Schwinger effect of a relativistic boson entangled with a qubit

Yujie Li,[1] Qingqing Mao,[1] and Yu Shi[1,2,*]
[1]*Department of Physics & State Key Laboratory of Surface Physics, Fudan University, Shanghai 200433, China*
[2]*Collaborative Innovation Center of Advanced Microstructures, Fudan University, Shanghai 200433, China*

(Received 2 January 2019; published 25 March 2019)

We use the concept of quantum entanglement to analyze the Schwinger effect on an entangled state of a qubit and a boson mode coupled with the electric field. As a consequence of the Schwinger production of particle-antiparticle pairs, the electric field decreases both the correlation and the entanglement between the qubit and the particle mode. This work exposes a profound difference between bosons and fermions. In the boson case, entanglement between the qubit and the antiparticle mode cannot be caused by the Schwinger effect on the preexisting entanglement between the qubit and the particle mode, though correlation can. In the fermion case, both can.

DOI: 10.1103/PhysRevA.99.032340

I. INTRODUCTION

In recent years we have witnessed the application of the concepts and measures of quantum entanglement to various areas of quantum sciences. The measures of quantum entanglement can well characterize quantum correlations without using any observable. Studies of quantum entanglement have been mostly in the realm of nonrelativistic quantum mechanics, in which the particles are eternal. New features of quantum entanglement appear in relativistic quantum field theory, in which a particle is not eternal. In field theory, one method is to partition the system in terms of the modes [1].

Known as the Schwinger effect, in a strong electromagnetic field, the vacuum decays into particle-antiparticle pairs [2], and likewise, a particle becomes a superposition state involving both particles and antiparticles. Many experimental efforts have been made to observe the Schwinger effect [3], but they have not yet been successful, because the rate is very low.

In this paper, the question we address is how the correlation and the entanglement between a qubit and a bosonic particle are inherited by those between the qubit on one hand and the particles and antiparticles generated by the Schwinger effect on the other. Here the qubit is a simple representation of another particle uncoupled with the electric field.

Quantum entanglement in the Schwinger effect of the Dirac or Klein-Gordon field, between a subsystem and the rest of the system, as measured by the von Neumann entropy of the reduced density matrix, was calculated [4,5]. Pairwise correlation and entanglement were also studied for the Dirac field [6], by using mutual information and logarithmic negativity as the measures.

Pairwise correlation and entanglement are between two parts A and B, generically described in terms of the density matrix ρ_{AB}, obtained by tracing out other parts sharing a pure state. ρ_{AB} usually represents a mixed state, with the pure state being a special case. The reduced density matrix of A is $\rho_A \equiv \mathrm{Tr}_B(\rho_{AB})$; similarly, $\rho_B \equiv \mathrm{Tr}_A(\rho_{AB})$. The mutual information in ρ_{AB} is [7]

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \tag{1}$$

where $S(\rho) \equiv -\mathrm{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy of ρ. $I(\rho_{AB}) = 0$ if ρ_{AB} is a product pure state. Hence $I(\rho_{AB})$ measures a kind of distance from product pure states, containing both quantum entanglement and classical correlation. The logarithmic negativity $N(\rho_{AB})$, which is a measure of the quantum entanglement between A and B in ρ_{AB}, is defined as [8]

$$N(\rho_{AB}) \equiv \log_2 \left\| \rho_{AB}^{T_A} \right\|, \tag{2}$$

where $\|\rho_{AB}^{T_A}\|$ is the sum of the absolute values of the eigenvalues of the partial transpose ρ^{T_A} of the original density matrix ρ_{AB} with respect to subsystem A. The partial transpose can also be made with respect to B, without changing the result of $N(\rho_{AB})$.

In this paper, we consider an initial state of a boson entangled with a qubit, which is then transformed, via the Schwinger effect, to a superposition of different number states of the particle and antiparticle modes. We study pairwise mutual information and quantum entanglement in the final state, between a qubit and the particle out-mode and those between the qubit and the antiparticle out-mode. An introduction to the Schwinger effect is made in Sec. II. The quantum state is described in Sec. III. The correlation and the entanglement between the qubit and the particle mode $\mathbf{q}$ are calculated in Sec. IV. The correlation and the entanglement between the qubit and the antiparticle mode $-\mathbf{q}$ are calculated in Sec. V. Then the effect of a pulsed electric field is discussed in Sec. VI. A summary is made in Sec. VII.

II. SCHWINGER EFFECT IN A CONSTANT ELECTRIC FIELD

Consider a scalar field $\phi(t, x)$ describing the bosons of mass m and charge q, coupled with a constant electric field E_0 along the z direction, satisfying the Klein-Gordon equation

*yushi@fudan.edu.cn

2469-9926/2019/99(3)/032340(8)

in the four-dimensional Minkowski space,

$$[(\partial_\mu - ieA_\mu)(\partial^\mu - ieA^\mu) + m^2]\phi(t,x) = 0, \quad (3)$$

where $A_\mu = (0,0,0,-E_0 t)$, and $\phi(t,x)$ is the scalar field and can be expanded in terms of the mode functions as $\phi(t,x) = \sum_{\mathbf{k}}[a_{\mathbf{k}}\phi_{\mathbf{k}}(t,x) + b^\dagger_{\mathbf{k}}\phi^*_{\mathbf{k}}(t,x)]$, where $\mathbf{k}$ denotes the momentum, $a_{\mathbf{k}}$ is the annihilation operator of the particle, and $b^\dagger_{\mathbf{k}}$ is the creation operator of the antiparticle. Practically the constant electric field can be regarded as the infinite-duration limit of a pulsed electric field, which is discussed in Sec. VI.

The Bogoliubov transformation between the in-modes and the out-modes, for $t_{\rm in} = -\infty$ and $t_{\rm out} = +\infty$, respectively, is

$$\phi^{\rm in}_{\mathbf{k}} = \alpha_{\mathbf{k}}\phi^{\rm out}_{\mathbf{k}} + \beta_{\mathbf{k}}\phi^{\rm out*}_{-\mathbf{k}}, \quad (4)$$

where $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}}$ are Bogoliubov coefficients [4,9,10],

$$\alpha_{\mathbf{k}} = \frac{\sqrt{2\pi}}{\Gamma(-\nu)}e^{\frac{-i\pi(\nu+1)}{2}}, \quad \beta_{\mathbf{k}} = e^{-i\pi\nu}, \quad (5)$$

with $\nu = -\frac{1}{2} - i\frac{\mu}{2}$, $\mu = \frac{k_\perp^2+m^2}{eE_0}$, satisfying $|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2 = 1$. The corresponding annihilation and creation operators of the in-modes and the out-modes are related as

$$a^{\rm in}_{\mathbf{k}} = \alpha^*_{\mathbf{k}}a^{\rm out}_{\mathbf{k}} - \beta^*_{\mathbf{k}}b^{\rm out\dagger}_{-\mathbf{k}}, \quad (6)$$

$$b^{\rm in}_{\mathbf{k}} = \alpha^*_{\mathbf{k}}b^{\rm out}_{\mathbf{k}} - \beta^*_{\mathbf{k}}a^{\rm out\dagger}_{-\mathbf{k}}. \quad (7)$$

Consequently the in-vacuum state for each mode becomes a superposition state of the out-modes [4,9],

$$|0_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle^{\rm in} = \frac{1}{\alpha_{\mathbf{k}}}\sum_{n=0}^{\infty}\left(\frac{\beta^*_{\mathbf{k}}}{\alpha^*_{\mathbf{k}}}\right)^n |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle^{\rm out}, \quad (8)$$

where $0_{\mathbf{k}}$ represents that the particle number of mode $\mathbf{k}$ is 0, $0_{-\mathbf{k}}$ represents that the particle number of mode $-\mathbf{k}$ is 0, $n_{\mathbf{k}}$ represents that the number of particles occupying mode $\mathbf{k}$ is n, and $n_{-\mathbf{k}}$ represents that the number of antiparticles occupying mode $-\mathbf{k}$ is n. Similar notations are used throughout the paper. Equation (8) indicates the distribution of the created particles and antiparticles due to the Schwinger effect when an electric field is applied.

Similarly, from $|1_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle^{\rm in} = a^{\rm in\dagger}_{\mathbf{k}}|0_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle^{\rm in}$, one obtains

$$|1_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle^{\rm in} = \frac{1}{|\alpha_{\mathbf{k}}|^2}\sum_{n=0}^{\infty}\left(\frac{\beta^*_{\mathbf{k}}}{\alpha^*_{\mathbf{k}}}\right)^n \sqrt{n+1}|(n+1)_{\mathbf{k}}, n_{-\mathbf{k}}\rangle^{\rm out}, \quad (9)$$

which indicates the distribution of the created particles and antiparticles resulting from the effect of the electric field on the one-particle state. We refer to this also as the Schwinger effect.

III. THE INITIAL ENTANGLED STATE

Now we investigate the influence of an electric field on the state of a qubit σ entangled with a boson of momentum $\mathbf{q}$, which is an excitation of the scalar field discussed above,

$$|\Phi_{\sigma,\mathbf{q}}\rangle = \varepsilon|\uparrow\rangle|0_{\mathbf{q}}\rangle^{\rm in} + \sqrt{1-\varepsilon^2}|\downarrow\rangle|1_{\mathbf{q}}\rangle^{\rm in}, \quad (10)$$

where ε is a coefficient and the basis states of the qubit are denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$. Obviously, the von Neumann entropies of the reduced matrices ρ_σ and $\rho_{\mathbf{q}}$ are both equal to

$$S(\varepsilon) = -\varepsilon^2\log_2\varepsilon^2 - (1-\varepsilon^2)\log_2(1-\varepsilon^2). \quad (11)$$

Being a pure state, the von Neumann entropy of $|\Phi_{\sigma,\mathbf{q}}\rangle$ is 0; therefore the mutual information is

$$I(\Phi_{\sigma,\mathbf{q}}) = 2S(\varepsilon). \quad (12)$$

The entanglement entropy, characterizing the entanglement between the qubit σ and the in-mode $\mathbf{q}$, is just $S(\varepsilon)$.

With the mode $-\mathbf{q}$ also considered, in terms of the in-modes, $|\Phi_{\sigma,\mathbf{q}}\rangle$ can be rewritten as

$$|\Phi_{\sigma,\mathbf{q},-\mathbf{q}}\rangle^{\rm in} = (\varepsilon|\uparrow\rangle|0_{\mathbf{q}}\rangle^{\rm in} + \sqrt{1-\varepsilon^2}|\downarrow\rangle|1_{\mathbf{q}}\rangle^{\rm in})|0_{-\mathbf{q}}\rangle^{\rm in}. \quad (13)$$

Because of the Bogoliubov transformation given in Eqs. (8) and (9), one obtains

$$|\Phi_{\sigma,\mathbf{q},-\mathbf{q}}\rangle^{\rm in} = \frac{\varepsilon}{\alpha_{\mathbf{q}}}\sum_{n=0}^{\infty}\frac{\beta^{*n}_{\mathbf{q}}}{\alpha^{*n}_{\mathbf{q}}}|\uparrow\rangle|n_{\mathbf{q}}, n_{-\mathbf{q}}\rangle^{\rm out} + \frac{\sqrt{1-\varepsilon^2}}{|\alpha_{\mathbf{q}}|^2} \times \sum_{n=0}^{\infty}\frac{\beta^{*n}_{\mathbf{q}}}{\alpha^{*n}_{\mathbf{q}}}\sqrt{n+1}|\downarrow\rangle|(n+1)_{\mathbf{q}}, n_{-\mathbf{q}}\rangle^{\rm out}. \quad (14)$$

The density matrix $\rho_{\sigma,\mathbf{q},-\mathbf{q}} = |\Phi_{\sigma,\mathbf{q},-\mathbf{q}}\rangle^{\rm in\,in}\langle\Phi_{\sigma,\mathbf{q},-\mathbf{q}}|$ is thus

$$\begin{aligned}\rho_{\sigma,\mathbf{q},-\mathbf{q}} = {}& \frac{\varepsilon^2}{|\alpha_{\mathbf{q}}|^2}\sum_{n,m=0}^{\infty}\frac{\beta^{*n}_{\mathbf{q}}\beta^m_{\mathbf{q}}}{\alpha^{*n}_{\mathbf{q}}\alpha^m_{\mathbf{q}}}|\uparrow, n_{\mathbf{q}}, n_{-\mathbf{q}}\rangle\langle\uparrow, m_{\mathbf{q}}, m_{-\mathbf{q}}| + \frac{\varepsilon\sqrt{1-\varepsilon^2}}{|\alpha_{\mathbf{q}}|^2\alpha_{\mathbf{q}}}\sum_{n,m=0}^{\infty}\frac{\beta^{*n}_{\mathbf{q}}\beta^m_{\mathbf{q}}}{\alpha^{*n}_{\mathbf{q}}\alpha^m_{\mathbf{q}}}\sqrt{m+1}|\uparrow, n_{\mathbf{q}}, n_{-\mathbf{q}}\rangle\langle\downarrow, (m+1)_{\mathbf{q}}, m_{-\mathbf{q}}| \\ & + \frac{\varepsilon\sqrt{1-\varepsilon^2}}{|\alpha_{\mathbf{q}}|^2\alpha^*_{\mathbf{q}}}\sum_{n,m=0}^{\infty}\frac{\beta^{*n}_{\mathbf{q}}\beta^m_{\mathbf{q}}}{\alpha^{*n}_{\mathbf{q}}\alpha^m_{\mathbf{q}}}\sqrt{n+1}|\downarrow, (n+1)_{\mathbf{q}}, n_{-\mathbf{q}}\rangle\langle\uparrow, m_{\mathbf{q}}, m_{-\mathbf{q}}| + \frac{1-\varepsilon^2}{|\alpha_{\mathbf{q}}|^4}\sum_{n,m=0}^{\infty}\frac{\beta^{*n}_{\mathbf{q}}\beta^m_{\mathbf{q}}}{\alpha^{*n}_{\mathbf{q}}\alpha^m_{\mathbf{q}}}\sqrt{(n+1)(m+1)} \\ & \times|\downarrow, (n+1)_{\mathbf{q}}, n_{-\mathbf{q}}\rangle\langle\downarrow, (m+1)_{\mathbf{q}}, m_{-\mathbf{q}}|, \end{aligned} \quad (15)$$

which indicates that the Bogoliubov transformation causes the in-mode $\mathbf{q}$ to be replaced by the out-modes $\mathbf{q}$ and $-\mathbf{q}$. How the original correlation and entanglement are inherited between the qubit and these out-modes is investigated below. For brevity, we have omitted the superscript "out."

IV. CORRELATION AND ENTANGLEMENT BETWEEN THE QUBIT σ AND THE OUT-MODE q

We first study the correlation and the entanglement between the qubit σ and the out-mode $\mathbf{q}$. Tracing out the out-mode $-\mathbf{q}$, we obtain the reduced density matrix of the qubit σ and $\mathbf{q}$, $\rho_{\sigma,\mathbf{q}} = \mathrm{Tr}_{-\mathbf{q}}(\rho_{\sigma,\mathbf{q},-\mathbf{q}})$, as

$$\rho_{\sigma,\mathbf{q}} = \frac{\varepsilon^2}{|\alpha_\mathbf{q}|^2}\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}|\uparrow,n_\mathbf{q}\rangle\langle\uparrow,n_\mathbf{q}| + \frac{\varepsilon\sqrt{1-\varepsilon^2}}{|\alpha_\mathbf{q}|^2\alpha_\mathbf{q}} \times\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\sqrt{n+1}|\uparrow,n_\mathbf{q}\rangle\langle\downarrow,(n+1)_\mathbf{q}| + \frac{\varepsilon\sqrt{1-\varepsilon^2}}{|\alpha_\mathbf{q}|^2\alpha^*_\mathbf{q}} \times\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\sqrt{n+1}|\downarrow,(n+1)_\mathbf{q}\rangle\langle\uparrow,n_\mathbf{q}| + \frac{1-\varepsilon^2}{|\alpha_\mathbf{q}|^4} \times\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}(n+1)|\downarrow,(n+1)_\mathbf{q}\rangle\langle\downarrow,(n+1)_\mathbf{q}|. \quad (16)$$

In the subspace of $\{|\uparrow,n_\mathbf{q}\rangle, |\downarrow,(n+1)_\mathbf{q}\rangle\}$ $(n=0,1,2,\ldots)$, $\rho_{\sigma,\mathbf{q}}$ is a block matrix, with nonzero eigenvalues:

$$\frac{1}{|\alpha_\mathbf{q}|^2}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\left[\varepsilon^2 + \frac{(n+1)(1-\varepsilon^2)}{|\alpha_\mathbf{q}|^2}\right]. \quad (17)$$

Tracing out the out-mode $\mathbf{q}$ in $\rho_{\sigma,\mathbf{q}}$ yields $\rho_\sigma = \mathrm{Tr}_\mathbf{q}(\rho_{\sigma,\mathbf{q}})$, which is

$$\rho_\sigma = \varepsilon^2|\uparrow\rangle\langle\uparrow| + (1-\varepsilon^2)|\downarrow\rangle\langle\downarrow|, \quad (18)$$

with eigenvalues ε^2 and $1-\varepsilon^2$. This remains unchanged from the reduced density matrix of the qubit σ obtained from $|\Phi_{\sigma,\mathbf{q}}\rangle$ in Eq. (10), as nothing is done on the qubit σ.

Tracing out the qubit σ in $\rho_{\sigma,\mathbf{q}}$ yields $\rho_\mathbf{q} = \mathrm{Tr}_\sigma\rho(\sigma,\mathbf{q})$, which is

$$\rho_\mathbf{q} = \frac{1}{|\alpha_\mathbf{q}|^2}\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\left[\varepsilon^2 + \frac{n(1-\varepsilon^2)}{|\beta_\mathbf{q}|^2}\right]|n_\mathbf{q}\rangle\langle n_\mathbf{q}|, \quad (19)$$

with eigenvalues

$$\frac{1}{|\alpha_\mathbf{q}|^2}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\left[\varepsilon^2 + \frac{n(1-\varepsilon^2)}{|\beta_\mathbf{q}|^2}\right] \ (n=0,1,2,\ldots). \quad (20)$$

Then we obtain the mutual information $I(\rho_{\sigma,\mathbf{q}}) = S(\rho_\sigma) + S(\rho_\mathbf{q}) - S(\rho_{\sigma,\mathbf{q}})$ as

$$I(\rho_{\sigma,\mathbf{q}}) = -\varepsilon^2\log_2\varepsilon^2 - (1-\varepsilon^2)\log_2(1-\varepsilon^2) - \sum_{n=0}^{\infty}\frac{|\beta_\mathbf{q}|^{2n}}{|\alpha_\mathbf{q}|^{2(n+1)}}\left[\varepsilon^2 + \frac{n(1-\varepsilon^2)}{|\beta_\mathbf{q}|^2}\right]\log_2\left[\frac{|\beta_\mathbf{q}|^{2n}}{|\alpha_\mathbf{q}|^{2(n+1)}}\left(\varepsilon^2 + \frac{n(1-\varepsilon^2)}{|\beta_\mathbf{q}|^2}\right)\right] + \sum_{n=0}^{\infty}\frac{|\beta_\mathbf{q}|^{2n}}{|\alpha_\mathbf{q}|^{2(n+1)}}\left[\varepsilon^2 + \frac{(n+1)(1-\varepsilon^2)}{|\alpha_\mathbf{q}|^2}\right]\log_2\left[\frac{|\beta_\mathbf{q}|^{2n}}{|\alpha_\mathbf{q}|^{2(n+1)}}\left(\varepsilon^2 + \frac{(n+1)(1-\varepsilon^2)}{|\alpha_\mathbf{q}|^2}\right)\right], \quad (21)$$

which depends on the coefficient parameter ε and the strength of the electric field E_0. When $E_0 = 0$, $I(\rho_{\sigma,\mathbf{q}})$ reduces to $S(\varepsilon)$.

The dependence of the mutual information $I(\rho_{\sigma,\mathbf{q}})$ on the electric field E_0 and the parameter ε is shown in Fig. 1. For a fixed value of ε, $I(\rho_{\sigma,\mathbf{q}})$ monotonically decreases with the increase of the electric field E_0 and asymptotically approaches a certain nonvanishing value independent of E_0. The closer to $\frac{1}{\sqrt{2}}$ ε is, the quicker $I(\rho_{\sigma,\mathbf{q}})$ decreases with the increase of E_0 when E_0 is small, and the larger the asymptotic value of $I(\rho_{\sigma,\mathbf{q}})$ is. For any given value of E_0, the farther to $\frac{1}{\sqrt{2}}$ ε is, the smaller $I(\rho_{\sigma,\mathbf{q}})$ is. The mutual information $I(\rho_{\sigma,\mathbf{q}})$ becomes zero as $\varepsilon = 0$ or 1, in which case the mutual information vanishes even in the absence of the electric field.

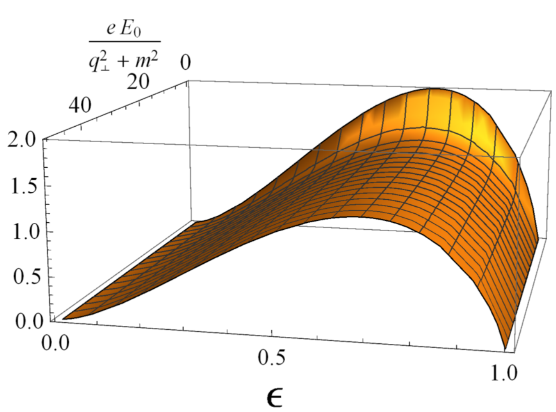

FIG. 1. The mutual information $I(\rho_{\sigma,\mathbf{q}})$ as a function of the dimensionless parameters $\frac{eE_0}{q_\perp^2+m^2}$ and ε, where E_0 is the strength of the constant electric field and ε is the coefficient parameter of the initial entangled state.

We use logarithmic negativity to measure the entanglement. After making the partial transpose of the density matrix $\rho_{\sigma,\mathbf{q}}$ with respect to σ, we obtain

$$\rho^{T_\sigma}_{\sigma,\mathbf{q}} = \frac{\varepsilon^2}{|\alpha_\mathbf{q}|^2}\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}|\uparrow,n_\mathbf{q}\rangle\langle\uparrow,n_\mathbf{q}| + \frac{\varepsilon\sqrt{1-\varepsilon^2}}{|\alpha_\mathbf{q}|^2\alpha_\mathbf{q}} \times\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\sqrt{n+1}|\downarrow,n_\mathbf{q}\rangle\langle\uparrow,(n+1)_\mathbf{q}| + \frac{\varepsilon\sqrt{1-\varepsilon^2}}{|\alpha_\mathbf{q}|^2\alpha^*_\mathbf{q}} \times\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}\sqrt{n+1}|\uparrow,(n+1)_\mathbf{q}\rangle\langle\downarrow,n_\mathbf{q}| + \frac{1-\varepsilon^2}{|\alpha_\mathbf{q}|^4} \times\sum_{n=0}^{\infty}\left|\frac{\beta_\mathbf{q}}{\alpha_\mathbf{q}}\right|^{2n}(n+1)|\downarrow,(n+1)_\mathbf{q}\rangle\langle\downarrow,(n+1)_\mathbf{q}|, \quad (22)$$

which is a block matrix in the subspace of $\{|\uparrow,(n+1)_\mathbf{q}\rangle, |\downarrow,n_\mathbf{q}\rangle\}$ $(n=0,1,2,\ldots)$. Therefore the eigenvalues of $\rho^{T_\sigma}_{\sigma,\mathbf{q}}$

are

$$\frac{\varepsilon^2}{|\alpha_{\mathbf{q}}|^2}, \quad \frac{1}{2|\alpha_{\mathbf{q}}|^2}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\left[\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^2\varepsilon^2+\frac{n(1-\varepsilon^2)}{|\beta_{\mathbf{q}}|^2}\pm\sqrt{\left(\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^2\varepsilon^2+\frac{n(1-\varepsilon^2)}{|\beta_{\mathbf{q}}|^2}\right)^2+\frac{4\varepsilon^2(1-\varepsilon^2)}{|\alpha_{\mathbf{q}}|^2}}\right] \quad (n=0,1,2,\ldots). \tag{23}$$

Thus the logarithmic negativity is

$$N(\rho_{\sigma,\mathbf{q}})=\log_2\left[\frac{\varepsilon^2}{|\alpha_{\mathbf{q}}|^2}+\sum_{n=0}^{\infty}\frac{|\beta_{\mathbf{q}}|^{2n}}{|\alpha_{\mathbf{q}}|^{2(n+1)}}\sqrt{\left(\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^2\varepsilon^2+\frac{n(1-\varepsilon^2)}{|\beta_{\mathbf{q}}|^2}\right)^2+\frac{4\varepsilon^2(1-\varepsilon^2)}{|\alpha_{\mathbf{q}}|^2}}\right]. \tag{24}$$

When $E_0=0$, $N(\rho_{\sigma,\mathbf{q}})=\log_2[1+2\varepsilon\sqrt{1-\varepsilon^2}]$.

Figure 2 shows how the logarithmic negativity $N(\rho_{\sigma,\mathbf{q}})$ depends on the strength of the electric field E_0 and the parameter ε. The variation trend of $N(\rho_{\sigma,\mathbf{q}})$ with respect to E_0 and ε is similar to that of $I(\rho_{\sigma,\mathbf{q}})$. But when E_0 is small, $N(\rho_{\sigma,\mathbf{q}})$ decreases more rapidly with the increase of E_0 than $I(\rho_{\sigma,\mathbf{q}})$ does, indicating that the entanglement is more sensitive to the coupling with the electric field. Like $I(\rho_{\sigma,\mathbf{q}})$, $N(\rho_{\sigma,\mathbf{q}})$ monotonically decreases with the increase of the electric field E_0 and approaches a certain nonzero asymptotic value as $E_0\to\infty$.

What if the boson is replaced as a fermion? The fermion counterpart of the present problem can be obtained from the previously studied Schwinger effect of two entangled fermions of momenta **p** and **q** [6], under the constraint that the electric field does not couple the mode **p**, thus fixing the Bogoliubov coefficients of the **p** mode to be $\alpha_{\mathbf{p}}=1$ and $\beta_{\mathbf{p}}=0$, thereby reducing mode **p** to our qubit σ. Then from the analytical expressions, it can be seen that with the increase of E_0 the mutual information and the entanglement between the qubit σ and the fermionic mode **q** both decrease towards 0, instead of the nonvanishing asymptotic values for bosons. The boson-fermion comparison is further discussed in the summary.

It is also interesting to make comparison with the bosons in the Unruh effect [11–13] and near a dilaton black hole [14], with the role of the electric field in our case replaced as the acceleration, but the Bogoliubov coefficient that is the counterpart of $|\beta_{\mathbf{k}}|^2$ can be arbitrarily large, making the entanglement disappear in the limiting cases. In contrast, in our present case, $|\beta_{\mathbf{k}}|^2<1$, consequently the entanglement in $\rho_{\sigma,\mathbf{q}}$ persists as $E_0\to\infty$. One could use the Schwinger effect to design analog gravity experiments.

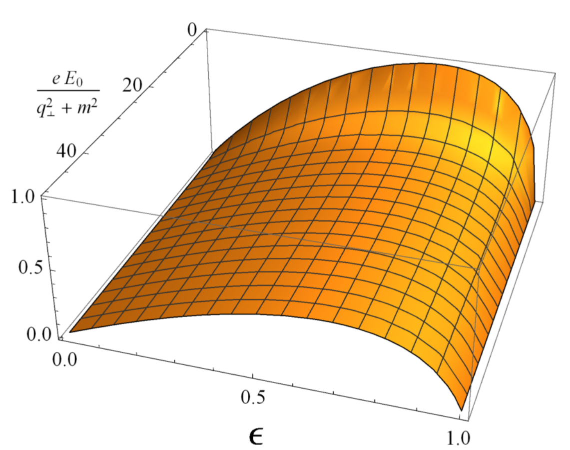

FIG. 2. The logarithmic negativity $N(\rho_{\sigma,\mathbf{q}})$ as a function of the dimensionless parameters $\frac{eE_0}{q_\perp^2+m^2}$ and ε, where E_0 is the strength of the constant electric field and ε is the coefficient parameter of the initial entangled state.

V. CORRELATION AND NONENTANGLEMENT BETWEEN THE QUBIT σ AND THE OUT-MODE $-\mathbf{q}$

Now we calculate the correlation and the entanglement between σ and $-\mathbf{q}$. Tracing out the out-mode **q**, we obtain the reduced density matrix of σ and $-\mathbf{q}$, $\rho_{\sigma,-\mathbf{q}}=\mathrm{Tr}_{\mathbf{q}}(\rho_{\sigma,\mathbf{q},-\mathbf{q}})$, as

$$\begin{aligned}\rho_{\sigma,-\mathbf{q}}=&\frac{\varepsilon^2}{|\alpha_{\mathbf{q}}|^2}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}|\uparrow,n_{-\mathbf{q}}\rangle\langle\uparrow,n_{-\mathbf{q}}|+\frac{\varepsilon\sqrt{1-\varepsilon^2}\beta_{\mathbf{q}}^*}{|\alpha_{\mathbf{q}}|^4}\\&\times\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\sqrt{n+1}|\uparrow,(n+1)_{-\mathbf{q}}\rangle\langle\downarrow,n_{-\mathbf{q}}|\\&+\frac{\varepsilon\sqrt{1-\varepsilon^2}\beta_{\mathbf{q}}}{|\alpha_{\mathbf{q}}|^4}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\sqrt{n+1}|\downarrow,n_{-\mathbf{q}}\rangle\\&\times\langle\uparrow,(n+1)_{-\mathbf{q}}|+\frac{1-\varepsilon^2}{|\alpha_{\mathbf{q}}|^4}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}(n+1)\\&\times|\downarrow,n_{-\mathbf{q}}\rangle\langle\downarrow,n_{-\mathbf{q}}|,\end{aligned} \tag{25}$$

which is a block matrix in the subspace of $\{|\uparrow,n_{-\mathbf{q}}\rangle,|\downarrow,(n-1)_{\mathbf{q}}\rangle\}$ $(n=1,2,3,\ldots)$. Thus the nonzero eigenvalues of $\rho_{\sigma,-\mathbf{q}}$ are

$$\frac{1}{|\alpha_{\mathbf{q}}|^2}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\left[\varepsilon^2+\frac{n(1-\varepsilon^2)}{|\beta_{\mathbf{q}}|^2}\right] \quad (n=0,1,2,\ldots). \tag{26}$$

Tracing out the out-mode $-\mathbf{q}$ in $\rho_{\sigma,-\mathbf{q}}$ yields $\rho_\sigma=\mathrm{Tr}_{-\mathbf{q}}(\rho_{\sigma,-\mathbf{q}})$, which is

$$\rho_\sigma=\varepsilon^2|\uparrow_\sigma\rangle\langle\uparrow_\sigma|+(\downarrow-\varepsilon^2)|\downarrow_\sigma\rangle\langle\downarrow_\sigma|, \tag{27}$$

with eigenvalues ε^2 and $1-\varepsilon^2$.

Tracing out σ in $\rho_{\sigma,-\mathbf{q}}$ yields $\rho_{-\mathbf{q}}=\mathrm{Tr}_\sigma(\rho_{\sigma,-\mathbf{q}})$, which is

$$\rho_{-\mathbf{q}}=\frac{1}{|\alpha_{\mathbf{q}}|^2}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\left[\varepsilon^2+\frac{(n+1)(1-\varepsilon^2)}{|\alpha_{\mathbf{q}}|^2}\right]|n_{-\mathbf{q}}\rangle\langle n_{-\mathbf{q}}|, \tag{28}$$

with eigenvalues

$$\frac{1}{|\alpha_{\mathbf{q}}|^2}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\left[\varepsilon^2+\frac{(n+1)(1-\varepsilon^2)}{|\alpha_{\mathbf{q}}|^2}\right]\ (n=0,1,2,\ldots). \tag{29}$$

According to the definition of the mutual information, we have

$$I(\rho_{\sigma,-\mathbf{q}})=-\varepsilon^2\log_2\varepsilon^2-(1-\varepsilon^2)\log_2(1-\varepsilon^2)+\sum_{n=0}^{\infty}\frac{|\beta_{\mathbf{q}}|^{2n}}{|\alpha_{\mathbf{q}}|^{2(n+1)}}\left[\varepsilon^2+\frac{n(1-\varepsilon^2)}{|\beta_{\mathbf{q}}|^2}\right]\log_2\left[\frac{|\beta_{\mathbf{q}}|^{2n}}{|\alpha_{\mathbf{q}}|^{2(n+1)}}\left(\varepsilon^2+\frac{n(1-\varepsilon^2)}{|\beta_{\mathbf{q}}|^2}\right)\right]$$
$$-\sum_{n=0}^{\infty}\frac{|\beta_{\mathbf{q}}|^{2n}}{|\alpha_{\mathbf{q}}|^{2(n+1)}}\left[\varepsilon^2+\frac{(n+1)(1-\varepsilon^2)}{|\alpha_{\mathbf{q}}|^2}\right]\log_2\left[\frac{|\beta_{\mathbf{q}}|^{2n}}{|\alpha_{\mathbf{q}}|^{2(n+1)}}\left(\varepsilon^2+\frac{(n+1)(1-\varepsilon^2)}{|\alpha_{\mathbf{q}}|^2}\right)\right]. \tag{30}$$

The dependence of the mutual information $I(\rho_{\sigma,-\mathbf{q}})$ on the electric field E_0 and the parameter ε is shown in Fig. 3. $I(\rho_{\sigma,-\mathbf{q}})$ monotonically increases with the increase of the electric field E_0 and asymptotically approaches a certain value independent of E_0. The closer to $\frac{1}{\sqrt{2}}$ ε is, the larger the asymptotic value is. Moreover, when E_0 is small, the closer to $\frac{1}{\sqrt{2}}$ ε is, the quicker $I(\rho_{\sigma,-\mathbf{q}})$ increases with the increase of E_0, and the larger the value of $I(\rho_{\sigma,-\mathbf{q}})$ is.

Now we calculate the logarithmic negativity of $\rho_{\sigma,-\mathbf{q}}$. After making the partial transpose of the density matrix $\rho_{\sigma,-\mathbf{q}}$ with respect to the qubit σ, one obtains

$$\rho^{\mathrm{T}_\sigma}_{\sigma,-\mathbf{q}}=\frac{\varepsilon^2}{|\alpha_{\mathbf{q}}|^2}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}|\uparrow,n_{-\mathbf{q}}\rangle\langle\uparrow,n_{-\mathbf{q}}|+\frac{\varepsilon\sqrt{1-\varepsilon^2}\beta^*_{\mathbf{q}}}{|\alpha_{\mathbf{q}}|^4}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\sqrt{n+1}|\downarrow,(n+1)_{-\mathbf{q}}\rangle\langle\uparrow,n_{-\mathbf{q}}|+\frac{\varepsilon\sqrt{1-\varepsilon^2}\beta_{\mathbf{q}}}{|\alpha_{\mathbf{q}}|^4}$$
$$\times\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\sqrt{n+1}|\uparrow,n_{-\mathbf{q}}\rangle\langle\downarrow,(n+1)_{-\mathbf{q}}|+\frac{1-\varepsilon^2}{|\alpha_{\mathbf{q}}|^4}\sum_{n=0}^{\infty}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}(n+1)|\downarrow,n_{-\mathbf{q}}\rangle\langle\downarrow,n_{-\mathbf{q}}|, \tag{31}$$

which is a block matrix in the subspace of $\{|\uparrow,n_{-\mathbf{q}}\rangle,|\downarrow,(n+1)_{-\mathbf{q}}\rangle\}$, with eigenvalues

$$\frac{1-\varepsilon^2}{|\alpha_{\mathbf{q}}|^4},\quad\frac{1}{2|\alpha_{\mathbf{q}}|^2}\left|\frac{\beta_{\mathbf{q}}}{\alpha_{\mathbf{q}}}\right|^{2n}\left[\varepsilon^2+\frac{(n+2)(1-\varepsilon^2)|\beta_{\mathbf{q}}|^2}{|\alpha_{\mathbf{q}}|^4}\pm\sqrt{\left(\varepsilon^2+\frac{(n+2)(1-\varepsilon^2)|\beta_{\mathbf{q}}|^2}{|\alpha_{\mathbf{q}}|^4}\right)^2-\frac{4\varepsilon^2(1-\varepsilon^2)|\beta_{\mathbf{q}}|^2}{|\alpha_{\mathbf{q}}|^4}}\right]$$
$$\times\ (n=0,1,2,\ldots). \tag{32}$$

Thus the logarithmic negativity of $\rho_{\sigma,-\mathbf{q}}$ is

$$N(\rho_{\sigma,-\mathbf{q}})=\log_2\left[\frac{1-\varepsilon^2}{|\alpha_{\mathbf{q}}|^4}+\sum_{n=0}^{\infty}\frac{|\beta_{\mathbf{q}}|^{2n}}{|\alpha_{\mathbf{q}}|^{2(n+1)}}\left(\varepsilon^2+\frac{(n+2)(1-\varepsilon^2)|\beta_{\mathbf{q}}|^2}{|\alpha_{\mathbf{q}}|^4}\right)\right]=\log_2 1=0, \tag{33}$$

which means that starting from the initial state (13) with any value of the parameter ε, which is entangled between σ and in-mode $\mathbf{q}$, under the action of the electric field, the entanglement between σ and out-mode $-\mathbf{q}$ is never generated.

From the expressions of the mutual information of $\rho_{\sigma,\mathbf{q}}$ and $\rho_{\sigma,-\mathbf{q}}$, we obtain

$$I(\rho_{\sigma,\mathbf{q}})+I(\rho_{\sigma,-\mathbf{q}})=2S(\varepsilon), \tag{34}$$

implying that the Schwinger effect redistributes the total correlation in the initial entangled state into $\rho_{\sigma,\mathbf{q}}$ and $\rho_{\sigma,-\mathbf{q}}$. However, there is no such identity for the logarithmic negativity; hence the Schwinger effect does not redistribute quantum entanglement.

In contrast, in the fermion model studied previously [6], even when reduced to the problem of an uncoupled qubit and the fermion mode coupled with the electric field, redistribution exists in both mutual information and logarithmic negativity. See Eqs. (38) and (39) in Ref. [6], where the fermion mode coupled with the electric field is denoted as $\mathbf{p}$, and the uncoupled mode $\mathbf{q}$ is equivalent to a qubit.

VI. EFFECT OF A PULSED ELECTRIC FIELD

Now we investigate the effect of a pulsed electric field. Consider a Sauter-type electric field $E(t)=E_0\mathrm{sech}^2(t/\tau)$ along the z direction, where τ is the width of the pulsed electric field [15]. The gauge potential A_μ can be chosen as

$$A_\mu=\left[0,0,0,-E_0\tau\tanh\left(\frac{t}{\tau}\right)\right], \tag{35}$$

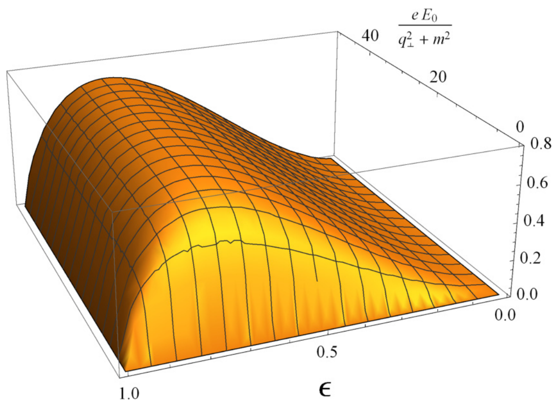

FIG. 3. The mutual information $I(\rho_{\sigma,-\mathbf{q}})$ as a function of the dimensionless parameters $\frac{eE_0}{q_\perp^2+m^2}$ and ε, where E_0 is the strength of the constant electric field and ε is the coefficient parameter of the initial entangled state.

for which the Bogoliubov transformation is given as [4,10]

$$|\alpha_{\mathbf{k}}|^2 = \frac{\cosh\left[\pi\tau\left(\omega_k^{\text{out}}+\omega_k^{\text{in}}\right)\right]+\cosh(2\pi\lambda)}{2\sinh\left(\pi\tau\omega_k^{\text{in}}\right)\sinh\left(\pi\tau\omega_k^{\text{out}}\right)}, \tag{36}$$

$$|\beta_{\mathbf{k}}|^2 = \frac{\cosh\left[\pi\tau\left(\omega_k^{\text{out}}-\omega_k^{\text{in}}\right)\right]+\cosh(2\pi\lambda)}{2\sinh\left(\pi\tau\omega_k^{\text{in}}\right)\sinh\left(\pi\tau\omega_k^{\text{out}}\right)}, \tag{37}$$

where

$$\lambda = \sqrt{(eE_0\tau^2)^2 - \tfrac{1}{4}}, \tag{38}$$

$$\omega_k^{\text{in}} = \sqrt{(k_z + eE_0\tau)^2 + k_\perp^2 + m^2}, \tag{39}$$

$$\omega_k^{\text{out}} = \sqrt{(k_z - eE_0\tau)^2 + k_\perp^2 + m^2}. \tag{40}$$

As $\tau \to 0$ and $E(t) \to 0$, then $|\alpha_{\mathbf{k}}|^2 \to 1$ and $|\beta_{\mathbf{k}}|^2 \to 0$, and the problem is reduced to the case without the electric field. As $\tau \to +\infty$, $E(t) \to E_0$, which means $|\alpha_{\mathbf{k}}|^2$ and $|\beta_{\mathbf{k}}|^2$ reduce to the values in the case of the constant electric field.

The analyses and calculations for $\rho_{\sigma,\mathbf{q}}$ and $\rho_{\sigma,-\mathbf{q}}$ above for the case of a constant electric field can be applied to a pulsed electric field, but with $|\alpha_{\mathbf{k}}|^2$ and $|\beta_{\mathbf{k}}|^2$ now given in Eqs. (36) and (37). Hence the mutual information and logarithmic negativity now depend on not only E_0 but also τ.

In parallel with the above study on a constant electric field, we first investigate the influence of a pulsed electric field on the entanglement and correlation between σ and mode $\mathbf{q}$.

The influence of the pulsed electric field on the mutual information $I(\rho_{\sigma,\mathbf{q}})$ is shown in Fig. 4, which indicates its dependence on the strength E_0 and the width τ of the pulsed electric field. For E_0 smaller than a certain value, with the increase of τ, $I(\rho_{\sigma,\mathbf{q}})$ monotonically decreases and approaches the asymptotic value dependent on E_0. For E_0 larger than a certain value, with the increase of τ, $I(\rho_{\sigma,\mathbf{q}})$ first decreases to a minimum and then increases to a maximum before it finally decreases and approaches asymptotically a value dependent on E_0. The larger E_0 is, the smaller the values of τ corresponding to the minimum and the maximum of $I(\rho_{\sigma,\mathbf{q}})$ are. When τ is small, with the increase of E_0, $I(\rho_{\sigma,\mathbf{q}})$ decreases to a minimum, then increases to a maximum, and finally decreases and approaches asymptotically a certain value independent of E_0. The variation trend of $I(\rho_{\sigma,\mathbf{q}})$ with respect to E_0 is opposite to that of $I(\rho_{\sigma,\mathbf{q}})$ with respect to τ. When τ is smaller than a certain value, the smaller τ is, the larger the values of E_0 corresponding to the minimum and the maximum of $I(\rho_{\sigma,\mathbf{q}})$ are. When τ is larger than a certain value, $I(\rho_{\sigma,\mathbf{q}})$ decreases monotonically with the increase of E_0 and asymptotically approaches a value independent of E_0. For $\varepsilon = 1/\sqrt{2}$, as $\tau \to \infty$, the dependence of $I(\rho_{\sigma,\mathbf{q}})$ on E_0 is the same as that of the case of the constant electric field, as shown in Fig. 1.

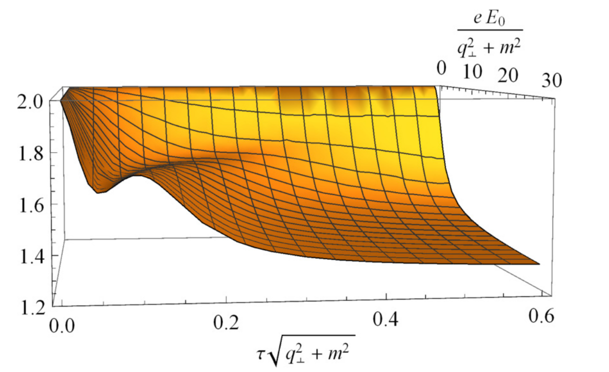

FIG. 4. The mutual information $I(\rho_{\sigma,\mathbf{q}})$ as a function of the dimensionless parameters $\frac{eE_0}{q_\perp^2+m^2}$ and $\tau\sqrt{q_\perp^2+m^2}$. It is assumed that $q_z = \sqrt{q_\perp^2+m^2}$ and $\varepsilon = 1/\sqrt{2}$.

As shown in Fig. 5, the dependence of $N(\rho_{\sigma,\mathbf{q}})$ on E_0 and τ is entirely similar to that of $I(\rho_{\sigma,\mathbf{q}})$, but the values of τ and E_0 corresponding to the minima and maxima of $N(\rho_{\sigma,\mathbf{q}})$ are different from those of $I(\rho_{\sigma,\mathbf{q}})$. For $\varepsilon = 1/\sqrt{2}$, as $\tau \to \infty$, the case of the constant electric field is also recovered, as shown in Fig. 2.

The correlation between σ and $-\mathbf{q}$ is exactly a complement of that between σ and $\mathbf{q}$, as indicated in Eq. (34). Therefore the dependence of $I(\rho_{\sigma,-\mathbf{q}})$ on the pulsed electric field is exactly opposite to that of $I(\rho_{\sigma,\mathbf{q}})$, as shown in Fig. 6.

In the expressions of Bogoliubov coefficients in Eqs. (36) and (37), when $eE_0\tau$ dominates the momenta and mass, E_0

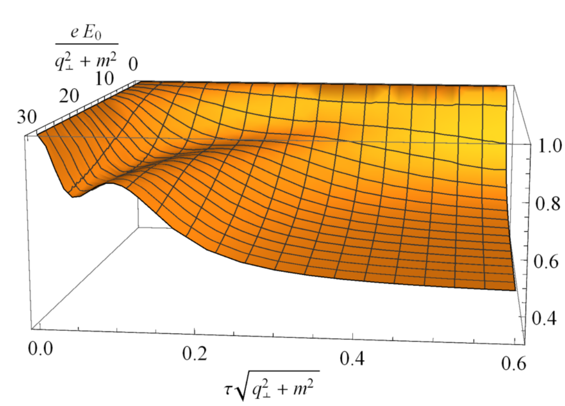

FIG. 5. The logarithmic negativity $N(\rho_{\sigma,\mathbf{q}})$ as a function of the dimensionless parameters $\frac{eE_0}{q_\perp^2+m^2}$ and $\tau\sqrt{q_\perp^2+m^2}$. It is assumed that $q_z = \sqrt{q_\perp^2+m^2}$ and $\varepsilon = 1/\sqrt{2}$.

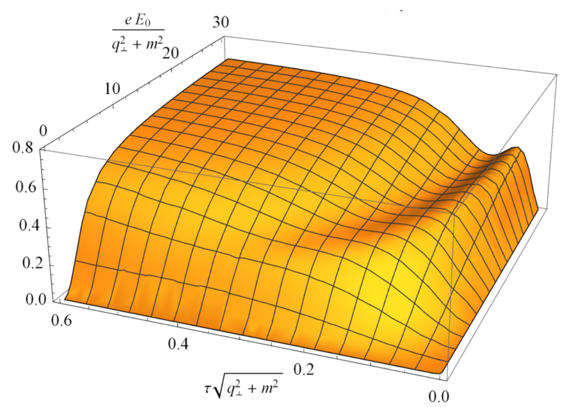

FIG. 6. The mutual information $I(\rho_{\sigma,-\mathbf{q}})$ as a function of the dimensionless parameters $\frac{eE_0}{q_\perp^2+m^2}$ and $\tau\sqrt{q_\perp^2+m^2}$. It is assumed that $q_z = \sqrt{q_\perp^2+m^2}$ and $\varepsilon = 1/\sqrt{2}$.

and τ appear only together as $E_0\tau^2$. Hence the maxima and minima in Figs. 4, 5, and 6 form ridges and valleys with $E_0\tau^2 =$ constant. This explains some features discussed above.

VII. SUMMARY AND DISCUSSION

In this paper, we have considered the Schwinger effect on a state in which a qubit is entangled with a boson mode. We have studied how the total correlation and the quantum entanglement in $\rho_{\sigma,\mathbf{q}}$ and $\rho_{\sigma,-\mathbf{q}}$ depend on the electric field. In the case of a constant electric field, the mutual information $I(\rho_{\sigma,\mathbf{q}})$ decreases with the increase of the strength of the electric field and approaches a certain nonvanishing value. Therefore the total correlation between qubit σ and the out-mode $\mathbf{q}$ never vanishes. Similarly, the logarithmic negativity $N(\rho_{\sigma,\mathbf{q}})$ decreases with the increase of the strength of the electric field and approaches a certain nonvanishing value, implying that the entanglement in $\rho_{\sigma,\mathbf{q}}$ never vanishes even if the strength of the electric field approaches infinity. For $\rho_{\sigma,-\mathbf{q}}$, the mutual information $I(\rho_{\sigma,-\mathbf{q}})$ increases with the increase of the strength of the electric field and asymptotically approaches a certain value. In fact, the sum of $I(\rho_{\sigma,\mathbf{q}})$ and $I(\rho_{\sigma,-\mathbf{q}})$ is a constant determined by the initial state, appearing as a conservation law. However no matter how strong the electric field is, the logarithmic negativity $N(\rho_{\sigma,-\mathbf{q}})$ remains zero, i.e., σ and the out-mode $-\mathbf{q}$ remain unentangled, though the entanglement between σ and mode $\mathbf{q}$ decreases.

We have also studied the Schwinger effect of a pulsed electric field, for which the pulse width plays a crucial role. The minima in the correlation and the entanglement between σ and $\mathbf{q}$, as well as the maxima in the correlation and the entanglement between σ and $-\mathbf{q}$, nearly satisfy $E_0\tau^2 =$ constant. This is well consistent with the case of the constant electric field, which is the limiting case of $\tau \to \infty$, in which the correlation and the entanglement vary monotonically with the electric field strength.

The correlation and the entanglement between a qubit and boson modes in an electric field are different from those between a qubit and fermion modes [6]. In the boson case, with the increase of the electric field strength, the correlation and the entanglement between the qubit and the original particle mode decrease towards nonzero asymptotic values, while the entanglement between the qubit and the antiparticle mode remains zero, though the correlation increases towards an asymptotic value. In the fermion case, with the increase of the electric field strength, the correlation and the entanglement between the qubit and the original particle mode both decrease towards zero, while both the correlation and the entanglement between the qubit and the antiparticle mode increases towards the original values of those between the qubit and the particle mode. In other words, in the fermion case, both correlation and entanglement satisfy conservation laws. Consequently, the electric field transforms the entanglement between the qubit and the fermion mode into that between the qubit and the antifermion mode. But there is no such transformation for bosons. Note that the entanglement transformation here is not entanglement swapping. In the initial state, the entanglement is between the qubit and an in-mode, while in the final state the entanglement is between this qubit and an out-mode. In-modes and out-modes are different sets of basis states, though they approach each other with the decrease of electric field strength.

Qualitatively speaking, this fermion-boson difference originates from the difference in statistics, which leads to different behaviors of Bogoliubov coefficients. In the fermion case, $|\alpha_{\mathbf{k}}|^2 + |\beta_{\mathbf{k}}|^2 = 1$, where $\alpha_{\mathbf{k}}$ is the amplitude of the out-particle-mode $\mathbf{k}$ in the in-mode $\mathbf{k}$, while $\beta_{\mathbf{k}}$ is the amplitude of the out-antiparticle-mode $-\mathbf{k}$ in the in-mode $\mathbf{k}$. Therefore, when $|\alpha_{\mathbf{k}}|^2$ decreases towards 0, $|\beta_{\mathbf{k}}|^2$ is constrained to increase towards 1. In the boson case, there is no such complementary relation, instead $|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2 = 1$. More detailed investigation is needed to expose how the boson-fermion difference emerges.

ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China (Grant No. 11574054).

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