Scaling law in laser cooling on narrow-line optical transitions

O. N. Prudnikov,[1,2,*] R. Ya. Il'enkov,[1,2] A. V. Taichenachev,[1,2] and V. I. Yudin[1,2,3]
[1]*Institute of Laser Physics, 630090 Novosibirsk, Russia*
[2]*Novosibirsk State University, 630090 Novosibirsk, Russia*
[3]*Novosibirsk State Technical University, 630073 Novosibirsk, Russia*

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The laser cooling of atoms with a narrow-line optical transition, i.e., in regimes of quantum nature of laser-light interactions resulting in a significant recoil effect, is studied. It is demonstrated that a minimum laser-cooling temperature for two-level atoms in a standing wave reached for red detuning close to three recoil frequencies is vastly different from the theory used for a semiclassical description of Doppler cooling. A set of dimensionless parameters uniquely characterizing the time evolution and the steady state of different atoms with narrow-line optical transitions in the laser field is introduced. The results can be used for analysis of optimal conditions for laser cooling of atoms with narrow lines such as Ca, Sr, and Mg, which are of great interest for atomic clocks.

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I. INTRODUCTION

Nowadays, laser cooling of neutral atoms is routinely used for a wide range of modern quantum physics investigations, including metrology and atom optics. There are well-known techniques for laser cooling below the Doppler limit, such as sub-Doppler polarization gradient cooling [1–4], velocity-selective coherent population trapping [5–7], or Raman cooling [8,9] restricted to atoms with energy levels degenerated over angular momentum or hyperfine structure. However, these techniques cannot be applied to atoms with a single ground state, such as ^{24}Mg, ^{40}Ca, ^{88}Sr, and ^{174}Yb, which are of great interest for atomic clocks.

For these atoms the well-known semiclassical theory predicts the Doppler laser-cooling temperature [10,11]

$$k_B T_D \approx \hbar\gamma/2 \tag{1}$$

proportional to the natural linewidth γ. One way of reaching a deeper cooling for these atoms is to use a narrow-line optical transition (clock transition) with a smaller γ. However, the basic semiclassical theory becomes no longer valid, since the main requirement $\omega_R/\gamma \ll 1$ (where $\omega_R = \hbar k^2/2M$ is the recoil frequency) is violated and the Doppler temperature (1) (Table I) may not be reached, as was clearly shown in [12] for an atom in the σ_+-σ_- light field.

There are several experimental realizations of laser cooling of ^{174}Yb [13] and ^{88}Sr atoms [14–16] on the intercombination line $^1S_0 \to {}^3P_1$. In order to increase the capture velocity range to larger than the Doppler velocity $v_D = \gamma/k$, it was proposed to use the broadband light sources [17] that were also used for laser cooling of ^{88}Sr atoms [18].

The ω_R/γ ratios for ^{40}Ca and ^{24}Mg atoms are larger and the laser cooling of these atoms on the intercombination line does not seem possible in monochromatic light. The deep laser cooling of Ca atoms at the narrow-line optical transition was reached by using the quenching cooling techniques [19–21] that effectively increase the spontaneous decay rate. However, no significant progress for ^{24}Mg atoms has been achieved so far. Thus, while for atoms with a strong optical transition $\omega_R/\gamma \ll 1$ the laser cooling is well described by semiclassical theory, the laser cooling of atoms with narrow lines $\omega_R/\gamma \geqslant 1$ (Table I) requires a separate study, which can be done by numerical simulation taking into account quantum recoil effects of atom-light interaction.

Here we present a quantum theory of laser cooling in a standing wave far beyond the semiclassical limit, i.e., for $\omega_R/\gamma \geqslant 1$. This allows us to clarify the laser-cooling mechanisms with narrow-line optical transitions and estimate the optimal parameters for minimum cooling temperatures and time, especially for a sufficiently strong light field intensity, resulting in the power broadening regime of laser cooling. We find that the cooling dynamics and steady-state momentum distribution can be uniquely characterized for various atoms with different ω_R/γ ratios, which allows us to identify the scaling law in laser cooling for the narrow-line optical transition. The proposed universal scalability allows us to transfer the results of laser cooling (momentum distribution of atoms, energy, cooling time, and light field parameters), once obtained for certain atoms, to various elements with $\omega_R/\gamma \geqslant 1$.

II. SCALING IN LASER COOLING

The evolution of atoms in resonant monochromatic light can be described by the density-matrix equation, taking into account the quantum recoil effects in the processes of absorption and emission of light field photons (see Fig. 1)

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \hat{\Gamma}\{\hat{\rho}\}. \tag{2}$$

Here $\hat{H}$ is the Hamiltonian and $\hat{\Gamma}\{\hat{\rho}\}$ is the relaxation operator due to spontaneous emission (see, for example, [10]). The Hamiltonian in the rotating-wave approximation has the

*oleg.nsu@gmail.com

TABLE I. Semiclassical parameter ω_R/γ intercombination line ${}^1S_0 \rightarrow {}^3P_1$ of various atoms and estimation of $k_BT_D \approx \hbar\gamma/2$.

Atom	T_D	λ (nm)	ω_R/γ
^{200}Hg	32 μK	254	0.01
^{174}Yb	4.5 μK	556	0.02
^{88}Sr	0.17 μK	689	0.635
^{40}Ca	10 nK	657	32.3
^{24}Mg	0.75 nK	457	1100

standard form

$$\hat{H} = \frac{\hat{p}^2}{2M} - \hbar\delta\hat{P}_e + \hat{V}_{\rm ed}. \tag{3}$$

The first term is the kinetic energy, $\hat{P}_e = |e\rangle\langle e|$ is the projection operator to the excited state $|e\rangle$, $\hat{V}_{\rm ed} = \hbar\Omega/2|e\rangle\langle g| +$ H.c. is the atom-light coupling of the ground (g) and excited (e) states, and Ω is the Rabi frequency. In the standing light field $\Omega(z) = 4\Omega_0\cos^2(kz)$, where Ω_0 is the Rabi frequency per wave. Here $\delta = \omega - \omega_0$ is the detuning of the laser light frequency ω from the atom optical transition frequency ω_0.

Let us focus on the light field and atomic parameters that define the time evolution and the steady state of atoms. The steady state of Eq. (2) for the two-level atom is determined by the set of parameters of the frequency dimension: γ is the natural linewidth, δ is the detuning, Ω is the Rabi frequency, and ω_R is the recoil frequency.

A. Semiclassical limit

Let us first briefly consider the well-known semiclassical limit

$$\omega_R/\gamma \ll 1, \tag{4}$$

which is valid for the majority of atoms with strong dipole transitions where laser cooling is realized (see, for example, [22]). Here, within the framework of a two-level model (an atom with a nondegenerate ground state), the steady-state momentum distribution in the limit (4) can be equally described by only two of the aforementioned dimensionless parameters:

$$\delta/\gamma, \quad \Omega/\gamma. \tag{5}$$

The steady-state momentum distribution for low saturation $|\Omega|^2/(\delta^2 + \gamma^2/4) \leqslant 1$ and red detuning scales in these parameters as

$$\mathcal{F}_p = \mathcal{F}_p\left(\frac{p}{\tilde{p}}, \frac{\Omega}{\gamma}, \frac{\delta}{\gamma}\right), \quad \tilde{p} = \hbar k\sqrt{\gamma/\omega_R}, \tag{6}$$

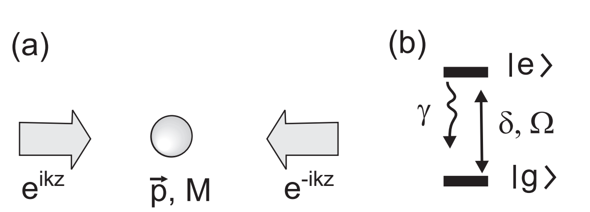

FIG. 1. Scheme of laser cooling in a standing light wave (a). Scheme of interaction of laser light with atomic energy levels (b).

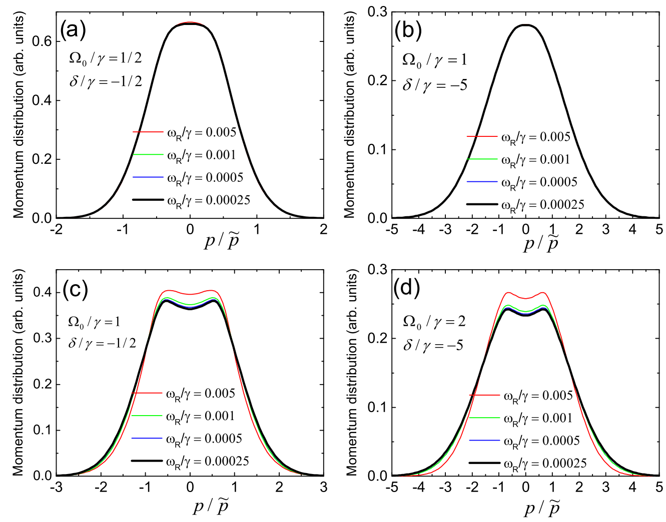

FIG. 2. Steady-state momentum distribution of cold atoms in the semiclassical limit (4) for the field parameters (a) $\Omega_0/\gamma = 1/2$ and $\delta/\gamma = -1/2$, (b) $\Omega_0/\gamma = 1$ and $\delta/\gamma = -5$, (c) $\Omega_0/\gamma = 1$ and $\delta/\gamma = -1/2$, and (d) $\Omega_0/\gamma = 2$ and $\delta/\gamma = -5$.

which represents the scaling law in the semiclassical limit (4). In particular, the steady-state average kinetic energy is scaled in units of γ and is a function of dimensionless parameters (5),

$$E_{\rm kin} = \langle p^2/2M\rangle = \hbar\gamma\mathcal{E}_S(\delta/\gamma, \Omega/\gamma), \tag{7}$$

where $\langle\cdots\rangle$ denotes the averaging with the $\mathcal{F}_p$ momentum distribution function.

The momentum distribution in Fig. 2 for various small ratios of ω_R/γ is obtained by numerical solution of Eq. (2) with the use of the method suggested by us in [23,24]. The momentum distribution function is well scaled for the set of parameters introduced, and for sufficiently small ω_R/γ the difference between the curves becomes less noticeable in low intensity [see Figs. 2(a) and 2(b)]. Here, for Figs. 2(c) and 2(d) we choose a larger field intensity to make the difference more visible.

As an example, the momentum distribution function $\mathcal{F}_p(p/\tilde{p}, \Omega/\gamma, \delta/\gamma)$ in the low-intensity limit, neglecting the atom localization in the optical potential of the standing light wave, can be represented by a Gaussian function with a width that determines the laser-cooling temperature. This is the ratio of diffusion D and friction κ coefficients (see, for example, [10])

$$k_BT = \left\langle\frac{p^2}{M}\right\rangle = \frac{D}{\kappa} = -\hbar\gamma\frac{7}{20}\frac{\delta^2/\gamma^2 + 1/4}{\delta/\gamma}. \tag{8}$$

In addition, the function $\mathcal{E}_S(\delta/\gamma,\ \Omega/\gamma)$ for this case has a minimum $\simeq 1/6$ at $\delta/\gamma = -1/2$ for $\Omega/\gamma \rightarrow 0$, which is the so-called Doppler cooling limit.

Note that the cooling rate of slow atoms with $p < \hbar k\gamma/\omega_R$ in the semiclassical limit (4) can also be represented in an equivalent form for various atoms by introducing a dimensionless time t/t_S,

$$t_S = \omega_R^{-1}\tau_S(\Omega/\gamma, \delta/\gamma), \tag{9}$$

which is scaled in units of ω_R^{-1} and is a function of the parameters Ω/γ and δ/γ only. As is well known, the cooling rate of slow atoms in the semiclassical limit is determined by the friction coefficient κ and the evolution of the averaged square momentum is described by

$$\langle p^2\rangle(t) = M\frac{D}{\kappa} + \exp\left(-\frac{2\kappa}{M}t\right)\left[\langle p^2\rangle_0 - M\frac{D}{\kappa}\right]. \qquad (10)$$

Note that the friction coefficient for two-level atoms in a low-intensity standing wave has the form [10,11,22]

$$\kappa = -\hbar k^2 \frac{\Omega_0^2}{\gamma^2}\frac{\delta/\gamma}{[\delta^2/\gamma^2 + 1/4 + \Omega_0^2/\gamma^2]^2}, \qquad (11)$$

which results in the cooling rate for the case considered in the form (9) for red detuning $\delta < 0$.

B. Quantum limit

In the quantum regime being considered,

$$\omega_R \geqslant \gamma, \qquad (12)$$

we may expect significant modification of the scaling law. The simple scheme of atom-light interaction [Fig. 1(b)] is significantly modified. The internal states can be written in a momentum representation as a set of families

$$|\psi(t)\rangle = \sum_n \alpha_n(t)|g, p_0 + 2n\hbar k\rangle + \beta_n(t)|e, p_0 + (2n+1)\hbar k\rangle, \qquad (13)$$

with quasimomentum p_0 in the range $-\hbar k \leqslant p_0 \leqslant \hbar k$. Coupling schemes for families for $p_0 = \hbar k$ and $p_0 = 0$ are shown in Fig. 3. In the regime (12) the processes of induced absorption and emission of light field photons are beyond the resonance contour γ and have different detuning

$$\delta_p = \delta - \omega_R(1 \mp 2p/\hbar k), \qquad (14)$$

depending on the atom momentum on the ground state p. The minus sign is related to induced emission of the photon to a copropagating wave or absorption from a counterpropagating wave. The plus sign is related to the reverse processes.

Let us define the scaling law in the regime (12). Here γ is the smallest parameter of Eq. (2) and its steady-state solution is governed by a different set of dimensionless parameters

$$\delta/\omega_R, \quad \Omega/\omega_R, \qquad (15)$$

which define the equivalence of laser cooling of various atoms for (12). The momentum distribution is a function of parameters scaled in units of ω_R, contrary to semiclassical limit

$$\mathcal{F}_p = \mathcal{F}_p\left(\frac{p}{\hbar k}, \frac{\Omega}{\omega_R}, \frac{\delta}{\omega_R}\right), \qquad (16)$$

which results in the average kinetic energy

$$E_{\rm kin} = \langle p^2/2M\rangle = \hbar\omega_R\mathcal{E}_Q(\delta/\omega_R, \Omega/\omega_R), \qquad (17)$$

with $\mathcal{E}_Q(\delta/\omega_R, \Omega/\omega_R)$ a dimensionless function of the parameters introduced.

To confirm this, we perform numerical simulations of the master equation (2) for various atoms with narrow-line optical transitions using the method in [23,24]. This method allows us to get a density matrix containing all the information on internal and external states of the atom without any restrictions or limits. The results in Fig. 4 show a strong equivalence of the steady-state momentum distribution of laser-cooled ^{24}Mg ($\omega_R/\gamma \simeq 1100$), ^{40}Ca ($\omega_R/\gamma \simeq 32.3$), and ^{88}Sr ($\omega_R/\gamma \simeq 0.635$) atoms for the $^1S_0 \rightarrow {}^3P_1$ optical transition, which confirms the scaling law (15)–(17) of laser cooling on narrow-line optical transitions.

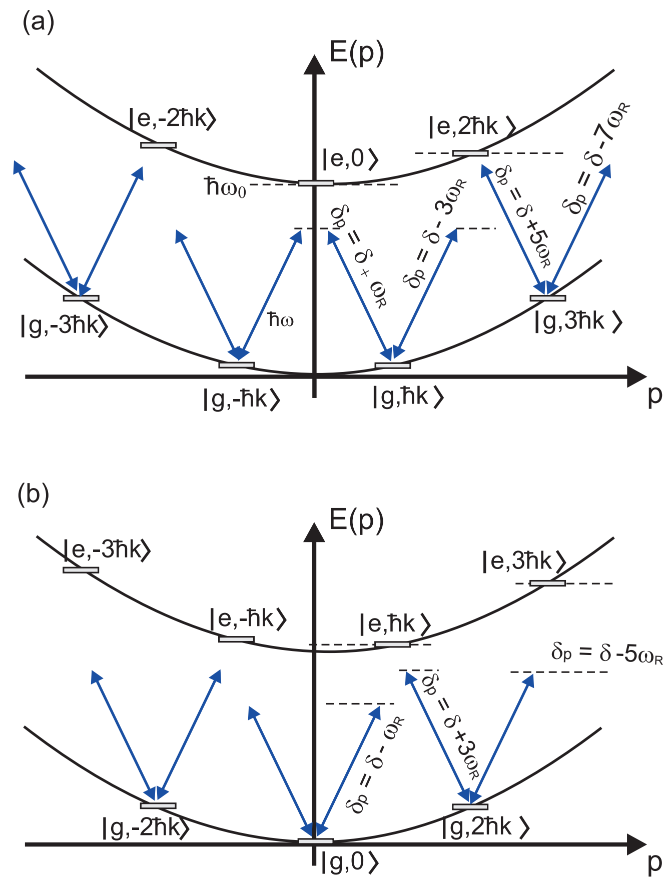

FIG. 3. Schemes of atomic energy-level coupling by light for quasimomentum (a) $p_0 = \hbar k$ and (b) $p_0 = 0$.

The sharp peaks in the momentum distribution at $p = \pm\hbar k$ represent the effects of velocity-selective coherent population trapping [5–7] for the Λ scheme of families (13) with $p_0 = \hbar k$ [Fig. 3(a)]. This effect was first demonstrated in two-level system for metastable helium [25] with $\omega_R/\gamma \simeq 0.22$.

The numerical simulation [Fig. 5(a)] shows that the minimum kinetic energy of laser cooling on narrow lines (12) in a low-intensity standing wave is reached for detuning about

$$\delta^* \simeq -3\omega_R, \qquad (18)$$

which is far beyond the optimal condition of laser cooling in a semiclassical limit and close to the one was shown in [12] for the σ_+-σ_- light field configuration. The momentum distribution is a significantly non-Gaussian function for the entire range of detuning considered [Fig. 5(b)].

The possible qualitative explanation of the optimal value of detuning (18) can be given as follows. For a low field intensity the population of the excited state is negligibly small. In this

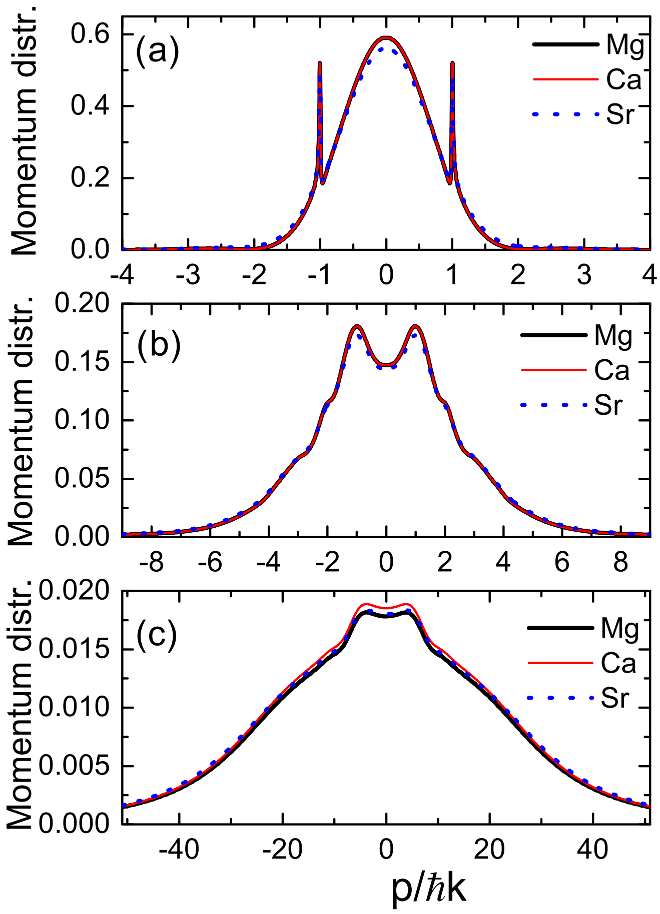

FIG. 4. Steady-state momentum distribution of laser-cooled atoms for different sets of equivalent parameters (15), including the light field detuning $\delta = -3\omega_R$ and Rabi frequency per wave (a) $\Omega_0 = 0.64\omega_R$, (b) $\Omega_0 = 6.4\omega_R$, and (c) $\Omega_0 = 64\omega_R$.

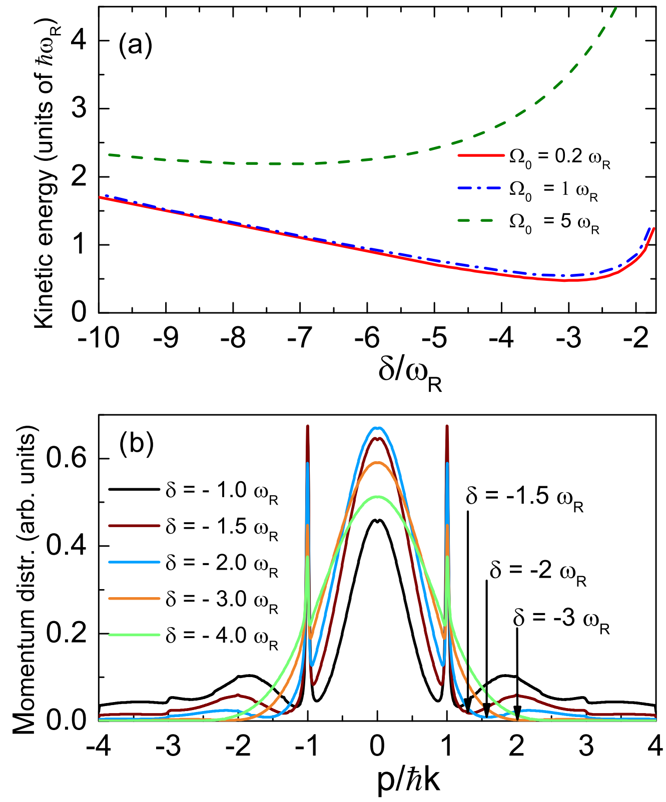

FIG. 5. (a) Kinetic energy and (b) steady-state momentum distribution of atoms for different detunings and small Rabi frequency $\Omega_0 = 0.64\,\omega_R$.

case, the atom distribution in the momentum space can be represented as a set of families (13) with nonzero amplitudes $\alpha_n(p_0)$ near $p_0 = 0$ (i.e., $|g, p = 0\rangle$ and $|g, p = \pm 2\hbar k\rangle$ states), which results in a main peak in the momentum distribution with $p = 0$ and two side peaks at $p = \pm 2\hbar k$ [Fig. 5(b)]. To obtain a minimum kinetic energy, it is necessary to suppress the amplitudes of these side peaks. The depopulation of $|g, p = \pm 2\hbar k\rangle$ states is reached for detuning δ_p, providing resonance for transitions $|g, p = \pm 2\hbar k\rangle \to |e, p = \pm \hbar k\rangle$, i.e., for $\delta_p = \delta + 3\omega_R = 0$ according to (14) [see Fig. 3(b)]. Thus the detuning $\delta = -3\omega_R$ results in the effective suppression of side peaks of the momentum distribution at $p = \pm 2\hbar k$ [Fig. 3(b)] and the effective population of the $|g, p\rangle$ state around $p = 0$.

Finally, an important question is the time evolution of the atomic distribution and cooling time. In the quantum regime (12), the atom cooling rate can be represented in an equivalent form that is different from the semiclassical case (4). Here the scaling law for the cooling rate can be written for dimensionless time t/t_Q,

$$t_Q = \gamma^{-1} \tau_Q(\Omega/\omega_R, \delta/\omega_R), \qquad (19)$$

where time t_Q scales in units of γ^{-1}, in contrast to the semiclassical limit (9), and is a function of the dimensionless parameters Ω/ω_R and δ/ω_R.

Figure 6 shows the time evolution of the momentum distribution obtained by the quantum Monte Carlo method [26] averaging over 3000 trajectories. Here we consider the laser cooling from an initial Gaussian momentum distribution with $p_\sigma = 20\hbar k$, which corresponds to the initial temperature of Mg atoms, $T \simeq 1.5$ mK, that is, the cooling temperature in a magneto-optical trap using the strong dipole optical transition ${}^1S_0 \to {}^1P_1$ [27], which can be treated as an initial stage of laser cooling.

The scaling law we discover for the quantum regimes is significantly different from the semiclassical one. In particular, the field dependence in the semiclassical regime Ω/γ

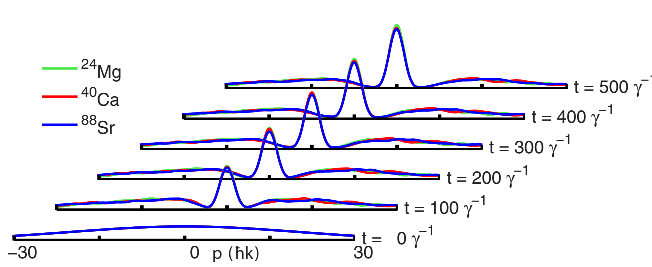

FIG. 6. Time evolution of the momentum distribution of various laser-cooled atoms for equivalent times in units of γ^{-1}. The light field parameters are $\Omega_0 = 2\omega_R$ and $\delta = -3\omega_R$.

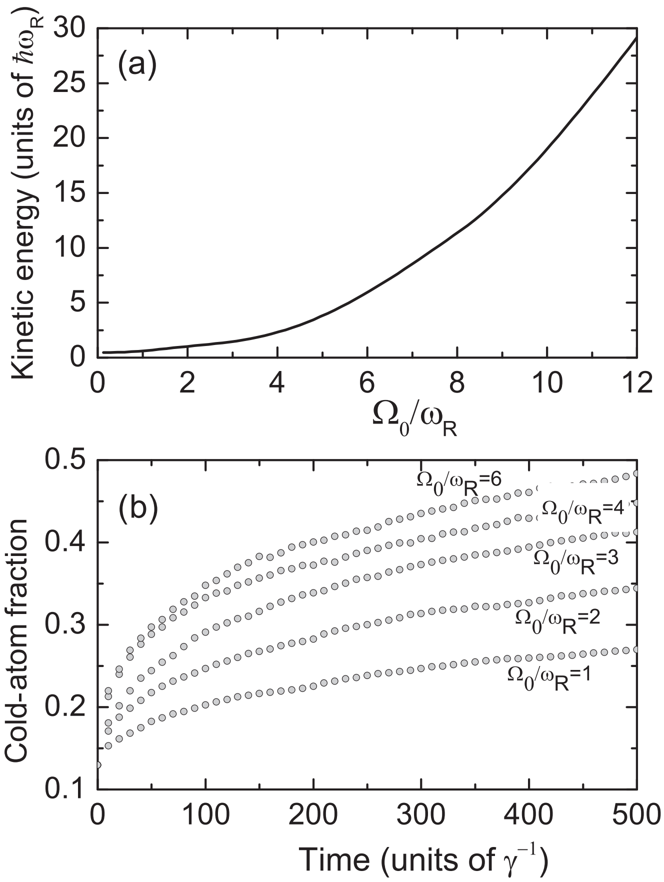

FIG. 7. (a) Steady-state kinetic energy of atoms as a function of Rabi frequency in units of ω_R and (b) time evolution of the cold-atom fraction with momentum $|p| < 3\hbar k$ in units of γ^{-1} for detuning $\delta = -3\omega_R$.

changes to Ω/ω_R, which makes it possible to use a more intense laser field without much of an increase in temperature of the cold atoms [Fig. 7(a)]. This power broadening allows a significant decrease in cooling time. The cooling time can be considered by analyzing the evolution of the cold-atom fraction. The evolution of the atom fraction with $|p| < 3\hbar k$ is shown in Fig. 7(b). The cold-atom fraction increases quickly for times $t \sim 100\gamma^{-1}$ and sufficiently large Rabi frequency $\Omega_0 = 6\omega_R$. As an example, for ^{24}Mg atoms cooling with the use of the intercombination line $^1S_0 \rightarrow {}^3P_1$, the required Rabi frequency corresponds to a sufficiently large field intensity $I \simeq 5\,\mathrm{W/cm^2}$. In addition, the cooling time $t \sim 100\gamma^{-1}$ corresponds to about 0.5 s, which may require an additional dipole trap to prevent the loss of atoms from the cooling region during this time.

III. CONCLUSION

In attempts to reach deep laser cooling of atoms with nondegenerate ground states, such as ^{24}Mg, ^{40}Ca, and ^{88}Sr, the researchers tried to use the narrow-line optical transitions. The laser cooling at the narrow-line optical transition requires a special study, which we have performed here. We have shown the equivalence of laser cooling of various atoms with $\omega_R \geqslant \gamma$, which refers to the momentum distribution, energy, and cooling rate for a set of dimensionless light field parameters (Ω/ω_R and δ/ω_R) that represent a scaling law in laser cooling on the narrow-line optical transition. The momentum distribution of various atoms is scaled for the light field parameters expressed in recoil frequency units and momentum in units of $\hbar k$. We have demonstrated that the kinetic energy of cold atoms in a low-intensity standing wave reaches a minimum for detuning close to $\delta \simeq -3\omega_R$, i.e., far from the Doppler theory of laser cooling.

The cooling time using narrow-line optical transition scales in units of γ^{-1} differs from the semiclassical limit. The efficient cooling rate can be achieved with power broadening for the Rabi frequency of few recoil frequency.

The scaling law introduced allows us to transfer the results of laser cooling (momentum distribution of atoms, energy, cooling time, and light field parameters), once obtained for certain atoms with a narrow-line optical transition, to various elements with $\omega_R \geqslant \gamma$ and predict appropriate light field parameters required for laser cooling.

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