Stabilizing Bell states of two separated superconducting qubits via quantum reservoir engineering

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(Received 19 January 2019; published 30 April 2019)

We propose a quantum reservoir engineering approach for stabilizing Bell states of two superconducting qubits. The system under consideration consists of two linearly coupled superconducting transmission line resonators and two separated flux qubits, one of which is interacted with one resonator. Applying external driving fields to tailor appropriate qubit-resonator interactions, we show that dissipative photons of the resonators can be exploited to autonomously drive and stabilize the two flux qubits into an approximate Bell state at the stationary state. Because of using the dissipative dynamical process, the present approach does not need to well prepare the initial state of the system and exactly monitor the evolution time. Compared with previous schemes, the present one has the remarkable features that the generation of the entangled state is implemented at a single-photon quantum level, and all four Bell states can be generated and stabilized on demand by changing the external driving parameters. Our result may have useful applications for the realization of quantum computation with superconducting quantum circuits.

DOI: 10.1103/PhysRevA.99.042336

I. INTRODUCTION

Circuit quantum electrodynamics (circuit QED) appears to be one of the most promising candidates for building the future quantum processor [1–3]. In contrast to the conventional cavity QED, the elements in circuit QED are fabricated on-chip with nanofabrication techniques, i.e., the photon is stored in a superconducting microwave resonator, and the information carrier is not a natural atom but an artificial one [4–8]. Depending on the excellent properties such as tunability, controllability, and scalability, the solid-state circuits provide a prominent platform for the study of quantum information processing [9–13]. Up to now, substantial progress in this field has been made for the improved qubit lifetimes, higher gate fidelities, and increasing circuit complexity [14–18].

For the realization of quantum information protocols with superconducting circuits, the crucial task is to prepare various kinds of quantum entangled states, which are key resources in the fields of fundamental quantum physics, quantum cryptography, and quantum computation [19]. By coupling individual superconducting qubits to a common coupler, such as a microwave resonator, a lot of experiments have achieved the two-qubit entanglement [20–22]. With the further improvement of qubit coherence time, recent experiments have reported the generation of multiqubit entangled states [23–27] and a 12-qubit linear cluster state [28]. Since those schemes are based on the unitary evolution process, however, one has to initialize the system in a pure state and obtains the target state at a specific time point.

On the other hand, it has been shown that a particular quantum state has to be stabilized for performing the error-correction protocols [29]. The major challenge to achieve this goal is to overcome the effect of decoherence. One choice is to apply error syndrome measurements and the associated feedback controls for protecting the quantum state from noise [30,31]. Alternatively, one can use the method of quantum reservoir engineering, where the environmental coupling can enforce the system into a nontrivial ground state without regard to its initial state [32–35]. Based on this approach, previous investigations have proposed the generation of stable entanglement of two superconducting qubits in a single microwave resonator [36–38], which has been demonstrated in experiment [39]. For scalability, it is desirable to produce steady-state entanglement of superconducting qubits in independent resonators [40]. Several scalable schemes have been presented for stabilizing a two-qubit entangled state in separated resonators [41–44], and the recent experiment has achieved this goal [45]. However, none of them are operated at a single-photon quantum level. The ideas of Refs. [41,42] are to first squeeze the two resonator modes and then transfer the entanglement to qubits. Other proposed schemes [43–45] need to displace the microwave fields of resonators into the coherent state with large average photon number. In addition, all of those schemes can only produce one or two kinds of the four Bell states.

In this work, we propose an efficient scheme for stabilizing entangled states of two superconducting qubits by means of quantum reservoir engineering. The system under consideration consists of two linearly coupled superconducting transmission line resonators, each of which is coupled to a superconducting flux qubit. By modulating energy gaps of the flux qubits via external driving fields, we can sculpt the well-designed couplings between qubits and resonators. It is shown that the resonators of fast photon decay can play the role of reservoir and steer the two flux qubits into an approximate Bell state at the steady state. The quantum state preparation is based on a dissipative dynamical process, eliminating the

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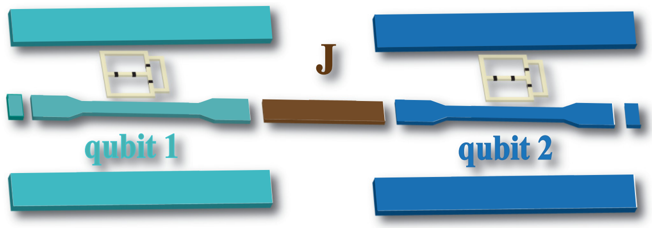

FIG. 1. Schematic of the experimental architecture. Two gap-tunable superconducting flux qubits are coupled to two neighboring interacting superconducting transmission line resonators, respectively.

needs such as precise control of the evolution time and specific initialization of the system. Hence, our scheme is insensitive to experimental noise and has significant advantages for practical feasibility. Compared with the previous schemes [41–45], the present one has the remarkable features that the quantum state engineering is implemented at a single quanta level and all four Bell states can be generated and stabilized on demand by adjusting the external driving fields. In general, the coupling of superconducting qubits to the environment cannot be avoided, and results in decoherence and decreases the fidelity of the generating state. In the quantum engineering scheme under consideration, however, the much stronger coupling between qubits and dissipative resonators is engineered, and the fast decay process of the resonators is employed to bring and stabilize the qubits into the desired states. Thus, the steady state of the system can be quickly reached within the coherence time of qubits, and the decoherence of qubits does not significantly affect the fidelity of the resulting states. The numerical simulations indicate that the high-fidelity Bell states can be achieved with current available technology. The present work may have potential applications for the realization of quantum information processing with superconducting quantum circuits.

II. MODEL

As illustrated in Fig. 1, the circuit QED framework under consideration consists of two linearly coupled superconducting transmission line resonators, each of which is interacted with a superconducting flux qubit. The superconducting resonators can be modeled as simple harmonic oscillators, in which two ground planes are placed on the two sides of a narrow central conductor with distributed inductor L and capacitance C [1]. With just the fundamental modes taken into account, the Hamiltonian of the two superconducting resonators is given by (in units of $\hbar = 1$)

$$H_r = \sum_{k=1}^{2} \omega a_k^\dagger a_k + J(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{1}$$

where $\omega = 2\pi/\sqrt{LC}$ is the associated eigenfrequency, a_k ($a_k^\dagger$) is the annihilation (creation) operator of the kth superconducting resonator, and J is the intraresonator coupling strength, similar to the photon hopping rate between optical cavities. In circuit QED, the photon exchange can be realized by connecting the independent resonators via a nonlinear coupler, such as a capacitor [46]. Experimentally, the strong coupling between two superconducting resonators has been demonstrated and can be dynamically adjusted by tuning the capacitance or inductance of the coupler [47–50].

The flux qubits are adopted with gradiometric configuration, as shown in Fig. 1, i.e., a pair of identical Josephson junctions with smaller critical current form a superconducting quantum interference device (SQUID) loop [51,52]. This specific symmetry of gradiometric design can greatly reduce environmental noise and make the flux qubit well tunable while still operating at the symmetry point [53]. The energy gap of this artificial atom can be dynamically tuned by altering the external magnetic fields threading the SQUID loop [54–57]. Operating the flux qubits at the optimal point, we have the Hamiltonian

$$H_q = \sum_{k=1}^{2} \left[\frac{\delta}{2}\sigma_{zk} + \Omega_1^k \cos\left(\omega_{d1}^k t\right)\sigma_{zk} + \Omega_2^k \cos\left(\omega_{d2}^k t + \theta_k\right)\sigma_{zk} \right], \tag{2}$$

where the Pauli matrix reads $\sigma_{zk} = |e\rangle_k\langle e| - |g\rangle_k\langle g|$, and δ is the the static energy gap; Ω_1^k and Ω_2^k represent the Rabi frequencies, ω_{d1}^k and ω_{d2}^k are the frequencies of the external driving fields, and θ_k is the relative phase of the external driving fields. In principle, this scheme can also be implemented with other kinds of Josephson-junction-based superconducting qubits for realizing the tunable coupling between qubits and resonators, such as charge qubit [58] and transmon qubit [14,59–61].

The interaction Hamiltonian between the flux qubits and superconducting resonators is given by

$$H_I = \sum_{k=1}^{2} g(a_k^\dagger + a_k)(\sigma_k^+ + \sigma_k^-), \tag{3}$$

where g describes the qubit-resonator coupling strength, and $\sigma_k^+ = |e\rangle_k\langle g|$, $\sigma_k^- = |g\rangle_k\langle e|$ are the spin-flip operators of flux qubits.

III. STABILIZATION OF BELL STATES

In this section, we study how to generate and stabilize entangled states of the two separated flux qubits with the engineered dissipation. The main idea of our work is to adequately design the qubit-resonator couplings via the external driving fields, and then use the photon decay of resonators to force the two flux qubits into an approximate Bell state at stationary state, i.e., the quantum superposition can be kept for an infinitely long time.

To detail the procedure of quantum state production, we first work out the effective interactions. The total Hamiltonian of the coupled system is given by

$$H = H_r + H_q + H_I. \tag{4}$$

We introduce the canonical transformations $A_1 = \frac{1}{\sqrt{2}}(a_1 + a_2)$ and $A_2 = \frac{1}{\sqrt{2}}(a_1 - a_2)$. In terms of A_1 and A_2, the Hamiltonian H_r can be rewritten to the form $H_r = \sum_{k=1}^{2} \omega_k A_k^\dagger A_k$, where $\omega_1 = \omega + J$ and $\omega_2 = \omega - J$ are the eigenfrequencies of the new modes A_1 and A_2, respectively. It is clear that the modes A_1 and A_2 will be spectrally well resolved with the large enough coupling strength J. By performing the unitary transformation $U_2(t) = T\exp[-i(H_r t + \int_0^t H_q(t')dt')]$, where T is the time-order operator,the interaction Hamiltonian between the qubits and resonators can be written in the form

$$H_I = \frac{g}{\sqrt{2}} e^{i\delta_1 t}\sigma_1^+ e^{2i[\xi_{11}\sin(\omega_{d1}^1 t)+\xi_{12}\sin(\omega_{d2}^1 t+\theta_1)]}(A_1 e^{-i\omega_1 t} + A_1^\dagger e^{i\omega_1 t} + A_2 e^{-i\omega_2 t} + A_2^\dagger e^{i\omega_2 t}) + \frac{g}{\sqrt{2}} e^{i\delta_2 t}\sigma_2^+ e^{2i[\xi_{21}\sin(\omega_{d1}^2 t)+\xi_{22}\sin(\omega_{d2}^2 t+\theta_2)]}(A_1 e^{-i\omega_1 t} + A_1^\dagger e^{i\omega_1 t} - A_2 e^{-i\omega_2 t} - A_2^\dagger e^{i\omega_2 t}) + \text{H.c.}, \tag{5}$$

where we have defined the parameters $\xi_{kl} = \Omega_l^k/\omega_{dl}^k$ $(l = 1, 2)$. For sufficiently small ξ_{kl}, we can apply the Taylor expansion and only keep ξ_{kl} up to the first order in the above equation. Thus, the Hamiltonian can be approximated to be

$$H_I = \frac{g}{\sqrt{2}} e^{i\delta_1 t}\sigma_1^+\{1 + \xi_{11}(e^{i\omega_{d1}^1 t} - e^{-i\omega_{d1}^1 t}) + \xi_{12}[e^{i(\omega_{d2}^1 t+\theta_1)} - e^{-i(\omega_{d2}^1 t+\theta_1)}]\} \times (A_1^\dagger e^{i\omega_1 t} + A_1 e^{-i\omega_1 t} + A_2^\dagger e^{i\omega_2 t} + A_2 e^{-i\omega_2 t}) + \frac{g}{\sqrt{2}} e^{i\delta_2 t}\sigma_2^+\{1 + \xi_{21}(e^{i\omega_{d1}^2 t} - e^{-i\omega_{d1}^2 t}) + \xi_{22}[e^{i(\omega_{d2}^2 t+\theta_2)} - e^{-i(\omega_{d2}^2 t+\theta_2)}]\}(A_1^\dagger e^{i\omega_1 t} + A_1 e^{-i\omega_1 t} - A_2^\dagger e^{i\omega_2 t} - A_2 e^{-i\omega_2 t}) + \text{H.c.} \tag{6}$$

To select the desired couplings between qubits and resonators, we choose $\delta_1 = \omega_{d1}^1 + \omega_1 = \omega_{d2}^1 - \omega_2$ and $\delta_2 = \omega_{d1}^2 + \omega_2 = \omega_{d2}^2 - \omega_1$. Meanwhile, if the conditions $\{\delta_k, \omega_k, \omega_{dl}^k\} \gg g/\sqrt{2}$ and $2J \gg \xi_{kl} g/\sqrt{2}$ are satisfied, we can retain the resonant terms and discard those fast oscillating terms under the rotating-wave approximation. Thus, we can obtain the effective Hamiltonian,

$$H_{\text{eff}}^1 = \Theta_1 A_1^\dagger(\sigma_1^- + \epsilon e^{-i\theta_2}\sigma_2^+) + \Theta_2 A_2^\dagger(\sigma_2^- - \epsilon e^{-i\theta_1}\sigma_1^\dagger) + \text{H.c.}, \tag{7}$$

where $\Theta_1 = -g\xi_{11}/\sqrt{2}$, $\Theta_2 = g\xi_{21}/\sqrt{2}$, and $\epsilon = \xi_{22}/\xi_{11} = \xi_{12}/\xi_{21}$.

Alternatively, if the resonance conditions $\delta_1 = \omega_{d1}^1 - \omega_1 = \omega_{d2}^1 + \omega_2$ and $\delta_2 = \omega_{d1}^2 + \omega_2 = \omega_{d2}^2 - \omega_1$ are satisfied, we can apply the rotating-wave approximation and get the effective Hamiltonian,

$$H_{\text{eff}}^2 = \Theta_1 A_1^\dagger(\sigma_1^+ + \epsilon e^{-i\theta_2}\sigma_2^+) + \Theta_2 A_2^\dagger(\sigma_2^- - \epsilon e^{i\theta_1}\sigma_1^-) + \text{H.c.} \tag{8}$$

In the following, we will show that the specially engineered Hamiltonians H_{eff}^1 and H_{eff}^2 can be exploited to produce and stabilize the Bell states of the separated flux qubits with the assistance of photon damping of the superconducting resonators. By considering the system coupled to the harmonic-oscillator environment in the Markovian approximation, the dynamics of density matrix ρ of the whole system is governed by the master equation

$$\frac{d\rho}{dt} = -i[H_{\text{eff}}^k, \rho] + \frac{\kappa}{2}D[A_1]\rho + \frac{\kappa}{2}D[A_2]\rho, \tag{9}$$

where $D[o]\rho = 2o\rho o^\dagger - \rho o o^\dagger - o o^\dagger\rho$ is the standard Lindblad operator, and κ is the photon leakage rate of the resonator. To make the quantum state generation mechanism clearer, we first neglect the damping terms such as $\frac{\gamma}{2}D[\sigma_-^k]$ and $\frac{\gamma_\phi}{2}D[\sigma_z^k]$ of the qubits, where γ is the energy-relaxation rate and γ_ϕ is the pure dephasing rate of the qubit. The effect of the detrimental factors of the qubits will later be analyzed in the numerical simulation.

The dynamics of master equation (9) describes a dissipative evolution process, i.e., the appropriately designed interaction with dissipative photons represents a bath engineering resource for the flux qubits. The superconducting resonators play the role of cold baths, which will continuously extract quanta from the flux qubits, and then radiate into the electromagnetic environment via the fast photon decay. Hence, the whole system will be eventually cooled down to the unique dark state with $d\rho_S/dt = 0$, i.e., $\rho_S = |\Psi_s\rangle\langle\Psi_s|$ and $|\Psi_s\rangle = |\phi\rangle\otimes|\varphi\rangle$, where $|\phi\rangle$ and $|\varphi\rangle$ are states of the qubits and the resonators, respectively. It is noted that the steady state of the system is unique without regard to its initial state, which can be mathematically guaranteed. Since the superconducting resonators have the large photon decay rates, their steady state will be readily damped to the tensor product of vacuum states $|\varphi\rangle = |0\rangle_1\otimes|0\rangle_2 = |0,0\rangle_{1,2}$. Thus, the steady state of flux qubits should obey the equation $H_{\text{eff}}^k(|\phi\rangle\otimes|0,0\rangle_{1,2}) = 0$.

With the effective Hamiltonian H_{eff}^1, it is straightforward to obtain the equations $(\sigma_1^- + \epsilon e^{-i\theta_2}\sigma_2^+)|\phi\rangle = 0$ and $(\sigma_2^- - \epsilon e^{-i\theta_1}\sigma_1^\dagger)|\phi\rangle = 0$. Thus, the unique steady state of the qubits can be analytically worked out,

$$|\phi_1\rangle = \frac{1}{\sqrt{1+\epsilon^2}}(|g,g\rangle_{1,2} + \epsilon|e,e\rangle_{1,2}), \tag{10}$$

with the choice of $\theta_1 = 0$ and $\theta_2 = \pi$. On the contrary, if we set $\theta_1 = \pi$ and $\theta_2 = 0$, the steady state will be

$$|\phi_2\rangle = \frac{1}{\sqrt{1+\epsilon^2}}(|g,g\rangle_{1,2} - \epsilon|e,e\rangle_{1,2}). \tag{11}$$

Furthermore, with the engineered Hamiltonian H_{eff}^2, we have the equations $(\sigma_1^+ + \epsilon e^{-i\theta_2}\sigma_2^+)|\phi\rangle = 0$ and $(\sigma_2^- - \epsilon e^{i\theta_1}\sigma_1^-)|\phi\rangle = 0$, which should be simultaneously satisfied. In the case of $\theta_1 = 0$ and $\theta_2 = \pi$, we can produce the state

$$|\phi_3\rangle = \frac{1}{\sqrt{1+\epsilon^2}}(|e,g\rangle_{1,2} + \epsilon|g,e\rangle_{1,2}). \tag{12}$$

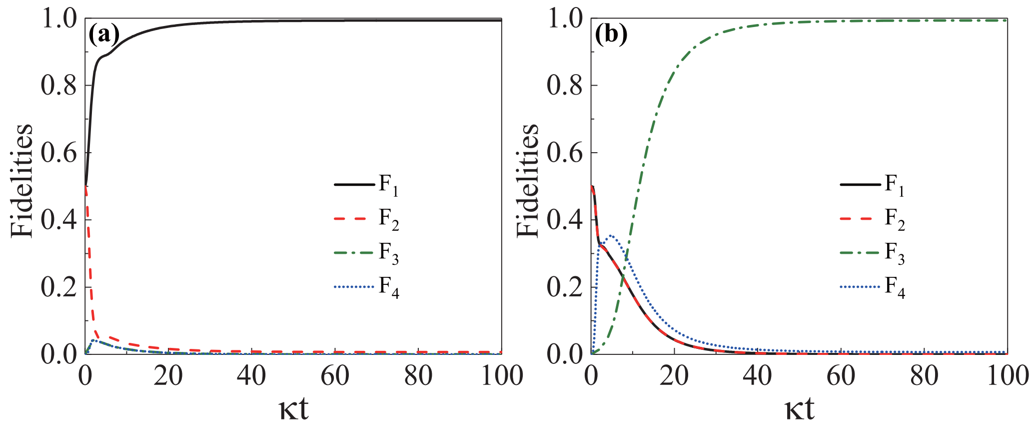

FIG. 2. The time evolution of the fidelities of F_j vs the dimensionless time κt obtained by numerically solving the master equation (9), (a) with the engineered Hamiltonian $H_{\rm eff}^1$ and (b) with the engineered Hamiltonian $H_{\rm eff}^2$ with $\theta_1 = 0$ and $\theta_2 = \pi$. The relevant parameters are chosen as $\delta/2\pi = 6$ GHz, $\omega/2\pi = 3$ GHz, $J/2\pi = 0.2$ GHz, $g/2\pi = 50$ MHz, $\kappa/2\pi = 4$ MHz, $\xi_{12} = \xi_{22} = 0.051$, and $\xi_{11} = \xi_{21} = 0.06$.

If we choose $\theta_1 = \pi$ and $\theta_2 = 0$, the dark state of the qubits will become

$$|\phi_4\rangle = \frac{1}{\sqrt{1+\epsilon^2}}(|e, g\rangle_{1,2} - \epsilon|g, e\rangle_{1,2}). \tag{13}$$

With ϵ approaching 1, it is seen that the states $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle$ are approximately to be the four Bell states of $|\Phi_1\rangle = (|g, g\rangle_{1,2} + |e, e\rangle_{1,2})/\sqrt{2}$, $|\Phi_2\rangle = (|g, g\rangle_{1,2} - |e, e\rangle_{1,2})/\sqrt{2}$, $|\Phi_3\rangle=(|e, g\rangle_{1,2}+|g, e\rangle_{1,2})/\sqrt{2}$, $|\Phi_4\rangle=(|e, g\rangle_{1,2} - |g, e\rangle_{1,2})/\sqrt{2}$, respectively. By means of quantum reservoir engineering, the photon decay of resonators as a resource is utilized to actively propel the qubits into a highly entangled state at the stationary state, i.e., the coherence can be stabilized for an arbitrarily long time. The long-lived entangled states have wide applications for the fundamental test of quantum theory and quantum information science [39]. Since our scheme is realized via a dissipative steady-state process, it does not require unitary dynamics and specific initialization of the system. Compared with the previous methods [41–45], the present work has the following remarkable features. First, the quantum state preparation is operated at the single-photon level, which renders the squeezing and displacement of the resonator fields unnecessary. Second, all four Bell states can be generated and stabilized on demand by tuning the oscillating frequencies and relative phases of the external driving fields.

To check the validity of our results, we numerically solve the master equation (9). Here, we exploit the fidelities $F_j = {\rm tr}(\rho|\Phi_j\rangle\langle\Phi_j|)$ to quantify the overlap between the Bell state $|\Phi_j\rangle$ and the resulting state $|\phi_j\rangle$ ($j = 1, 2, 3, 4$). When performing the numerical simulations, the chosen realistic parameters are shown in the caption of Fig. 2 and the system is initially prepared in the ground state $|0, 0\rangle_{1,2}\otimes\ |g, g\rangle_{1,2}$. In Figs. 2(a) and 2(b), the numerical results for the time evolutions of fidelities F_j are displayed with the engineered Hamiltonians $H_{\rm eff}^1 = \Theta_1 A_1^\dagger(\sigma_1^- - \epsilon\sigma_2^+) + \Theta_2 A_2^\dagger(\sigma_2^- - \epsilon\sigma_1^\dagger) + {\rm H.c.}$ and $H_{\rm eff}^2 = \Theta_1 A_1^\dagger(\sigma_1^+ - \epsilon\sigma_2^+) + \Theta_2 A_2^\dagger(\sigma_2^- - \epsilon\sigma_1^-) + {\rm H.c.}$, respectively. It can be observed that, as the time goes on, the fidelities of F_1 in Fig. 2(a) and F_3 in Fig. 2(b) eventually converge to the constant value of 0.993, but with the others F_j to be zero. This means that the states of the flux qubits are driven into the approximate Bell states of $|\phi_1\rangle = \frac{1}{\sqrt{1+\epsilon^2}}(|g, g\rangle_{1,2} + \epsilon|e, e\rangle_{1,2})$

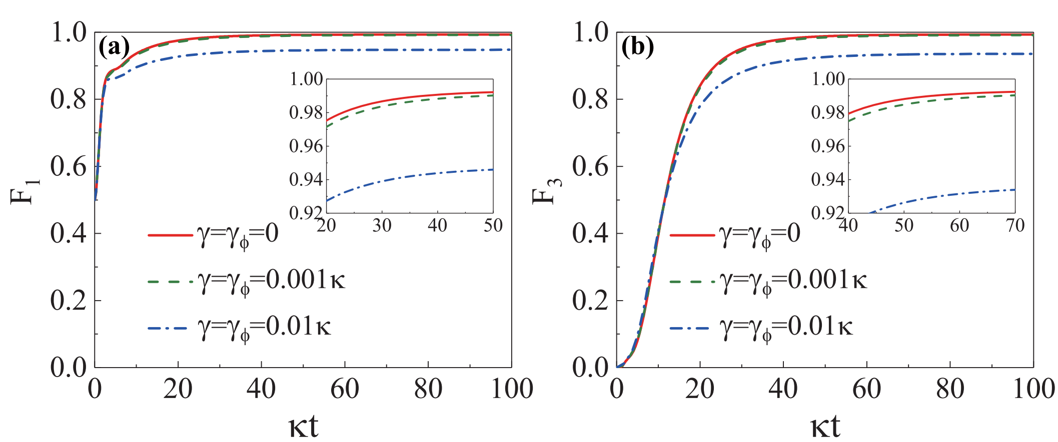

FIG. 3. The time evolution of the fidelities of F_1 and F_3 by including the different energy-relaxation rate γ and pure dephasing γ_ϕ of qubits into the master equation (9), (a) with the engineered Hamiltonian $H_{\rm eff}^1$ and (b) with the engineered Hamiltonian $H_{\rm eff}^2$ with $\theta_1 = 0$ and $\theta_2 = \pi$. The relevant parameters are chosen to be the same as in Fig. 2.

and $|\phi_3\rangle = \frac{1}{\sqrt{1+\epsilon^2}}(|e, g\rangle_{1,2} + \epsilon|g, e\rangle_{1,2})$ with $\epsilon = 0.85$, respectively. Therefore, the numerical simulation proves our scheme is valid.

On the other hand, the coupling of the qubits to the environment will inevitably damage the coherence. To investigate the effect of dissipation of the qubits, we add the dissipative terms of the qubits into the master equation (9). The time evolution of F_1 and F_3 with different energy-relaxation rates γ and pure dephasing rates γ_ϕ are shown in Figs. 3(a) and 3(b), respectively. For $\gamma = \gamma_\phi = 0.001\kappa = 2\pi \times 4$ kHz, which corresponds to the coherence times $T_\gamma = 1/\gamma \simeq 40$ μs and $T_\phi = 1/\gamma_\phi \simeq 40$ μs, the fidelities have almost no deviation from the case of $\gamma = \gamma_\phi = 0$. For $\gamma = \gamma_\phi = 0.01\kappa = 2\pi \times 40$ kHz and the corresponding coherence times $T_\gamma = T_\phi \simeq 4$ μs, the stable value above 0.94 of the F_1 and F_3 at the steady state can still be achieved, implying that the entanglement can be stabilized even in the presence of decoherence. Experimentally, the coherence times of the superconducting qubits have been raised to the range 10–100 μs, and steadily increased with the improved technologies [62–67]. Therefore, our scheme can generate and stabilize a highly entangled state with the currently available parameters.

IV. CONCLUSION

We consider a system consisting of two linearly coupled superconducting transmission line resonators, each of which is interacted with a superconducting flux qubit. We have proposed an approach to generate and stabilize the steady-state entanglement of the two superconducting flux qubits. By virtue of the mechanism of quantum reservoir engineering, it is shown that the photon decay of the resonators as resource can be utilized to deterministically steer the two flux qubits into an approximate Bell state at the final state, which can be sustained for a long time. Since the scheme is implemented via a dissipative dynamical process, it does not require the initial-state preparation and the unitary dynamics control of the system. The distinct advantages of the present work are that the process of quantum state production is operated at a single-quanta level and all four Bell states can be deterministically produced and stabilized on demand by tuning the external driving fields. By considering the rapid development in superconducting quantum circuits, the present scheme can be implemented in a realistic experiment.

ACKNOWLEDGMENTS

The work was partly supported by the National Nature Science Foundation of China (Grants No. 11704306 and No. 11534008), the National Key R&D Project (Grant No. 2016YFA0301404), and the China Postdoctoral Science Foundation (Grant No. 2016M602795).

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