Leggett mode in a two-component Fermi gas with dipolar interactions

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We develop an effective field theory to understand collective modes of a three-dimensional two-component Fermi superfluid with dipolar interparticle interactions, which are modeled by an idealized separable potential. We first examine the phase transition of the system at zero temperature, as the fermionic superfluidity is known to be characterized by two competing order parameters. We find that for strong interactions there exists a regime where the two order parameters are out of phase and coupled, giving rise to an undamped massive Leggett mode. This is in addition to the well-known gapless phonon mode. We show that the Leggett mode can be seen in the spectral function of the in-medium Cooper pairs, and in principle could be measured through Bragg spectroscopy.

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I. INTRODUCTION

Owing to the rapid experimental progress on the control of ultracold gases over the last decade [1–3], there has been significant work done on creating ultracold dipolar atomic gases with large magnetic moments [4–9] and polar molecules with large electronic dipole moments [10–15]. The long-range and anisotropic nature of the dipole-dipole interaction in these systems leads to many fascinating quantum phenomena, such as self-bound droplets in Bose systems [16–18], p-wave and topological superfluidity in Fermi systems [19–21], and quantum chaos [6,22,23].

There are extensive many-body calculations of Fermi dipolar systems [24–27], which mainly focus on the mean-field regime. To deal with the short-range divergence of the dipolar interaction, the most common method of using the two-body T matrix as a way to renormalize the interaction [28] is not tractable in the many-body calculations, since the dipolar interaction couples different partial wave channels [29–31]. A useful strategy for renormalization is to take the Born approximation [32–34], which unfortunately is appropriate in the weakly interacting regime only [35–37]. In this work, we consider an effective *separable* interaction potential that couples different angular momentum channels (i.e., $|l - l'| \leqslant 2$) in the strongly interacting regime, as was used in Ref. [38]. Using a separable potential captures the low-energy physics of the dipolar interaction and allows us to account for the effect of the coupling between different partial wave channels. In particular, it provides us a convenient framework to compute the order parameters for each scattering channel and to explore the behavior of these order parameters.

For systems with *multiple* superfluid order parameters, there can exist an additional collective mode other than the well-known phonon mode: the so-called Leggett mode [39]. This mode is characterized as the out-of-phase coupling of different superfluid order parameters. It has been long predicted to occur in two-band superconductors, for example in MgB_2 [40], and in nonequilibrium systems [41]. Most recently, an ultracold atomic Fermi gas near an orbital Feshbach resonance has been thought to be a possible candidate for exhibiting the Leggett mode [42–44]. The purpose of this work is to show that a two-component Fermi gas with dipolar interactions provides an excellent platform to observe the long-sought Leggett mode.

The rest of the paper is set out as follows. In Sec. II we consider the many-body thermodynamic potential and derive the mean-field equations for the density and order parameters. We determine the order parameters in the different phases of the system as we sweep over scattering lengths, and examine the symmetry of the associated momentum distribution. In Sec. III we calculate the collective modes by expanding the thermodynamic potential to second order, which correspond to the pair fluctuations at the Gaussian level. We show that the Leggett mode is undamped for a range of interaction strengths and how the collective modes can be seen in the spectral function of the Cooper pairs. In Sec. IV we discuss and summarize our findings.

II. MANY-BODY THERMODYNAMIC POTENTIAL

We consider a many-body two-component Fermi gas with dipolar interactions in three dimensions, described by the model Hamiltonian (we set $\hbar = 1$ and the volume $V = 1$) [38,45],

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \mathcal{H}_{\text{int}}, \tag{1}$$

where the single-particle dispersion is $\xi_{\mathbf{k}} = \mathbf{k}^2/2M - \mu$ with the chemical potential μ, $a^{\dagger}_{\mathbf{k}\sigma} \equiv a^{\dagger}_{\mathbf{k}\sigma}(\tau)$ and $a_{\mathbf{k}\sigma} \equiv a_{\mathbf{k}\sigma}(\tau)$ are creation and annihilation operators respectively, for atoms with spin σ and mass M, and the interaction Hamiltonian is given by

$$\mathcal{H}_{\text{int}} = \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} U(\mathbf{k} - \mathbf{k}') a^{\dagger}_{\mathbf{q}/2-\mathbf{k}\uparrow} a^{\dagger}_{\mathbf{q}/2+\mathbf{k}\downarrow} a_{\mathbf{q}/2+\mathbf{k}'\downarrow} a_{\mathbf{q}/2-\mathbf{k}'\uparrow}, \tag{2}$$

where the dipolar interaction is $U(\mathbf{k}) = 4\pi d^2(\cos^2\theta_{\mathbf{k}} - 1)/3$, with d being the dipole moment of the two dipoles polarized along the z axis and $\theta_{\mathbf{k}}$ the angle between $\mathbf{k}$ and the z axis. We can write the interaction in the following separable form [38,46–48]:

$$U(\mathbf{k}-\mathbf{k}')=4\pi\sum_j g_j w_j(\hat{\mathbf{k}})w_j^*(\hat{\mathbf{k}}'), \tag{3}$$

where the coupling constants g_j satisfy the renormalization condition for the effective scattering lengths λ_j [49]:

$$\frac{M}{4\pi\lambda_j}=\frac{1}{g_j}+\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{M}{\mathbf{k}^2}. \tag{4}$$

Truncating the sum in Eq. (3) to the two lowest-order terms as was done in Ref. [38], the effective scattering lengths are given by

$$\lambda_{1,2}=\left[a_{00}\pm\mathrm{sgn}(a_{02})\sqrt{a_{00}^2+4a_{02}^2}\right]/2, \tag{5}$$

where a_{00} and a_{02} are the scattering lengths of the s and d partial wave channels. For a set of scattering lengths (a_{00},a_{02}), either λ_1 or λ_2 will be positive, supporting a bound state energy of $E_b=-1/M\lambda_j^2$ [38]. Throughout this work we set $a_{00}^{-1}>0$ and sweep across a_{02}^{-1}, thus there will be a phase transition as the bound state changes from λ_1 to λ_2 as a_{02} changes sign. The orthogonal basis vectors in Eq. (3) are given by [49],

$$w_{1,2}(\hat{\mathbf{k}})=\frac{s_{1,2}Y_{00}(\hat{\mathbf{k}})+Y_{20}(\hat{\mathbf{k}})}{\sqrt{s_{1,2}^2+1}}, \tag{6}$$

where $s_{1,2}=-(y\pm\sqrt{y^2+4})/2$ and $y=a_{00}/a_{02}$, and $Y_{lm}(\hat{\mathbf{k}})$ are the spherical harmonics.

The Hamiltonian with the separable potential in Eq. (3) then becomes

$$\mathcal{H}=\sum_{\mathbf{k}\sigma}\xi_{\mathbf{k}}a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma}+4\pi\sum_{\mathbf{q},j=(1,2)}g_j b_j^\dagger(\mathbf{q},\tau)b_j(\mathbf{q},\tau), \tag{7}$$

where $b_j(\mathbf{q},\tau)=\sum_{\mathbf{k}}w_j(\hat{\mathbf{k}})a_{-\mathbf{k}+\mathbf{q}/2\uparrow}a_{\mathbf{k}+\mathbf{q}/2\downarrow}$. In the imaginary time formalism we can write the partition function as $\mathcal{Z}=\int Da^\dagger Da\exp(-S)$, where the action S is given by ($\beta\equiv 1/k_BT$)

$$S=\int_0^\beta d\tau\left[\sum_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}^\dagger(\tau)\partial_\tau a_{\mathbf{k}\sigma}(\tau)+\mathcal{H}(\tau)\right]. \tag{8}$$

Using the standard Hubbard-Stratonovich transformation, we may decouple the interaction term by introducing auxiliary complex pairing fields ($j=1,2$), $\Phi_{\mathbf{q}}^j(\tau)$. Physically, each pairing field roughly describes a Cooper pair consisting of two fermions, i.e.,

$$\Phi_{\mathbf{q}}^j(\tau)\sim 4\pi g_j b_j(\mathbf{q},\tau). \tag{9}$$

Using the Nambu spinor representation $\Psi_{\mathbf{k}}^\dagger=(a_{\mathbf{k}\uparrow}^\dagger,a_{-\mathbf{k}\downarrow})$ for a two-component Fermi gas, we can rewrite the action as

$$S=\int_0^\beta d\tau\left[-\sum_{\mathbf{q},j}\frac{\left|\Phi_{\mathbf{q}}^j(\tau)\right|^2}{4\pi g_j}+\frac{1}{2}\sum_{\mathbf{k}\mathbf{k}'}\left(\xi_{\mathbf{k}}\delta_{\mathbf{k}\mathbf{k}'}-\Psi_{\mathbf{k}}^\dagger\mathcal{G}_{\mathbf{k}\mathbf{k}'}^{-1}\Psi_{\mathbf{k}'}\right)\right], \tag{10}$$

where the inverse fermionic Green's function takes the form ($\mathbf{p}\equiv\frac{\mathbf{k}+\mathbf{k}'}{2}$)

$$\mathcal{G}_{\mathbf{k}\mathbf{k}'}^{-1}=\begin{bmatrix}-(\partial_\tau+\xi_{\mathbf{k}})\delta_{\mathbf{k}\mathbf{k}'} & \sum_j\Phi_{\mathbf{k}-\mathbf{k}'}^j(\tau)w_j(\hat{\mathbf{p}})\\ \sum_j\Phi_{-\mathbf{k}+\mathbf{k}'}^{j*}(\tau)w_j^*(\hat{\mathbf{p}}) & -(\partial_\tau-\xi_{\mathbf{k}})\delta_{\mathbf{k}\mathbf{k}'}\end{bmatrix}. \tag{11}$$

By integrating out the fermionic degrees of freedom from the partition function and taking the Fourier transform from imaginary time to Matsubara frequencies, we obtain the effective action

$$S_{\mathrm{eff}}=-\beta\sum_{Q,j}\frac{\left|\Phi_Q^j\right|^2}{4\pi g_j}+\sum_{K,K'}\left[\beta\xi_{\mathbf{k}}\delta_{KK'}-\mathrm{Tr}\ln\mathcal{G}_{KK'}^{-1}\right], \tag{12}$$

where $Q\equiv(i\nu_n,\mathbf{q})$ with bosonic Matsubara frequencies $\nu_n=2\pi n/\beta$ and $K\equiv(i\omega_m,\mathbf{k})$ with fermionic Matsubara frequencies $\omega_m=(2m+1)\pi/\beta$. We have also used the shorthand notations, $\sum_Q\equiv k_BT\sum_{i\nu_n}\sum_{\mathbf{q}}$ and $\sum_K\equiv k_BT\sum_{i\omega_m}\sum_{\mathbf{k}}$.

In the following, we make a saddle-point approximation and expand the action in orders of the fluctuation fields $\hat{\phi}_j(Q)$ around the order parameters Δ_j,

$$\Phi_Q^j=\Delta_j\delta_{Q0}+\hat{\phi}_j(Q), \tag{13}$$

and we can obtain $S_{\mathrm{eff}}=S_{\mathrm{MF}}+S_{\mathrm{GF}}+\cdots$, where S_{MF} is the mean-field action and S_{GF} is the Gaussian fluctuation action.

A. Mean-field theory

First looking at the mean-field contribution to the action, we have

$$S_{\mathrm{MF}}=-\beta\sum_j\frac{|\Delta_j|^2}{4\pi g_j}+\sum_K\left[\beta\xi_{\mathbf{k}}-\mathrm{Tr}\ln\mathcal{G}_{\mathrm{sp}}^{-1}\right], \tag{14}$$

where the saddle-point Green's function is given by

$$\mathcal{G}_{\mathrm{sp}}^{-1}(K)=\begin{bmatrix}i\omega_m-\xi_{\mathbf{k}} & \Delta(\mathbf{k})\\ \Delta^*(\mathbf{k}) & i\omega_m+\xi_{\mathbf{k}}\end{bmatrix}, \tag{15}$$

the quasiparticle dispersion is $E_{\mathbf{k}}=\sqrt{\xi_{\mathbf{k}}^2+|\Delta(\mathbf{k})|^2}$, and we have defined $\Delta(\mathbf{k})=\sum_j\Delta_j\omega_j(\hat{\mathbf{k}})$. We thus obtain the mean-field thermodynamic potential,

$$\Omega_{\mathrm{MF}}=-\sum_j\frac{|\Delta_j|^2}{4\pi g_j}+\sum_{\mathbf{k}}\left[\xi_{\mathbf{k}}-E_{\mathbf{k}}-\frac{2}{\beta}\ln(1+e^{-\beta E_{\mathbf{k}}})\right], \tag{16}$$

and from the condition $\delta\Omega_{\mathrm{MF}}/\delta\Delta_j^*=0$ we get the coupled gap equations at finite temperature,

$$-\frac{\Delta_j}{4\pi g_j}=\sum_{\mathbf{k},j'}\frac{\Delta_{j'}\omega_{j'}(\hat{\mathbf{k}})\omega_j^*(\hat{\mathbf{k}})}{2E_{\mathbf{k}}}\tanh\frac{\beta E_{\mathbf{k}}}{2}. \tag{17}$$

Using the renormalization condition, Eq. (4), to replace the bare coupling constants g_j, and using the fact that the basis functions are orthogonal, we rewrite the gap equation at zero temperature:

$$-\frac{M\Delta_j}{16\pi^2\lambda_j}=\sum_{\mathbf{k},j'}\Delta_{j'}\omega_{j'}(\hat{\mathbf{k}})\omega_j^*(\hat{\mathbf{k}})\left(\frac{1}{2E_{\mathbf{k}}}-\frac{M}{\mathbf{k}^2}\right). \tag{18}$$

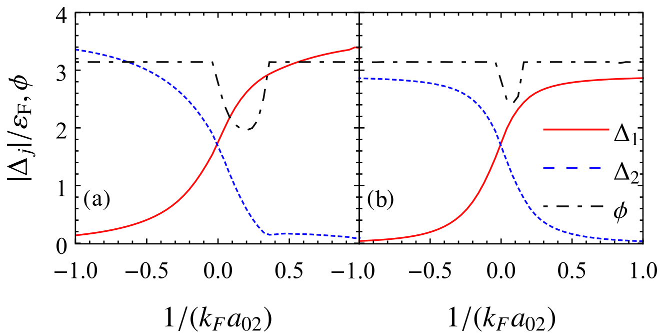

FIG. 1. Plots of the order parameters $|\Delta_j|$ in units of the Fermi energy, ε_F, for a range of a_{02} scattering lengths and (a) $k_F a_{00} = 2$ and (b) $k_F a_{00} = 5$. The relative phase of the order parameters, $\phi = \arg(\Delta_1) - \arg(\Delta_2)$, is shown in both plots.

The number equation at the mean-field level is easily found from the relation $n = -\partial\Omega_{MF}/\partial\mu$:

$$n = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(1 - \frac{\varepsilon_\mathbf{k}}{E_\mathbf{k}}\right). \tag{19}$$

Together the above two equations form a closed set and we can solve for the chemical potential μ and order parameters Δ_j. As we take only the first two partial wave channels, the thermodynamic potential only depends on the absolute values of the order parameters, $|\Delta_1|$ and $|\Delta_2|$, and the relative phase between the two order parameters, $\phi = \arg(\Delta_1) - \arg(\Delta_2)$. There are several solutions to the number and gap equations for a given set of scattering lengths, which correspond to different local minima. The true ground state should be determined by minimizing the energy density, $\mathcal{E} \equiv \Omega + \mu n$:

$$\mathcal{E}(\Delta_1, \Delta_2, \mu) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\varepsilon_\mathbf{k} - E_\mathbf{k} + \frac{|\Delta(\hat{\mathbf{k}})|^2}{2E_\mathbf{k}}\right] + \frac{\mu k_F^3}{3\pi^2}. \tag{20}$$

For our units of numerical calculations, we take the Fermi wave vector, $k_F \equiv (3\pi^2 n)^{1/3}$, as the units of the wave vectors and the Fermi energy $\varepsilon_F = \hbar^2 k_F^2/(2M)$ as the units of energy. This is equivalent to setting $2M = \hbar = 1$. In Fig. 1 we plot the order parameters for a range of the scattering length a_{02}, where we set $k_F a_{00} = 2$ in Fig. 1(a) and $k_F a_{00} = 5$ in Fig. 1(b). We see the nontrivial behavior of the order parameters, depending on the sign of a_{02} and the associated two-body bound state [38]. The change of the dominant order parameter implies that the condensate can have two different symmetries and therefore there exists a quantum phase transition, as discussed in the previous work [38]. Near $(k_F a_{02})^{-1} \simeq 0$, the two order parameters become comparable. The relative phase of the order parameters also changes with $k_F a_{02}$, taking a nontrivial value near $(k_F a_{02})^{-1} \simeq 0$ for both values of $k_F a_{00}$. It is always out of phase for $k_F a_{02} < 0$.

B. Momentum distribution

The momentum dependence in the different interaction regimes is nontrivial due to the mixing of order parameters and angular dependence of the dipolar interaction. This has already been investigated in the previous work, by considering the quasiparticle spectral function [38]. Here, we show that the momentum distribution can also exhibit different underlying symmetry, depending on the sign of $k_F a_{02}$ and the resulting two-body bound state.

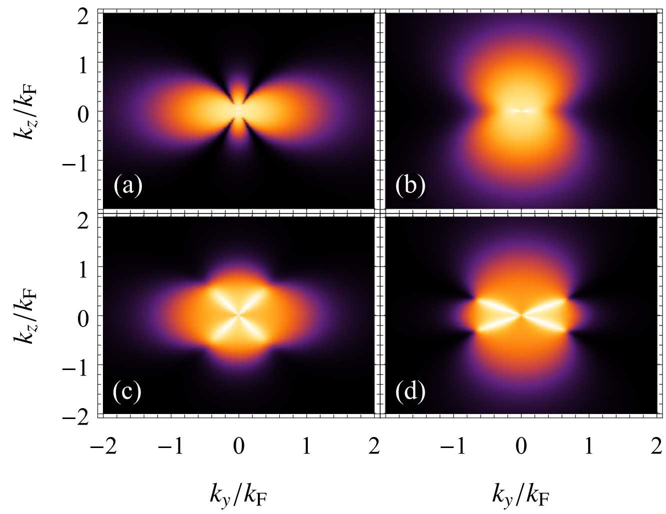

FIG. 2. Density plots of the momentum distribution $n(k_x = 0, k_y, k_z) = 1 - \varepsilon_\mathbf{k}/E_\mathbf{k}$ in units of the Fermi momentum k_F for different sets of interaction parameters $(k_F a_{00}, k_F a_{02})$: (a) (1,1), (b) (1, −1), (c) (1,5), and (d) (1, −5).

In Fig. 2 we show the zero-temperature density plots of the momentum distribution

$$n(k_x = 0, k_y, k_z) = 1 - \frac{\varepsilon_\mathbf{k}}{E_\mathbf{k}} \tag{21}$$

at $k_F a_{00} = 1$ and at different values of $k_F a_{02}$: (a) $k_F a_{02} = 1$, (b) $k_F a_{02} = -1$, (c) $k_F a_{02} = 5$, and (d) $k_F a_{02} = -5$. We note that the rotational symmetry of the system in the x-y plane ensures $n(k_x = 0, k_y, k_z) = n(k_x, k_y = 0, k_z)$.

As the scattering length $k_F a_{20}$ changes, we see how the underlying symmetry of the momentum distribution is changing. In Fig. 2(a) the distribution has a $s - d_{z^2}$ like symmetry and in Fig. 2(b) the symmetry is $s + d_{z^2}$. For both interactions the momentum distribution is dominated by the contribution from the bound-state dominated order parameter in $\Delta(\mathbf{k})$: $\Delta_1 w_1(\hat{\mathbf{k}})$ for (a) and $\Delta_2 w_2(\hat{\mathbf{k}})$ for (b).

As we increase $|k_F a_{02}|$, in Figs. 2(c) and 2(d) we see a higher-order nontrivial symmetry. The distribution is no longer dominated by the bound-state order parameter and the mixing of order parameters becomes important. For the negative scattering length, $k_F a_{02} = -5$, in Fig. 2(c) the coupling of the order parameters is out of phase. This is where we expect the dipolar superfluid to support an additional collective mode; we will soon see that the system has two collective modes in this regime. For the positive scattering length, $k_F a_{02} = 5$, in Fig. 2(d) the relative phase of order parameters becomes nontrivial, and the time-reversal symmetry has been broken due to the order-parameter mixing [38]. In this interaction regime we expect there to be no Leggett mode as the order parameters are not out of phase.

III. COLLECTIVE MODES

To study the behavior of the collective modes, we calculate the Gaussian fluctuation contribution to the effective action, Eq. (12). This can be taken into account by expanding the action to the second order of the bosonic fields $\hat{\phi}_j(Q)$ and $\hat{\phi}_j^*(Q)$ [50]:

$$S_{\rm GF} = \sum_Q \left[-\sum_j \frac{|\hat{\phi}_j(Q)|^2}{4\pi g_j} \right] + \frac{\beta}{2} \sum_{QK} {\rm Tr}\left[\mathcal{G}\left(K - \frac{Q}{2}\right) \times \Phi(-Q) \mathcal{G}\left(K + \frac{Q}{2}\right) \Phi(Q) \right], \tag{22}$$

where we have the saddle-point fermionic Green's function and fluctuation fields,

$$\mathcal{G}(K) = \frac{1}{(i\omega_m)^2 - E_{\mathbf{k}}^2} \begin{bmatrix} i\omega_m + \xi_{\mathbf{k}} & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^*(\hat{\mathbf{k}}) & i\omega_m - \xi_{\mathbf{k}} \end{bmatrix}, \tag{23}$$

$$\Phi(Q) = \begin{bmatrix} 0 & \sum_j \hat{\phi}_j(-Q)\omega_j(\hat{\mathbf{k}}) \\ \sum_j \hat{\phi}_j^*(Q)\omega_j^*(\hat{\mathbf{k}}) & 0 \end{bmatrix}. \tag{24}$$

The subscript "sp" in the saddle-point Green's function has been suppressed for a better presentation. From this we can then rearrange the terms to obtain the final form,

$$S_{\rm GF} = \frac{\beta}{2} \sum_{Q,jj'} [\hat{\phi}_j{}^*(Q), \hat{\phi}_j(-Q)] M_{jj'}(Q) \begin{bmatrix} \hat{\phi}_{j'}(Q) \\ \hat{\phi}_{j'}^*(-Q) \end{bmatrix}, \tag{25}$$

where we have defined the elements $M_{jj'}$ (each of which is a 2 by 2 matrix),

$$[M_{jj'}]_{11}(Q) = \sum_K \mathcal{G}_{11}\left(\frac{Q}{2} + K\right) \mathcal{G}_{22}\left(\frac{Q}{2} - K\right) \omega_j(\hat{\mathbf{k}})\omega_{j'}^*(\hat{\mathbf{k}}) - \frac{\delta_{j,j'}}{4\pi g_j}, \tag{26}$$

$$[M_{jj'}]_{12}(Q) = \sum_K \mathcal{G}_{12}\left(\frac{Q}{2} + K\right) \mathcal{G}_{12}\left(\frac{Q}{2} - K\right) \omega_j(\hat{\mathbf{k}})\omega_{j'}^*(\hat{\mathbf{k}}), \tag{27}$$

$[M_{jj'}]_{21}(Q) = [M_{jj'}]_{12}(Q)$, and $[M_{jj'}]_{22}(Q) = [M_{jj'}]_{11}(-Q)$. We then complete the sums over the Matsubara frequencies to arrive at the zero-temperature result:

$$[M_{jj'}]_{11}(Q) = -\frac{\delta_{jj'}}{4\pi g_j} + \sum_{\mathbf{k}} \left(\frac{u_-^2 u_+^2}{i\nu_n - E_+ - E_-} - \frac{v_+^2 v_-^2}{i\nu_n + E_+ + E_-} \right) \omega_j(\hat{\mathbf{k}})\omega_{j'}^*(\hat{\mathbf{k}}), \tag{28}$$

$$[M_{jj'}]_{12}(Q) = -\sum_{\mathbf{k}} \left(\frac{u_+ v_+ u_- v_-}{i\nu_n - E_+ - E_-} - \frac{u_+ v_+ u_- v_-}{i\nu_n + E_+ + E_-} \right) \omega_j(\hat{\mathbf{k}})\omega_{j'}^*(\hat{\mathbf{k}}), \tag{29}$$

where we define the BCS parameters $u_\pm^2 = (1 + \xi_\pm/E_\pm)/2$ and $v_\pm^2 = (1 - \xi_\pm/E_\pm)/2$, and the shorthand notations $\xi_\pm = \xi_{\mathbf{k}\pm\mathbf{q}/2}$ and $E_\pm = E_{\mathbf{k}\pm\mathbf{q}/2}$. We renormalize the bare coupling constants g_j again using Eq. (4) and this also cures the divergences in the integrals of M_{11}. We can then write the *inverse* boson propagator for Cooper pairs, $\mathbf{M}(Q)$, as a $2N_j \times 2N_j$ matrix, where N_j is the number of channels and in this work $N_j = 2$.

Analytically continuing the Matsubara frequencies, $i\nu_n \to \omega + i0^+$, the phonon and Leggett collective mode dispersions are determined by the equation $\det[\mathbf{M}(\mathbf{q}, \omega)] = 0$. As the scattering potential we have used for the dipole-dipole interaction has an angular dependency, the bosonic propagator $\mathbf{\Gamma}(\mathbf{q}, \omega) = \mathbf{M}^{-1}(\mathbf{q}, \omega)$ has an angular dependence and is a function of three parameters: $\mathbf{\Gamma}(\mathbf{q}, \omega) \equiv \mathbf{\Gamma}(q, \theta, \omega)$, where $q \equiv |\mathbf{q}|$ and $\theta \equiv \theta_{\mathbf{q}}$.

Results

We plot in Fig. 3 the collective modes for scattering lengths $k_{\rm F}a_{00} = 2$ and (a) $(k_{\rm F}a_{02}, \theta) = (-2, \pi/2)$, (b) $(k_{\rm F}a_{02}, \theta) = (-5, 0)$, (c) $(k_{\rm F}a_{02}, \theta) = (-5, \pi/4)$, and (d) $(k_{\rm F}a_{02}, \theta) = (-5, \pi/2)$. The two-particle continuum is shown as the blue shaded region, the phonon mode is the blue dashed line, and the Leggett mode is the black dot-dashed line.

We see in Fig. 3(a) that the dipolar superfluid supports only the phonon mode when the channel coupling a_{02} is weak, which becomes damped once it enters the two-particle continuum as we increase the momentum $q/k_{\rm F}$, indicating that for this interaction regime the system is BCS like [38,51]. Looking at Fig. 1(a) for scattering lengths $(k_{\rm F}a_{00}, k_{\rm F}a_{02}) = (2, -2)$, the order parameters are out of phase and the superfluid is mainly characterized by the order parameter Δ_2, thus

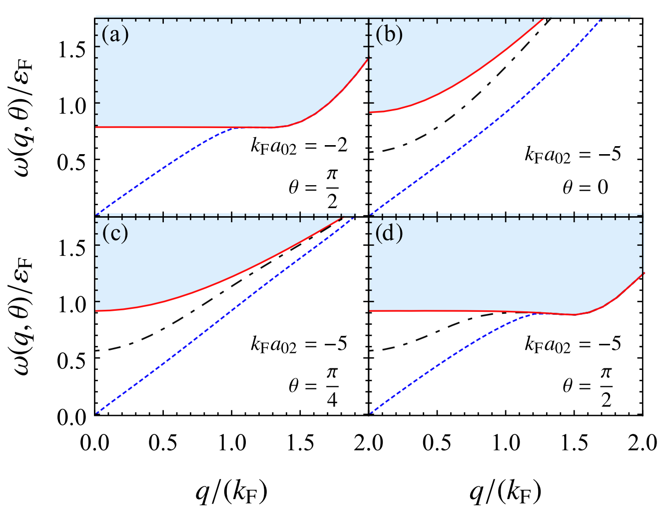

FIG. 3. Plots of the two-particle continuum (blue shaded region), phonon mode (blue dashed), and Leggett mode (black dash-dotted) in units of the Fermi energy for scattering lengths $k_{\rm F}a_{00} = 2$ and (a) $(k_{\rm F}a_{02}, \theta) = (-2, \pi/2)$, (b) $(k_{\rm F}a_{02}, \theta) = (-5, 0)$, (c) $(k_{\rm F}a_{02}, \theta) = (-5, \pi/4)$, and (d) $(k_{\rm F}a_{02}, \theta) = (-5, \pi/2)$.

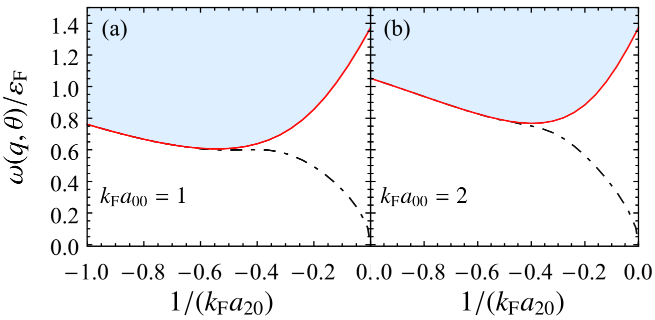

FIG. 4. Plots of the two particle continuum (blue shaded region) and Leggett mode (black dash-dotted) for $q = 0$ as a function of the scattering length $k_{\mathrm{F}}a_{02}$ at (a) $k_{\mathrm{F}}a_{00} = 1$ and (b) $k_{\mathrm{F}}a_{00} = 2$.

we would expect the mixing between the order parameters to be negligible and there is no Leggett mode.

In Figs. 3(b)–3(d) we increase the channel coupling to $k_{\mathrm{F}}a_{02} = -5$, and we now see two undamped collective modes, the Leggett and phonon modes, at low momentum. In this interaction regime the order parameters are approximately at the same order of magnitude and are out of phase [see Fig. 1(a), where $1/(k_{\mathrm{F}}a_{02}) = -0.2$], satisfying Leggett's original picture of two well-defined and coupled condensates [39]. The Leggett mode merges into the two-particle continuum and becomes damped for large momentum at each θ. The phonon mode is always undamped for $\theta = 0$, but it merges into the two-particle continuum for large momenta when θ becomes sufficiently large; see, for example, Fig. 3(d).

In Figs. 4(a) and 4(b) we plot only the Leggett mode (black dash-dotted line) and the two-particle continuum (blue shaded region) for a range of $k_{\mathrm{F}}a_{02}$ at zero momentum $q = 0$, and set $k_{\mathrm{F}}a_{00} = 1$ and $k_{\mathrm{F}}a_{00} = 2$, respectively. For negative scattering length, $(k_{\mathrm{F}}a_{02})^{-1} < 0$, we see in both figures the Leggett mode becomes undamped for large enough $|k_{\mathrm{F}}a_{02}|$, and disappears as the scattering length changes sign. Here, the system undergoes a quantum phase transition as the bound state changes its character and the relative phase between the two order parameters starts to deviate from π. We find for positive scattering lengths, $(k_{\mathrm{F}}a_{02})^{-1} > 0$, there are no longer two collective modes and the Leggett mode always lies in the two-particle continuum (not shown in the figure). This can be understood from Fig. 1: for large positive $k_{\mathrm{F}}a_{02}$, the relative phase of the order parameters exhibits non trivial dependence on $(k_{\mathrm{F}}a_{02})^{-1}$ and is not completely out of phase. As $(k_{\mathrm{F}}a_{02})^{-1}$ increases further, the two order parameters become out of phase again, however Δ_2 becomes dominant and leaves no room for the Leggett mode.

Experimentally, the collective modes of a strongly interacting Fermi gas can be probed by measuring the density dynamic structure factor via Bragg spectroscopy [52,53]. We would expect that, if the regimes where the Leggett mode is undamped can be reached, we should be able to measure the phonon and Leggett modes. To support this idea, in Fig. 5 we show a typical spectral function of the in-medium Cooper pairs, i.e., $-\mathrm{Im}\boldsymbol{\Gamma}_{11}(q,\theta,\omega)$, in arbitrary units for a range of momenta, where for clarity we have shifted each curve to be visible. We have chosen an interaction strength of $k_{\mathrm{F}}a_{00} = 1$ and $k_{\mathrm{F}}a_{02} = -5$. We can clearly see how the phonon mode and the Leggett mode evolve as the momentum increases.

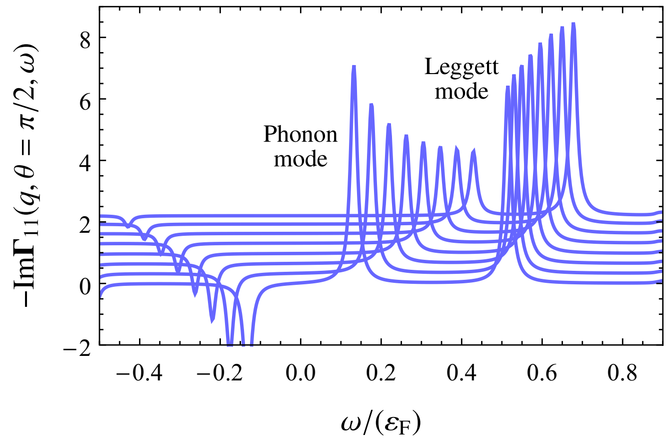

FIG. 5. Plot of the spectral function of Cooper pairs, $-\mathrm{Im}\boldsymbol{\Gamma}_{11}(q,\theta,\omega)$, in arbitrary units, scattering lengths $(k_{\mathrm{F}}a_{00}, k_{\mathrm{F}}a_{02}) = (1, -5)$, and $\theta = \pi/2$. From bottom to top the momentum q increases from $q = 0.1k_{\mathrm{F}}$ to $0.5k_{\mathrm{F}}$.

IV. DISCUSSION AND SUMMARY

We have found that an undamped Leggett mode requires interactions in the $k_{\mathrm{F}}a_{00}$ and $k_{\mathrm{F}}a_{02}$ channels to be such that both order parameters are significant and out of phase. Practically, such an interaction regime could be achieved with a multichannel resonance [30,38,54], changing the scattering lengths by sweeping across the shape resonances induced by the dipolar interaction. For polar molecule systems, the large electronic dipole moments can be adjusted such that the interaction regime to observe the Leggett mode could be reached [11,15]. For atomic species with a magnetic dipole moment, the interaction is fixed but Feshbach resonances can be used to tune the background s-wave interaction, i.e., the scattering length a_{00}. However, as we require a large $k_{\mathrm{F}}a_{02}$ as well to have a significant coupling between the two channels, a direct observation of the Leggett mode would be difficult. The addition of higher-order channels would not significantly alter our results, since the higher-order channel coupling will most likely be weak [30].

In summary, through an effective separable form of the dipolar interaction we have investigated the collective modes of a dipolar Fermi gas, in which the superfluid is described by two order parameters. We have found for strong interactions, where the order parameters are strongly coupled and out of phase, an additional collective mode—the Leggett mode—emerges, on top of the phonon mode. We have determined the interaction regime where this mode persists and have shown that the Leggett mode can be seen through the spectral function of the Cooper pairs, indicating that in principle it could be measured through Bragg spectroscopy.

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APPENDIX: TWO-BODY SCATTERING

To renormalize the many-body equations we need to calculate the two-body T matrix. The dipolar interaction is nonseparable and this makes the many-body calculation intractable. We separate the dipolar interaction using the effective potential in Ref. [38], which was introduced to model a multichannel resonance, and here we briefly derive the effective potential. The scattering amplitude for the dipolar interaction is given by [29,31]

$$f(\mathbf{k}',\mathbf{k})|_{k=k'} = 4\pi \sum_{lml'm'} i^{l'-l} k^{-1} \left(\frac{1}{\mathcal{K}^{-1} - i} \right)_{lm}^{l'm'} Y_{lm}(\hat{\mathbf{k}}) Y^{*}_{l'm'}(\hat{\mathbf{k}}') \tag{A1}$$

where $\mathcal{K}_{lm}^{l'm}$ is the K matrix and can be calculated as in Refs. [30,55]. The K matrix is related to the T matrix by $\mathcal{T} = 2(\mathcal{K}^{-1} - i)^{-1}$ and in the small k limit the scattering lengths are given by the K-matrix elements, $a_{ll'}^{(m)} = -\lim_{k\to 0} \mathcal{K}_{lm}^{l'm}/k$. Introducing a matrix $\mathcal{A}$ whose elements are defined by the scattering lengths as $\mathcal{A}_{ll'}^{(m)} = i^{l-l'} a_{ll'}^{(m)}$, we diagonalize the matrix $\mathcal{A}$ in an orthonormal basis, $w_{jm}(\hat{\mathbf{k}}) = \sum_l d_{jl} Y_{lm}(\hat{\mathbf{k}})$ [56]. We can write the scattering amplitude as [57]

$$f(\mathbf{k}',\mathbf{k})|_{k=k'\to 0} = 4\pi \sum_{jm} f_{jm} w_{jm}(\hat{\mathbf{k}}) w^{*}_{jm}(\hat{\mathbf{k}}'), \tag{A2}$$

where $f_{jm} = -1/(\lambda_{jm}^{-1} + ik)$. We can find a separable potential which reproduces this scattering amplitude as

$$U(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = 4\pi \sum_{jm} g_{jm} w_{jm}(\hat{\mathbf{k}}) w^{*}_{jm}(\hat{\mathbf{k}}'), \tag{A3}$$

where the coupling constants g_{jm} satisfy the renormalization condition,

$$\frac{M}{4\pi\lambda_{jm}} = \frac{1}{g_{jm}} + \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{M}{\mathbf{k}^2}. \tag{A4}$$

Taking the separable potential to second order as the minimal model to describe the dipolar interaction, we set the scattering matrix to the following form for the multichannel resonance:

$$A_{sc} = \begin{pmatrix} a_{00} & -a_{02} \\ -a_{02} & 0 \end{pmatrix}. \tag{A5}$$

The eigenvalues of this matrix are given by

$$\lambda_{1,2} = \left[a_{00} \pm \mathrm{sgn}(a_{02})\sqrt{a_{00}^2 + 4a_{02}^2}\right]/2, \tag{A6}$$

and for any set of values (a_{00}, a_{02}) either λ_1 or λ_2 will be positive with a bound-state energy of $E_b = -1/M\lambda_i^2$. This will mean that as we sweep across a_{00}^{-1} or a_{02}^{-1} there will be a phase transition, since the bound state changes from λ_1 to λ_2. The choice of Eq. (A5) is not unique and we can change the sign of the off diagonal elements; this would have the effect of changing the sign of the λ_1 and λ_2 and would not qualitatively change any of the results here. The orthogonal basis vectors are given by

$$w_{1,2}(\hat{\mathbf{k}}) = \frac{s_{1,2} Y_{00}(\hat{\mathbf{k}}) + Y_{20}(\hat{\mathbf{k}})}{\sqrt{s_{1,2}^2 + 1}}, \tag{A7}$$

with $s_{1,2} = -(y \pm \sqrt{y^2+4})/2$ and $y = a_{00}/a_{02}$.

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