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QED and relativistic nuclear recoil corrections to the 413-nm tune-out wavelength for the $2\,^3S_1$ state of helium

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Comparison of high-accuracy calculations with precision measurement of the 413-nm tune-out wavelength of the He($2\,^3S_1$) state provides a unique test of quantum electrodynamics (QED). We perform large-scale relativistic-configuration-interaction (RCI) calculations of the tune-out wavelength that include the mass-shift operator and fully account for leading relativistic nuclear recoil terms in the Dirac-Coulomb-Breit (DCB) Hamiltonian. We obtain the QED correction to the tune-out wavelength using perturbation theory, and the effect of finite nuclear size is also evaluated. The resulting tune-out wavelengths for the $2\,^3S_1(M_J = 0)$ and $2\,^3S_1(M_J = \pm1)$ states are 413.084 26(4) nm and 413.090 15(4) nm, respectively. When we incorporate the retardation correction of 0.000 560 0236 nm obtained by Drake *et al.* [Hyperfine Interact **240**, 31 (2019)] to compare results with the only current experimental value of 413.0938($9_{\rm stat}$)($20_{\rm syst}$) nm for the $2\,^3S_1(M_J = \pm1)$ state, there is 1.4σ discrepancy between theory and experiment, which stimulates further theoretical and higher precision experimental investigations on the 413-nm tune-out wavelength. In addition, we also determine the QED correction for the static dipole polarizability of the He($2\,^3S_1$) state to be 22.5 ppm, which may enable a new test of QED in the future.

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Bound-state quantum electrodynamics (QED) is one of the most successful theories in modern physics, having been tested through precision measurement over a diverse spectrum of experimental realisations. For example, measurement of the bound-state g factor in the hydrogen-like $^{28}Si^{13+}$ and $^{12}C^{5+}$ at the sub-part-per-billion (sub-ppb) level [1–3] has provided one of the strictest QED tests.

In order to test QED theory in many-electron systems, calculations and measurements for helium, the simplest multielectron atom, are of great importance. Measurements of the fine-structure splitting in the $2\,^3P$ manifold have yielded a test of QED predictions with a precision at the sub-ppb (10^{-9}) level [4–7]. The Lamb shift of the $2\,^1S_0$ and $2\,^3S_1$ states has been determined, respectively, using the $2\,^1S_0 \rightarrow 3\,^1D_2$ [8] and $2\,^3S_1 \rightarrow 2\,^3D_1$ two-photon transitions [9]. However, four standard deviations in the discrepancy between measurements for the helium nuclear charge radius, which are determined by two different methods (the $2\,^3S \rightarrow 2\,^3P$ [10–12] and $2\,^3S \rightarrow 2\,^1S$ [13,14] transition frequencies combined with calculations of the QED and recoil corrections [15–17]), pose significant challenges to QED theory.

QED tests that do not rely on energy-level determinations can potentially provide important independent verification, such as the experimental and theoretical determination of transition rates, but these are both inherently difficult and of much lower precision [18–20]. Therefore, further experiments probing other nonenergy properties of helium are important to deliver an independent validation of QED, provided that the corresponding progress in theory can be achieved.

QED contributions play an important role in the atomic polarizability of helium. The most accurate theoretical calculation of the ground-state static dipole polarizability of helium has now reached an accuracy of 0.2 ppm [21], which provides a nonenergy QED test when compared with high-precision experimental measurements [22,23]. It is difficult to further improve this experimental accuracy, since a measurement of polarizability depends on precisely measuring the electric field strength.

However, the same QED effects are also reflected in the dynamic polarizability [24,25]. The 413-nm tune-out wavelength for the He($2\,^3S_1$) state, where the dynamic polarizability equals to zero, provides a further nonenergy scheme to test QED [26]. Since the position of the tune-out wavelength does not depend on the details of the laser power or beam profile, a measurement of the tune-out wavelength can potentially achieve higher sensitivity to test QED calculations than a measurement of the static dipole polarizability.

This application of the 413-nm tune-out wavelength of metastable helium to test QED theory has sparked great interest in high-precision measurement and high-accuracy calculations [27–32]. The first hybrid calculations were carried out by

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Mitroy and Tang [26]. In 2015, Henson *et al.* [27] performed the first experimental measurement utilizing a highly sensitive technique and reported a value of 413.0938(9stat)(20syst) nm ($\approx$5 ppm accuracy) for the $2\,^3S_1(M_J=1)$ state of ^{4}He, two orders of magnitude more precise than the value of 413.02(9) nm first predicted in Ref. [26]. Recently, Zhang *et al.* performed an *ab initio* calculation of the tune-out wavelength by extending nonrelativistic and relativistic configuration interaction (NRCI and RCI) methods [28,29]. The RCI value of 413.085 9(4) nm, which includes the finite nuclear mass and relativistic corrections, reduced the discrepancy between the theoretical value and measurement result from 134 to 19 ppm. The remaining 19-ppm discrepancy was mainly due to neglected QED corrections, which provides motivation for the more detailed QED and higher order nuclear recoil investigations in the present work.

In this paper, we improve on previous B-spline RCI method by self-consistently taking into account the nuclear recoil correction in the Dirac-Coulomb-Breit (DCB) framework and perform the QED correction with perturbation calculation. We obtain the individual contributions of the nuclear recoil effect, QED, and finite nuclear-size corrections to the 413-nm tune-out wavelength and static dipole polarizability of the ^{4}He$(2\,^3S_1)$ state. The uncertainty in the static dipole polarizability has achieved an accuracy of 0.1 ppm. The present values of the tune-out wavelength will set a benchmark for future measurements to seriously test QED calculations at a higher level of accuracy.

It is convenient to efficiently calculate dynamic polarizabilities at off-resonance frequencies using a power series expansion, such as employed in determining the ground-state polarizability at the He-Ne laser wavelength of helium [24,33]. However, since the 413-nm tune-out wavelength is located near the $2\,^3S_1 \to 3\,^3P_J$ resonance line, the power series expansion cannot be used. In this work, we employ the sum-over-states method [26,29] to obtain dynamic dipole polarizablities, then extract the tune-out wavelength from making $\alpha_1(\omega)=0$. Under linear polarized light with laser frequency ω, the dynamic dipole polarizability for a state with angular momentum J and magnetic quantum number M_J is

$$\alpha_1(\omega) = \alpha_1^S(\omega) + \frac{3M_J^2 - J(J+1)}{J(2J-1)}\alpha_1^T(\omega)\,, \tag{1}$$

where $\alpha_1^S(\omega)$ and $\alpha_1^T(\omega)$ are, respectively, the scalar and tensor dipole polarizabilities [29].

In order to take account of the nuclear recoil correction, the mass shift (MS) operator $H_{\rm MS}$, which explicitly includes the nonrelativistic and leading relativistic components, $H_{\rm NRMS}$ and $H_{\rm RMS}$ [34], respectively, has been added directly into the DCB Hamiltonian,

$$H = H_{\rm DCB} + H_{\rm MS} = H_{\rm DCB} + H_{\rm NRMS} + H_{\rm RMS}, \tag{2}$$

$$H_{\rm DCB} = \sum_{i=1}^{2}\left[c\boldsymbol{\alpha}_i\cdot\boldsymbol{p}_i + \beta m_e c^2 - \frac{Z}{r_i}\right] + \frac{1}{r_{12}} - \frac{1}{2r_{12}}[\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2 + (\boldsymbol{\alpha}_1\cdot\hat{\boldsymbol{r}}_{12})(\boldsymbol{\alpha}_2\cdot\hat{\boldsymbol{r}}_{12})], \tag{3}$$

$$H_{\rm NRMS} = \frac{1}{2m_0}\sum_{i,j}^{2}\boldsymbol{p}_i\cdot\boldsymbol{p}_j, \tag{4}$$

$$H_{\rm RMS} = -\frac{1}{2m_0}\sum_{i,j}^{2}\frac{\alpha Z}{r_i}\left[\boldsymbol{\alpha}_i + \frac{(\boldsymbol{\alpha}_i\cdot\boldsymbol{r}_i)\boldsymbol{r}_i}{r_i^2}\right]\cdot\boldsymbol{p}_j, \tag{5}$$

where c is the speed of light, Z is the nuclear charge, m_e is the mass of the electron, $\boldsymbol{\alpha}_i$ and β are the 4×4 Dirac matrices, $\boldsymbol{p}_i$ is the momentum operator, r_i represents the distance of the ith electron from the nucleus, $\hat{\boldsymbol{r}}_{12}$ is the unit vector of the electron-electron distance $\boldsymbol{r}_{12}$, α is the fine structure constant, and $m_0 = 7294.2995361\, m_e$ [35] is the nuclear mass of ^{4}He.

The wave function of helium for a state is expanded as a linear combination of the configuration-state wave functions. The configuration-state wave functions $|\phi_{ij}(JM_J)\rangle$ are constructed by $a^+_{im_i}|0\rangle$ and $a^+_{jm_j}|0\rangle$ with the angular quantum numbers ℓ_i and ℓ_j less than the maximum number of partial wave $\ell_{\rm max}$,

$$|\phi_{ij}(JM_J)\rangle = \eta_{ij}\sum_{m_im_j}\langle j_im_i; j_jm_j|JM_J\rangle a^+_{im_i}a^+_{jm_j}|0\rangle\,, \tag{6}$$

where η_{ij} is a normalization constant, $\langle j_im_i; j_jm_j|JM_J\rangle$ represents the Clebsch-Gordan coefficient of jj coupling, $|0\rangle$ is the vacuum state, and $a^+_{im_i}|0\rangle$ represents the ith single-electron wave function, which can be obtained by solving the single-electron Dirac equation using the Notre Dame basis sets [36] of N number of B-spline functions with order of $k=7$ [37].

QED corrections to polarizability and tune-out wavelength are obtained by the perturbation theory using accurate energies and wave functions of previous NRCI calculations [28]. According to the calculation of QED correction to static polarizability [38], the following expression of QED correction to the dynamic dipole polarizability can be derived,

$$\begin{aligned}\delta\alpha_1^{\rm QED}(\omega) = 2\Bigg[&\sum_n \frac{\langle g|D|n\rangle\langle n|D|g\rangle\langle g|\delta H_{\rm QED}|g\rangle[(E_n-E_g)^2+\omega^2]}{[(E_n-E_g)^2-\omega^2]^2} - 2\sum_{nm}\frac{\langle g|D|n\rangle\langle n|D|m\rangle\langle m|\delta H_{\rm QED}|g\rangle(E_n-E_g)}{[(E_n-E_g)^2-\omega^2](E_m-E_g)}\\ &-\sum_{nm}\frac{\langle g|D|n\rangle\langle n|\delta H_{\rm QED}|m\rangle\langle m|D|g\rangle[(E_n-E_g)(E_m-E_g)+\omega^2]}{[(E_n-E_g)^2-\omega^2][(E_m-E_g)^2-\omega^2]}\Bigg],\end{aligned} \tag{7}$$

where $|g\rangle$ represents the nonrelativistic wave function of the initial state, $|n\rangle$ and $|m\rangle$ represent nonrelativistic wave functions of intermediate states, and D is the electric dipole transition operator. The QED operator, $\delta H_{\rm QED} = H^{(3)}_{\rm QED} + H^{(4)}_{\rm QED}$, expanded to α^3 and α^4 order for the He$(2\,^3S)$ state are

TABLE I. Convergence of the energy (in a.u.) for the ^{4}He($2\,^3S_1$) state.

$(\ell_{\max}, N)$	DCB	DCB+NRMS	DCB+MS
(7, 40)	− 2.175 344 5653	− 2.175 045 2572	− 2.175 045 3806
(8, 40)	− 2.175 344 5952	− 2.175 045 2851	− 2.175 045 4098
(9, 40)	− 2.175 344 6132	− 2.175 045 3011	− 2.175 045 4224
(10, 40)	− 2.175 344 6157	− 2.175 045 3020	− 2.175 045 4282
(10, 50)	− 2.175 344 6220	− 2.175 045 3083	− 2.175 045 4270
Extrap.	− 2.175 344 64(2)	− 2.175 045 31(1)	− 2.175 045 43(1)
Ref. [29]		− 2.175 045 3(2)	
Ref. [15]			− 2.175 045 451

defined respectively as [15]

$$H_{\mathrm{QED}}^{(3)} = \frac{4Z\alpha^3}{3}\left\{\frac{19}{30} + \ln[(Z\alpha)^{-2}] - \ln\left(\frac{k_0}{Z^2}\right)\right\} \times [\delta^3(r_1) + \delta^3(r_2)] - \frac{14\alpha^3}{3}\left(\frac{1}{4\pi r_{12}^3}\right), \quad (8)$$

$$H_{\mathrm{QED}}^{(4)} = \alpha^4\left\{\left[-\frac{9\zeta(3)}{4\pi^2} - \frac{2179}{648\pi^2} + \frac{3\ln(2)}{2} - \frac{10}{27}\right]\pi Z + \left[\frac{427}{96} - 2\ln(2)\right]\pi Z^2\right\}[\delta^3(r_1) + \delta^3(r_2)], \quad (9)$$

where $\ln k_0$ is the Bethe logarithm and $\zeta(x)$ is the Riemann ζ function.

When an atom is in the external electric field $\mathcal{E}$, the Bethe logarithm involves the electric-field dependence term $\partial_{\mathcal{E}}^2 \ln k_0$, which introduces about 0.6% of the total QED correction to the ground-state polarizability [21]. In our calculation, we use the value of $\ln k_0 = 4.364\,036\,82(1)$ [39] for a free atom. The correction from the electric-field derivative of Bethe logarithm is evaluated by indicating 1% of the α^3-order QED correction to the dynamic dipole polarizability. The α^4-order QED includes the one-loop and two-loop radiative effects. The nonradiative component is neglected since the contribution to helium $2\,^3S_1$ ionization energy from the nonradiative component accounts for less than 5% of total α^4-order QED correction [40]. The Araki-Sucher correction [last term in Eq. (8)] contributes -5.6×10^{-9} a.u. to helium $2\,^3S_1$ energy [41], which is four orders of magnitude smaller than 1.67×10^{-5} a.u. from the first term of Eq. (8), and two orders of magnitude smaller than the α^4-order QED contribution of 2.91×10^{-7} a.u. So, we omit the Araki-Sucher correction in the determination of the 413-nm tune-out wavelength.

The calculations of the nuclear recoil corrections on the energies, polarizabilities, and tune-out wavelengths are performed using our improved RCI method. Table I gives a convergence test of the energy for the $2\,^3S_1$ state of ^{4}He. The extrapolation was done by assuming that the ratio between two successive differences in energies stays constant as the $\ell_{\max}$ and N become infinitely large. The DCB energies in the second column do not include the nuclear recoil correction. The DCB+MS and DCB+NRMS columns present energies with and without relativistic nuclear recoil effects, respectively. Comparing the extrapolated results between DCB+MS and DCB+NRMS columns, it is found that the relativistic nuclear recoil effect of H_{RMS} reduces 1.2×10^{-7} a.u. to the energy of the $2\,^3S_1$ state. The present DCB+NRMS value is in reasonable agreement with the previous RCI energy [29], where the relativistic nuclear recoil correction is not taken into account. Compared with the perturbation calculation [15], which includes the leading α^2-order relativistic correction, our DCB+MS energy agrees well with the result of −2.175 045 451 a.u. of Ref. [15]. The same energy accuracy for other $n\,^3S_1$ and $n\,^{1,3}P_J$ states with n up to 8 is maintained in our calculations.

Table II gives a convergence test of the static dipole polarizability and the 413-nm tune-out wavelength for the ^{4}He($2\,^3S_1$) state. For $\alpha_1(0)$, present RCI values have seven convergent digits, which improves on previous RCI values [29] by one order of magnitude. For λ_t, the convergence is very smooth as $\ell_{\max}$ and N increased. The tune-out wavelengths for the $2\,^3S_1(M_J = 0)$ and $2\,^3S_1(M_J = \pm 1)$ states are 413.080 00(1) and 413.085 89(1) nm, respectively. The present value of 413.085 89(1) nm is more accurate than the previous RCI result of 413.085 9(4) nm [29] by one order of magnitude. The relativistic nuclear recoil correction decreases the tune-out wavelength by 0.02 picometers (pm).

Recently, Drake and Manalo carried out an independent calculation of the tune-out wavelength by solving the Schrödinger equation with Hylleraas basis sets, and the relativistic effects of relative $O(Z\alpha^2)$ were obtained by perturbation theory. They obtained the tune-out wavelengths of 413.079 958(2) and 413.085 828(2) nm for the magnetic sublevels of $M_J = 0$ and $M_J = \pm 1$ [31,32], respectively, which are in good agreement with our RCI values. It is worth mentioning that the method of Hylleraas coordinates allows accurate calculation of electron correlation effects, while

TABLE II. Convergence of the static dipole polarizability $\alpha_1(0)$ (in a.u.) and the tune-out wavelength λ_t (in nm) for the $2\,^3S_1(M_J = 0, \pm 1)$ states of ^{4}He.

$(\ell_{\max}, N)$	$\alpha_1(0)(M_J = 0)$	$\alpha_1(0)(M_J = \pm 1)$	$\lambda_t(M_J = 0)$	$\lambda_t(M_J = \pm 1)$
(7, 40)	315.715 818 07	315.724 122 42	413.079 716 23	413.085 585 95
(8, 40)	315.715 993 59	315.724 290 09	413.079 899 85	413.085 764 03
(9, 40)	315.716 037 70	315.724 343 39	413.079 963 29	413.085 832 75
(10, 40)	315.716 053 51	315.724 366 77	413.079 994 33	413.085 867 66
(10, 50)	315.716 050 67	315.724 377 89	413.080 000 16	413.085 882 02
Extrap.	315.716 05(1)	315.724 38(1)	413.080 00(1)	413.085 89(1)
Ref. [29]	315.716 5(4)	315.724 8(4)	413.080 1(4)	413.085 9(4)

TABLE III. Convergence of QED correction to the static dipole polarizability $\alpha_1(0)$ (in a.u.) and the 413-nm tune-out wavelength λ_t (in nm) for the ^{4}He($2\,^3S_1$) state. The number of B-splines $N = 40$ is fixed. $\delta\alpha_1^{\rm QED}(0)(\alpha^3)$ and $\delta\alpha_1^{\rm QED}(0)(\alpha^4)$ represent the α^3- and α^4-order QED corrections to $\alpha_1(0)$ respectively. $\delta\lambda_t^{\rm QED}(\alpha^3) = \lambda_t(\text{NRCI} + \alpha^3\ \text{QED}) - \lambda_t(\text{NRCI})$ and $\delta\lambda_t^{\rm QED}(\alpha^4) = \lambda_t(\text{NRCI} + \alpha^4\ \text{QED}) - \lambda_t(\text{NRCI})$ represent the α^3- and α^4-order QED corrections to λ_t.

$\ell_{\max}$	$\delta\alpha_1^{\rm QED}(0)(\alpha^3)$	$\delta\alpha_1^{\rm QED}(0)(\alpha^4)$	$\delta\lambda_t^{\rm QED}(\alpha^3)$	$\delta\lambda_t^{\rm QED}(\alpha^4)$
7	0.006 899 132 62	0.000 119 945 10	0.004 147 699 87	0.000 072 114 31
8	0.006 899 146 48	0.000 119 945 35	0.004 147 716 05	0.000 072 114 59
9	0.006 899 152 88	0.000 119 945 46	0.004 147 723 72	0.000 072 114 72
10	0.006 899 156 22	0.000 119 945 52	0.004 147 727 74	0.000 072 114 79
Extrap.	0.006 899 158(2)	0.000 119 946(1)	0.004 147 729(2)	0.000 072 115(1)

present RCI calculations automatically include higher order one-electron relativistic corrections and electron-electron correlation of relative order $Z\alpha^2$.

As pointed out in our previous paper [29], the main discrepancy between the earlier theory [28] and experiment [27] for the 413-nm tune-out wavelength comes from omission of QED contributions to the theoretical value. In Table III, we present the convergence test for the α^3- and α^4-order QED corrections to the static dipole polarizability and the 413-nm tune-out wavelength of the $2\,^3S_1$ state. The numerical results of $\delta\alpha_1^{\rm QED}(0)(\alpha^3)$ and $\delta\alpha_1^{\rm QED}(0)(\alpha^4)$ converge fairly smoothly and monotonically to an extrapolated values of 0.006 899 158(2) a.u. and 0.000 119 946(1) a.u., with at least five converged digits. The α^3-order QED correction contributes 4.147 729(2) pm to the tune-out wavelength, which is two orders of magnitude greater than the α^4-order QED correction. The α^4-order QED correction has four significant digits, which is more than satisfactory for our purposes.

In addition, we also evaluate the finite-nuclear-size effect on the static dipole polarizability and the tune-out wavelength by adopting the operator of $\frac{4\pi}{3} r_{^4\text{He}}^2[\delta^3(r_1) + \delta^3(r_2)]$ [25], where $r_{^4\text{He}} = 1.6755$ fm is the nuclear charge radius of ^{4}He [42]. The corrections due to finite nuclear size on $\alpha_1(0)$ and λ_t are respectively 4.58×10^{-6} a.u. and 2.75 fm, which are negligible in the present work. But in the future, if a measurement of the 413-nm tune-out wavelength can reach 10^{-9} level of accuracy, it would have potential for the determination of the nuclear charge radius of helium, which is comparable with most of the precision spectroscopy methods [12,14,16,17].

The individual and relative contributions from the QED, relativistic nuclear recoil, and finite-nuclear-size effects to static dipole polarizability and the 413-nm tune-out wavelength for the $2\,^3S_1(M_J = \pm 1)$ state can be seen clearly from Table IV and Fig. 1. The largest contribution to $\alpha_1(0)$ and λ_t comes from the α^3-order QED correction without $\partial_\varepsilon^2 \ln k_0$. The α^3-QED correction from the electric-field dependence of the Bethe logarithm is hard to compute but has been confirmed to be relatively small (≈0.6%) to the total QED correction in Ref. [21,25]. So in order to give a conservative estimation of this correction, we assume a 1% of the α^3-order QED correction [25] to reflect the contribution from $\partial_\varepsilon^2 \ln k_0$ term, which results in 0.000 07(1) a.u. correction to $\alpha_1(0)$. Combined with the α^3- and α^4-order QED corrections, the total QED contribution of 0.007 09(1) a.u. is added to the RCI values of 315.716 05(1) and 315.724 38(1) a.u., which gives 315.723 14(4) and 315.731 47(4) a.u. for the $2\,^3S_1(M_J = 0)$ and $2\,^3S_1(M_J = \pm 1)$ states, respectively. The uncertainties, which are mainly from the $\partial_\varepsilon^2 \ln k_0$ term, have been doubled to be conservative. The total QED correction on the polarizability is 22.5 ppm. Like the ground-state polarizability, which has a similar QED contribution (22 ppm) [38], the contribution to the $2\,^3S_1$ state could also be measured as a test of QED.

For the 413-nm tune-out wavelength, seen from the Table IV, the α^3-order QED correction without $\partial_\varepsilon^2 \ln k_0$ has about 10-ppm effect on λ_t, and 1% of the α^3-order QED correction is assumed to estimate the QED contribution from $\partial_\varepsilon^2 \ln k_0$ term. The total QED correction on the tune-out wavelength is then 0.004 26(1) nm. Adding this correction to our RCI values of 413.080 00(1) and 413.085 89(1) nm, we obtain the final tune-out wavelengths of 413.084 26(4) nm for $M_J = 0$ and 413.090 15(4) nm for $M_J = \pm 1$ magnetic sublevel of the $2\,^3S_1$ state, respectively. Comparison of calculations with measurement [27] is displayed in Fig. 2. The result of 413.085

TABLE IV. Contributions to the static dipole polarizability (in a.u.) and the 413-nm tune-out wavelength (in nm) for the $2\,^3S_1(M_J = 0, \pm 1)$ states of ^{4}He.

Contribution	M_J	$\alpha_1(0)$(a.u.)	λ_t(nm)
RCI + nuclear recoil	0	315.716 05(1)	413.080 00(1)
RCI + nuclear recoil	±1	315.724 38(1)	413.085 89(1)
α^3 QED without $\partial_\varepsilon^2 \ln k_0$		0.006 899 158(2)	0.004 147 729(2)
α^4 QED		0.000 119 946(1)	0.000 072 115(1)
α^3 QED from $\partial_\varepsilon^2 \ln k_0$		0.000 07(1)	0.000 04(1)
Finite nuclear size		0.000 004 58	0.000 002 75
Total	0	315.723 14(4)	413.084 26(4)
Total	±1	315.731 47(4)	413.090 15(4)

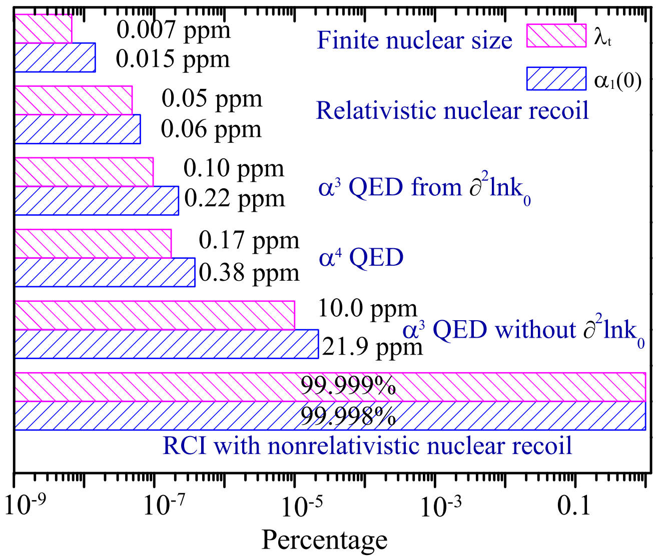

FIG. 1. Relative contributions of various corrections to the static dipole polarizability $\alpha_1(0)$ and the tune-out wavelength λ_t for the $2\,^3S_1(M_J = \pm 1)$ state of ^{4}He.

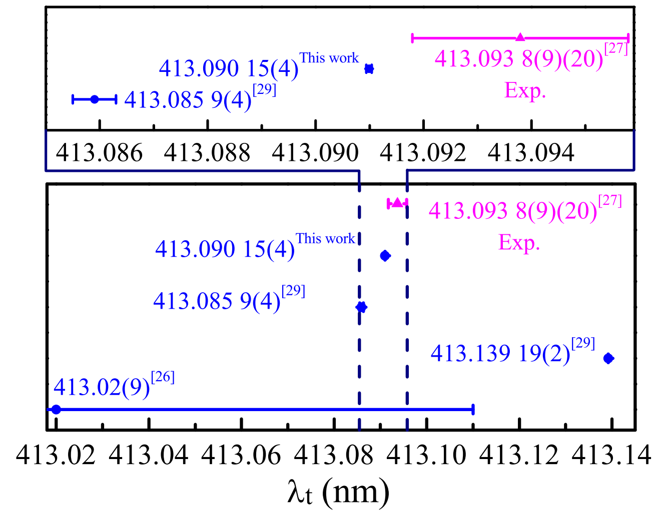

FIG. 2. Comparisons of the tune-out wavelength λ_t (in nm) for the $2\,^3S_1(M_J = \pm 1)$ state of ^{4}He.

9(4) nm [29], which does not includes the relativistic nuclear recoil and QED corrections, agrees with the measured value of 413.0938(9_{stat})(20_{syst}) nm [27] at the level of 19 ppm. The present result of 413.090 15(4) nm for the $M_J = \pm 1$ sublevel has included QED and relativistic nuclear recoil corrections. In order to make a meaningful comparison with the measurement [27] which probed the polarizability by using a traveling wave, the retardation correction to the tune-out wavelength needs to be taken into account. We incorporate Drake *et al.*'s retardation correction of 0.000 560 0236 [43] in our result of 413.090 15(4) nm to give 413.090 71(4) nm. Therefore, a 1.4σ discrepancy still exists in the tune-out wavelength between theory and experiment, so the present work provides considerable motivation for future experimental improvements to seriously test QED calculations at a higher level of accuracy.

In summary, we have calculated the dynamic dipole polarizability of the metastable helium under the DCB framework with the relativistic nuclear recoil effect included. The QED correction on the polarizability is taken into account using perturbation theory, and the finite-nuclear-size effect is also estimated. We precisely determine the tune-out wavelengths for the ^{4}He($2\,^3S_1$) state for $M_J = 0$ and $M_J = \pm 1$ magnetic sublevels as 413.084 26(4) nm and 413.090 15(4) nm, respectively. We find that the relativistic nuclear recoil effect decreases the tune-out wavelength by ≈0.02 pm, and the QED correction increases the tune-out wavelength by ≈4.26 pm. Our theoretical prediction for the 413-nm tune-out wavelength can be improved by introducing larger scale configuration calculations with higher order relativistic nuclear recoil effects included and by calculating contributions from the field-dependent Bethe logarithm in detail. We anticipate that this work will stimulate new high-precision measurements of the helium 413-nm tune-out wavelength to test QED calculations. In addition, we also obtained the static dipole polarizabilities for the $M_J = 0$ and $M_J = \pm 1$ magnetic sublevels of the ^{4}He($2\,^3S_1$) state as 315.723 14(4) and 315.731 47(4) a.u. respectively. We determined QED corrections for these polarizabilities of 22.5 ppm, which suggests that sensitive experimental measurements of static dipole polarizabilities of the $2\,^3S_1$ state might also be future test of QED.

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