Robust generation of entangled fields against inhomogeneous parameters of coupled superconducting resonators and qubits

Sheng-li Ma, Ji-kun Xie, Xin-ke Li, and Fu-li Li*
Shaanxi Province Key Laboratory of Quantum Information and Quantum Optoelectronic Devices and Department of Applied Physics, School of Science, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China

(Received 21 December 2018; published 10 April 2019)

We propose an efficient approach for the generation of a stable entangled state of microwave fields in circuit quantum electrodynamics. The system under consideration consists of two linearly coupled superconducting resonators, each of which is coupled to a superconducting flux qubit. By individually driving the qubits to engineer the desired interactions with the resonators, we show that the energy relaxation of qubits can be exploited to steer the two resonators into the two-mode entangled state via a dissipative dynamical process. Since our protocol is based on quantum reservoir engineering, neither specific initialization nor unitary dynamics is required. Moreover, the distinct advantage of our scheme is that the quantum state preparation is robust in relation to inhomogeneous parameters, i.e., there is no need for identical qubit-field couplings, as well as the same resonance frequencies of the resonators. These features make the present scheme pretty feasible for experimental implementation.

DOI: 10.1103/PhysRevA.99.042317

I. INTRODUCTION

Circuit quantum electrodynamics is an on-the-chip counterpart of cavity QED systems, i.e., it has already become one of the most promising platforms for studying fundamental quantum physics and applications of quantum information processing [1–3]. These solid-state superconducting circuits possess the advantages of flexibility, tunability, and scalability with nanofabrication techniques [4–6]. Due to the strong coupling between superconducting elements and electromagnetic fields, circuit QED provides robust quantum manipulation, storage, and readout [7–9]. Up to now, significant progress on this subject has been made in demonstrating the improved qubit lifetimes, higher gate fidelities, and increasing circuit complexity [10–16]. Generally, the superconducting qubit is regarded as an artificial atom. Due to the nonlinearity of Josephson junctions, the energy-level separation of an artificial atom becomes nonuniform, where the two lowest levels can be used as a qubit to encode quantum information [17–21]. In contrast, recent experiments have demonstrated the ultrahigh-quality factor of superconducting resonators [22–24]. These harmonic oscillators with a theoretically infinite ladder of states can also be efficiently addressed and manipulated [25–30]. Therefore, circuit QED provides an alternative appealing way to realize quantum information processing involving microwave photons.

For realization of quantum computation, the prerequisite is to synthesize the various kinds of nonclassical quantum states of superconducting resonators [31]. By utilizing the nonlinearity of a superconducting qubit, experiments have demonstrated the robust control of a single superconducting resonator, including the efficient preparation of Fock states [32], Schrödinger cat states [33], and quantum superposition states [34,35]. Further progress requires the efficient quantum control of microwave photons in coupled networks of resonators. For this goal, coupled networks have been proposed to interconnect the separated resonators via superconducting qubits [36–44]. Experimentally, the implementation of NOON states [45], entangled coherent states [46], quantum logic gates [47], and quantum state transfer [48–50] between two separated resonators has been achieved. However, it has been shown that unwanted intercavity crosstalk is inevitable and it is also difficult to guarantee the homogenous couplings between qubit and resonators, as well as the same frequencies of the resonators [51,52]. These fluctuations will degrade the performance of quantum operations and affect the final fidelity substantially.

In this paper we propose an efficient method for robust generation of stable entangled state of microwave fields in relation to inhomogeneous parameters. We consider the system of two directly coupled superconducting transmission line resonators [53–55], each of which is coupled to a superconducting flux qubit. The photon exchange can occur between two adjacent microwave resonators and thus avoid the uncontrolled crosstalk interaction [56–59]. By modulating the energy gaps of artificial atoms via external driving fields, the appropriate qubit-resonator interactions can be engineered. We let each of the qubits individually be coupled to one of the two combined resonator modes via squeezing-type interactions, but the squeezing parameters have a π phase difference. It is shown that the qubits can play the role of a dissipative bath and help drive the two resonators into the stable two-mode squeezed state via the quantum interference effect. Since each of the qubits is only coupled to one of the combined modes, it does not require the identical qubit-resonator couplings. Moreover, the effect of the resonance frequency difference between resonators can be greatly inhibited. Compared with

*flli@mail.xjtu.edu.cn

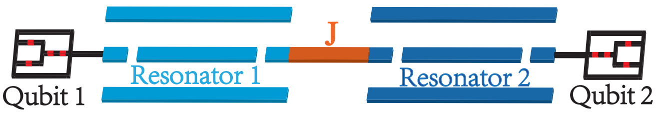

FIG. 1. Schematic of two directly coupled superconducting transmission line resonators, each coupled to a gap-tunable superconducting flux qubit. The flux qubit is made up of four Josephson junctions.

previous work [60], the present scheme has the distinct advantage that the quantum state preparation is robust against these inhomogeneous parameters. Therefore, it is more feasible in a realistic experiment. The present proposal provides significant advantages for practical feasibility and may have useful applications for realization of quantum information processing with superconducting circuits.

II. MODEL

As sketched in Fig. 1, we consider a system composed of two linearly coupled superconducting transmission line resonators, each of which is magnetically coupled to a gap-tunable superconducting flux qubit. The flux qubit consists of four Josephson junctions, where two identical Josephson junctions with smaller critical current form a superconducting quantum interference device (SQUID) loop [61–63]. The external magnetic fields threading the SQUID loop can tune the energy levels [64]. In principle, the generic model can also be implemented with other kinds of Josephson-junction-based superconducting qubits, such as charge qubits [65] or transmon qubits [66]. Operating the flux qubit at the degeneracy point, we have the free Hamiltonian of the system (set $\hbar = 1$ hereafter)

$$H_0 = \sum_{\lambda=1}^{2}\left(\frac{\delta_\lambda}{2}\sigma_\lambda^z + \omega_\lambda a_\lambda^\dagger a_\lambda\right), \tag{1}$$

where the parameter λ ($\lambda = 1, 2$) indicates the λth qubit and superconducting resonator, δ_λ denotes the static qubit energy gap between the excited state $|e\rangle_\lambda$ and ground state $|g\rangle_\lambda$, $\sigma_\lambda^z = |e_\lambda\rangle\langle e_\lambda| - |g_\lambda\rangle\langle g_\lambda|$ is the Pauli operator, ω_λ is the eigenfrequency of the superconducting resonator, and $a_\lambda^\dagger$ (a_λ) is the corresponding creation (annihilation) operator. The interaction Hamiltonian between qubits and superconducting resonators is written as

$$H_{qr} = \sum_{\lambda=1}^{2} g_\lambda(\sigma_\lambda^+ + \sigma_\lambda^-)(a_\lambda^\dagger + a_\lambda), \tag{2}$$

where g_λ is the coupling strength and $\sigma_\lambda^+ = |e_\lambda\rangle\langle g_\lambda|$ and $\sigma_\lambda^- = |g_\lambda\rangle\langle e_\lambda|$ are the spin-flip operators of qubits. The interaction between the two resonators is given by

$$H_{rr} = J(a_1^\dagger a_2 + a_1 a_2^\dagger), \tag{3}$$

where J is the coupling constant. Experimentally, the strong coupling between two superconducting resonators has been achieved by connecting them with a nonlinear coupler such as a capacitor or Josephson-junction circuits of the SQUID loop [67–69]. The coupling strength can be dynamically tuned by adjusting the capacitance or inductance of the coupler [70,71]. Furthermore, we also apply a pair of σ^z drivings to each flux qubit, which is described by the Hamiltonian

$$H_d = \sum_{\lambda=1}^{2}\left[\xi_1^\lambda\omega_{d1}^\lambda \cos\left(\omega_{d1}^\lambda t\right)\sigma_\lambda^z + \xi_2^\lambda\omega_{d2}^\lambda \cos\left(\omega_{d2}^\lambda t + \theta^\lambda\right)\sigma_\lambda^z\right], \tag{4}$$

where ω_{dl}^λ ($l = 1, 2$) is the driving frequency, ξ_l^λ ($l = 1, 2$) is the ratio between the amplitude and frequency, and θ^λ is the phase difference between the bichromatic fields. This Hamiltonian can be realized by penetrating the SQUID loop of flux qubits with time-dependent magnetic fields [72–74], which is crucial for mediating the suitable interactions between flux qubits and superconducting resonators.

III. GENERATION OF TWO-MODE ENTANGLED STATES

In this section we will show how to create the entangled state of two superconducting resonators in relation to inhomogeneous parameters. By taking into account the coupling of the whole system with a harmonic-oscillator environment in the Markovian approximation, the time evolution ρ of the system is governed by the master equation

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_{\lambda=1}^{2}\Gamma_\lambda D[\sigma_\lambda^-]\rho, \tag{5}$$

where $H = H_0 + H_{qr} + H_{rr} + H_d$ is the total Hamiltonian of whole system, $D[o]\rho = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o$ is the standard Lindblad operator, and Γ_λ represents the energy relaxation rate of the λth qubit. Here the photon leakage of resonators is neglected, which will be investigated in the following numerical simulation.

To detail the process of quantum state preparation, we introduce two normal boson modes $A_1 = (a_1 + a_2)/\sqrt{2}$ and $A_2 = (a_1 - a_2)/\sqrt{2}$. In terms of A_1 and A_2, the Hamiltonian in Eq. (5) can be rewritten as

$$H = H_d + \sum_{\lambda=1}^{2}\left(\frac{\delta_\lambda}{2}\sigma_\lambda^z + \Omega_\lambda A_\lambda^\dagger A_\lambda\right) + \left[\frac{\Delta}{2}A_1^\dagger A_2 + \frac{g_1\sigma_1^+}{\sqrt{2}}(A_1 + A_1^\dagger + A_2 + A_2^\dagger) + \frac{g_2\sigma_2^+}{\sqrt{2}}(A_1 + A_1^\dagger - A_2 - A_2^\dagger) + \text{H.c.}\right], \tag{6}$$

with $\Omega_1 = (\omega_1 + \omega_2)/2 + J$, $\Omega_2 = (\omega_1 + \omega_2)/2 - J$, and $\Delta = (\omega_1 - \omega_2)$ the resonance frequency difference between two resonators. We can see that the new boson modes A_1 and A_2 are spectrally well resolved with large enough coupling strength J.

We now proceed to perform the unitary transformation $U_1(t) = e^{-iH_0' t}$, with $H_0' = \sum_{\lambda=1}^{2}(\frac{\delta_\lambda}{2}\sigma_\lambda^z + \Omega_\lambda A_\lambda^\dagger A_\lambda)$, to the above equation. Then the Hamiltonian will become

$$H_I = H_d + \left[\frac{\Delta}{2}A_1^\dagger A_2 e^{i2Jt} + \frac{g_1}{\sqrt{2}}\sigma_1^+ e^{i\delta_1 t}(A_1^\dagger e^{i\Omega_1 t} + A_1 e^{-i\Omega_1 t} + A_2^\dagger e^{i\Omega_2 t} + A_2 e^{-i\Omega_2 t}) + \frac{g_2}{\sqrt{2}}\sigma_2^+ e^{i\delta_2 t}(A_1^\dagger e^{i\Omega_1 t} + A_1 e^{-i\Omega_1 t} - A_2^\dagger e^{i\Omega_2 t} - A_2 e^{-i\Omega_2 t}) + \text{H.c.}\right]. \tag{7}$$

To get the desired couplings between qubits and resonators, we perform another unitary transformation $U_2(t) = T\exp[-i\int_0^t H_d(t')dt']$, where T denotes the time-ordering operator. Assuming that the parameters ξ_l^λ $(\lambda, l = 1, 2)$ are small enough, they can be kept only up to the first order. Then we will have

$$H_I = \frac{\Delta}{2}A_1^\dagger A_2 e^{i2Jt} + \frac{g_1}{\sqrt{2}}\sigma_1^+ e^{i\delta_1 t}\left[1 - \xi_1^1\left(e^{i\omega_{d1}^1 t} - e^{-i\omega_{d1}^1 t}\right) - \xi_2^1\left(e^{i(\omega_{d2}^1 t+\theta^1)} - e^{-i(\omega_{d2}^1 t+\theta^1)}\right)\right]\left(A_1^\dagger e^{i\Omega_1 t} + A_1 e^{-i\Omega_1 t} + A_2^\dagger e^{i\Omega_2 t} + A_2 e^{-i\Omega_2 t}\right) + \frac{g_2}{\sqrt{2}}\sigma_2^+ e^{i\delta_2 t}\left[1 - \xi_1^2\left(e^{i\omega_{d1}^2 t} - e^{-i\omega_{d1}^2 t}\right) - \xi_2^2\left(e^{i(\omega_{d2}^2 t+\theta^2)} - e^{-i(\omega_{d2}^2 t+\theta^2)}\right)\right](A_1^\dagger e^{i\Omega_1 t} + A_1 e^{-i\Omega_1 t} - A_2^\dagger e^{i\Omega_2 t} - A_2 e^{-i\Omega_2 t}) + \text{H.c.} \tag{8}$$

If the parameters are chosen to be $\theta^1 = 0$ and $\theta^2 = \pi$, as well as $\omega_{d1}^\lambda = \delta_\lambda + \Omega_\lambda$ and $\omega_{d2}^\lambda = \delta_\lambda - \Omega_\lambda$ $(\lambda = 1, 2)$, we can apply the rotating-wave approximation to neglect those fast oscillating terms under the conditions $\{\delta_\lambda, \omega_{dl}^\lambda, \Omega_\lambda\} \gg g_\lambda$ $(\lambda, l = 1, 2)$ and $2J \gg \Delta$. Thus, we will obtain the effective Hamiltonian

$$H_{\text{eff}} = \sigma_1^+\Theta_1(A_1 + \varepsilon A_1^\dagger) + \sigma_2^\dagger\Theta_2(A_2 - \varepsilon A_2^\dagger) + \text{H.c.},$$

where we have set $\Theta_1 = g_1\xi_2^1/\sqrt{2}$, $\Theta_2 = g_2\xi_2^2/\sqrt{2}$, and $\varepsilon = \xi_1^1/\xi_2^1 = \xi_1^2/\xi_2^2$.

To make the two-mode squeezing generation mechanism clear, we apply the transformations $S = S_1 \otimes S_2$ to the master equation, in which $S_1 = \exp\{\frac{r}{2}[A_1^2 - (A_1^\dagger)^2]\}$ and $S_2 = \exp\{-\frac{r}{2}[A_2^2 - (A_2^\dagger)^2]\}$, where $r = \tanh^{-1}(\varepsilon)$ is single-mode squeezing degree. Under the squeezing representation, the master equation will take the form

$$\frac{d\tilde{\rho}}{dt} = -i[\tilde{H}_{\text{eff}}, \tilde{\rho}] + \sum_{\lambda=1}^{2}\Gamma_\lambda D[\sigma_\lambda^-]\tilde{\rho}, \tag{9}$$

with

$$\tilde{H}_{\text{eff}} = \sqrt{1-\varepsilon^2}\sum_{\lambda=1}^{2}\Theta_\lambda\sigma_\lambda^+ A_\lambda + \text{H.c.}, \tag{10}$$

where $\tilde{\rho} = S_2^\dagger(r)S_1^\dagger(r)\rho S_1(r)S_2(r)$. We can see that the Hamiltonian $\tilde{H}_{\text{eff}}$ describes a pair of Jaynes-Cummings-type interactions, thus it conserves the total energy excitations of the system. However, due to the participation of energy relaxation of qubits, they will continuously extract photons from the modes A_1 and A_2 and then decay back to the ground states. According to this dissipative dynamical process, the whole system will eventually be cooled down to the ground state at the stationary state, i.e., $|0,0\rangle_{1,2}\otimes|g,g\rangle_{1,2}$. Reversing the squeezing transformation S, we will obtain the unique steady state of the system

$$|\Psi_S\rangle = \exp\left[\frac{r}{2}\left(A_1^2 - A_1^{\dagger 2} - A_2^2 + A_2^{\dagger 2}\right)\right]|0,0\rangle_{1,2}\otimes|g,g\rangle_{1,2}. \tag{11}$$

If we replace the modes A_1 and A_2 with $a_1 = (A_1 + A_2)/\sqrt{2}$ and $a_2 = (A_1 - A_2)/\sqrt{2}$ in Eq. (11), the steady state of the system will yield

$$|\Psi_S\rangle = \exp[r(a_1a_2 - a_1^\dagger a_2^\dagger)]|0,0\rangle_{1,2}\otimes|g,g\rangle_{1,2}. \tag{12}$$

We see that the two superconducting resonators are prepared in the two-mode squeezed vacuum state, i.e., it is a quantum interference effect. Based on a dissipative steady-state production process, there is no need for exact control of the time evolution and specific initialization of the system [75–78].

Moreover, we note that the steady state of the master equation (10) is unique, even if the parameters Θ_1 and Θ_2 are different from each other. Hence, the inhomogeneous qubit-resonator couplings have no effect on the stationary state. Additionally, our scheme allows a certain range of fluctuation of the resonance frequency difference Δ between the resonators. Provided $\Delta \ll 2J$ is satisfied, the term $\frac{\Delta}{2}(A_1^\dagger A_2 e^{i2Jt} + A_1A_2^\dagger e^{-i2Jt})$ in Eq. (8) appears to be trivial, which can be safely omitted via the rotating-wave approximation. In practice, it is hardly possible to fabricate a resonator with equal resonance frequencies. Although the eigenfrequency of the superconducting resonator inserted with SQUID loops can be easily tuned within a few nanoseconds in experiment [79,80], the coherence time will be considerably reduced. Therefore, the present scheme is very resilient to the inhomogeneous parameters, i.e., it eliminates the need for equal couplings between qubits and resonators, and the effect of the frequency difference Δ can be sufficiently suppressed. In addition, because the steady state of qubits is the ground state, our protocol is also immune to the dephasing effect of them. These remarkable features greatly loosen the requirement for experimental implementation and make our proposal quite feasible in a realistic experiment. The engineered continuous-variable entangled state has important applications for implementing various quantum information protocols such as quantum key distribution, entanglement swapping, error correction, and quantum computing [81–84].

To check the validity of our result, we numerically solve the master equation by including the photon damping of

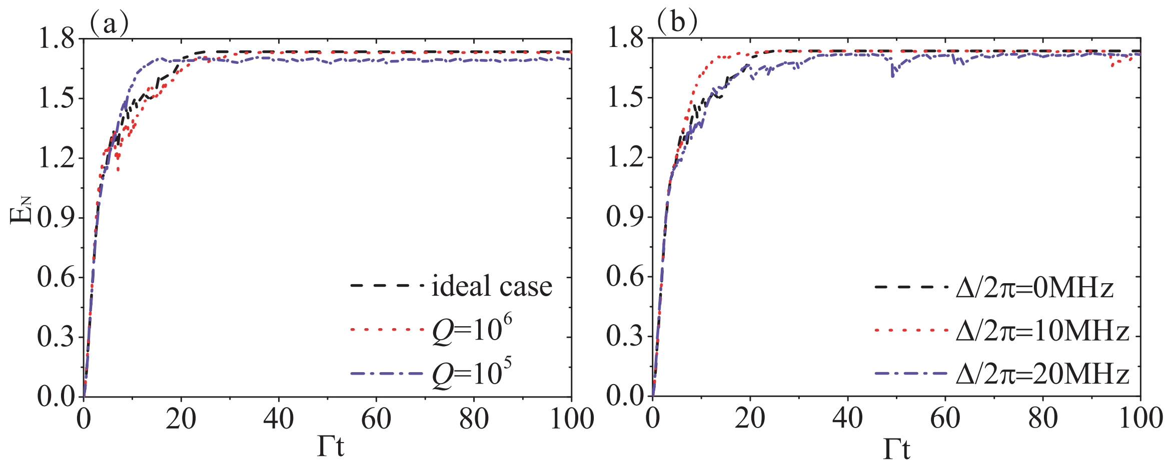

FIG. 2. Logarithmic negativity E_N vs the dimensionless time Γt by numerically solving the master equation. The parameters are $\delta_1/2\pi = 10$ GHz, $\delta_2/2\pi = 9.5$ GHz, $\omega_1/2\pi = 6$ GHz, $\omega_2/2\pi = \omega_1/2\pi - \Delta$, $J/2\pi = 0.3$ GHz, $g_1/2\pi = 50$ MHz, $g_2/2\pi = 60$ MHz, $\Gamma/2\pi = \Gamma_1/2\pi = \Gamma_2/2\pi = 10$ MHz, $\xi_1^1 = \xi_1^2 = 0.14$, and $\xi_2^1 = \xi_2^2 = 0.2$ for (a) different quality factors Q and (b) different Δ.

resonators [85]

$$\frac{d\rho}{dt} = -i[H_{\text{eff}}, \rho] + \sum_{\lambda=1}^{2} \Gamma_\lambda D[\sigma_\lambda^-]\rho + \sum_{\lambda=1}^{2} \kappa_\lambda D[a_\lambda]\rho, \quad (13)$$

where $\kappa_\lambda = \frac{\omega_\lambda}{Q_\lambda}$ is the photon leakage rate, with Q_λ the quality factor of the resonator. Here we exploit the logarithmic negativity to quantify the entanglement between two superconducting resonators [86]. The logarithmic negativity can be computed from the reduced 4×4 covariance matrix V for the resonator modes a_1 and a_2, defined as $V_{jk} = \frac{1}{2}\langle \Delta\zeta_j \Delta\zeta_k + \Delta\zeta_k \Delta\zeta_j \rangle$, where $\Delta\zeta_j = \zeta_j - \langle \zeta_j \rangle$ and $\vec{\zeta} = \{x_1, p_1, x_2, p_2\}$, with $x_\lambda = (a_\lambda + a_\lambda^\dagger)/\sqrt{2}$ and $p_\lambda = -i(a_\lambda - a_\lambda^\dagger)/\sqrt{2}$ ($\lambda = 1, 2$). If we write V in the form

$$V = \begin{pmatrix} V_1 & V_c \\ V_c^T & V_2 \end{pmatrix}, \quad (14)$$

in which V_1, V_2, and V_c are 2×2 subblock matrices of V, then the logarithmic negativity E_N is defined as

$$E_N = \max[0, -\ln(2\eta)], \quad (15)$$

with

$$\eta = \frac{1}{\sqrt{2}}[\Sigma - (\Sigma^2 - 4 \det V)^{1/2}]^{1/2}, \quad (16)$$

$$\Sigma = \det V_1 + \det V_2 - 2 \det V_c. \quad (17)$$

In our simulation, the system is initially set in the ground state $|0, 0\rangle_{1,2} \otimes |g, g\rangle_{1,2}$ and the parameters are given in the caption of Fig. 2. A plot of E_N versus the dimensionless time Γt is shown in Fig. 2. In the ideal case of $Q = Q_1 = Q_2 = \infty$, we can observe that, as time proceeds, the logarithmic negativity eventually converges to a constant value $E_N = 1.73$, which means that the steady state of two resonators is the two-mode entangled state. Under the situation of $Q = 10^6$ and 10^5, it can be seen that E_N has only a small divergence from the ideal case. Actually, the superconducting resonator with a quality factor $\sim 10^6$ is the common value in the present experiment. The transmission line resonator with $Q > 10^7$ has been fabricated [22] and a quality factor beyond 10^8 has been demonstrated in the three-dimensional resonator [23,24]. Additionally, we also add the term $\frac{\Delta}{2}(A_1^\dagger A_2 e^{i2Jt} + A_1 A_2^\dagger e^{-i2Jt})$ to the master equation (13). For $\Delta/2\pi = 10$ and 20 MHz, there is no obvious variation of the logarithmic negativity E_N as seen in Fig. 2(b). In the circuit QED experiment, the heterodyne state tomography techniques can be explored to measure the continuous-variable entanglement. It can detect all the elements of the quadrature covariance matrix via a two-channel heterodyne setup using amplitude detectors [87]. Thus, the full covariance matrix can be reconstructed to quantify the entanglement between the two superconducting resonators. Furthermore, we also calculate the purity of the entangled state, which is defined as

$$\mu = \frac{1}{4\sqrt{\det V}}. \quad (18)$$

The continuous-variable entangled state with high purity is required for practical applications in the field of quantum information [88]. The high purity typically corresponds to high fidelity. As plotted in Figs. 3(a) and 3(b), we also investigate the time evolution of purity μ with respect to the photon decay and frequency difference. In the ideal case, the stable value of purity $\mu = 1$ is attained in the stationary state, implying that the perfect two-mode squeezed vacuum state of the microwave fields in two superconducting resonators has been established. For the situation of $Q = 10^6$ and $\Delta/2\pi = 10$ MHz, it is clearly shown that the imperfect factors have negligible disturbance on the purity μ. Therefore, our scheme can generate a strongly entangled and highly pure microwave field in superconducting quantum circuits.

IV. CONCLUSION

We have studied a system of two directly interacting superconducting transmission line resonators, each coupled to a superconducting flux qubit. We proposed a method for dissipative generation of stable entangled fields of the two superconducting resonators. By means of the appropriately designed interaction between qubits and resonators, we showed that the energy relaxation of qubits plays a positive role and assists the two resonators in evolving into an entangled state at steady state. The remarkable feature of our scheme is that

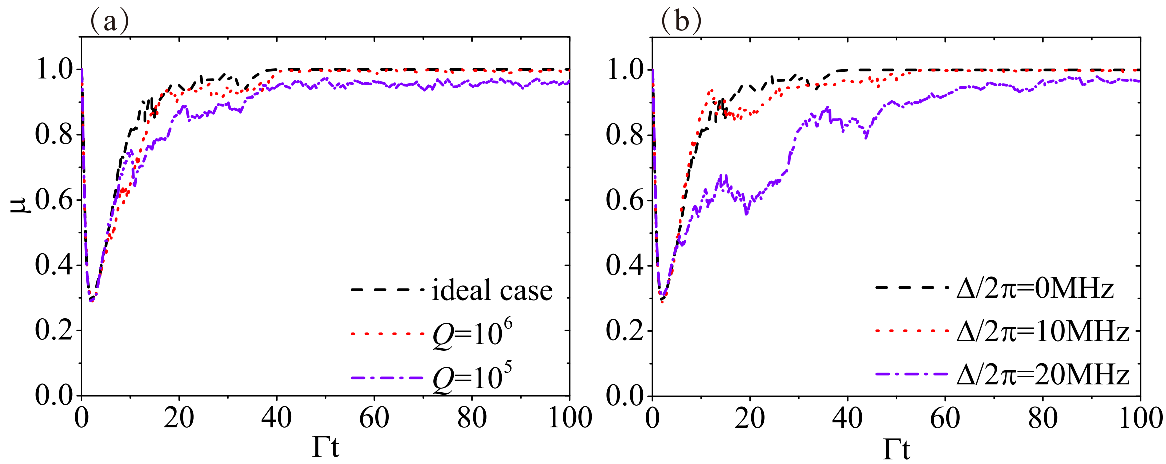

FIG. 3. Time evolution of purity μ by numerically solving the master equation for (a) different quality factors Q and (b) different Δ. The parameters are the same as in Fig. 2.

the quantum state engineering is robust against the inhomogeneous parameters, i.e., it does not require identical qubit-resonator couplings, and allows a certain range of central frequency difference between the resonators. By considering the great progress in circuit quantum electrodynamics, the present scheme can be experimentally implemented with currently available technology.

ACKNOWLEDGMENTS

This work was partly supported by the National Nature Science Foundation of China (Grants No. 11704306 and No. 11534008), the National Key R&D Project (Grant No. 2016YFA0301404), and the China Postdoctoral Science Foundation (Grant No. 2016M602795).

[1] A. Wallraff, D. I. Schuster, A. Blais, J. M. Gambetta, J. Schreier, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **75**, 032329 (2007).
[2] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008).
[3] J. Q. You and F. Nori, Phys. Today **58**(11), 42 (2005).
[4] P. Forn-Díaz, J. J. García-Ripoli, B. Peropadre, M. A. Yurtalan, J.-L. Orgiazzi, R. Belyansky, C. M. Wilson, and A. Lupascu, Nat. Phys. **13**, 39 (2017).
[5] T. Roy, S. Kundu, M. Chand, S. Hazra, N. Nehra, R. Cosmic, A. Ranadive, M. P. Patankar, K. Damle, and R. Vijay, Phys. Rev. Appl. **7**, 054025 (2017).
[6] A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J. M. Chow, and J. M. Gambetta, Nature (London) **549**, 242 (2017).
[7] J. Kelly *et al.*, Nature (London) **519**, 66 (2014).
[8] N. K. Langford, R. Sagastizabal, M. Kounalakis, C. Dickel, A. Bruno, F. Luthi, D. J. Thoen, A. Endo, and L. DiCarlo, Nat. Commun. **8**, 1715 (2017).
[9] X. Tan, D.-W. Zhang, Q. Liu, G. Xue, H.-F. Yu, Y.-Q. Zhu, H. Yan, S.-L. Zhu, and Y. Yu, Phys. Rev. Lett. **120**, 130503 (2018).
[10] M. Reagor *et al.*, Sci. Adv. **4**, 3603 (2018).
[11] C. Song *et al.*, Phys. Rev. Lett. **119**, 180511 (2017).
[12] H. Paik *et al.*, Phys. Rev. Lett. **117**, 250502 (2016).
[13] Y. Xu *et al.*, Phys. Rev. Lett. **121**, 110501 (2018).
[14] R. Versluis, S. Poletto, N. Khammassi, B. Tarasinski, N. Haider, D. J. Michalak, A. Bruno, K. Bertels, and L. DiCarlo, Phys. Rev. Appl. **8**, 034021 (2017).
[15] F. Yan *et al.*, Nat. Commun. **7**, 12964 (2016).
[16] Y. Makhlin, G. Schon, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[17] Y.-H. Lin, L. B. Nguyen, N. Grabon, J. San Miguel, N. Pankratova, and V. E. Manucharyan, Phys. Rev. Lett. **120**, 150503 (2018).
[18] Y. A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. V. Averin, and J. S. Tsai, Nature (London) **421**, 823 (2003).
[19] P. Kumar, S. Sendelbach, M. A. Beck, J. W. Freeland, Z. Wang, H. Wang, C. C. Yu, R. W. Wu, D. P. Pappas, and R. McDermott, Phys. Rev. Appl. **6**, 041001 (2016).
[20] F. Yoshihara, T. Fuse, S. Ashhab, K. Kakuyanagi, S. Saito, and K. Semba, Nat. Phys. **13**, 44 (2017).
[21] P. V. Klimov *et al.*, Phys. Rev. Lett. **121**, 090502 (2018).
[22] A. Megrant *et al.*, Appl. Phys. Lett. **100**, 113510 (2012).
[23] M. Reagor *et al.*, Phys. Rev. B **94**, 014506 (2016).
[24] A. Romanenko, P. Pilipenko, S. Zorzetti, D. Frolov, M. Awida, S. Posen, and A. Grassellino, arXiv:1810.03703.
[25] M. Sillanpää, J. I. Park, and R. W. Simmonds, Nature (London) **449**, 438 (2007).
[26] S. E. Nigg, Phys. Rev. A **89**, 022340 (2014).
[27] R. W. Heeres, P. Reinhold, N. Ofek, L. Frunzio, L. Jiang, M. H. Devoret, and R. J. Schoelkopf, Nat. Commun. **8**, 94 (2017).
[28] H. Goto, Phys. Rev. A **93**, 050301(R) (2016).
[29] N. Bartolo, F. Minganti, W. Casteels, and C. Ciuti, Phys. Rev. A **94**, 033841 (2016).
[30] N. Ofek *et al.*, Nature (London) **536**, 441 (2016).
[31] X. Gu, A. F. Kockum, A. Miranowicz, Y.-x. Liu, and F. Nori, Phys. Rep. **718**, 1 (2017).
[32] M. Hofheinz, E. M. Weig, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, H. Wang, J. M. Martinis, and A. N. Cleland, Nature (London) **454**, 310 (2008).

[33] B. Vlastakis, G. Kirchmair, Z. Leghtas, S. E. Nigg, L. Frunzio, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Science **342**, 607 (2013).
[34] M. Hofheinz *et al.*, Nature (London) **459**, 546 (2009).
[35] E. Z. Bajjani, Nat. Phys. **7**, 599 (2011).
[36] M. Mariantoni, F. Deppe, A. Marx, R. Gross, F. K. Wilhelm, and E. Solano, Phys. Rev. B **78**, 104508 (2008).
[37] M. Hua, M.-J. Tao, and F.-G. Deng, Phys. Rev. A **90**, 012328 (2014).
[38] P.-B. Li, S.-Y. Gao, and F.-L. Li, Phys. Rev. A **86**, 012318 (2012).
[39] F. W. Strauch, K. Jacobs, and R. W. Simmonds, Phys. Rev. Lett. **105**, 050501 (2010).
[40] E. A. Sete, E. Mlinar, and A. N. Korotkov, Phys. Rev. B **91**, 144509 (2015).
[41] S.-L. Ma, Z. Li, A.-P. Fang, P.-B. Li, S.-Y. Gao, and F.-L. Li, Phys. Rev. A **90**, 062342 (2014).
[42] C.-P. Yang, Q.-P. Su, S.-B. Zheng, F. Nori, and S. Han, Phys. Rev. A **95**, 052341 (2017).
[43] M. D. Kim and J. Kim, Phys. Rev. A **93**, 012321 (2016).
[44] F. Wang, W. Nie, X. Feng, and C. H. Oh, Phys. Rev. A **94**, 012330 (2016).
[45] H. Wang *et al.*, Phys. Rev. Lett. **106**, 060401 (2011).
[46] C. Wang *et al.*, Science **352**, 1087 (2016).
[47] S. Rosenblum *et al.*, Nat. Commun. **9**, 652 (2018)
[48] M. Mariantoni *et al.*, Nat. Phys. **7**, 287 (2011).
[49] S. J. Srinivasan, N. M. Sundaresan, D. Sadri, Y. Liu, J. M. Gambetta, T. Yu, S. M. Girvin, and A. A. Houck, Phys. Rev. A **89**, 033857 (2014).
[50] C. Axline *et al.*, Nat. Phys. **14**, 705 (2018).
[51] C.-P. Yang, Q.-P. Su, S.-B. Zheng, and F. Nori, Phys. Rev. A **93**, 042307 (2016).
[52] A. Baust *et al.*, Phys. Rev. B **91**, 014515 (2015).
[53] J. Khatibi Moqadam, M. C. de Oliveira, and R. Portugal, Phys. Rev. B **95**, 144506 (2017).
[54] J.-Q. Liao, Z. R. Gong, L. Zhou, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **81**, 042304 (2010).
[55] A. A. Houck, H. E. Tüeci, and J. Koch, Nat. Phys. **8**, 292 (2012).
[56] M. Leib, F. Deppe, A. Marx, R. Gross, and M. Hartmann, New J. Phys. **14**, 075024 (2012).
[57] Y. Hu and L. Tian, Phys. Rev. Lett. **106**, 257002 (2011).
[58] D. Z. Rossatto, S. Felicetti, H. Eneriz, E. Rico, M. Sanz, and E. Solano, Phys. Rev. B **93**, 094514 (2016).
[59] B. Peropadre, D. Zueco, F. Wulschner, F. Deppe, A. Marx, R. Gross, and J. J. García-Ripoll, Phys. Rev. B **87**, 134504 (2013).
[60] S.-l. Ma, X.-k. Li, J.-k. Xie, and F.-l. Li, Phys. Rev. A **99**, 012325 (2019).
[61] T. Hime, P. A. Reichardt, B. L. T. Plourde, T. L. Robertson, C.-E. Wu, A. V. Ustinov, and J. Clarke, Science **314**, 1427 (2006).
[62] S. H. W. van der Ploeg, A. Izmalkov, A. M. van den Brink, U. Hübner, M. Grajcar, E. Il'ichev, H.-G. Meyer, and A. M. Zagoskin, Phys. Rev. Lett. **98**, 057004 (2007).
[63] M. Stern, G. Catelani, Y. Kubo, C. Grezes, A. Bienfait, D. Vion, D. Esteve, and P. Bertet, Phys. Rev. Lett. **113**, 123601 (2014).
[64] X. Zhu, Appl. Phys. Lett. **97**, 102503 (2010).
[65] A. Wallraff, D. I. Schuster, A. Blais, J. M. Gambetta, J. Schreier, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. **99**, 050501 (2007).
[66] M. D. Hutchings, J. B. Hertzberg, Y. Liu, N. T. Bronn, G. A. Keefe, M. Brink, J. M. Chow, and B. L. T. Plourde, Phys. Rev. Appl. **8**, 044003 (2017).
[67] M. Fitzpatrick, N. M. Sundaresan, A. C. Y. Li, J. Koch, and A. A. Houck, Phys. Rev. X **7**, 011016 (2017).
[68] M. C. Collodo, A. Potočnik, S. Gasparinetti, J.-C. Besse, M. Pechal, M. Sameti, M. J. Hrtmann, A. Wallraff, and C. Eichler, arXiv:1808.00889v1.
[69] M. E. Kimchi-Schwartz, L. Martin, E. Flurin, C. Aron, M. Kulkarni, H. E. Tureci, and I. Siddiqi, Phys. Rev. Lett. **116**, 240503 (2016).
[70] D. L. Underwood, W. E. Shanks, J. Koch, and A. A. Houck, Phys. Rev. A **86**, 023837 (2012).
[71] F. Wulschner *et al.*, EPJ Quantum Technol. **3**, 10 (2015).
[72] A. Fedorov, A. K. Feofanov, P. Macha, P. Forn-Díaz, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. Lett. **105**, 060503 (2010).
[73] Z. Chen *et al.*, Phys. Rev. A **96**, 012325 (2017).
[74] D. Porras and J. J. García-Ripoll, Phys. Rev. Lett. **108**, 043602 (2012).
[75] S. Shankar, M. Hatridge, Z. Leghtas, K. M. Sliwa, A. Narla, U. Vool, S. M. Girvin, L. Frunzio, M. Mirrahimi, and M. H. Devoret, Nature (London) **504**, 419 (2013).
[76] Y. Lu, S. Chakram, N. Leung, N. Earnest, R. K. Naik, Z. Huang, P. Groszkowski, E. Kapit, J. Koch, and D. I. Schuster, Phys. Rev. Lett. **119**, 150502 (2017).
[77] X.-P. Zhang, L.-T. Shen, Z.-Q. Yin, H.-Z. Wu, and Z.-B. Yang, Phys. Rev. A **91**, 013825 (2015).
[78] F. Reiter, L. Tornberg, G. Johansson, and A. S. Sørensen, Phys. Rev. A **88**, 032317 (2013).
[79] M. Sandberg, C. M. Wilson, F. Persson, T. Bauch, G. Johansson, V. Shumeiko, T. Duty, and P. Delsing, Appl. Phys. Lett. **92**, 203501 (2008).
[80] Z. L. Wang, Y. P. Zhong, L. J. He, H. Wang, J. M. Martinis, A. N. Cleland, and Q. W. Xie, Appl. Phys. Lett. **102**, 163503 (2013).
[81] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
[82] E. Flurin, N. Roch, J.-D. Pillet, F. Mallet, and B. Huard, Phys. Rev. Lett. **114**, 090503 (2015).
[83] P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, Nat. Commun. **7**, 12548 (2016).
[84] C. W. Sandbo Chang, M. Simoen, J. Aumentado, C. Sabín, P. Forn-Díaz, A. M. Vadiraj, F. Quijandría, G. Johansson, I. Fuentes, and C. M. Wilson, Phys. Rev. Appl. **10**, 044019 (2018).
[85] J. R. Johansson, P. D. Nation, and F. Nori, Comput. Phys. Commun. **183**, 1760 (2012).
[86] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002).
[87] C. Eichler, D. Bozyigit, C. Lang, M. Baur, L. Steffen, J. M. Fink, S. Filipp, and A. Wallraff, Phys. Rev. Lett. **107**, 113601 (2011).
[88] M. Wang, X.-Y. Lü, Y.-D. Wang, J. Q. You, and Y. Wu, Phys. Rev. A **94**, 053807 (2016).