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Relativistic chiral qubits, their time evolution, and correlations

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We introduce and discuss the concept of chiral relativistic qubit as an irreducible amount of quantum information related to a one-half spin relativistic chiral elementary system (carrier particle). We propose a Lorentz-covariant time evolution of the qubit which on the level of the density matrix is unitary. Next we investigate behavior of the Bloch vector as a function of time during the relativistic uniformly accelerated motion of the carrier particle. In particular, we select the same special evolutions which correspond to the hyperbolic, rotational, and structurally unstable motion. Finally, we consider two-qubit systems. We extend the proposed Lorentz-covariant and unitary evolution on this case in a way preserving tensor product structure of the two-particle space of states. We also discuss a correlation function in an Einstein-Podolsky-Rosen type experiment with uniformly accelerating particles; as an example we calculate correlations in the evolving Bell state.

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I. INTRODUCTION

Relativistic quantum information theory has been attracting growing attention during the past two decades (see, e.g., Refs. [1–14] and references therein). This is partially connected with some new potential applications in areas where the relativistic effects should be taken into account (see, e.g., Ref. [15]). However, the main reason lies in some fundamental unsolved questions on the border of the two pillars of the contemporary physics: quantum mechanics and relativity theory. One of such questions is the nature of the quantum nonlocality and its relation to the relativity [16]. From the experimental point of view, these problems are usually investigated with the use of photons [17–20]. However, as we know, helicity correlations of photons are momentum independent like spin correlations in nonrelativistic quantum mechanics. In contrast, the investigation of quantum correlations of relativistic massive spinning particles may offer some opportunities to deepen our understanding of the character of quantum nonlocality. The reason is that spin correlation functions in Einstein-Podolsky-Rosen (EPR) experiments with relativistic massive particles differs from its nonrelativistic counterparts—due to Wigner rotation they are usually momentum dependent [1,7,10,21–23]. These effects up to now have not been tested experimentally, although some preliminary steps in this direction were undertaken [24–26]. The last of these experiments was performed by Sakai *et al.* [26] at the RIKEN Accelerator Research Facility where the proton-proton spin correlations were measured with the proton energy ≅135 MeV ($v \cong 0.5c$). In all these experiments correlation functions were measured only for some special configurations and the results were in agreement with the nonrelativistic quantum mechanics predictions. Our arguments [7,21] show that a clear effect should appear when the kinetic energy of the EPR particles is at least of the order of the particles' rest mass. The experiments [24–26] did not meet this condition [27].

Analysis of quantum correlations of relativistic massive spinning particles is highly nontrivial and demands use of the full machinery of quantum field theory. Therefore, a notion of relativistic qubit could be a useful tool in analysis of correlation experiments taking into account the space-time degrees of freedom. One of the first findings in relativistic quantum information theory was the observation that the canonical model of a qubit—spin states of spin-1/2 particle—is inadequate in the relativistic case [28]. The reason is quite obvious: Lorentz transformations mix spin and space-time degrees of freedom. As a consequence, the standard definition of a reduced spin density matrix based on tracing over the space-time degrees of freedom from the full density matrix leads to an object with ill-defined transformation properties under Lorentz boosts. This issue has been discussed in many papers (see, e.g., Refs. [29–32]) but none of the proposed solutions is completely satisfactory.

In this paper we propose an approach to this problem. First of all we introduce a concept of a relativistic chiral qubit as an irreducible amount of quantum information related to a one-half spin relativistic chiral elementary system (carrier particle) in a sharp momentum state which can be treated classically. In the definition of relativistic qubits we employ $\mathsf{SL}(2,\mathbb{C})$ chiral spinors as simplest objects with definite transformation properties under Lorentz transformations. Next we propose a model of the time evolution for such qubits. This evolution is unitary and Lorentz covariant and corresponds to relativistic uniformly accelerated motion of a carrier particle. We also extend our formalism to systems of qubits. Our approach can be used as a tool in further investigations of correlations in systems of qubits that undergo different evolutions.

We should mention here that there exists a vast literature on quantum information theory in noninertial frames of reference; see, e.g., Refs. [33–36] and references therein. In these

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works authors consider correlations, discord, etc. in a scenario when at least one observer is uniformly accelerated. Notice that it is a passive point of view when the system (usually modes of a free quantum field) is observed by accelerated observers. On the contrary, in our approach observers are inertial but the quantum system (particles) is uniformly accelerated.

II. NONRELATIVISTIC QUBITS

In nonrelativistc quantum mechanics a qubit is represented by density matrices ρ belonging in the endomorphism's space of a two-dimensional complex Hilbert space $\mathbb{C}^2$. We can "equip" the qubit with space-time characteristics by identification of the qubit space $\mathbb{C}^2$ with a sharp momentum subspace of the entire Hilbert space spanned by fixed momentum eigenstates of the momentum operator. Such state has the momentum-dependent polarization vector $\boldsymbol{\xi} = \boldsymbol{\xi}(k)$ and transforms covariantly according to the law

$$\mathcal{G} \ni g : \rho'(\mathbf{k}') = \tfrac{1}{2}[I + \boldsymbol{\xi}'(\mathbf{k}') \cdot \boldsymbol{\sigma}], \quad \boldsymbol{\xi}' = R\boldsymbol{\xi}, \quad \mathbf{k}' = g\mathbf{k}. \tag{1}$$

We can treat the particle momentum as an external parameter characterizing the kinematical state of the carrier particle of the qubit provided that we do not measure any observables which do not commute with the momentum.

III. RELATIVISTIC CHIRAL QUBIT

In relativistic quantum theory a complete one-particle density matrix ρ given in the momentum representation in the spin basis transforms under Lorentz group according to the rule

$$\rho'(\Lambda r, \Lambda p) = \mathbb{W}(A, r)\rho(r, p)\mathbb{W}^\dagger(A, p), \tag{2}$$

where in the case of spin one-half $\rho(r, p) = [\rho(r, p)_{\sigma\lambda}]$, $\sigma, \lambda = \pm\frac{1}{2}$ is a 2×2 momentum-dependent matrix satisfying the hermiticity condition $\rho(r, p)^*_{\sigma\lambda} = \rho(p, r)_{\lambda\sigma}$, spectrum $(\rho) \geqslant 0$, $\mathrm{Tr}\,\rho = 1$, and trace is given also over momentum variables (r, p). Here $\mathbb{W}(A, k) = \mathbb{L}^{-1}_{\Lambda k} A \mathbb{L}_k$ is the Wigner-Thomas matrix; $\mathbb{L}_k$ is the Lorentz boost defined by the relation $\mathbb{L}_k q\sigma \mathbb{L}^\dagger_k = k\sigma$, where $k\sigma = k^\mu \sigma_\mu$, $k^2 = m^2$, and $q\sigma = m\sigma_0 = mI$. Finally, $A \in \mathsf{SL}(2, \mathbb{C})$ and the Lorentz group elements $\Lambda = \Lambda(A)$ are obtained via canonical homomorphism from $\mathsf{SL}(2, \mathbb{C})$. The explicit form of the boost matrix is the following:

$$\mathbb{L}_k = \frac{1}{\sqrt{2\left(1 + \frac{k^0}{m}\right)}}\left(I + \frac{k\sigma}{m}\right). \tag{3}$$

Taking into account the momentum-dependent transformation rule (2) one can show [28] that the reduction procedure is inadequate to this case: the reduced density matrix is not Lorentz covariant. This suggests the use of the sharp momentum densities $\rho(r, p) = 4r^0 p^0 \delta^3(\mathbf{r} - \mathbf{k})\delta^3(\mathbf{p} - \mathbf{k})\rho(k)$. Consequently, the only nonzero part of the matrix $\rho(r, p)$ is $\rho(k)$ located in the sharp momentum subspace $\mathcal{H}(k) \sim \mathbb{C}^2$ of the entire Hilbert space $\mathcal{H}$ and thus $\rho(k)$ has the form

$$\rho(k) = \tfrac{1}{2}[I + \boldsymbol{\sigma} \cdot \boldsymbol{\xi}(k)] \equiv \rho(k, \boldsymbol{\xi}) \tag{4}$$

with the unitary transformation law

$$\rho'(\Lambda k, \boldsymbol{\xi}') = \mathbb{W}(A, k)\rho(k, \boldsymbol{\xi})\mathbb{W}^\dagger(A, k). \tag{5}$$

Taking into account the form of $\rho(k, \boldsymbol{\xi})$ we obtain that the polarization vector transforms according to the rule $\boldsymbol{\xi}'(\Lambda k) = R(\Lambda(A), k)\boldsymbol{\xi}(k)$, where $R(\Lambda, k)$ is the Wigner rotation.

Now, the spin operator $\hat{\mathbf{S}}$ has in the spin basis the standard form $\mathbf{S} = \frac{1}{2}\boldsymbol{\sigma}$, while its average value $\langle \mathbf{S} \rangle = \frac{1}{2}\boldsymbol{\xi}(k)$. We observe that the transformation rule (5) of the density $\rho(k, \boldsymbol{\xi})$ is not a manifestly Lorentz-covariant one. Moreover, it is nonlocal in the configuration representation because of the momentum dependence of the Wigner matrix $\mathbb{W}$. Fortunately, we can omit the above difficulty with help of the so-called intertwining operators connecting the transformation law (5) with spinor representations of the Lorentz group. As is well known there exists two fundamental complex-conjugated chiral representations of the group $\mathsf{SL}(2, \mathbb{C})$, left handed (L) and right handed (R) (for details see [37] and references therein).

The corresponding chiral spinors, say wave functions $\psi_{L\alpha}$ and $\psi_{R\dot{\beta}}$, $\alpha = 1, 2$ and $\dot{\beta} = \dot{1}, \dot{2}$, transform under the $\mathsf{SL}(2, \mathbb{C})$ group action according to the law $\psi'_L = A\psi_L$ and $\psi'_R = A^*\psi_R$, where we follow the Van der Waerden formalism [37]. Because chiral representations are conjugated it is enough to find only one intertwining matrix denoted here as $\mathsf{s}(k)_{\alpha\sigma}$. We postulate the relationship of $\rho(k, \boldsymbol{\xi})$ (given in the spin basis) with its counterpart $\theta(k, \boldsymbol{\xi})$ (given in the spinorial basis), in the following form:

$$\theta(k, \boldsymbol{\xi}) = \mathsf{s}(k)\rho(k, \boldsymbol{\xi})\mathsf{s}^\dagger(k), \tag{6}$$

and we will assume the following Lorentz-covariant transformation law for $\theta(k, \boldsymbol{\xi})$:

$$\theta'(k', \boldsymbol{\xi}') = A\theta(k, \boldsymbol{\xi})A^\dagger. \tag{7}$$

The transformation rules (5) and (7) are compatible only if the Weinberg consistency condition [38] holds for the intertwining matrix $\mathsf{s}(k)$

$$A\mathsf{s}(k) = \mathsf{s}(\Lambda k)\mathbb{W}(A, k). \tag{8}$$

The solution to the condition (8) has, up to a factor, the simple form

$$\mathsf{s}(k) = \mathbb{L}_k. \tag{9}$$

Notice that the Weinberg condition (8) reduces to the definition of the Wigner-Thomas matrix. The complex conjugated matrix $\mathsf{s}^*(k) = \mathbb{L}^{\dagger T}_k$ connects the spin and the right-handed bases. We remark that $\mathbb{L}_k$ as matrix is Hermitian; however, its above form expresses change of the undotted into dotted index *via* the complex conjugation.

By means of (4), (6), and (9) we finally obtain the surprisingly simple covariant form of the relativistic qubit as

$$\theta(k, \boldsymbol{\xi}) = (k^\mu + 2w^\mu)\sigma_\mu, \tag{10}$$

where the spacelike four-vector

$$w^\mu(k, \boldsymbol{\xi}) = \left\{ w^0 = \frac{\mathbf{k} \cdot \boldsymbol{\xi}}{2}, \mathbf{w} = \frac{m}{2}\left(\boldsymbol{\xi} + \frac{(\mathbf{k} \cdot \boldsymbol{\xi})\mathbf{k}}{m(m + k^0)}\right)\right\} \tag{11}$$

is the classical counterpart of the Pauli-Lubanski pseudovector and is known as polarization four-vector [39].

Now, taking into account that underparity operation $\boldsymbol{\xi}^\pi = \boldsymbol{\xi}$, $k^{\pi\mu} = (k^0, -\mathbf{k})$ we get the dual qubit:

$$\theta^\pi(k^\pi, \boldsymbol{\xi}^\pi) = \sigma_2\theta(k^\pi, \boldsymbol{\xi})\sigma_2 = (k^\mu - 2w^\mu)\sigma_\mu. \qquad (12)$$

As in the nonrelativistic case we will treat the four-momentum k^μ as an external kinematical attribute of the carrier particle; quantumness is affiliated to the spin degrees of freedom only.

To calculate the average value of an observable we can use a matrix $\rho(k, \boldsymbol{\xi})$ or $\theta(k, \boldsymbol{\xi})$ but we should remember that, due to the nonunitarity of the matrix $\mathsf{s}(k)$ [Eq. (9)], the chiral basis is nonorthogonal. Consequently, calculating averages with the help of matrices in chiral basis we have to insert a Gram matrix under trace. For example, for a spin observable we have

$$\langle\hat{\mathbf{S}}\rangle = \mathrm{Tr}\left[\rho(k, \boldsymbol{\xi})\frac{\boldsymbol{\sigma}}{2}\right] = \mathrm{Tr}[(\mathsf{s}(k)\mathsf{s}^\dagger(k))^{-1}\theta(k, \boldsymbol{\xi})\mathbf{S}_c], \qquad (13)$$

where $\mathbf{S}_c = \mathsf{s}(k)\frac{\sigma}{2}\mathsf{s}^{-1}(k)$ is a matrix of a spin observable in the chiral basis, while the term $(\mathsf{s}(k)\mathsf{s}^\dagger(k))^{-1} = \mathbb{L}_k^{-2} = \frac{1}{m}(k^\pi\sigma)$ is the Gram matrix.

IV. LORENTZ-COVARIANT EVOLUTION

There exists a simple way of introducing an interesting Lorentz covariant, unitary dynamics of the above qubit, and its carrier particle. Below we use the proper time τ of the spinning particle as the evolution parameter. Let us consider one parameter subgroup of the $\mathsf{SL}(2, \mathbb{C})$ group of the form

$$\mathbb{K}(\tau) = \exp\left[-i\tau(\mathbf{h}\cdot\boldsymbol{\sigma} + i\mathbf{e}\cdot\boldsymbol{\sigma})\right], \qquad (14)$$

with fixed real vectors $\mathbf{h}$ and $\mathbf{e}$. By means of (7) and with the initial conditions $k^\mu(0) = \kappa^\mu$, $w^\mu(0) = w^\mu(\kappa, \boldsymbol{\xi}_0)$ we define the Lorentz-covariant proper time evolution of the qubit by the formula

$$\theta(k(\tau), \boldsymbol{\xi}(\tau)) = \mathbb{K}(\tau)\theta(\kappa, \boldsymbol{\xi}_0)\mathbb{K}(\tau)^\dagger. \qquad (15)$$

Taking into account the form (10) of the qubit we have

$$k(\tau)^\mu\sigma_\mu = \mathbb{K}(\tau)[\kappa^\mu\sigma_\mu]\,\mathbb{K}(\tau)^\dagger, \qquad (16)$$

$$w(\tau)^\mu\sigma_\mu = \mathbb{K}(\tau)[w(\kappa, \boldsymbol{\xi}_0)^\mu\sigma_\mu]\,\mathbb{K}(\tau)^\dagger. \qquad (17)$$

Therefore, the evolution of four-momentum and polarization four-vector is given by

$$k^\mu(\tau) = K(\tau)^\mu{}_\nu\kappa^\nu, \quad w^\mu(\tau) = K(\tau)^\mu{}_\nu w^\nu(\kappa, \boldsymbol{\xi}_0), \qquad (18)$$

where $K(\tau) = \Lambda(\mathbb{K}(\tau))$. Now, by means of (6) and (15) the unitary evolution of the original density matrix $\rho(k, \boldsymbol{\xi})$ reads

$$\rho(k(\tau), \boldsymbol{\xi}(\tau)) = \mathbb{W}(\mathbb{K}(\tau), \kappa)\rho(\kappa, \boldsymbol{\xi}_0)\mathbb{W}^\dagger(\mathbb{K}(\tau), \kappa), \qquad (19)$$

where the Wigner matrix has the form

$$\mathbb{W}(\mathbb{K}(\tau), \kappa) = \mathbb{L}_{k(\tau)}^{-1}\mathbb{K}(\tau)\mathbb{L}_\kappa. \qquad (20)$$

Thus the relativistic qubit $\rho(k, \boldsymbol{\xi})$ evolves according to the unitary time evolution while evolution of its chiral counterpart $\theta(k, \boldsymbol{\xi})$ guarantees Lorentz covariance. Indeed, the $\mathsf{SL}(2, \mathbb{C})$ transformations $\mathbb{K}'(\tau) = A\mathbb{K}(\tau)A^\dagger$ are inner automorphisms of the $\mathsf{SL}(2, \mathbb{C})$, which leads to the Lorentz covariance of the above time evolution. Furthermore, the values of Casimir invariants $C_1 = \mathbf{e}^2 - \mathbf{h}^2$ and $C_2 = \mathbf{e}\cdot\mathbf{h}$ of those transformations determine the character of the motion of the qubit carrier particle and a classification of possible evolutions [40–42]. Finally, interpretation of the four-momentum $k^\mu(\tau)$ as the classical parametrization of a qubit enables us to calculate four-coordinates $x^\mu(\tau)$ by integration of the four-velocity $k^\mu(\tau)/m$ as well as four-acceleration by its differentiation. It is easy to prove that the obtained class of motions correspond to the constant square acceleration [containing also hyperbolic (Rindler) motion]. In particular, from the variety of possible evolution, we can distinguish the following special classes of motions.

(1) $\mathbf{h} = \mathbf{0}$, $\mathbf{e} \neq \mathbf{0}$ ($C_1 > 0$, $C_2 = 0$)—hyperbolic motion. In this case the trajectory is in a plane determined by the initial momentum $\mathbf{q}$ and the vector $\mathbf{e}$. The end of the polarization vector describes a fragment of a circle on the Bloch sphere. It is interesting that in this case it is possible to arrange initial conditions in such a way that the quantum spin state does not change during the evolution. Such a situation takes place when the initial momentum $\mathbf{q}$ is parallel to the vector $\mathbf{h}$.

(2) $\mathbf{h} \neq \mathbf{0}$, $\mathbf{e} = \mathbf{0}$ ($C_1 < 0$, $C_2 = 0$)—circular motion. In this case the trajectory in general is a helix, except the special case $\mathbf{q} \perp \mathbf{h}$ when the trajectory is a circle. The end of the Bloch vector moves on a circle on a Bloch sphere. Let us notice that when the initial momentum $\mathbf{q} = \mathbf{0}$ the particle stays at rest but the Bloch vector rotates.

(3) $|\mathbf{h}| = |\mathbf{e}| \neq 0$, $\mathbf{h} \perp \mathbf{e}$ ($C_1 = C_2 = 0$)—structurally unstable [43] polynomial in time motion. This case is structurally unstable, i.e., infinitesimal change of any of the parameters changes the character of the motion. In this case the trajectory is not plain; the end of the polarization vector describes a fragment of a circle on the Bloch sphere.

The case $\mathbf{e} = \mathbf{0}$, $\mathbf{h} = \mathbf{0}$ corresponds to a free motion. The general evolution corresponds to arbitrary values of C_1 and C_2. In Fig. 1 we present an exemplary evolution in the general situation that does not fall in any of the above cases.

It is worth stressing that (18) represents a class of solutions of the Bargmann-Michel-Telegdi equations [44]. Indeed, by the differentiation of (18) with respect to the proper time we obtain that

$$\frac{dk^\mu}{d\tau} = f^\mu{}_\nu k^\nu, \quad \frac{dw^\mu}{d\tau} = f^\mu{}_\nu w^\nu, \qquad (21)$$

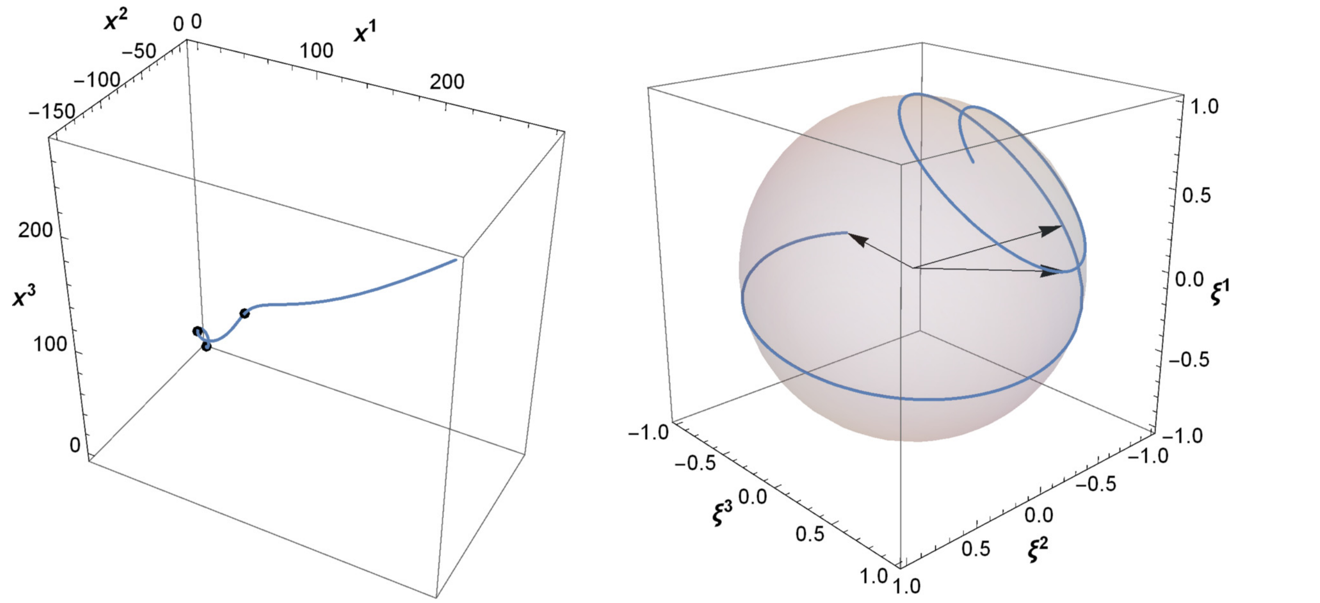
where $f_{0i} = -f_{i0} = e_i$ and $f_{ij} = \varepsilon_{ijk}h_k$. Thus the $\mathsf{SL}(2, \mathbb{C})$ group generator $\mathbf{h}\cdot\boldsymbol{\sigma} + i\mathbf{e}\cdot\boldsymbol{\sigma}$ plays the role analogous to an external homogenous electromagnetic field. The Bargmann-Michel-Telegdi equation of the above form describes motion of a one-half spin charged particle without radiation-reaction forces and with magnetic moment equal to 2. Thus the discussed evolution of chiral qubit can be experimentally realized by such a carrier particle.

V. TWO-PARTITE STATES

The density operator describing two particles, say A and B, with sharp momenta k and p can be defined in the spin or chiral basis. The corresponding matrices $\rho_{AB}(k, p)$ and $\theta_{AB}(k, p)$ are related *via* the equation

$$\theta_{AB}(k, p) = [\mathsf{s}(k)\otimes\mathsf{s}(p)]\rho_{AB}(k, p)[\mathsf{s}^\dagger(k)\otimes\mathsf{s}^\dagger(p)]. \qquad (22)$$

Properties of the original density matrix $\rho_{AB}(k, p)$ imply, by means of (9) and (26), that $\theta_{AB}(k, p)$ is



(a) (b)

FIG. 1. In (a) we present the trajectory of a carrier particle and in (b) the evolution of the Bloch vector. We assume the following parametrization: $m=1$, $q^\mu=(\sqrt{1+q^2},0,0,q)$, $\mathbf{h}=|\mathbf{h}|(\sin\alpha,0,\cos\alpha)$, $\mathbf{e}=|\mathbf{e}|(\sin\gamma\ \sin\alpha+\cos\alpha\ \cos\beta\ \sin\gamma, \sin\beta\ \sin\gamma, \cos\alpha\ \cos\gamma-\cos\beta\ \sin\alpha\ \sin\gamma)$, and $\boldsymbol{\xi}=(\cos\Phi\ \sin\Omega, \sin\Phi\ \sin\Omega, \cos\Omega)$, with $q=-6$, $|\mathbf{h}|=0.7$, $|\mathbf{e}|=0.4$, $\alpha=2\pi/5$, $\beta=4\pi/5$, $\gamma=2\pi/5$, $\Phi=2\pi/5$, and $\Omega=11\pi/10$. The curve is drawn for the proper time $\tau\in\langle 0,12\rangle$; exemplary Bloch vectors correspond to $\tau=0$, $\tau=4$, and $\tau=8$. We use natural units with $\hbar=c=1$.

Hermitian, $\mathrm{Tr}[((k^\pi\sigma)\otimes(p^\pi\sigma))\theta_{AB}(k,p)]=m^2$, and equation $\det[m^2\theta_{AB}(k,p)-\lambda(k\sigma)\otimes(p\sigma)]=0$ has nonnegative solutions for λ. Hermiticity of $\rho_{AB}(k,p)$ and $\theta_{AB}(k,p)$ allows us to write

$$\rho_{AB}(k,p)=\sum_{\mu,\nu}R(k,p)^{\mu\nu}\sigma_\mu\otimes\sigma_\nu \tag{23}$$

and

$$\theta_{AB}(k,p)=\sum_{\mu,\nu}\Omega(k,p)^{\mu\nu}\sigma_\mu\otimes\sigma_\nu \tag{24}$$

with real $R(k,p)^{\mu\nu}$ and $\Omega(k,p)^{\mu\nu}$. Moreover, from Eqs. (9) and (26) it follows that

$$\Omega(k,p)=L(k)R(k,p)L(p)^T, \tag{25}$$

where $L(k)$ is a four-dimensional standard boost taking $(m,\mathbf{0})$ to k, i.e., $L(k)=\Lambda(\mathbb{L}_k)$.

Under Lorentz transformations the matrix $\theta_{AB}(k,p)$ transforms according to

$$\theta'_{AB}(k',p')=(A\otimes A)\theta_{AB}(k,p)(A^\dagger\otimes A^\dagger). \tag{26}$$

In general, each particle from a two-particle state can evolve according to a different rule (for example, we can consider a situation when only one particle is accelerating while the second one moves with a constant velocity). In such a case evolution parameters are proper times τ_A and τ_B of corresponding particles. However, proper times τ_A and τ_B depend on the same coordinate time t. Indeed, as we have mentioned before, $k(\tau_A)$ and $p(\tau_B)$ can be integrated and in result we obtain parametric equations of classical trajectories $x_A^\mu(\tau_A)$ and $x_B^\mu(\tau_B)$. Therefore, solving equations $x_A^0(\tau_A)=x_B^0(\tau_B)=t$, we can express τ_A and τ_B by a single coordinate time t. We write $k(t)$ instead of $k(\tau_A(t))$ and analogously for other quantities. Thus

$$\theta'_{AB}(k(t),p(t),t)=[\mathbb{K}_1(t)\otimes\mathbb{K}_2(t)]\theta^{\mathrm{in}}_{AB}[\mathbb{K}_1^\dagger(t)\otimes\mathbb{K}_2^\dagger(t)], \tag{27}$$

where $\theta^{\mathrm{in}}_{AB}=\theta_{AB}(k(0),p(0),0)$.

On the level of density matrices $\rho_{AB}(k,p)$ the manifestly covariant evolution (27) corresponds to unitary transformation

$$\begin{aligned}\rho'_{AB}&(k(t),p(t),t)\\ &=[\mathbb{W}(\mathbb{K}_1(t),k(0))\otimes\mathbb{W}(\mathbb{K}_2(t),p(0))]\\ &\quad\times\rho^{\mathrm{in}}_{AB}[\mathbb{W}^\dagger(\mathbb{K}_1(t),k(0))\otimes\mathbb{W}^\dagger(\mathbb{K}_2(t),p(0))],\end{aligned} \tag{28}$$

where $\rho^{\mathrm{in}}_{AB}=\rho_{AB}(k(0),p(0),0)$.

It should be stressed here that (28) is a local unitary transformation; therefore, it preserves entanglement and other quantum correlations (like quantum discord). Thus we have a unique model of unitary (on the level of the matrix ρ_{AB}) and manifestly Lorentz-covariant (on the level of the matrix θ_{AB}) evolution of two spin-1/2 particles preserving entanglement of the quantum spin state. At the same time in our model the carrier particles undergo uniformly accelerated motion.

VI. EINSTEIN-PODOLSKY-ROSEN CORRELATIONS

Let Alice and Bob measure spin components along vectors $\mathbf{a}$ and $\mathbf{b}$, respectively. Then the correlation function reads

$$C_{\rho_{AB}}(\mathbf{a},\mathbf{b})=\mathrm{Tr}[\rho_{AB}(\mathbf{a}\cdot\boldsymbol{\sigma}\otimes\mathbf{b}\cdot\boldsymbol{\sigma})]. \tag{29}$$

Now, assuming that the state ρ_{AB} is the state $\rho'_{AB}(k(t),p(t),t)$ evolving according to the rule (28) we get for the correlation

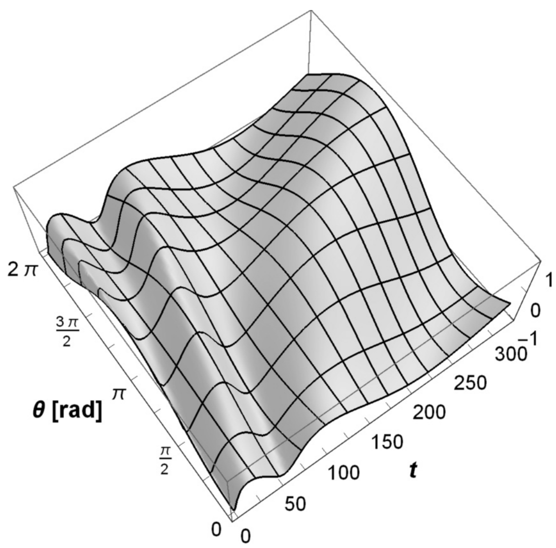


FIG. 2. Correlation function in the Bell state evolving in the following way: one particle stays at rest while the second one moves according to rules given in the caption of Fig. 1. We have utilized the following parametrization: $\mathbf{b} = (\cos\varphi\,\sin\psi, \sin\varphi\,\sin\psi, \cos\psi)$ and $\mathbf{a} = (\cos\alpha\cos\varphi\cos\psi\sin\theta + \cos\theta\cos\varphi\sin\psi - \frac{1}{2}\sin\alpha\sin\varphi[\sin\theta + 2\sin(\theta+\phi) - \sin(\theta+2\psi)],\ \cos\varphi\sin\alpha\sin\theta + \sin\varphi(\cos\alpha\cos\psi\sin\theta + \cos\theta\sin\psi),\ \cos\theta\cos\psi - \cos\alpha\sin\theta\sin\psi)$, with $\alpha = \pi/10$, $\varphi = \pi/3$, and $\psi = \pi/4$. θ is an angle between $\mathbf{a}$ and $\mathbf{b}$. $t = x^0(\tau)$ denotes coordinate time. In the considered motion $x^0(\tau = 4) = 47.76$, $x^0(\tau = 8) = 143.12$, and $x^0(\tau = 10.5) = 299.32$.

function the following formula:

$$C_{\rho'_{AB}(k(t),p(t),t)}(\mathbf{a}, \mathbf{b}, t) = C_{\rho^{\text{in}}_{AB}}(\mathbf{a}(t), \mathbf{b}(t)). \tag{30}$$

Here $\mathbf{a}(t) = R^T(\mathbb{K}_1(t), k(0))\mathbf{a}$ and $\mathbf{b}(t) = R^T(\mathbb{K}_2(t), p(0))\mathbf{b}$, where $R(\mathbb{K}_1(t), k(0))$ and $R(\mathbb{K}_2(t), p(0))$ are rotations corresponding to $\mathbb{W}(\mathbb{K}_1(t), k(0))$ and $\mathbb{W}(\mathbb{K}_2(t), p(0))$, respectively.

As an example let us consider two particles that are initially in the Bell (rotational singlet) spin state $\rho^{\text{in}}_{AB} = \rho_{\text{Bell}}(k(0), p(0), 0)$. Therefore, the correlation function in EPR experiment in this state reads $C_{\rho_{\text{Bell}}(k(0),p(0),0)}(\mathbf{a}, \mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}$. Now, let us assume that one particle stays at rest while the second one evolves in a way we have considered in Fig. 1. The correlation function, according to (30), is equal to $C_{\rho'_{\text{Bell}}(k(0),p(t),t)}(\mathbf{a}, \mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}(t)$, where $\mathbf{b}(t)$ is defined after Eq. (30). We have presented this function in Fig. 2. As we can see the strength and form of correlations depend on the motion of the carrier particles—acceleration influences correlations significantly.

VII. CONCLUSIONS

We have introduced the notion of a relativistic chiral qubit and described its unitary and Lorentz-covariant evolution during relativistic uniformly accelerated motion of the carrier particle. We have also indicated the connection of our evolution model with the Bargmann-Michel-Telegdi equation to point out possible physical realization of the proposed evolution. In our model the kinematical state of the carrier particles can be controlled during an experiment. This allows us to consider qualitatively and quantitatively different scenarios with distant observers sharing entangled particles like, e.g., the EPR-type experiment discussed above. Our model deals with sharp momentum states but in realistic experiments the prepared states (usually created in scattering processes) are almost sharp in the momentum. Moreover, the influence of the particle localization on the correlation function can be neglected when localization regions are larger than tens of the Compton wavelength. The detectors (pixel arrays) used in such experiments are able to localize particles in regions of linear size $\sim 100\lambda_e$ (the electron Compton wavelength). See [45] for an exhaustive discussion of this problem.

Let us stress that the presented formalism employs a unified framework that applies also equally well to Dirac and Majorana qubits. We believe that the presented results open possibilities in the description of experiments with observers in a relative motion (including accelerated ones).

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