} N^2. \quad (18)$$

The computation of the fidelity for Fermi gases in higher dimensions, $d > 1$, is more complicated. The ground state of N -particle gases is again given by the Slater determinant associated with the lowest N one-particle states, such as Eq. (8). They can be obtained by filling the lowest one-particle states (A2) with $\sum_i n_i \leq n_e$. The number N of particles or states is a

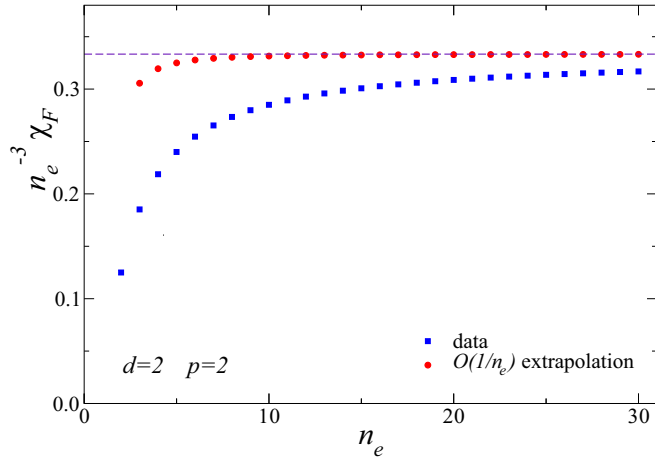


FIG. 1. The fidelity susceptibility for two-dimensional harmonic traps with respect to a variation of the trap size ℓ . We show (practically exact) data of $n_e^{-3}\chi_F$ versus n_e , and also the corresponding linear extrapolation $a + b/n_e$ using the data for n_e and $n_e - 1$. They clearly appear to approach the large- n_e limit $n_e^{-3}\chi_F \approx 1/3$ shown by the dashed line. Recalling that $n_e \approx \sqrt{2N}$ asymptotically, we obtain the large- N behavior (20).

function of n_e , which asymptotically behaves as $N \approx n_e^2/2$ in two dimensions, and $N \approx n_e^3/6$ in three dimensions.

The ground-state fidelity for different trap sizes is formally given by the integral of two N -particle Slater determinants. To compute matrix elements between states expressed in terms of Slater determinants, such as $\Psi^{(\kappa)}(\mathbf{x}_1, \dots, \mathbf{x}_N) = \det[\psi_i^{(\kappa)}(\mathbf{x}_j)]/\sqrt{N!}$, we may use the notable formula (see Ref. [30] and references therein)

$$\begin{aligned} & \langle \Psi^{(1)}(\mathbf{x}_1, \dots, \mathbf{x}_N) | \Psi^{(2)}(\mathbf{x}_1, \dots, \mathbf{x}_N) \rangle \\ &= \int \prod_{i=1}^N d\mathbf{x}_i \Psi^{(1)}(\mathbf{x}_1, \dots, \mathbf{x}_N)^* \Psi^{(2)}(\mathbf{x}_1, \dots, \mathbf{x}_N) \\ &= \det \left[\int d\mathbf{x} \psi_i^{(1)}(\mathbf{x})^* \psi_j^{(2)}(\mathbf{x}) \right]. \end{aligned} \quad (19)$$

We compute the fidelity associated with N particles (in practice this can be done exactly) by using the above formula with the one-particle eigenfunctions associated with different trap sizes, ℓ_0 and ℓ_1 , so that $\delta_\ell \ll 1$. Then, to evaluate the fidelity susceptibility $\chi_F(N)$ for N particles, we perform the $\delta_\ell \rightarrow 0$ extrapolation of the quantity $2(1 - F)/\delta_\ell^2$ at fixed N . This can be achieved with high accuracy. Results for two-dimensional Fermi gases are shown in Fig. 1. The large- N power law of χ_F is then obtained by analyzing the behavior of the data with increasing N . This analysis shows that the large- N power law depends on the spatial dimensions. Indeed, the fidelity susceptibility shows the asymptotic behavior

$$\begin{aligned} \chi_F(N) &= b_d n_e^{d+1} [1 + O(n_e^{-1})] \\ &= c_d N^{(d+1)/d} [1 + O(N^{-1/d})], \end{aligned} \quad (20)$$

for d -dimensional harmonic traps. This is clearly supported by the data shown in Fig. 1 for two-dimensional gases up to $n_e = 30$ corresponding to $N = 435$. We estimate $b_2 \approx 1/3$ with high accuracy, see Fig. 1, thus $c_2 \approx \sqrt{8/9}$. An analogous

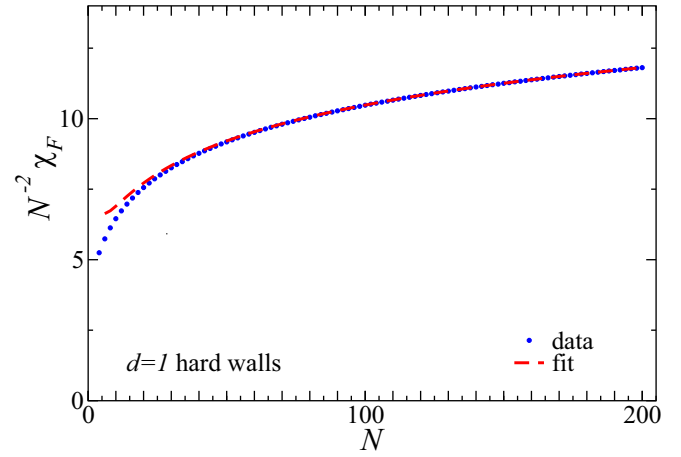


FIG. 2. The fidelity susceptibility for one-dimensional hard-wall traps with respect to a variation of the trap size ℓ . We show data of $N^{-2}\chi_F$ up to $N = 200$. Their large- N behavior nicely fits the function $f(N) = 2 \ln N + b + c/N$ as shown by the dashed line, supporting the asymptotic behavior (21).

analysis of three-dimensional data confirms the large- N behavior (20) with $b_3 \approx 1/8$, thus $c_3 \approx (81/32)^{1/3}$.

We now consider the hard-wall limit $p \rightarrow \infty$ of the confining potential. To compute the ground-state fidelity associated with two different trap sizes, we may use the one-particle eigenfunctions (A3) and the formula (19). As shown by Fig. 2, the results for one-dimensional hard-wall traps show the asymptotic large- N behavior

$$\chi_F(N) \approx a N^2 \ln N, \quad (21)$$

with $a \approx 2$. Therefore, it appears to increase faster than that associated with the harmonic traps.

IV. QUANTUM WORK ASSOCIATED WITH CHANGES OF THE TRAPPING POTENTIAL

A. Quantum work distribution

In this section we focus on the statistics of the work done on the Fermi gas, when this is driven out of equilibrium by suddenly switching the control parameter associated with the external potential. Several issues related to the definition and computation of the work statistics in quantum systems have been already discussed in a variety of physical implementations [31,32], including spin chains [33–41], fermionic, and bosonic systems [41–47].

We consider the quantum dynamics of a ground-state Fermi gas initially constrained within a trap of size ℓ_0 , that is subject to a sudden variation of the trap size from ℓ_0 to ℓ_1 . In this section we analyze the particle-number scaling of the quantum work average and square fluctuations associated with this quench protocol.

The quantum work W associated with out-of-equilibrium dynamic protocols do not generally have a definite value. More specifically, this quantity can be defined as the difference of two projective energy measurements [31]. The first one at $t = 0$ projects onto the eigenstates of the initial

Hamiltonian $H(\ell_0)$ with a probability $p_{m,N}^{\ell_0}$ given by the density matrix of the initial state, for example, given by the equilibrium Gibbs distribution. Then the system evolves, driven by the unitary operator $U(t, 0) = e^{-iH(\ell_1)t}$, and the second energy measurement projects onto the eigenstates of the many-body Hamiltonian $H(\ell_1)$. The work probability distribution can thus be written as [31,48,49]:

$$P(W) = \sum_{n,m} \delta[W - (E_{n,N}^{\ell_1} - E_{m,N}^{\ell_0})] | \langle n_{\ell_1,N} | m_{\ell_0,N} \rangle |^2 p_{m,N}^{\ell_0}, \quad (22)$$

where $E_{n,N}^{\ell}$ and $|n_{\ell,N}\rangle$ are the eigenvalues and corresponding eigenstates of the many-body Hamiltonian with trap size ℓ . The zero-temperature limit corresponds to a quench protocol starting from the ground state of $H(\ell_0)$ (we assume that the ground state is not degenerate). The work probability (22) reduces to

$$P(W) = \sum_n \delta[W - (E_{n,N}^{\ell_1} - E_{0,N}^{\ell_0})] | \langle n_{\ell_1,N} | 0_{\ell_0,N} \rangle |^2. \quad (23)$$

Assuming that both ℓ_0 and ℓ_1 are large, thus in the continuum or trap-size scaling limit, we conjecture that the work probability develops the asymptotic behavior

$$P(W, \ell_0, \ell_1, N) \approx \ell_0^{-2\theta} \mathcal{P}(w, \delta_\ell, N), \quad (24)$$

where we introduced the scaling variable

$$w = \ell_0^{-2\theta} W, \quad (25)$$

associated with the quantum work, and $\delta_\ell = \ell_1/\ell_0 - 1$. The power law of the prefactor of the work distribution and that of the rescaling of the quantum work are related to the scaling behavior of the gap $\Delta(\ell_0) \sim \ell_0^{-z\theta}$, so that

$$\int dW P(W, \ell_0, \ell_1, N) = \int dw \mathcal{P}(w, \delta_\ell, N) = 1. \quad (26)$$

The scaling behavior (24) implies that the moments $\langle W^k \rangle$ of the work distribution develop the asymptotic behavior

$$\langle W^k \rangle = \int dW W^k P(W) \approx \ell_0^{-z\theta k} \mathcal{W}_k(\delta_\ell, N), \quad (27)$$

and so on. These scaling relations will be supported by explicit calculations.

We also mention that within the same scaling framework we may also consider the more general case when the initial condition is represented by a Gibbs distribution with temperature T , thus the quantum work distribution is given by the more general expression (22), with $p_{m,N}^{\ell_0} \sim e^{-E_{m,N}^{\ell_0}/T}$. For sufficiently small T , the temperature dependence can be taken into account by adding a further scaling variable associated with T to the arguments of the scaling functions. The corresponding scaling variable is $T_r \sim T/\Delta(\ell_0)$ where $\Delta(\ell_0) \sim \ell_0^{-z\theta}$ is the gap, cf. Eq. (4). In the following we limit our calculations to the zero-temperature limit.

B. Average work

Let us first determine the average work. We compute it in the trap-size scaling or continuum limit. Using Eqs. (23) and

(27), we write it as

$$\begin{aligned} \langle W \rangle &= \langle 0_{\ell_0,N} | H(\ell) - H(\ell_0) | 0_{\ell_0,N} \rangle \\ &= \langle 0_{\ell_0,N} | \sum_{\mathbf{x}} [V(\mathbf{x}, \ell) - V(\mathbf{x}, \ell_0)] n_{\mathbf{x}} | 0_{\ell_0,N} \rangle \\ &= \int d\mathbf{x} [V(\mathbf{x}, \ell) - V(\mathbf{x}, \ell_0)] \rho(\mathbf{x}, \ell_0, N), \end{aligned} \quad (28)$$

where

$$\rho(\mathbf{x}, \ell_0, N) = \langle 0_{\ell_0,N} | n(\mathbf{x}) | 0_{\ell_0,N} \rangle. \quad (29)$$

Therefore, the trap-size and particle-number dependencies of the average work can be inferred from those of the ground-state particle density. For N -particle Fermi gases, confined by a generic power-law potential (2) with trap size ℓ_0 , the trap-size scaling of the particle density can be obtained from the corresponding continuum limit, i.e., [50]

$$\begin{aligned} \rho(\mathbf{x}, \ell_0, N) &\approx \ell_0^{-d\theta} S(\mathbf{X}, N), \\ \mathbf{X} \equiv \mathbf{x}/\ell_0^\theta, \quad S(\mathbf{X}, N) &= \sum_{k=1}^N \psi_k(\mathbf{X})^2, \end{aligned} \quad (30)$$

where ψ_k are the one-particle eigenfunctions of the one-particle Hamiltonian (7). The large- N behavior of S_p turns out to asymptotically behave as [17,50]

$$S(\mathbf{X}, N) \approx N^\theta S_r(\mathbf{X}/N^{(1-\theta)/d}). \quad (31)$$

In particular, for a one-dimensional harmonic trap [10,51,52]

$$S_r(z) = \frac{1}{\pi} \sqrt{2 - z^2} \quad \text{for } |z| \leq z_b = \sqrt{2}, \quad (32)$$

and $S_r(z) = 0$ for $|z| \geq z_b$. The corrections to this large- N behavior are [10] $O(N^{-1})$ (relatively to the leading term) for $z < z_b$. We also mention that at the boundary of the cloud, i.e., for $z = z_b$ where $\rho(x)$ vanishes, a different large- N scaling behavior sets in [10,53], where the particle density increases as $N^{1/6}$, instead of the $N^{1/2}$ behavior for $z < z_b$. This is related to the fact that the $z \rightarrow z_b$ limit of the $O(1/N)$ corrections is singular.

Using Eq. (30), we straightforwardly obtain

$$\begin{aligned} \langle W \rangle &\approx \ell_0^{-2\theta} \mathcal{W}_1(\delta_\ell, N), \\ \mathcal{W}_1(\delta_\ell, N) &= B(\delta_\ell) I_1(N), \\ B(x) &= \frac{1 - (1+x)^p}{p(1+x)^p} = -x + O(x^2), \\ I_1(N) &= \int d\mathbf{x} |\mathbf{x}|^p S(\mathbf{x}, N). \end{aligned} \quad (33)$$

Note that Eq. (33) agrees with the trap-size scaling reported in Eq. (27).

Moreover, using Eq. (31), we obtain the asymptotic large- N behavior

$$\begin{aligned} \mathcal{W}_1(\delta_\ell, N) &\approx B(\delta_\ell) \mathcal{I}_1 N^{1+2\theta/d}, \\ \mathcal{I}_1 &= \int d\mathbf{x} |\mathbf{x}|^p S_r(\mathbf{x}). \end{aligned} \quad (34)$$

Note that the above scaling equations imply first the trap-size scaling limit, and then the large- N limit, thus always remaining within the dilute regime.

For one-dimensional harmonic traps, using Eq. (32), we obtain

$$\mathcal{W}_1(\delta_\ell, N) = \frac{1}{2} B(\delta_\ell) N^2. \quad (35)$$

Note that, since the ground-state energy is given by

$$E_0^\ell = \ell^{-1} \sum_{i=1}^N (i - 1/2) = \frac{N^2}{2\ell}, \quad (36)$$

Eq. (35) implies

$$\langle W \rangle \geq E_0^{\ell_1} - E_0^{\ell_0} = -\frac{1}{2} \ell_0^{-1} N^2 \frac{\delta_\ell}{1 + \delta_\ell}, \quad (37)$$

as expected.

C. Work fluctuations

We now consider the second moment of the work distribution, and in particular,

$$\langle W^2 \rangle_c = \langle W^2 \rangle - \langle W \rangle^2. \quad (38)$$

We obtain its scaling behavior, and in particular, its large- N power law, by arguments similar to those for the average work. Using Eqs. (23) and (27), we write

$$\begin{aligned} \langle W^2 \rangle_c &= \langle 0_{\ell_0, N} | [H(\ell) - H(\ell_0)]^2 | 0_{\ell_0, N} \rangle_c \\ &= \ell_0^{-2p} B(\delta_\ell)^2 \int d\mathbf{x}_1 d\mathbf{x}_2 |\mathbf{x}_1|^p |\mathbf{x}_2|^p G(\mathbf{x}_1, \mathbf{x}_2), \end{aligned} \quad (39)$$

where

$$\begin{aligned} G(\mathbf{x}_1, \mathbf{x}_2) &= \langle 0_{\ell_0, N} | n(\mathbf{x}_1) n(\mathbf{x}_2) | 0_{\ell_0, N} \rangle \\ &\quad - \langle 0_{\ell_0, N} | n(\mathbf{x}_1) | 0_{\ell_0, N} \rangle \langle 0_{\ell_0, N} | n(\mathbf{x}_2) | 0_{\ell_0, N} \rangle. \end{aligned} \quad (40)$$

Therefore, the trap-size and particle-number dependencies of the work fluctuations can be inferred from those of the equilibrium density-density connected correlation $G(\mathbf{x}_1, \mathbf{x}_2)$.

For free fermions, $G(\mathbf{x}_1, \mathbf{x}_2)$ can be written as

$$G(\mathbf{x}_1, \mathbf{x}_2) = -|C(\mathbf{x}_1, \mathbf{x}_2)|^2 + \delta(\mathbf{x}_1 - \mathbf{x}_2) C(\mathbf{x}_1, \mathbf{x}_2), \quad (41)$$

where $C(\mathbf{x}_1, \mathbf{x}_2)$ is the one-particle correlation function, which is [17]

$$\begin{aligned} C(\mathbf{x}_1, \mathbf{x}_2) &= \langle 0_{\ell_0, N} | c(\mathbf{x}_1)^\dagger c(\mathbf{x}_2) | 0_{\ell_0, N} \rangle \\ &= \ell_0^{-d\theta} E(\mathbf{X}_1, \mathbf{X}_2), \end{aligned} \quad (42)$$

where

$$E(\mathbf{X}_1, \mathbf{X}_2) = \sum_{k=1}^N \psi_k(\mathbf{X}_1)^* \psi_k(\mathbf{X}_2), \quad \mathbf{X}_i = \mathbf{x}_i / \ell_0^\theta. \quad (43)$$

Of course, $C(\mathbf{x}, \mathbf{x}) = \rho(\mathbf{x})$. Therefore, we have

$$\begin{aligned} G(\mathbf{x}_1, \mathbf{x}_2) &= \ell_0^{-2d\theta} Z(\mathbf{X}_1, \mathbf{X}_2), \\ Z(\mathbf{X}_1, \mathbf{X}_2) &= -E(\mathbf{X}_1, \mathbf{X}_2)^2 + \delta(\mathbf{X}_1 - \mathbf{X}_2) E(\mathbf{X}_1, \mathbf{X}_2). \end{aligned} \quad (44)$$

Note that the trap-size scaling function Z develops the large- N behavior [17,50]

$$Z(\mathbf{X}_1, \mathbf{X}_2, N) \approx N^\theta Z_r(N^{\theta/d} \mathbf{X}_1, N^{\theta/d} \mathbf{X}_2) \quad (45)$$

for $\mathbf{X}_1 \neq \mathbf{X}_2$ (this scaling behavior does not hold for $|\mathbf{X}_1 - \mathbf{X}_2| \rightarrow 0$).

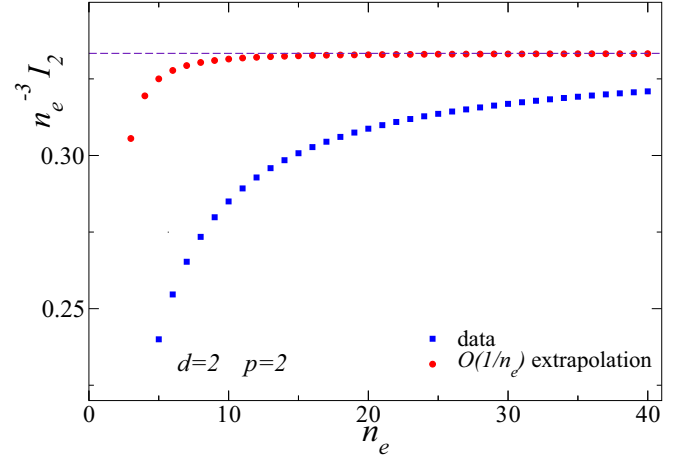


FIG. 3. N -dependence of the function $I_2(N)$ entering the expression (46) for the square work fluctuations $\langle W^2 \rangle_c$, associated with a sudden quench of the trap size of two-dimensional gases, from ℓ_0 to ℓ_1 . We show data of $n_e^{-3} I_2$ versus n_e , and also the corresponding linear extrapolation $a + b/n_e$ using the data for n_e and $n_e - 1$. They approach the large- n_e limit $n_e^{-3} I_2 \approx 1/3$ shown by the dashed line. Recalling that asymptotically $n_e \approx \sqrt{2N}$, we obtain the large- N behavior (50). Concerning the units, we recall that we set the lattice spacing $a = 1$, the kinetic constant $t = 1$, and $\hbar = 1$.

Using the above results for the connected density-density correlation function, we arrive at

$$\begin{aligned} \langle W^2 \rangle_c &\approx \ell_0^{-4\theta} \mathcal{W}_2(\delta_\ell, N), \\ \mathcal{W}_2(\delta_\ell, N) &= B(\delta_\ell)^2 I_2(N), \\ I_2(N) &= \int d\mathbf{X}_1 d\mathbf{X}_2 |\mathbf{X}_1|^p |\mathbf{X}_2|^p Z(\mathbf{X}_1, \mathbf{X}_2). \end{aligned} \quad (46)$$

Again, Eq. (46) agrees with the trap-size scaling put forward in Eq. (27).

The large- N dependence can hardly be inferred from the large- N scaling of the two-point function $G(\mathbf{x}_1, \mathbf{x}_2)$, such as Eq. (45), because the integral I_2 in Eq. (46) includes the contribution for $|\mathbf{x}_2 - \mathbf{x}_1| \rightarrow 0$, where Eq. (45) does not apply. To determine it, we write $I_2(N)$ as

$$I_2(N) = \text{Tr} M_{2p} - \text{Tr} M_{1p}^\dagger M_{1p}, \quad (47)$$

$$M_{k,p,i,j} = \int d\mathbf{x} |\mathbf{x}|^{kp} \psi_i(\mathbf{x})^* \psi_j(\mathbf{x}), \quad (48)$$

and compute it for several values of N . The analysis of $I_2(N)$ with increasing N shows that the leading contributions of the two terms in Eq. (47) asymptotically cancel. For one-dimensional harmonic traps we obtain the exact result

$$I_2(N) = \frac{1}{2} N^2 \quad \text{for } d = 1, \quad p = 2. \quad (49)$$

For two-dimensional harmonic traps, the large- N extrapolation of fixed- N results shows the asymptotic behavior

$$I_2(N) = e N^{3/2} [1 + O(N^{-1/2})] \quad \text{for } d = 2, \quad p = 2, \quad (50)$$

with $e \approx \sqrt{8/9}$, see Fig. 3. These results may hint at the general large- N behavior $I_2(N) \approx e N^{1+2\theta/d}$ when extending the results to confining potential with generic powers p .

We finally note that the higher moments (higher than the second one) of the work distribution (23) cannot be written as expectation values of powers of the difference of the final and initial Hamiltonians, such as the cases of the first and second moments, cf. Eqs. (28) and (39). In these cases more complicated expressions must be evaluated. In this paper we only report results for the first two moments.

D. Quantum work associated with a breathing Fermi gas

We now consider a noninteracting one-dimensional Fermi gas of N particles trapped by a harmonic potential. The initial state at $t = 0$ is its ground state within a trap of size $\ell_0 \equiv \omega_0^{-1}$ centered at a distance x_c : $\Psi(x, t = 0) = \frac{1}{\sqrt{N!}} \det[\psi_i(x_j - x_c)]$ where ψ_i are the one-particle eigenstates in a harmonic potential. Then the gas is released within a larger trap of size $\ell_1 \equiv \omega_1^{-1} > \ell_0$. The time-dependent many-body function describing the motion is given by

$$\begin{aligned} \Psi(x_1, \dots, x_N, t) &= \frac{1}{\sqrt{N!}} \det[\psi_i(x_j, t)], \\ \psi_i(x, t) &= \int_{-\infty}^{\infty} dy P(x, t; y, 0) \phi_i(y - x_c; \ell_0), \\ P(x, t; y, 0) &= \left[\frac{m\omega_1}{2\pi i \sin(\omega_1 t)} \right]^{1/2} \exp \left\{ \frac{im\omega_1}{2 \sin(\omega_1 t)} \right. \\ &\quad \left. \times [(x^2 + y^2) \cos(\omega_1 t) - 2xy] \right\}, \end{aligned} \quad (51)$$

where $\phi_i(y - x_c, \ell_0)$ are the eigenfunctions (A3) for a harmonic trap centered at x_0 with trap size ℓ_0 , $P(x_2, t_2; x_1, t_1)$ is the one-particle propagator associated with an oscillator of frequency $\omega_1 = \ell_1^{-1}$. The particle density oscillates as a pendulum, as one can easily infer by computing the time dependence of the particle density

$$\rho(x, t) = \sum_{i=1}^N |\psi_i(x, t)|^2 = \rho(x, t + 2\pi/\omega_1), \quad (52)$$

and $\rho(x, t) = \rho(-x, t + \pi/\omega_1)$.

We now compute the average quantum work associated with the sudden shift of the center moving from $x = x_c$ to $x = 0$, and enlargement of the harmonic trap. For this purpose we must evaluate

$$\begin{aligned} \langle W \rangle &= \langle 0_{x_c, \ell_0, N} | \frac{1}{2\ell_1^2} \int dx x^2 n(x) | 0_{x_c, \ell_0, N} \rangle \\ &\quad - \langle 0_{x_c, \ell_0, N} | \frac{1}{2\ell_0^2} \int dx (x - x_c)^2 n(x) | 0_{x_c, \ell_0, N} \rangle, \end{aligned} \quad (53)$$

where $|0_{x_c, \ell_0, N}\rangle$ indicates the ground state of the N -particle Fermi gas in a harmonic trap of size ℓ_0 centered at x_c . After some manipulations, we may write it as

$$\begin{aligned} \langle W \rangle &= \ell_0^{-2} \langle 0_{x_c, \ell_0, N} | B(\delta_\ell) \int dx (x - x_c)^2 n(x) | 0_{x_c, \ell_0, N} \rangle \\ &\quad + \ell_0^{-2} \langle 0_{x_c, \ell_0, N} | \frac{x_c}{(1 + \delta_\ell)^2} \int dx (x - x_c) n(x) | 0_{x_c, \ell_0, N} \rangle \\ &\quad + \ell_0^{-2} \langle 0_{x_c, \ell_0, N} | \frac{x_c^2}{2(1 + \delta_\ell)^2} \int dx n(x) | 0_{x_c, \ell_0, N} \rangle, \end{aligned} \quad (54)$$

where $\delta_\ell = \ell_1/\ell_0 - 1$. Thus

$$\begin{aligned} \langle W \rangle &= \ell_0^{-2} B(\delta_\ell) \int dx (x - x_c)^2 \rho(x - x_c, \ell_0) \\ &\quad + \ell_0^{-2} \frac{x_c}{(1 + \delta_\ell)^2} \int dx (x - x_c) \rho(x - x_c, \ell_0) \\ &\quad + \ell_0^{-2} \frac{x_c^2}{2(1 + \delta_\ell)^2} \int dx \rho(x - x_c, \ell_0), \end{aligned} \quad (55)$$

where $\rho(x, \ell_0)$ is the particle density for a trap size ℓ_0 . We note that the first term corresponds to the average work for the variation of the trap size from ℓ_0 to ℓ_1 , cf. Eq. (28), and the second one vanishes because of the reflection symmetry of the particle density. We obtain

$$\begin{aligned} \langle W \rangle &= \ell_0^{-1} \mathcal{W}_1(\delta_\ell, X_c, N), \\ \mathcal{W}_1(\delta_\ell, X_c, N) &= \frac{B(\delta_\ell)}{2} N^2 + \frac{X_c^2}{2(1 + \delta_\ell)^2} N, \end{aligned} \quad (56)$$

where $X_c = x_c/\ell_0$. The first term is essentially related to the change of the trap size, while the second one to the shift of the trap. Note that their N -dependence power law differs; the dominant one is that related to the change of the trap size.

We may also compute the average fluctuations $\langle W^2 \rangle_c$. We only report the results for the case the trap size is unchanged, thus $\ell_1 = \ell_0$, and we only shift the trap center from x_c to the origin. We obtain

$$\begin{aligned} \langle W^2 \rangle_c &= \ell_0^{-2} \mathcal{W}_2(\delta_\ell, X_c, N), \\ \mathcal{W}_2 &= X_c^2 \int dX_1 dX_2 X_1 X_2 Z(X_1, X_2) = X_c^2 \frac{N}{2}. \end{aligned} \quad (57)$$

V. QUANTUM OVERLAP BETWEEN INITIAL AND EVOLVED STATES

A. Loschmidt echo

To characterize the quantum dynamics arising from variations of the trapping potential, we study how the out-of-equilibrium states arising from the change of the trapping potential depart from the initial one, which is the ground state associated with the trap size ℓ_0 . This issue can be quantitatively analyzed by considering the *overlap* between the initial state and the evolving N -particle states during the out-of-equilibrium quantum evolution. This provides nontrivial information on the nature of the quantum dynamics associated with the quenches considered in this paper, extending earlier studies focusing on the correlation functions and spatial entanglement at fixed time [12, 14].

The evolution of the overlap with the initial ground state can be quantified by the so-called Loschmidt echo

$$L_E = |\langle \Psi(\mathbf{x}_1, \dots, \mathbf{x}_N; t) | \Psi(\mathbf{x}_1, \dots, \mathbf{x}_N; t = 0) \rangle|, \quad (58)$$

and the related echo function

$$Q(t, N) = -\ln L_E(t, N), \quad (59)$$

where the initial $t = 0$ state $|\Psi(\mathbf{x}_1, \dots, \mathbf{x}_N; t = 0)\rangle$ is the ground state for a system constrained within a trap of size ℓ_0 . Therefore, the echo function Q becomes larger and larger when the overlap measured by the Loschmidt echo gets more and more suppressed.

We consider again a sudden quench of the potential, corresponding to the variation of the trap size from ℓ_0 to ℓ_1 , including $\ell_1 \rightarrow \infty$ corresponding to a free expansion of the gas. We generally expect the following scaling behavior:

$$Q(t, \ell_0, \ell_1, N) \approx Q(\tau, \delta_\ell, N), \quad (60)$$

where

$$\tau = \ell_0^{-2\theta} t \quad (61)$$

is a scaling variable associated with the time t , so that $\tau \sim t \Delta(\ell_0)$ since $\Delta(\ell_0) \sim \ell_0^{-2\theta}$ is the gap for the trap of size ℓ_0 . The dynamic trap-size scaling behavior (60) is analogous to that put forward, and numerically checked, for the dynamic finite-size scaling of the Loschmidt echo in out-of-equilibrium conditions arising from quenches at quantum transitions [54].

The Loschmidt echo $L_E(t)$ of the out-of-equilibrium dynamic arising from a sudden variation of the trap size from ℓ_0 to ℓ_1 (starting from the ground state associated with ℓ_0) can be related to the corresponding work distribution $P(W)$, cf. Eq. (23). Indeed one can show that [33]

$$L_E(t) = |C_P(t)|, \quad C_P(t) = \int dW e^{iW} P(W), \quad (62)$$

relating the Loschmidt echo with the characteristic function $C_P(t)$ of the work distribution. Of course, the trap-size scaling behavior (60) put forward for the Loschmidt echo is consistent with that for the work distribution, cf. Eq. (24).

In the following we focus on one-dimensional systems trapped by harmonic and hard-wall potentials. Extensions to higher dimensions may be straightforwardly considered, but they require more cumbersome calculations. We present calculations in the continuum limit, which are valid in the trap-size scaling limit of the lattice gas model.

B. Harmonic traps

1. Quantum dynamics when changing one-dimensional harmonic traps

We consider Fermi gases in general time-dependent confining harmonic potential, Eq. (9) with $p = 2$, starting from an equilibrium ground-state configuration with initial trap size ℓ_0 , as outlined in Sec. II.

As shown in Ref. [55], see also Ref. [6], the time-dependent many-body wave function $\Psi(x_1, \dots, x_N; t)$ of the system can be derived from the solutions $\psi_j(x, t)$ of the one-particle Schrödinger equation

$$i\partial_t \psi_j(x, t) = \left[-\frac{1}{2} \partial_x^2 + \frac{1}{2} \kappa(t) x^2 \right] \psi_j(x, t), \quad (63)$$

with the initial condition $\psi_j(x, 0) = \phi_j(x)$ where $\phi_j(x)$ are the eigensolutions of the Hamiltonian at $t = 0$, characterized by a trap size ℓ_0 , with eigenvalue $E_j = \ell_0^{-1}(j - 1/2)$. The solution can be obtained introducing a time-dependent function $s(t)$, writing [55,56]

$$\psi_j(x, t) = s^{-1/2} \phi_j(x/s) \exp\left(i \frac{\dot{s} x^2}{2s} - i E_j \int_0^t s^{-2} dt' \right), \quad (64)$$

where $\phi_j(x)$ is the j th eigenfunction of the Schrödinger equation of the one-particle Hamiltonian at $t = 0$, thus with trap

size ℓ_0 . The function $s(t)$ satisfies the nonlinear differential equation

$$\ddot{s} + \kappa(t)s = \kappa_0 s^{-3} \quad (65)$$

with initial conditions $s(0) = 1$ and $\dot{s}(0) = 0$. Then, using Eq. (64), one can write the time-dependent many-body wave function as [6]

$$\begin{aligned} \Psi(x_1, \dots, x_N; t) &= \frac{1}{\sqrt{N!}} \det[\psi_j(x_i, t)] \\ &= s^{-N/2} \Psi(x_1/s, \dots, x_N/s; 0) \\ &\quad \times \exp\left(\frac{i\dot{s}}{2s} \sum_j x_j^2 - i \sum_j E_j \int_0^t s^{-2} dt' \right), \end{aligned} \quad (66)$$

where $\Psi(x_1, \dots, x_N; 0)$ is the wave function of the ground state for the Hamiltonian at $t = 0$.

In the case of an instantaneous change to a confining potential with trap size ℓ_1 , so that $\kappa(t) = \ell_1^{-2}$ for $t > 0$, the solution of Eq. (65) reads

$$s(t) = \sqrt{1 + (\ell_1^2/\ell_0^2 - 1)[\sin(t/\ell_1)]^2}. \quad (67)$$

Notice that, assuming $\ell_1/\ell_0 > 1$,

$$1 \leq s(t) \leq \ell_1/\ell_0, \quad (68)$$

and $\dot{s} = 0$ when $s(t) = 1$ and $s(t) = \ell_1/\ell_0$. Interestingly, the many-body quantum states at times corresponding to $s(t) = 1$ and $s(t) = \ell_1/\ell_0$ turn out to coincide with the ground states associated with the trap sizes $\ell = \ell_0$ and $\ell = \ell_1^2/\ell_0$, respectively (notice that $\ell_1^2/\ell_0 > \ell_1$). In the case $\ell_1 \rightarrow \infty$, corresponding to an instantaneous drop of the trap, so that $\kappa(t) = 0$ for $t > 0$, the solution of Eq. (65) is a monotonically increasing function, given by

$$s(t) = \sqrt{1 + (t/\ell_0)^2}. \quad (69)$$

Further analytic results for a linear time dependence of $\kappa(t)$ in Eq. (9) can be found in Ref. [10].

2. Loschmidt echo for an instantaneous change of the trap size

Using the above results, we may write the Loschmidt echo as

$$\begin{aligned} L_E(t, N) &= \left| s^{-N/2} \int \prod_{i=1}^N dx_i \Psi(x_i/s; 0)^* \Psi(x_i; 0) \right. \\ &\quad \times \exp\left(\frac{-i\dot{s}}{2s} \sum_j x_j^2 \right) \left. \right| \\ &= s^{-N/2} \left| \det \int dx \psi_i(x/s; 0)^* \psi_j(x; 0) \exp\left(\frac{-i\dot{s}}{2s} x^2 \right) \right|, \end{aligned} \quad (70)$$

where we used Eq. (19). Then, noting that the function $s(t)$, cf. Eq. (67), can be rewritten as

$$s(t) \equiv S(\tau, \delta_\ell), \quad (71)$$

where $\tau = t/\ell_0$, $\delta_\ell = \ell_1/\ell_0 - 1$, and

$$\begin{aligned} S(\tau, \delta_\ell) &= \left[1 + (2\delta_\ell + \delta_\ell^2) \left(\sin \frac{\tau}{1 + \delta_\ell} \right)^2 \right]^{1/2}, \\ S(\tau, \delta_\ell = \infty) &= \sqrt{1 + \tau^2}, \end{aligned} \quad (72)$$

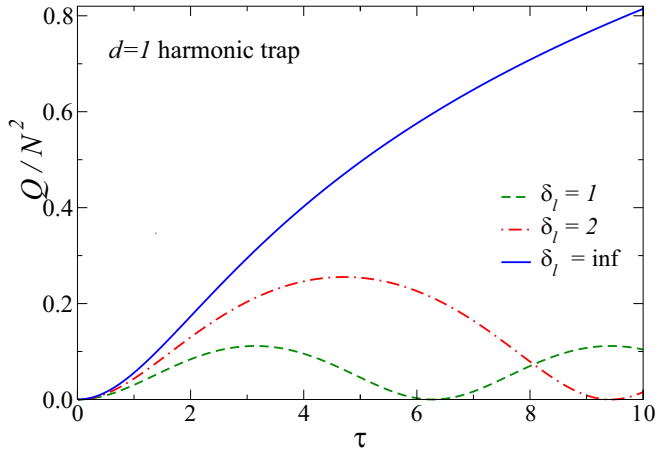


FIG. 4. The echo function Q , cf. Eqs. (58) and (59), associated with changes of one-dimensional harmonic traps, whose trap size suddenly varies from ℓ_0 to ℓ_1 , for $\delta_\ell = 1, 2, \infty$, as given by Eq. (75).

we obtain

$$L_E(t, \ell_0, \ell_1, N) = |\det A_{ij}(\tau, \delta_\ell, N)|,$$

$$A_{ij} = \int dZ e^{-iS'Z^2/2} \phi_i(Z/\sqrt{S}) \phi_j(Z\sqrt{S}), \quad (73)$$

where $S' = dS/d\tau$, and the eigenfunctions $\phi_n(X)$ are those reported in Eq. (A3) with $\xi = 1$.

By further developing the above equations, we arrive at the final expression

$$L_E(t, \ell_0, \ell_1, N) = F(S, S')^{N^2/2},$$

$$F(S, S') = \frac{2S}{\sqrt{(1+S^2)^2 + S^2 S'^2}}. \quad (74)$$

This can be derived by straightforward manipulations of the expression (70). We also checked it numerically using Eq. (73).

Finally, for the echo function $Q = -\ln L_E$ we obtain

$$Q(t, \ell_0, \ell_1, N) = \frac{N^2}{4} \ln \left[\frac{(1+S^2)^2 + S^2 S'^2}{4S^2} \right], \quad (75)$$

where $S(\tau, \delta_\ell)$ is reported in Eq. (72). The above expression is in agreement with the general scaling behavior put forward in Eq. (60). In Fig. 4 we show the echo function for some values of δ_ℓ , including that for the free expansion $\delta_\ell \rightarrow \infty$. Note that when $Q(t) = 0$, the quantum state coincides with the initial one, apart from a trivial phase; this occurs periodically, when $\tau = k\pi \ell_1/\ell_0$ and $k = 0, 1, 2, \dots$. In the case of a free expansion, $\ell_1/\ell_0 = \infty$, we have

$$Q(t, \ell_0, \infty, N) \approx \frac{N^2}{2} \ln \tau \quad (76)$$

in the large-time limit.

We finally note that, by exploiting Eq. (62), the behavior of the Loschmidt echo can also provide information on the work distribution of the same dynamic protocol.

C. Free expansion from a hard-wall trap

We now consider an N -particle Fermi gas constrained within hard walls, in the corresponding ground state, and study the out-of-equilibrium dynamics arising from the sudden drop of the hard walls, allowing the Fermi gas to expand freely.

The free expansion of the gas after the instantaneous drop of the walls is described by the time-dependent wave function

$$\Psi(x_1, \dots, x_N; t) = \frac{1}{\sqrt{N!}} \det[\psi_i(x_j, t)], \quad (77)$$

where $\psi_i(x, t)$ are the one-particle wave functions with initial condition $\psi_i(x, 0) = \phi_i(x)$, cf. Eq. (A4), which can be written in terms of the free-particle propagator $P(x_2, t_2; x_1, t_1)$, as

$$\psi_i(x, t) = \int_{-\ell_0}^{\ell_0} dy P(x, t; y, 0) \phi_i(y),$$

$$P(x_2, t_2; x_1, t_1) = \frac{1}{\sqrt{i2\pi(t_2 - t_1)}} \exp \left[\frac{i(x_2 - x_1)^2}{2(t_2 - t_1)} \right]. \quad (78)$$

Then the Loschmidt echo can be written as

$$L_E(\ell_0, t, N) = |\langle \Psi_0 | \Psi(t) \rangle| = |\det F_{kq}(t)|, \quad (79)$$

where

$$F_{kq}(t) = \int_{-\ell_0}^{\ell_0} dx \phi_k(x)^* \psi_q(x, t). \quad (80)$$

One can easily check that L_E , thus $Q = -\ln L_E$, can be written as a function of the scaling variable $\tau = \ell_0^{-2}t$ and the particle number N , in agreement with the scaling behavior predicted by Eq. (60).

The Loschmidt echo is expected to vanish in the large-time limit due to the fact that the particles escape from the trap in their free expansion. This is also formally obtained by noting that the large- t behavior of the one-particle wave

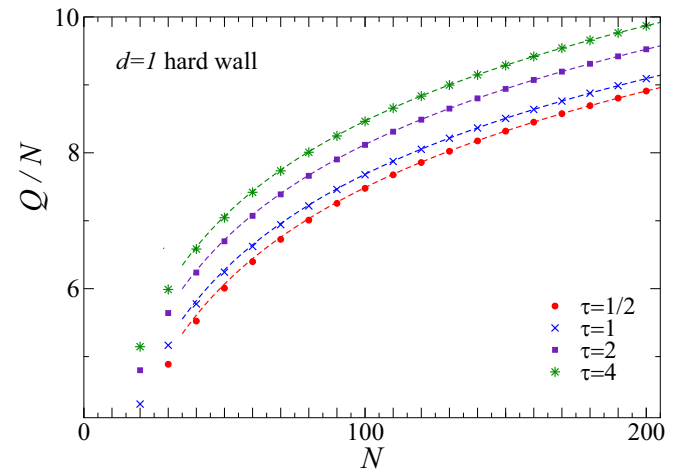


FIG. 5. The echo function Q , cf. Eqs. (58) and (59), for a free expansion of the Fermi gas after the sudden drop of the hard walls trapping the gas, for some values of $\tau = \ell_0^{-2}t$. The data are consistent with the asymptotic behavior $Q \approx aN(\ln N + b)$ (represented by the dashed lines).

functions, cf. Eq. (78), have the following asymptotic behavior:

$$\psi_n(x, t) \approx \sqrt{\frac{2}{\pi^3 t}} \frac{1 - (-1)^n}{n}, \quad (81)$$

i.e., they tend to be independent of x , corresponding to the fact that when $x \ll v_F t$ (where v_F is the Fermi velocity) the one-particle wave functions within the space occupied initially can be approximated by a constant.

Results for the Loschmidt echo are shown in Fig. 5, up to $N = 200$ particles, for some values of τ . As expected, the echo function Q increases with increasing τ , thus implying a rapid suppression of the Loschmidt echo. The results also show that in the large- N limit the echo function increases as

$$Q(\ell_0, t, N) \sim N \ln N. \quad (82)$$

Therefore, in the case of hard-wall trap, the echo function Q turns out to increase more slowly than the case of harmonic traps, cf. Eq. (75).

VI. INTERACTING FERMION GASES

A. Hubbard model

We now discuss the effects of short-ranged particle interactions on the particle-number scaling behaviors obtained for free fermions, in particular for the quantum work statistics. For this purpose, we consider the Hubbard model describing lattice gases of spinful fermions. The Hamiltonian of the Hubbard model reads

$$H_h = -t \sum_{\sigma, \langle xy \rangle} (c_{\sigma x}^\dagger c_{\sigma y} + \text{H.c.}) + U \sum_x n_{\uparrow x} n_{\downarrow x}, \quad (83)$$

where \mathbf{x} are the sites of a cubic lattice, $\langle xy \rangle$ indicates nearest-neighbor sites, $c_{\sigma x}$ is a fermionic operator, $\sigma = \uparrow \downarrow$ labels the spin states, and $n_{\sigma x} \equiv c_{\sigma x}^\dagger c_{\sigma x}$. Again we set $t = 1$. Analogously to noninteracting lattice Fermi gases, cf. Eq. (1), the external force trapping the particles is taken into account by adding a potential term, i.e.,

$$H = H_h + H_v, \quad H_v = \sum_{\sigma, \mathbf{x}} V(\mathbf{x}, \ell) n_{\sigma \mathbf{x}}. \quad (84)$$

The particle number operators $\hat{N}_\sigma = \sum_{\mathbf{x}} n_{\sigma \mathbf{x}}$ are conserved, i.e., $[H, \hat{N}_\sigma] = 0$. For simplicity, in the following we consider balanced Fermi systems, thus $N_\uparrow = N_\downarrow = N/2$ where N is the total number of particles.

We define the particle density

$$\rho(\mathbf{x}) = \langle n_x \rangle \equiv \langle n_{\uparrow x} + n_{\downarrow x} \rangle, \quad (85)$$

the one-particle correlation

$$C(\mathbf{x}, \mathbf{y}) = \sum_{\sigma} \langle c_{\sigma \mathbf{x}}^\dagger c_{\sigma \mathbf{y}} + \text{H.c.} \rangle, \quad (86)$$

and the connected density-density correlation

$$G(\mathbf{x}, \mathbf{y}) = \langle n_x n_y \rangle - \langle n_x \rangle \langle n_y \rangle. \quad (87)$$

In the symmetric case $N_\uparrow = N_\downarrow$, we have that $\langle n_{\uparrow x} \rangle = \langle n_{\downarrow x} \rangle$ and $\langle c_{\uparrow x}^\dagger c_{\uparrow y} \rangle = \langle c_{\downarrow x}^\dagger c_{\downarrow y} \rangle$.

We again consider an out-of-equilibrium dynamics arising from the sudden change of the trap size, from ℓ_0 to $\ell_1 > \ell_0$,

in the dilute regime. Our purpose is to discuss the particle-number dependence of the average work and its fluctuations, associated with this process.

B. Dilute regime of the Hubbard model

To investigate the particle-number dependence of the work fluctuations, we need to summarize a number of known results concerning the equilibrium correlation functions of N -particle interacting lattice fermions, associated with the ground state in the presence of an external power-law potential. They show a corresponding equilibrium trap-size scaling.

1. Three-dimensional systems

In the language of the renormalization-group theory, the power-law scaling behaviors in the dilute regime are controlled by a corresponding dilute fixed point, related to the vacuum-to-metal quantum transition [20]. The renormalization-group analysis of the effects of the interactions shows that the U term is irrelevant at the dilute fixed point for $d > 2$ because its renormalization-group dimension $y_U = 2 - d$ is negative. Therefore, the asymptotic trap-size dependence in the dilute regime turns out to be the same as that of a free Fermi gases of N particles with $N_\uparrow = N_\downarrow = N/2$, independently of U , at least for $U > U^*$ with $U^* < 0$ [20]. The corresponding trap-size scaling reads [17]

$$\begin{aligned} \rho(\mathbf{x}, \ell, U, N) &\approx \ell^{-3\theta} 2S(\mathbf{X}, N/2), \\ C(\mathbf{x}_1, \mathbf{x}_2, \ell, U, N) &\approx \ell^{-3\theta} 2E(\mathbf{X}_1, \mathbf{X}_2, N/2), \\ G(\mathbf{x}_1, \mathbf{x}_2, \ell, U, N) &\approx \ell^{-6\theta} 2Z(\mathbf{X}_1, \mathbf{X}_2, N/2), \\ \mathbf{X}_i &\equiv \mathbf{x}_i / \ell^\theta, \quad \theta = \frac{p}{p+2}, \end{aligned} \quad (88)$$

for the particle density, the one-particle and connected density-density correlations, respectively. The scaling functions S , E , and Z are the same trap-size scaling functions of the free spinless fermion theory, introduced in Secs. IV B and IV C. The presence of the on-site interaction associated with the parameter U induces $O(\ell^{-(d-2)\theta})$ scaling corrections. They dominate the scaling corrections expected within the lattice model of free spinless fermion, i.e., the Hubbard model with $U = 0$, which are relatively suppressed as $O(\ell^{-2\theta})$ [10].

2. Lower-dimensional systems

The on-site coupling U becomes marginal in two dimensions, indeed its renormalization-group dimension $y_U = 2 - d$ vanishes, thus a residual weak dependence on U is expected in the asymptotic regime. More precisely we expect [17,18]

$$\begin{aligned} \rho(\mathbf{x}, \ell, U, N) &\approx \ell^{-2\theta} \mathcal{R}(\mathbf{X}, U, N), \\ C(\mathbf{x}_1, \mathbf{x}_2, \ell, U, N) &\approx \ell^{-2\theta} \mathcal{C}(\mathbf{X}_1, \mathbf{X}_2, U, N), \\ G(\mathbf{x}_1, \mathbf{x}_2, \ell, U, N) &\approx \ell^{-4\theta} \mathcal{G}(\mathbf{X}_1, \mathbf{X}_2, U, N). \end{aligned} \quad (89)$$

Finally, in one dimension the U term turns out to be relevant, since $y_U = 1$, therefore the asymptotic behaviors are expected to change. The relevance of the U term in one dimension gives rise to nontrivial asymptotic trap-size scaling limits, requiring an appropriate rescaling of the parameter U .

This is taken into account by introducing the scaling variable

$$U_r = U \ell^\theta, \quad (90)$$

where θ is the same exponent of Eq. (3). Indeed, the system develops the trap-size scaling behavior [17]

$$\begin{aligned} \rho(x) &\approx \ell^{-\theta} \mathcal{R}(X, U_r, N), \\ C(x_1, x_2) &\approx \ell^{-\theta} \mathcal{C}(X_1, X_2, U_r, N), \\ G(x_1, x_2) &\approx \ell^{-2\theta} \mathcal{G}(X_1, X_2, U_r, N), \end{aligned} \quad (91)$$

where $X_i = x_i/\ell^\theta$. These scaling behaviors are expected to be approached with power-law suppressed corrections. Of course, for $U_r = 0$, i.e., for a strictly vanishing U , we must recover the scaling functions of the free Fermi gas, taking into account that an unpolarized free Fermi gases of N particles is equivalent to two independent spinless Fermi gases of $N/2$ particles.

3. Continuum limit

It is important to note again that the trap-size scaling limit corresponds to a continuum limit in the presence of the trap, i.e., it generally realizes a continuum quantum field theory in the presence of an inhomogeneous external field [17,19]. In particular, in the trap-size scaling limit the observables of the one-dimensional trapped Hubbard model can be written in terms of the solutions of the continuum Hamiltonian [17,18]

$$H_c = \sum_{i=1}^N \left[\frac{p_i^2}{2m} + V(x_i) \right] + g \sum_{i=1}^{N_\uparrow} \sum_{j=1}^{N_\downarrow} \delta(x_i - x_j), \quad (92)$$

describing N fermions, with $N_\uparrow = N_\downarrow = N/2$, interacting through a local δ -like term. In particular, in one dimension we recover the so-called Gaudin-Yang model [58,59]. Indeed, the trap-size scaling limit of the one-dimensional Hubbard model at fixed N is related to the Gaudin-Yang model with $g \sim U_r \equiv U \ell^\theta$. More precisely, the trap-size scaling functions entering formulas (91) are exactly given by corresponding quantities of the Gaudin-Yang problem with a trap of unit size. Analogously, in two dimensions we recover the continuum interacting model with $g \sim U$ [18,57]. Finally, in three dimensions, the continuum limit is given by the trap-size scaling of the free Fermi theory, for any value of the lattice coupling $U > U^*$ with $U^* < 0$ [20].

C. Particle-number dependence of the quantum work

We now discuss the particle-number dependence of the quantum work associated with a sudden quench of the trap size, from ℓ_0 to $\ell_1 > \ell_0$, starting the ground state of the Fermi gas in the trap of size ℓ_0 . We again consider the definition of work distribution given by Eq. (22), in particular Eq. (23). To compute the average work we follow the same line of reasoning used in the case of free Fermi gases, see Sec. IV B. This leads us to the following general formula for the trap-size scaling limit of the average work:

$$\begin{aligned} \langle W \rangle &\approx \ell_0^{-2\theta} B(\delta_\ell) A_1(U_r, N), \\ A_1(U_r, N) &= \int d\mathbf{X} |\mathbf{X}|^p \mathcal{R}(\mathbf{X}, U_r, N), \end{aligned} \quad (93)$$

where $\mathbf{X} = \mathbf{x}/\ell_0^\theta$, $\mathcal{R}(\mathbf{X}, U_r, \ell_0)$ is the rescaled particle density of the ground state with trap size ℓ_0 , and $B(\delta_\ell)$ is defined in Eq. (33). Analogously for the square work fluctuations, following the initial steps outlined in Sec. IV C, we obtain

$$\begin{aligned} \langle W^2 \rangle_c &\approx \ell_0^{-4\theta} B(\delta_\ell)^2 A_2(U_r, N), \\ A_2(U_r, N) &= \int d\mathbf{X}_1 d\mathbf{X}_2 |\mathbf{X}_1|^p |\mathbf{X}_2|^p \mathcal{G}(\mathbf{X}_1, \mathbf{X}_2, U_r, N), \end{aligned} \quad (94)$$

where \mathcal{G} is the rescaled density-density connected correlation function. As shown in Sec. VI B, the effective on-site coupling U_r is given by

$$\begin{aligned} U_r &= U \ell_0^\theta & \text{for } d = 1, \\ U_r &= U & \text{for } d = 2, \\ U_r &= 0 & \text{for } d = 3. \end{aligned} \quad (95)$$

We now argue that the power laws associated with the particle-number dependence of the average work and its square fluctuations are generally analogous to those of the d -dimensional free Fermi gases.

In the case of three-dimensional Fermi gases this claim is clearly a consequence of the fact that in the trap-size scaling functions \mathcal{R} and \mathcal{G} coincide with those of the free Fermi gases, cf. Eqs. (88), when the onsite coupling U is larger than a negative value U^* . Therefore Eqs. (34), (46), and (50) are expected to hold as well.

On the other hand, as shown by Eqs. (89) and (91), the trap-size scaling, or continuum limit, of lower-dimensional models is more complicated. Let us first discuss the apparently more complicated case of one-dimensional systems, whose continuum limit corresponds to the Gaudin-Yang model. As shown in Ref. [17], for a large number of particles (still remaining in the dilute regime), the trap-size scaling function of the particle density behaves asymptotically as

$$\mathcal{R}(X, U_r, N) \approx N^{1/2} \mathcal{R}_\infty(X/N^{1/2}, U_r/N^{1/2}), \quad (96)$$

where $\mathcal{R}_\infty(z, u)$ is a nontrivial scaling function, and power-law-suppressed corrections are neglected. This already suggests that the effect of a finite continuum coupling U_r gets suppressed in the large- N limit. As we shall see, this is also confirmed by arguments based on the relation between the trap-size scaling of the trapped Hubbard model and the continuum Gaudin-Yang model, which allows us to determine the trap-size scaling functions of the particle density and its correlation, i.e., $\mathcal{R}(X, U_r, N)$ and $\mathcal{G}(X_1, X_2, U_r, N)$ respectively, in the strongly repulsive and attractive limits, i.e., $U_r \rightarrow \infty$ and $U_r \rightarrow -\infty$.

The equation of state of the homogenous Gaudin-Yang model is exactly known for both repulsive and attractive zero-range interaction [58,59]. It is characterized by different asymptotic regimes with respect to the effective dimensionless coupling $\gamma \equiv g/\rho$, where ρ is the particle density. At weak coupling $\gamma \ll 1$ it behaves as a perfect Fermi gas; in the strongly repulsive regime $\gamma \gg 1$ the equation of state approaches that of a spinless Fermi gas; in the strongly attractive regime $\gamma \rightarrow -\infty$ and for unpolarized gases it matches that of a one-dimensional gas of impenetrable bosons [60], more precisely hard-core bosonic molecules of fermion pairs [61,62]. We know that in the $g \rightarrow \infty$ limit the particle density and its

correlations of the Gaudin-Yang model become identical to those of a gas of N spinless fermions [63–65]. This would imply that the $U_r \rightarrow \infty$ limit of the trap-size scaling functions is

$$\begin{aligned}\mathcal{R}(X, U_r \rightarrow \infty, N) &= S(X, N), \\ \mathcal{G}(X_1, X_2, U_r \rightarrow \infty, N) &= Z(X_1, X_2, N),\end{aligned}\quad (97)$$

where S and Z are the same functions entering the spinless free-fermion trap-size scaling.

In the $g \rightarrow -\infty$ limit the density properties of the Gaudin-Yang model are expected to match that of an ensemble of hard-core $N/2$ bosonic molecules constituted by up and down fermions. Indeed, with increasing attraction, the pairing becomes increasingly localized in space, and eventually the paired fermions form a tightly bound bosonic molecule. Actually, the results of Ref. [61] for harmonic traps, see also Ref. [17], show that these bound states get trapped in a smaller region, with an effective trap size $\ell_b = \ell/2$ in the strongly attractive limit. Thus we expect that in the $g \rightarrow -\infty$ limit the particle density of the unpolarized Gaudin-Yang model with a harmonic trap matches that of $N/2$ hard-core doubly charged bosons with an effective trap size $\ell_b = \ell/2$, which in turn can be mapped into a free gas of $N/2$ spinless doubly-charged fermions in a harmonic trap of size ℓ_b . On the basis of these arguments, the $U_r \rightarrow -\infty$ limit of the trap-size scaling functions for harmonic traps is expected to be

$$\begin{aligned}\mathcal{R}(X, U_r \rightarrow -\infty, N) &= 2^{3/2} S(\sqrt{2}X, N/2), \\ \mathcal{G}(X_1, X_2, U_r \rightarrow -\infty, N) &= 8Z(\sqrt{2}X_1, \sqrt{2}X_2, N/2).\end{aligned}\quad (98)$$

These results for the Gaudin-Yang model imply that, if we compute the average work (93) and its fluctuations (94) in the limits $U_r \rightarrow \pm\infty$, we obtain formulas analogous to those for the free Fermi theory when inserting them into the corresponding Eqs. (97) and (98). In particular, we obtain the same large- N power laws, with trivial changes of their coefficients. These arguments suggest that the large- N behavior of one-dimensional systems is essentially the same of the free Fermi particles, at least in the regime of trap-size scaling, or equivalently for the continuum Gaudin-Yang model.

Another important issue concerns the degree of universality of the above claims, with respect to further local interaction terms extending the Hubbard model (83). This can be inferred by the universality of the behavior of the particle density, and particle density correlations [17]. We expect that they are universal with respect to a large class of further short-ranged interaction terms, such as

$$H_{nn} = \sum_{\sigma, \sigma'} w_{\sigma\sigma'} \sum_{\langle xy \rangle} n_{\sigma x} n_{\sigma' y}.\quad (99)$$

Indeed, H_{nn} may only give rise to a change of the effective quartic coupling U (when adding H_{nn} to the Hubbard Hamiltonian, the effective relevant quartic coupling becomes $U + 2w_{\uparrow\downarrow}$), and to further $O(\ell^{-\theta})$ corrections due to the fact that they introduce other irrelevant perturbations of renormalization-group dimension $y_w = -d$ at the dilute fixed point.

In conclusion, the above arguments show that the large- N power laws of the work fluctuations remain unchanged when

we consider three-dimensional Fermi gases with short-ranged interactions with positive on-site couplings (more precisely for $U > U^*$ with $U^* < 0$). We also conjecture that this property extends to one-dimensional systems, in the regime where trap-size scaling holds, and in particular in the continuum Gaudin-Yang model for any interaction coupling. We believe that the same conclusion applies to two-dimensional systems for any value of the on-site coupling U , for which the relation between the trap-size scaling and continuum limit does not require a rescaling of the coupling.

VII. SUMMARY AND CONCLUSION

We investigate the particle-number scaling behaviors characterizing the out-of-equilibrium quantum dynamics of dilute d -dimensional Fermi gases, in the limit of a large number N of particles. We consider protocols entailing variations of the external potential constraining them within a limited spatial region, such as those giving rise to a change of the size ℓ of the trap. We consider generic traps arising from external power-law potential, in particular the case of harmonic traps and hard-wall traps. We mostly consider lattice gas models of noninteracting Fermi particles in the dilute regime, $\ell/a \gg 1$ (where a is the lattice spacing) and $N/\ell^d \ll 1$, corresponding to the large trap-size limit keeping N fixed. In the framework of the trap-size scaling, the asymptotic large- ℓ behavior can be related to that of a continuum many-body theory of Fermi particles in an external confining potential [17, 18]. Therefore, our results apply to lattice Fermi gases in the dilute limit, and also to continuum Fermi models such as the Gaudin-Yang model [58, 59].

We determine the asymptotic large- N power laws of some features characterizing the out-of-equilibrium dynamics of Fermi gases, arising from the change of the trap features, starting from the equilibrium ground state for the initial trap size ℓ_0 . We focus on a number of global quantities, providing information on the evolution of the quantum state with respect to the initial one. We consider the ground-state fidelity associated with adiabatic changes of the trap size, the quantum work average and its fluctuations associated with a sudden change of the trap size, and the overlap of the quantum state at a given time t with the initial ground state as measured by the so-called Loschmidt echo. In the case of the quantum work statistics, we also discuss the effects of short-ranged particle interactions, in the framework of the Hubbard model and its continuum limit realized in the trap-size scaling limit.

We show that the N dependence of the first few moments of the work statistics, associated with the sudden change of the trap size, can be obtained from the scaling behaviors of the ground-state particle density and its correlations, see Secs. IV and VI. Our main results concern the asymptotic large- N power laws for d -dimensional Fermi gases in the dilute regime, confined by a generic power-law potential. The large- N behavior of the average work turns out to be

$$\langle W \rangle \sim N^{1+2\theta/d},\quad (100)$$

where $\theta = p/(p+2)$ and p is the power law of the spatial dependence of the confining potential, cf. Eq. (2). Analogous power laws are obtained for the square work fluctuations. It is important to note that the asymptotic large- N behaviors that

we consider should be always meant within the dilute regime of the lattice gas models, i.e., when the condition $N/\ell^d \ll 1$ is satisfied. The order of the limits $\ell \rightarrow \infty$ and then $N \rightarrow \infty$ is essential, they cannot be interchanged.

We also argue that short-ranged particle interactions, such as those described by the Hubbard model and the Gaudin-Yang model, do not change the large- N power laws in the dilute regime, within appropriate ranges of their coupling values, depending on the spatial dimensions, see Sec. [VIC](#). In particular, for three-dimensional systems the large- N behavior is expected to be the same as the free Fermi gases for on-site couplings U larger than a negative value $U^* < 0$, thus including an interval around $U = 0$ and for any repulsive interaction. For one-dimensional models we argue that the large- N behaviors remain unchanged in the regime of trap-size scaling, thus for the corresponding continuum Gaudin-Yang model.

We note that, in the case of one-dimensional systems, the results for noninteracting Fermi gases extends to one-dimensional Bose gases in the limit of strong short-ranged repulsive interactions. The basic model to describe the many-body features of a boson gas confined to an effective one-dimensional geometry is the Lieb-Liniger model with an effective two-particle repulsive contact interaction [\[66\]](#). The limit of infinitely strong repulsive interactions corresponds to a one-dimensional gas of impenetrable bosons [\[60\]](#), the Tonks-Girardeau gas. One-dimensional Bose gases with repulsive two-particle short-ranged interactions become more and more nonideal with decreasing the particle density, acquiring fermion-like properties, so that the one-dimensional gas of impenetrable bosons is expected to provide an effective description of the low-density regime of confined one-dimensional bosonic gases [\[67\]](#). Due to the mapping between one-dimensional gases of impenetrable bosons and spinless fermions, the particle density of hard-core bosons, and its correlations, are identical to those of free fermion gases. Therefore, the results of this paper for the work statistics apply to one-dimensional repulsively interacting Bose gases as well, subject to analogous dynamic protocols.

For one-dimensional Fermi gases we also study the quantum evolution arising from the change of the trap size, including the extreme case of the free expansion of the gas after the drop of the trap. In the case of harmonic traps, we present results for generic time dependencies of the trap size. We show that the particle-number dependence of the echo function $Q = -\ln L_E$, where L_E is the Loschmidt echo, is generally characterized by the power-law behavior

$$Q = -\ln |\langle \Psi(\mathbf{x}_1, \dots, \mathbf{x}_N; t) | \Psi(\mathbf{x}_1, \dots, \mathbf{x}_N; t = 0) \rangle| \sim N^2, \quad (101)$$

independently of the particular protocol varying the trap. This is compared to the asymptotic behavior obtained when dropping a hard wall, which turns out to increase more slowly, i.e., $Q \sim N \ln N$.

Quite remarkably, the particle-number scaling behaviors outlined in this paper can be observed for systems with a relatively small number of particles, i.e., $O(10^2)$ or even less. Therefore, even systems with relatively few particles may show definite signatures of the scaling laws derived in

this work. In this respect, present-day quantum-simulation platforms already demonstrated their capability to reproduce and control the dynamics of ultracold atoms in optical lattices. Therefore the properties of the quantum many-body physics discussed here may be tested with a small number of controllable objects. In particular, the work statistics may be accessible experimentally in ultracold-atom systems, see, e.g., Refs. [\[68–70\]](#).

APPENDIX: GROUND STATE OF FERMI GASES

The ground state of a Fermi gas constituted by N particles is given by a Slater determinant

$$\Psi(\mathbf{x}_1, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \det[\psi_i(\mathbf{x}_j)], \quad (A1)$$

where $\psi_i(\mathbf{x})$ are the lowest N eigensolutions of the one-particle Schrödinger equation.

In the case of the harmonic potential, the one-particle energy spectrum in harmonic traps is discrete. The eigensolutions can be written as a product of eigenfunctions of corresponding one-dimensional Schrödinger problems

$$\begin{aligned} \psi_{n_1, n_2, \dots, n_d}(\mathbf{x}) &= \prod_{i=1}^d \phi_{n_i}(x_i), \\ E_{n_1, n_2, \dots, n_d} &= \sum_{i=1}^d e_{n_i}, \end{aligned} \quad (A2)$$

where the subscript n_i labels the eigenfunctions along the d directions, which are

$$\begin{aligned} \phi_n(x) &= \xi^{-1/2} \frac{H_{n-1}(X)}{\pi^{1/4} 2^{(n-1)/2} (n-1)!^{1/2}} e^{-X^2/2}, \\ \xi &= \ell^{1/2}, \quad X = x/\xi, \\ e_n &= \ell^{-1}(n-1/2), \quad n = 1, 2, \dots, \end{aligned} \quad (A3)$$

where $H_n(x)$ are the Hermite polynomials. Note, however, that, although the spatial dependence of the one-particle eigenfunctions is decoupled along the various directions, fermion gases in different dimensions present some differences due to the nontrivial filling of the lowest N states which provides the ground state of the N -particle system. Exploiting the properties of the Hermite polynomials, the ground state [\(77\)](#) of one-dimensional systems with N particles can be written as in Eq. [\(17\)](#).

In the case of a hard-wall trap, corresponding to finite-volume systems with open boundary conditions, the eigensolutions can be written as a product of eigenfunctions of the corresponding one-dimensional Schrödinger problem, analogously to Eqs. [\(A2\)](#) with

$$\phi_n(x) = \ell^{-1/2} \sin\left(n\pi \frac{x+\ell}{2\ell}\right), \quad e_n = \ell^{-2} \frac{\pi^2}{8} n^2, \quad (A4)$$

for $n = 1, 2, \dots$

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