Collective radiance effects in the ultrastrong-coupling regime

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We investigate the collective radiance characteristics of qubits in the ultrastrong-coupling regime, where the radiance witness is defined based on the resonator-qubit dressed basis. An ultrastrong hyperradiance effect is demonstrated when the dressed state of the system is resonantly driven. Interestingly, we show that, besides the resonator-qubit coupling strength, the parity-symmetry-breaking-induced cascade transition can significantly enhance the collective radiance of the qubits, which allows us to manipulate the transitions between subradiance, superradiance, and hyperradiance via adjusting the parity symmetry of the system with an external magnetic field. This work extends the collective radiance theory to the ultrastrong-coupling regime, and offers potential applications in the engineering of laser devices.

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I. INTRODUCTION

The theory of collective radiance is of great importance in quantum optics, and has important applications in lasing engineering [1–3], precision measurements [4–6], and quantum information [7,8]. One of the very intriguing phenomena exhibiting collective radiance behavior is superradiance, discovered by Dicke in 1954 [9]. Specifically, the radiance intensity from an atomic ensemble can be enhanced with a factor of N^2 (N is the atom number). Recently, an enhanced radiance factor that is larger than N^2 has also been present in a cavity quantum electrodynamics (QED) system, called hyperradiance [10–12], which has a stronger collective radiance effect than superradiance. The above theoretical results have pushed the progress in the corresponding experiments, including superradiance lasers [2,13], the measurement of collective Lamb shifts [5,14], the research of the coherence properties of Bose-Einstein condensates [15,16], and the realization of superradiance in quantum dots [17] and artificial atomic systems [18]. However, the present collective radiance theories are confined to the weak- and strong-coupling regimes.

Recently, an ultrastrong-coupling regime, where the light-matter coupling rate reaches an order of 10% of the bare resonance frequency of photons or the transition frequency of quantum emitters, has been reached experimentally in a variety of solid state quantum systems [19–29]. In this regime, the counter-rotating terms in the interaction Hamiltonian are nonignorable [30], and in some cases the parity symmetry of the system cannot be conserved approximately [20,31–33]. Thus many novel quantum effects emerge in the ultrastrong-coupling regime, such as vacuum degeneracy [34], the generation of correlated photon pairs from the initial polariton vacuum state [35], nonclassical states [36], Casimir-like photons [37], and so on [38–56]. Then, extending the collective radiance theory to the ultrastrong-coupling regime becomes interesting in the exploration of novel effects and the application of the lasing theory.

In the ultrastrong-coupling regime, the usual radiance witness [10–12], defined by an independent system operator, fails to describe the collective radiance characteristics of the system. Here, we rewrite the radiance witness in terms of a resonator-qubit dressed basis, which is valid for any resonator-qubit coupling strength. We find that ultrastrong resonator-qubit coupling could significantly enhance the collective radiance of qubits under the condition of resonantly driving the dressed state of the system, which leads to the emergence of enhanced hyperradiance. The parameter ranges for different radiance effects, such as subradiance and hyperradiance, become more distinguishable. Moreover, we also show that the resonator-qubit detuning could change the property of the radiance, allowing for transitions between different radiance effects.

More interestingly, when the coupling strength is fixed, the collective radiance of qubits can also be enhanced by breaking the parity symmetry of the system, e.g., making the radiance property of the system go from subradiance and superradiance to hyperradiance. This originally comes from the symmetry-breaking-induced cascade transition of decay during the dressed states. Note that, in some cases (e.g., the superconducting circuits), the system parity symmetry could be controlled by an external magnetic field [20,31–33,57]. Then our results allow us to realize controllable transitions between the subradiance, superradiance, and hyperradiance via adjusting the system parity symmetry, which might inspire different laser technologies. Our work is also fundamentally interesting in building the collective radiance theory in the ultrastrong-coupling regime.

II. SYSTEM AND COLLECTIVE RADIANCE WITNESS

We consider a circuit-QED system that consists of a single-mode resonator coupled to two qubits driven by a coherent

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microwave field. The Hamiltonian of the system can be given by ($\hbar = 1$)

$$H = H_0 + H_d, \tag{1}$$

with

$$H_0 = \omega_c a^\dagger a + \omega_\sigma \sum_j \sigma_j^+ \sigma_j^- + \lambda(a^\dagger + a) \sum_{j=1}^{2} \left(\cos\theta \sigma_z^j - \sin\theta \sigma_x^j\right), \tag{2}$$

where a ($a^\dagger$) is the annihilation (creation) operator of the resonator with resonance frequency ω_c, σ_j^+ (σ_j^-) is the raising (lowering) operator for the jth qubit with transition frequency ω_σ, and $\sigma_x^j = \sigma_j^- + \sigma_j^+$, $\sigma_z^j = \sigma_j^+\sigma_j^- - \sigma_j^-\sigma_j^+$. The constant λ is the coupling rate between the resonator mode and each qubit. The term $H_d = \Omega \cos(\omega_d t) \sum_{j=1}^{2} (\sigma_j + \sigma_j^+)$ describes coherent driving with a driving frequency ω_d and amplitude Ω.

Here, the mixing angle θ, describing the relative contribution of the longitudinal and transverse couplings, can be controlled by adjusting an external magnetic flux Φ_x threading the qubit loop, i.e., $\sin\theta = \Delta/\omega_\sigma$, where Δ is the qubit energy gap [20,31–33,57,58]. The value of θ can influence the transition of radiance via changing the parity symmetry of the system, and will further produce an effect on the collective radiance property of the system. Here, the parity operator of the system is defined as $\Pi = \exp[i\pi N] = \exp[i\pi(a^\dagger a + \sigma_1^+\sigma_1^- + \sigma_2^+\sigma_2^-)]$ [59–62]. For $\theta = \pi/2$, with $[H_0, \Pi] = 0$, the parity of the number of excitations in the Hamiltonian H_0 is conserved. However, the parity symmetry of H_0 is broken when $\theta \neq \pi/2$, i.e., $[H_0, \Pi] \neq 0$. This enables a cascade transition between adjacent dressed states, e.g., the transition $|\varphi_3\rangle \to |\varphi_1\rangle$ in Fig. 1(d), which is forbidden for the case of parity symmetry conservation. Here, the dressed states are given approximately, i.e., $|\varphi_1\rangle \approx (|e, g, 0\rangle + |g, e, 0\rangle)/2 + |g, g, 1\rangle/\sqrt{2}$ and $|\varphi_3\rangle \approx (|e, g, 0\rangle + |g, e, 0\rangle)/2 - |g, g, 1\rangle/\sqrt{2}$. Note that the term leading parity symmetry breaking can be safely ignored by the rotating-wave approximation (RWA) in the weak- and strong-coupling regimes.

To describe the system more realistically, the influence of dissipation on the system needs to be taken into account. The system coupled to a zero-temperature environment can be studied by a quantum optical master equation. However, the standard master equation fails to provide a correct description of the dynamics of the system in the case of $\lambda \sim \omega_c, \omega_\sigma$, because in the ultrastrong-coupling regime, the qubits and resonator mode can form an inseparable system with the new dressed states. We thus write the system Hamiltonian operators in terms of the resonator-qubit dressed basis $|\varphi_n\rangle$ ($n = 0, 1, 2, \dots$), where $H_0|\varphi_n\rangle = E_n|\varphi_n\rangle$. By applying the Born-Markov approximation and tracing out the environment degrees of freedom, the master equation for the reduced density matrix of the system reads [33,46,51]

$$\frac{d\rho}{dt} = i[\rho, H] + \kappa\mathcal{L}[X^+] + \gamma_\sigma \sum_{j=1}^{N} \mathcal{L}[D_j^+], \tag{3}$$

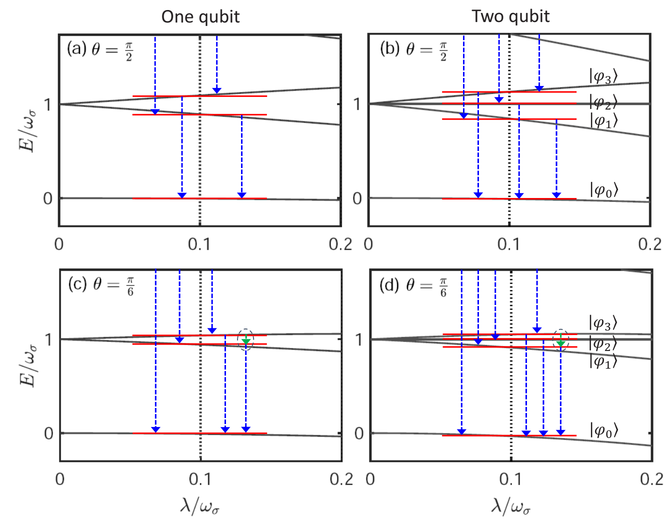

FIG. 1. Energy spectrum of Hamiltonian H_0 vs the coupling strength λ/ω_σ for (a), (b) $\theta = \pi/2$ and (c), (d) $\theta = \pi/6$. Moreover, (a), (c) and (b), (d) correspond to the case of one qubit and two qubits, respectively. Here, $|\varphi_n\rangle$ ($n = 0, 1, 2, \dots$) are the corresponding eigenstates of H_0. The arrows show the possible transitions of radiative decay between these eigenstates. Especially, the green arrow indicates the symmetry-breaking-induced cascade transition of decay. Here, the system parameters are chosen as $\omega_c = \omega_\sigma$ and $\lambda/\omega_\sigma = 0.1$.

where the Liouvillian superoperator $\mathcal{L}$ is defined as $\mathcal{L}[O] = (2O\rho O^\dagger - \rho O^\dagger O - O^\dagger O\rho)/2$. The constants κ and γ_σ describe the damping rates of the cavity and the qubits, respectively. Here, $X^+ = \sum_{E_n, E_m > E_n} X_{nm}|\varphi_n\rangle\langle\varphi_m|$ and $D_j^+ = \sum_{E_n, E_m > E_n} D_{nm}^j |\varphi_n\rangle\langle\varphi_m|$, with $X_{nm} = \langle\varphi_n|(a + a^\dagger)|\varphi_m\rangle$ and $D_{nm}^j = \langle\varphi_n|(\sigma_j^- + \sigma_j^+)|\varphi_m\rangle$, are positive frequency components of the cavity photon and the jth qubit operators, respectively. Note that $a|\varphi_0\rangle \neq 0$ for the ground state of the Hamiltonian H_0, and $X^+|\varphi_0\rangle = 0$. Under the condition of including the RWA or neglecting the resonator-qubit coupling rate, X^+ and $X^- = (X^+)^\dagger$ correspond approximately to a and $a^\dagger$. Similarly, D_j^+ and $D_j^- = (D_j^+)^\dagger$ coincide with σ_j^- and σ_j^+.

In this article, we explore the collective radiance characteristics of qubits in the steady-state limit. According to the input-output theory, the photon emission from the qubits can be measured by detecting the average photon number from the cavity. The output photon rate of the cavity is expressed as $\Psi_{\text{out}} = \kappa\langle X^- X^+\rangle$, obtained by the input-output relation $a_{\text{out}}(t) = a_{\text{in}}(t) - \sqrt{\kappa} X^+(t)$ in the case of $\omega_c \approx \omega_\sigma$, where the input is in the vacuum. The photon emission could be detected in the photodetection experiment by coupling the qubit to a microwave antenna [63]. The radiance characteristics of two qubits can be described by a radiance witness,

$$R = \frac{\langle X^- X^+\rangle_2 - 2\langle X^- X^+\rangle_1}{2\langle X^- X^+\rangle_1}. \tag{4}$$

Here, $\langle X^- X^+\rangle_2$ is the average photon number when a cavity is coupled to two qubits, and $\langle X^- X^+\rangle_1$ corresponds to the case of coupling the cavity to only one qubit. Under this definition, $R = 0$ indicates an uncorrelated radiance between two qubits.

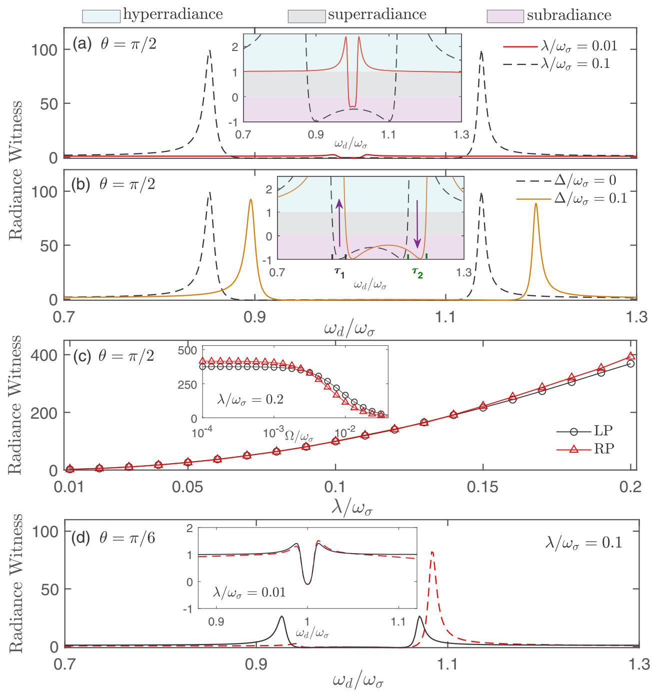

FIG. 2. Radiance witness R vs ω_d/ω_σ for different (a) λ/ω_σ and (b) Δ/ω_σ. Insets: Enlarged region of the small value of R. The blue, gray, and pink areas indicate $R>1$ (hyperradiance), $0<R\leqslant 1$ (superradiance), and $-1<R<0$ (subradiance), respectively. Here, $\tau_1\in(0.875, 0.917)$ and $\tau_2\in(1.12, 1.178)$. (c) The values of the left peak (LP) and right peak (RP) in (a) vs λ/ω_σ and Ω/ω_σ (the inset). (d) R vs ω_d/ω_σ for different λ/ω_σ when the term including σ_z^j is neglected (black solid lines) and kept (red dashed lines). The other system parameters used here are $\gamma/\omega_\sigma=\kappa/\omega_\sigma=0.01$, $\Omega/\omega_\sigma=0.001$, and (a), (c), (d) $\omega_c=\omega_\sigma$, and (b) $\lambda/\omega_\sigma=0.1$.

Specifically, the emission photons of two qubits are the sum of that of two isolated qubits, i.e., $\langle X^-X^+\rangle_2=2\langle X^-X^+\rangle_1$. $R<0$ corresponds to the subradiance of two qubits, i.e., $\langle X^-X^+\rangle_2<2\langle X^-X^+\rangle_1$, indicating the suppression of radiance. The range of $0<R\leqslant 1$ corresponds to the regime of superradiance, and $R=1$ means that the radiance strength being proportional to the square of the number of qubits, i.e., $\langle X^-X^+\rangle_2=2^2\langle X^-X^+\rangle_1$. $R>1$, i.e., $\langle X^-X^+\rangle_2>2^2\langle X^-X^+\rangle_1$, is the hyperradiance, which has a stronger radiance effect than the superradiance behavior [10–12].

III. RADIANCE WITHOUT PARITY SYMMETRY BREAKING

To clearly show the influence of the resonator-qubit coupling strength on the radiance effect of qubits, we investigate the case $\theta=\pi/2$ (holding the parity symmetry of the system) in Figs. 2(a)–2(c). It shows that the collective radiance effect of the qubits is significantly influenced in the ultrastrong-coupling regime. Under different driving frequencies, one can obtain the subradiance, superradiance, and hyperradiance, respectively. For example, when we resonantly drive the dressed state $|\varphi_1\rangle$ (or $|\varphi_3\rangle$) of the system including two qubits, strong hyperradiance is obtained, which corresponds to the peaks in Fig. 2(a). This comes from the emission of photons between

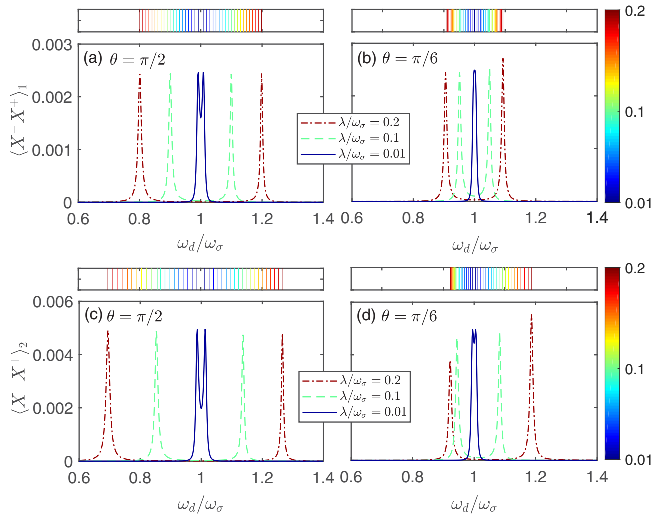

FIG. 3. The excitation spectrum of the systems consisting of (a), (b) one qubit and (c), (d) two qubits. Here, we present the change of the excitation spectrum with the increase of λ/ω_σ, and the top stripes indicate the positions of the peaks for different λ/ω_σ. The other system parameters used here are $\omega_c=\omega_\sigma$, $\gamma/\omega_\sigma=\kappa/\omega_\sigma=0.01$, and $\Omega/\omega_\sigma=0.001$.

the two states $|\varphi_1\rangle$ (or $|\varphi_3\rangle$) and $|\varphi_0\rangle$ with different transition paths [10]. For the case of resonantly driving the dressed state of the system that consists of one qubit, the subradiance can be obtained, corresponding to the dips in Fig. 2(a). These imply that there are different optimal radiance frequencies for systems containing different numbers of qubits, as shown in Figs. 1(a) and 1(b).

The distance between the peak and the dip is getting farther and farther away when increasing the resonator-qubit coupling strength λ. This result can be understood from the energy spectrum [see Figs. 1(a) and 1(b)] and the corresponding excitation spectrum [see Figs. 3(a) and 3(c)]. We see that, as the resonator-qubit coupling strength increases, a system consisting of a resonator coupled to two qubits has a faster splitting speed between two adjacent dressed states than that coupled to one qubit. In other words, when we fix the value of λ, the splitting between two peaks is larger than that of two dips, which leads to an increase in the distance between the peak and the dip. Then, in the ultrastrong-coupling regime, the regions of different radiance effects (e.g., superradiance and hyperradiance) become more distinguishable.

In Fig. 2(c), we plot the dependence of the maximum radiance strength on λ and Ω. The collective radiance effect is enhanced when the value of λ increases, whereas too large a drive strength will decrease the collective radiance of the qubits. This is because it is difficult to neglect the higher-order dressed states when Ω has a higher value. So, the possible radiative transitions could lead to the occurrence of a destructive quantum path interference in the system. Moreover, we also show the influence of resonator-qubit detuning Δ ($\Delta=\omega_c-\omega_\sigma$) on the radiance effect of qubits in Fig. 2(b). In the presence of detuning, enhanced collective radiance effects still persist, but the curve has an obvious shift due to the shift of the dressed states. This allows for transitions between subradiance, superradiance, and hyperradiance

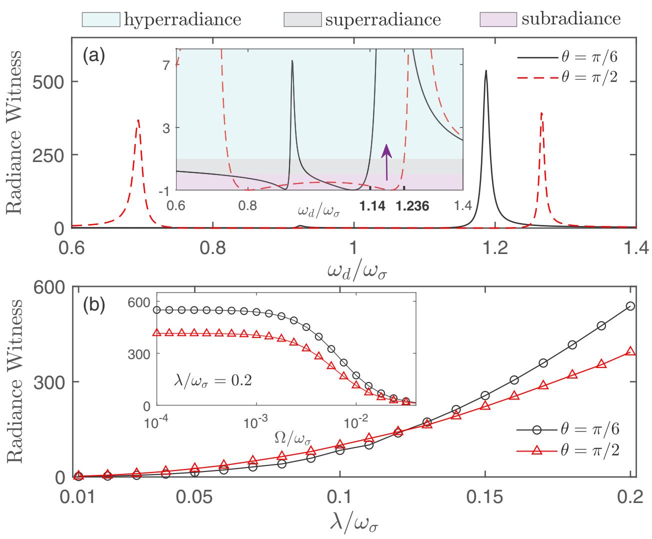

FIG. 4. (a) Radiance witness R vs ω_d/ω_σ for different θ when $\lambda/\omega_\sigma = 0.2$. Inset: Enlarged region of the small value of R. The blue, gray, and pink areas indicate $R > 1$ (hyperradiance), $0 < R \leqslant 1$ (superradiance), and $-1 < R < 0$ (subradiance), respectively. (b) The maximum value of R in (a) and Fig. 2(a) vs λ/ω_σ and Ω/ω_σ (the inset). The other system parameters used here are $\omega_c = \omega_\sigma$, $\gamma/\omega_\sigma = \kappa/\omega_\sigma = 0.01$, and $\Omega/\omega_\sigma = 0.001$.

within the proper parameter ranges [see the arrows in the ranges $0.875 < \omega_d/\omega_\sigma < 0.917$ and $1.12 < \omega_d/\omega_\sigma < 1.178$ in Fig. 2(b)].

Even for the case $\theta \neq \pi/2$, the system can also hold the parity symmetry in the weak-coupling regime when we ignore the term $\lambda(a^\dagger + a)\sum_{j=1}^{2}\cos\theta\sigma_z^j$ under RWA. However, in the ultrastrong-coupling regime, the RWA becomes invalid and the parity symmetry of the system is broken. This property is clearly shown in Fig. 2(d), which indicates that the system parity symmetry can significantly influence the collective radiance.

IV. RADIANCE WITH PARITY SYMMETRY BREAKING

Now let us investigate in detail the influence of parity symmetry of the system on the collective radiance effect. First, in Fig. 4(a), we plot the radiance witness R as a function of ω_d/ω_σ when $\theta = \pi/2$ (holding parity symmetry) and $\theta = \pi/6$ (breaking parity symmetry). It is shown that parity symmetry breaking will significantly enhance the collective radiance when one resonantly drives the upper dressed state $|\varphi_3\rangle$. Physically, this enhancement comes from the cascade transition of decay during the dressed states, i.e., $|\varphi_3\rangle \rightarrow |\varphi_1\rangle$, induced by the counter-rotating terms $a^\dagger\sigma_z^j$ and $a\sigma_z^j$. Specifically, when the upper dressed state $|\varphi_3\rangle$ is resonantly excited, besides the radiance transition $|\varphi_3\rangle \rightarrow |\varphi_0\rangle$, the parity-symmetry-breaking-induced radiance transitions $|\varphi_3\rangle \rightarrow |\varphi_1\rangle \rightarrow |\varphi_0\rangle$ will also emit photons, which significantly enhance the radiance effect of the qubits. However, this cascade-radiance transition does not exist in the case of driving the lower dressed state $|\varphi_1\rangle$. Together with the reduced resonator-qubit interaction strength $\lambda\sin\theta$ for $\theta \neq \pi/2$, the radiance effect is suppressed by parity symmetry breaking when we resonantly drive the lower dressed state $|\varphi_1\rangle$ [see the left peaks in Fig. 4(a)].

Second, from Fig. 4(a), we also see that the positions of the peaks and dips have some shifts when the parity symmetry of the system is broken. This leads to a parity-symmetry-breaking-induced transition from subradiance and superradiance to hyperradiance during a proper parameter range [see the arrow in the range $1.14 < \omega_d/\omega_\sigma < 1.263$ in Fig. 4(a)]. Then our results allow the realization of a controllable radiance transition in a system with controllable parity symmetry. For example, in a superconducting circuit, one could obtain a transition from subradiance to hyperradiance by breaking system parity symmetry with an external magnetic field. To understand the above result, we plot the excitation spectrum of the systems including one qubit and two qubits, respectively, in Fig. 3. Note that the resonant excitation frequencies for the cases of one qubit and two qubits correspond to the positions of dips and peaks, respectively, in Fig. 4(a). Comparing the cases of $\theta = \pi/6$ and $\theta = \pi/2$, we see that parity symmetry breaking destroys the symmetry of the excitation spectrum, which ultimately leads to the shifts of the peaks and dips of R in Fig. 4(a).

Lastly, in Fig. 4(b), we plot the dependence of the maximum radiance strength on λ and Ω for $\theta = \pi/6$ and $\theta = \pi/2$. It shows that parity symmetry breaking can enhance the radiance effect in the ultrastrong-coupling regime. Note that this enhancement effect can be ignored approximately in the weak- and strong-coupling regimes. This result is consistent with Fig. 2(d). Physically, parity symmetry breaking can enhance the radiance of the qubits by inducing the cascade transition of decay ($|\varphi_3\rangle \rightarrow |\varphi_1\rangle \rightarrow |\varphi_0\rangle$). However, the presence of the term $\sin\theta$ in Eq. (2) decreases the collective radiance effect by reducing the effective resonator-qubit coupling strength when $\theta \neq \pi/2$. The change in the radiance effect is the result of the competition between the symmetry-breaking-induced cascade transition of decay and the decreasing effective coupling strength. Thus, in Fig. 4(b), we see that the maximum radiance strength in the parity-symmetry-breaking system is greater than that in the parity-symmetry-conserving system when $\lambda/\omega_\sigma < 0.12$. For a weaker-coupling strength, the effect from a decrease in coupling strength is also larger than that of the cascade transition in the parity-symmetry-breaking system.

V. CONCLUSIONS AND DISCUSSIONS

We have investigated the influences of resonator-qubit coupling strength, resonator-qubit detuning, and system parity symmetry on the collective radiance characteristics of a circuit-QED system in the ultrastrong-coupling regime. We have shown that, besides the ultrastrong-coupling strength, parity symmetry breaking will also significantly enhance the collective radiance effect by inducing a cascade transition between two adjacent dressed states of the system. Moreover, resonator-qubit detuning and parity symmetry breaking of the system will also largely shift the positions of subradiance, superradiance, and hyperradiance. This result provides potential methods to manipulate the transitions between subradiance, superradiance, and hyperradiance via adjusting the resonator-qubit detuning or system parity symmetry. Note that usually when referring to the superradiance, it is easy to associate it with the superradiance phase and superradiance phase transition. However, the model we are considering is dynamical, so the superradiance (and hyperradiance) phenomenon in our investigations is different from the superradiance involving the ground state in the usual phase transition approach [64,65].

In addition, we discuss that the superconducting circuit is an ideal experimental platform for our study. A possible implementation is in a system that consists of a superconducting coplanar waveguide resonator galvanically coupled to two flux qubits threaded by an external flux bias. In this system, the mixing angle θ can be adjusted by the flux bias threading the qubit loop [20,31–33,57]. In principle, our results are also fit for an acoustic system, and then different types of photon and phonon laser devices might be inspired by our work.

In future work, it will be interesting to extend the collective radiance to the deep strong-coupling regime. In this coupling regime, although it is not easy to calculate the collective radiance effect by numerical and analytical methods in a resonator-qubit resonance system, we can consider a solvable or quasisolvable model of large detunings [59–61,66–68]. This will bring more interesting results to the study of collective radiance in systems without RWA.

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[1] A. A. Svidzinsky, L. Yuan, and M. O. Scully, Quantum Amplification by Superradiant Emission of Radiation, Phys. Rev. X **3**, 041001 (2013).

[2] J. G. Bohnet, Z. Chen, J. M. Weiner, D. Meiser, M. J. Holland, and J. K. Thompson, A steady-state superradiant laser with less than one intracavity photon, Nature (London) **484**, 78 (2012).

[3] Q. Baudouin, N. Mercadier, V. Guarrera, W. Guerin, and R. Kaiser, A cold-atom random laser, Nat. Phys. **9**, 357 (2013).

[4] W.-J. Kim, J. H. Brownell, and R. Onofrio, Detectability of Dissipative Motion in Quantum Vacuum via Superradiance, Phys. Rev. Lett. **96**, 200402 (2006).

[5] R. Röhlsberger, K. Schlage, B. Sahoo, S. Couet, and R. Rüffer, Collective Lamb shift in single-photon superradiance, Science **328**, 1248 (2010).

[6] M. A. Norcia, J. R. K. Cline, J. A. Muniz, J. M. Robinson, R. B. Hutson, A. Goban, G. E. Marti, J. Ye, and J. K. Thompson, Frequency Measurements of Superradiance from the Strontium Clock Transition, Phys. Rev. X **8**, 021036 (2018).

[7] A. Kuzmich, W. P. Bowen, A. D. Boozer, A. Boca, C. W. Chou, L.-M. Duan, and H. J. Kimble, Generation of nonclassical photon pairs for scalable quantum communication with atomic ensembles, Nature (London) **423**, 731 (2003).

[8] A. Asenjo-Garcia, M. Moreno-Cardoner, A. Albrecht, H. J. Kimble, and D. E. Chang, Exponential Improvement in Photon Storage Fidelities Using Subradiance and "Selective Radiance" in Atomic Arrays, Phys. Rev. X **7**, 031024 (2017).

[9] R. H. Dicke, Coherence in spontaneous radiation processes, Phys. Rev. **93**, 99 (1954).

[10] M.-O. Pleinert, J. Von Zanthier, and G. S. Agarwal, Hyperradiance from collective behavior of coherently driven atoms, Optica **4**, 779 (2017).

[11] J. P. Xu, S. L. Chang, Y. P. Yang, S. Y. Zhu, and G. S. Agarwal, Hyperradiance accompanied by nonclassicality, Phys. Rev. A **96**, 013839 (2017).

[12] Y. F. Han, C. J. Zhu, J. P. Xu, and Y. P. Yang, Electromagnetic control of the collective radiations with three-photon blockade, arXiv:1809.09826.

[13] M. A. Norcia and J. K. Thompson, Cold-Strontium Laser in the Superradiant Crossover Regime, Phys. Rev. X **6**, 011025 (2016).

[14] J. Keaveney, A. Sargsyan, U. Krohn, I. G. Hughes, D. Sarkisyan, and C. S. Adams, Cooperative Lamb Shift in an Atomic Vapor Layer of Nanometer Thickness, Phys. Rev. Lett. **108**, 173601 (2012).

[15] S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, J. Stenger, D. E. Pritchard, and W. Ketterle, Superradiant Rayleigh scattering from a Bose-Einstein condensate, Science **285**, 571 (1999).

[16] D. Schneble, Y. Torii, M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, The onset of matter-wave amplification in a superradiant Bose-Einstein condensate, Science **300**, 475 (2003).

[17] M. Scheibner, T. Schmidt, L. Worschech, A. Forchel, G. Bacher, T. Passow, and D. Hommel, Superradiance of quantum dots, Nat. Phys. **3**, 106 (2007).

[18] J. A. Mlynek, A. A. Abdumalikov, C. Eichler, and A. Wallraff, Observation of Dicke superradiance for two artificial atoms in a cavity with high decay rate, Nat. Commun. **5**, 5186 (2014).

[19] G. Günter, A. A. Anappara, J. Hees, A. Sell, G. Biasiol, L. Sorna, S. D. Liberota, C. Ciuti, A. Tredicucci, A. Leitenstorfer, and R. Huber, Sub-cycle switch-on of ultrastrong light-matter interaction, Nature (London) **458**, 178 (2009).

[20] T. Niemczyk, F. Deppe, H. Huebl, E. P. Menzel, F. Hocke, M. J. Schwarz, J. J. García-Ripoll, D. Zueco, T. Hümmer, E. Solano, A. Marx, and R. Gross, Circuit quantum electrodynamics in the ultrastrong-coupling regime, Nat. Phys. **6**, 772 (2010).

[21] Y. Todorov, A. M. Andrews, R. Colombelli, S. De Liberato, C. Ciuti, P. Klang, G. Strasser, and C. Sirtori, Ultrastrong Light-Matter Coupling Regime with Polariton Dots, Phys. Rev. Lett. **105**, 196402 (2010).

[22] P. Forn-Díaz, J. Lisenfeld, D. Marcos, J. J. García-Ripoll, E. Solano, C. J. P. M. Harmans, and J. E. Mooij, Observation of the Bloch-Siegert Shift in a Qubit-Oscillator System in the Ultrastrong Coupling Regime, Phys. Rev. Lett. **105**, 237001 (2010).

[23] T. Schwartz, J. A. Hutchison, C. Genet, and T. W. Ebbesen, Reversible Switching of Ultrastrong Light-Molecule Coupling, Phys. Rev. Lett. **106**, 196405 (2011).

[24] A. J. Hoffman, S. J. Srinivasan, S. Schmidt, L. Spietz, J. Aumentado, H. E. Türeci, and A. A. Houck, Dispersive Photon Blockade in a Superconducting Circuit, Phys. Rev. Lett. **107**, 053602 (2011).

[25] C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, Observation of the dynamical Casimir effect in a superconducting circuit, Nature (London) **479**, 376 (2011).
[26] G. Scalari, C. Maissen, D. Trčinková, D. Hagenmüller, S. De Liberato, C. Ciuti, C. Reichl, D. Schuh, W. Wegscheider, M. Beck, and J. Faist, Ultrastrong Coupling of the cyclotron transition of a 2D electron gas to a THz metamaterial, Science **335**, 1323 (2012).
[27] S. Gambino, M. Mazzeo, A. Genco, O. D. Stefano, S. Savasta, S. Patanè, D. Ballarini, F. Mangione, G. Lerario, D. Sanvitto, and G. Gigli, Exploring light-matter interaction phenomena under ultrastrong coupling regime, ACS Photonics **1**, 1042 (2014).
[28] P. Forn-Díaz, J. J. García-Ripoll, B. Peropadre, J.-L. Orgiazzi, M. A. Yurtalan, R. Belyansky, C. M. Wilson, and A. Lupascu, Ultrastrong coupling of a single artificial atom to an electromagnetic continuum in the nonperturbative regime, Nat. Phys. **13**, 39 (2017).
[29] A. F. Kockum, A. Miranowicz, S. De Liberato, S. Savasta, and F. Nori, Ultrastrong coupling between light and matter, Nat. Rev. Phys. **1**, 19 (2019).
[30] C. Ciuti and I. Carusotto, Input-output theory of cavities in the ultrastrong coupling regime: The case of time-independent cavity parameters, Phys. Rev. A **74**, 033811 (2006).
[31] F. Deppe, M. Mariantoni, E. P. Menzel, A. Marx, S. Saito, K. Kakuyanagi, H. Tanaka, T. Meno, K. Semba, H. Takayanagi, E. Solano, and R. Gross, Two-photon probe of the Jaynes-Cummings model and controlled symmetry breaking in circuit QED, Nat. Phys. **4**, 686 (2008).
[32] A. Fedorov, A. K. Feofanov, P. Macha, P. Forn-Díaz, C. J. P. M. Harmans, and J. E. Mooij, Strong Coupling of a Quantum Oscillator to a Flux Qubit at Its Symmetry Point, Phys. Rev. Lett. **105**, 060503 (2010).
[33] A. Ridolfo, M. Leib, S. Savasta, and M. J. Hartmann, Photon Blockade in the Ultrastrong Coupling Regime, Phys. Rev. Lett. **109**, 193602 (2012).
[34] P. Nataf and C. Ciuti, Vacuum Degeneracy of a Circuit QED System in the Ultrastrong Coupling Regime, Phys. Rev. Lett. **104**, 023601 (2010).
[35] C. Ciuti, G. Bastard, and I. Carusotto, Quantum vacuum properties of the intersubband cavity polariton field, Phys. Rev. B **72**, 115303 (2005).
[36] S. Ashhab and F. Nori, Qubit-oscillator systems in the ultrastrong-coupling regime and their potential for preparing nonclassical states, Phys. Rev. A **81**, 042311 (2010).
[37] F. Beaudoin, J. M. Gambetta, and A. Blais, Dissipation and ultrastrong coupling in circuit QED, Phys. Rev. A **84**, 043832 (2011).
[38] Z. H. Wang, Y. Li, D. L. Zhou, C. P. Sun, and P. Zhang, Single-photon scattering on a strongly dressed atom, Phys. Rev. A **86**, 023824 (2012).
[39] A. P. Hines, C. M. Dawson, R. H. McKenzie, and G. J. Milburn, Entanglement and bifurcations in Jahn-Teller models, Phys. Rev. A **70**, 022303 (2004).
[40] E. K. Irish, J. Gea-Banacloche, I. Martin, and K. C. Schwab, Dynamics of a two-level system strongly coupled to a high-frequency quantum oscillator, Phys. Rev. B **72**, 195410 (2005); E. K. Irish, Generalized Rotating-Wave Approximation for Arbitrarily Large Coupling, Phys. Rev. Lett. **99**, 173601 (2007).
[41] J. Larson, Dynamics of the Jaynes-Cummings and Rabi models: Old wine in new bottles, Phys. Scr. **76**, 146 (2007).
[42] J. Bourassa, J. M. Gambetta, A. A. Abdumalikov, Jr., O. Astafiev, Y. Nakamura, and A. Blais, Ultrastrong coupling regime of cavity QED with phase-biased flux qubits, Phys. Rev. A **80**, 032109 (2009).
[43] D. Zueco, G. M. Reuther, S. Kohler, and P. Hänggi, Qubit-oscillator dynamics in the dispersive regime: Analytical theory beyond the rotating-wave approximation, Phys. Rev. A **80**, 033846 (2009).
[44] C. P. Meaney, T. Duty, R. H. McKenzie, and G. J. Milburn, Jahn-Teller instability in dissipative quantum systems, Phys. Rev. A **81**, 043805 (2010).
[45] I. Lizuain, J. Casanova, J. J. García-Ripoll, J. G. Muga, and E. Solano, Zeno physics in ultrastrong-coupling circuit QED, Phys. Rev. A **81**, 062131 (2010).
[46] R. Stassi, A. Ridolfo, O. Di Stefano, M. J. Hartmann, and S. Savasta, Spontaneous Conversion from Virtual to Real Photons in the Ultrastrong-Coupling Regime, Phys. Rev. Lett. **110**, 243601 (2013).
[47] C. Maissen, G. Scalari, F. Valmorra, M. Beck, J. Faist, S. Cibella, R. Leoni, C. Reichl, C. Charpentier, and W. Wegscheider, Ultrastrong coupling in the near field of complementary split-ring resonators, Phys. Rev. B **90**, 205309 (2014).
[48] E. Sanchez-Burillo, D. Zueco, J. J. Garcia-Ripoll, and L. Martin-Moreno, Scattering in the Ultrastrong Regime: Nonlinear Optics with One Photon, Phys. Rev. Lett. **113**, 263604 (2014).
[49] G. Scalari, C. Maissen, S. Cibella, R. Leoni, P. Carelli, F. Valmorra, M. Beck, and J. Faist, Superconducting complementary metasurfaces for THz ultrastrong light-matter coupling, New J. Phys. **16**, 033005 (2014).
[50] K. K. W. Ma and C. K. Law, Three-photon resonance and adiabatic passage in the large-detuning Rabi model, Phys. Rev. A **92**, 023842 (2015).
[51] L. Garziano, V. Macrì, R. Stassi, O. Di Stefano, F. Nori, and S. Savasta, One Photon Can Simultaneously Excite Two or More Atoms, Phys. Rev. Lett. **117**, 043601 (2016).
[52] Q. Bin, X.-Y. Lü, S.-W. Bin, G.-L. Zhu, and Y. Wu, Single-photon-induced two qubits excitation without breaking parity symmetry, Opt. Express **25**, 31718 (2017).
[53] P. Nataf and C. Ciuti, Protected Quantum Computation with Multiple Resonators in Ultrastrong Coupling Circuit QED, Phys. Rev. Lett. **107**, 190402 (2011).
[54] N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, Open quantum systems with local and collective incoherent processes: Efficient numerical simulations using permutational invariance, Phys. Rev. A **98**, 063815 (2018).
[55] R. Stassi and F. Nori, Long-lasting quantum memories: Extending the coherence time of superconducting artificial atoms in the ultrastrong-coupling regime, Phys. Rev. A **97**, 033823 (2018).
[56] L.-L. Zheng, T.-S. Yin, Q. Bin, X.-Y. Lü, and Y. Wu, Single-photon-induced phonon blockade in a hybrid spin-optomechanical system, Phys. Rev. A **99**, 013804 (2019).
[57] Y. X. Liu, J. Q. You, L. F. Wei, C. P. Sun, and F. Nori, Optical Selection Rules and Phase-Dependent Adiabatic State Control in a Superconducting Quantum Circuit, Phys. Rev. Lett. **95**, 087001 (2005).

[58] R. Stassi, V. Macrì, A. F. Kockum, O. Di Stefano, A. Miranowicz, S. Savasta, and F. Nori, Quantum nonlinear optics without photons, Phys. Rev. A **96**, 023818 (2017).

[59] J. Casanova, G. Romero, I. Lizuain, J. J. García-Ripoll, and E. Solano, Deep Strong Coupling Regime of the Jaynes-Cummings Model, Phys. Rev. Lett. **105**, 263603 (2010).

[60] D. Braak, Integrability of the Rabi Model, Phys. Rev. Lett. **107**, 100401 (2011).

[61] M.-J. Hwang, R. Puebla, and M. B. Plenio, Quantum Phase Transition and Universal Dynamics in the Rabi Model, Phys. Rev. Lett. **115**, 180404 (2015).

[62] L.-L. Zheng, X.-Y. Lü, Q. Bin, Z.-M. Zhan, S. Li, and Y. Wu, Switchable dynamics in the deep-strong-coupling regime, Phys. Rev. A **98**, 023863 (2018).

[63] M. Hofheinz, H. Wang, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, D. Sank, J. Wenner, J. M. Martinis, and A. N. Cleland, Synthesizing arbitrary quantum states in a superconducting resonator, Nature (London) **459**, 546 (2009).

[64] K. Hepp and E. H. Lieb, On the superradiant phase transition for molecules in a quantized radiation field: The Dicke maser model, Ann. Phys. (NY) **76**, 360 (1973).

[65] Y. K. Wang and F. T. Hioe, Phase Transition in the Dicke Model of Superradiance, Phys. Rev. A **7**, 831 (1973).

[66] M. Bina, G. Romero, J. Casanova, J. J. García-Ripoll, A. Lulli, and E. Solano, Solvable model of dissipative dynamics in the deep strong coupling regime, Eur. Phys. J. Spec. Top. **203**, 207 (2012).

[67] J. Peng, Z. Z. Ren, D. Braak, G. J. Guo, G. X. Ju, X. Zhang, and X. Y. Guo, Solution of the two-qubit quantum Rabi model and its exceptional eigenstates, J. Phys. A: Math. Theor. **47**, 265303 (2014).

[68] M. Bina, S. M. Felis, and S. Olivares, Entanglement generation in the ultra-strongly coupled Rabi model, Int. J. Quantum Inf. **12**, 1560016 (2014).