Generation of superposition coherent states of microwave fields via dissipation of a superconducting qubit with broken inversion symmetry

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We present an efficient approach for dissipative generation of quantum superposition states in the microwave resonator, which is coupled to a superconducting qubit. We investigate the situation where the inversion symmetry of potential energy of the qubit is broken, and the strong two-photon nonlinear coupling between qubit and resonator can be realized via the transverse and longitudinal couplings. According to the two-photon dissipation and driving process, the dissipation of a qubit can be utilized to steer the microwave field into a quantum superposition of distinct coherent states, i.e., the Schrödinger cat state. In addition, we also extend the method to produce an entangled coherent state of two spatially separated resonators. Our scheme is based on quantum reservoir engineering and turns detrimental noise into a resource, which makes it feasible in experimental implementation. The present result may have potential applications in the field of quantum information processing with circuit QED systems.

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I. INTRODUCTION

Superconducting qubits have emerged as a promising architecture for building a quantum computer. It can be easily fabricated using techniques borrowed from conventional integrated circuits [1–3]. The key circuit element of a superconducting qubit is the Josephson junction, which can be operated at low temperature without dissipation and exhibits strong nonlinearity. This leads to the unequally spaced energy levels of the superconducting qubit, and the two lowest levels could be used to construct a quantum bit [4–7]. In contrast to microscopic entities such as atoms, superconducting qubits can easily be integrated and coupled to each other or to resonators of electromagnetic fields. These features make superconducting qubits become scalable and robust during gate operation and readout [8–11]. Up to now, remarkable progress has been made to explore superconducting qubits for performing quantum simulation [12–17] and realizing quantum information processing [18–23].

As is well known, the Hamiltonian of natural atoms has an inversion symmetry, which results in an electronic state with well-defined parity. Thus, single-photon induced electric-dipole transitions can only occur between electronic states with different parities. This is the well-known electric-dipole selection rule in quantum mechanics. The potential energy of artificial atoms can be adjusted by changing external parameters [24]. When the superconducting qubit has symmetric potential energy, such as the flux qubit at the optimal point, it has the same selection rule of the natural atom [24]. However, the inversion symmetry can be broken when external parameters are properly chosen, and the selection rule is quite different; i.e., single-photon transitions between any two eigenstates of the superconducting qubit are possible [24–26]. This greatly enriches the methods to implement quantum information protocols. Additionally, when the superconducting qubit with broken inversion symmetry is coupled to the superconducting resonator, there will coexist transverse and longitudinal couplings. This is different from the usual Jaynes-Cummings model that has only transverse atom-field coupling [27,28]. It has been recently shown that the longitudinal couplings are useful for realizing quantum state engineering and building a universal multiqubit architecture [11,29–32].

In this work, we propose a method for dissipative generation of a quantum superposition of coherent states in the microwave resonator, which is coupled to a superconducting qubit. We focus on the situation where the inversion symmetry of potential energies of the qubit is broken. It is shown that the strong two-photon nonlinear interaction between qubit and resonator can be realized via the transverse and longitudinal couplings, and the qubit can absorb a pair of photons from the resonator. It is further shown that when the qubit is resonantly driven by an external field, its energy relaxation can assist to drive the microwave field of the resonator into a quantum superposition of distinct coherent states, i.e., the Schrödinger cat state. We recall that the two-photon loss-engineered cat state has been theoretically proposed [33,34] and experimentally demonstrated in circuit QED [35,36]; i.e., the strong nonlinear coupling of hundreds of kilohertz has been achieved, which is limited by the cross-Kerr interaction. Since the single-photon decay readily causes decoherence, only the transient quantum coherence can be observed. In the present scheme, the nonlinear two-photon interaction induced by the transverse and longitudinal couplings is dispersive and has the coupling strength g^2/ω. It can be almost two orders of magnitude larger than that from the cross-Kerr effect.

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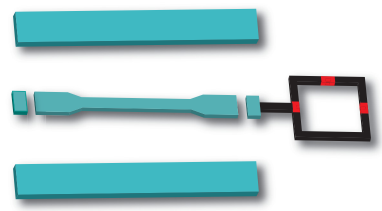

FIG. 1. Schematic of a superconducting flux qubit coupled to a transmission line resonator, where the flux qubit consisting of three Josephson junctions is operated away from the sweet point.

Therefore, the generation of cat states in our scheme can be greatly speeded up; i.e., it becomes more robust against single-photon decay, and the coherence can be stabilized for long enough time.

As an extension, we also consider the case where one qubit simultaneously is coupled to two spatially separated superconducting resonators, and the entangled coherent state between two microwave modes can be engineered via the dissipation of the qubit. Note that the preparation of an entangled coherent state has been investigated in previous schemes [37–40] and was achieved in a recent experiment [41]. However, all of the previous investigations are based on the unitary dynamical evolution process of the system, which requires the mapping of coherence from the qubit to the resonator modes. However, the present scheme is based on quantum reservoir engineering, in which the decay of a qubit is employed as a resource for quantum state preparation. Therefore, our scheme is more feasible in experimental implementation. The present result may have important applications for quantum information processing with solid-state superconducting quantum circuits.

II. MODEL

As illustrated in Fig. 1, the architecture under consideration consists of a superconducting transmission line resonator coupled to a superconducting flux qubit. The superconducting resonator with frequency ω is described by a harmonic oscillator, and its Hamiltonian is $\hat{H}_r = \hbar\omega a^\dagger a$, in which ω is the eigenfrequency and $a^\dagger$ (a) is the creation (annihilation) operator of the microwave field. We adapt the flux qubit with an inductance superconducting loop intersected by three Josephson junctions; i.e., two junctions have larger critical current than that of the third one by a factor α [42–44]. The Hamiltonian of the flux qubit is $H_q = \frac{\hbar}{2}(\delta_z\overline{\sigma}_z + \delta_x\overline{\sigma}_x)$, where $\overline{\sigma}_z$ and $\overline{\sigma}_x$ are the Pauli operators in the basis of clockwise $|\curvearrowright\rangle$ and anticlockwise $|\curvearrowleft\rangle$ persistent current states, $\delta_z = 2I_P(\Phi_{\rm ext} - \Phi_0/2)$ is the energy bias with the persistent current I_P ($\Phi_{\rm ext}$ is the external magnetic flux threading the qubit loop, and $\Phi_0 = \frac{h}{2e}$ is the flux quantum), and δ_x represents the coupling between the two circulating current states. The full interaction Hamiltonian between the flux qubit and the quantized field of the resonator is given by

$$H = \omega a^\dagger a + \tfrac{1}{2}(\delta_z\overline{\sigma}_z + \delta_x\overline{\sigma}_x) + g(a^\dagger + a)\overline{\sigma}_z, \qquad (1)$$

where g is the coupling strength and units of $\hbar = 1$ are used. It is noted that the energy gap of a flux qubit can be controlled by changing the external magnetic flux $\Phi_{\rm ext}$. For $\Phi_{\rm ext} = \Phi_0/2$ and $\delta_z = 0$, the qubit is operated at the optimal point. The above Hamiltonian is reduced to the extensively studied Jaynes-Cummings model after the rotating-wave approximation.

Here, we focus on the situation of $\Phi_{\rm ext} \neq \Phi_0/2$ and $\delta_z \neq 0$; i.e., the flux qubit is worked away from the optimal point and the inversion symmetry of its potential energy is broken. The two eigenstates of the Hamiltonian H_q are the ground state $|g\rangle = -\sin(\frac{\theta}{2})|\curvearrowright\rangle + \cos(\frac{\theta}{2})|\curvearrowleft\rangle$ and the excited state $|e\rangle = \cos(\frac{\theta}{2})|\curvearrowright\rangle + \sin(\frac{\theta}{2})|\curvearrowleft\rangle$ with $\theta = \arctan(\delta_x/\delta_z)$. Their energy difference is $\delta = \sqrt{\delta_z^2 + \delta_x^2}$. In the eigenstate representation, Eq. (1) has the form

$$H = \omega a^\dagger a + \frac{\delta}{2}\sigma_z + g_x(a^\dagger + a)\sigma_x + g_z(a^\dagger + a)\sigma_z, \qquad (2)$$

where $g_x = -g\sin\theta$, $g_z = g\cos\theta$, and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$. It is seen that both the transverse interaction with the coupling strength g_x and the longitudinal interaction with the coupling strength g_z appear between the qubit and the resonator, which is crucial for realization of the nonlinear two-photon coupling.

III. GENERATION OF SCHRÖDINGER CAT STATES

In this section, we discuss how to generate Schrödinger cat states of the superconducting resonators via a dissipative dynamical process. The key ingredient is to engineer a strong two-photon nonlinear coupling; i.e., photons are exchanged only in pairs between the flux qubit and the resonators.

To get the end, we use an external microwave field to resonantly drive the qubit. The corresponding Hamiltonian is $H_d = \Omega(\sigma_+ e^{-i\omega_p t} + \sigma_- e^{i\omega_p t})$, where Ω is the Rabi frequency. To work out the effective two-photon interaction, we first perform the unitary transformation $U = \exp[-i(a^\dagger a + \sigma_z)\omega t]$ to Eq. (2) and obtain the Hamiltonian

$$\begin{aligned} H_T = {} & g_x(a^\dagger\sigma_- e^{-i\omega t} + a\sigma_+ e^{i\omega t}) \\ & + g_z(a^\dagger e^{i\omega t} + a e^{-i\omega t})\sigma_z + \Omega(\sigma_+ + \sigma_-), \end{aligned} \qquad (3)$$

where we have discarded the antirotating terms and the relation $\omega_p = \delta = 2\omega$ is used. In the case of $\omega \gg g_x, g_y, \Omega$, applying the standard effective Hamiltonian theory for the time-averaged dynamics of highly detuned quantum systems [45], we obtain the effective Hamiltonian

$$\begin{aligned} H_{\rm eff} = {} & \frac{3g_x^2}{\omega}|e\rangle\langle e| + \frac{2g_x^2}{\omega}a^\dagger a|e\rangle\langle e| \\ & - \frac{2g_x g_z}{\omega}(\sigma_+ a^2 + \sigma_- a^{\dagger 2}) + \Omega(\sigma_+ + \sigma_-), \end{aligned} \qquad (4)$$

where we have used the condition $|e\rangle\langle e| + |g\rangle\langle g| = 1$. It is seen that the third term of $H_{\rm eff}$ represents a nonlinear two-photon coupling, which corresponds to the conversion of pairs of photons from resonator into the excitation of qubit, and vice versa.

By taking into account coupling of the qubit with a harmonic oscillator environment in the Markovian approximation, the time evolution of the density matrix ρ of the system is governed by the master equation

$$\frac{d\rho}{dt} = -i[H_{\rm eff}, \rho] + \Gamma[2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-], \qquad (5)$$

where Γ represents the energy relaxation rate of the qubit. The above master equation contains two-photon driving and dissipation processes. The qubit continuously extracts photon pairs from the resonator, and then decays back to its ground state via the energy relaxation. Meanwhile, the qubit is also resonantly driven by an external field, and it conversely sends photon pairs back to the resonator. Because of this dissipation-repumping process, the whole system would come into the steady state, i.e., $d\rho_S/dt = 0$ with $\rho_S = |\Psi_s\rangle\langle\Psi_s|$ and $|\Psi_s\rangle = |\phi_s\rangle \otimes |\psi_s\rangle$, where $|\phi_s\rangle$ and $|\psi_s\rangle$ are the states of the qubit and the resonator, respectively.

The steady state of the qubit $|\phi_s\rangle$ is obviously its ground state $|g\rangle$. The steady state of the resonator $|\psi_s\rangle$ satisfies the equation $H_{\rm eff}|\psi_s\rangle \otimes |g\rangle = 0$. Solving this equation, we have $|\psi_s\rangle = N_+|\alpha\rangle + N_-|-\alpha\rangle$, where $\alpha = i\sqrt{\frac{\Omega\omega}{g^2 \sin 2\theta}}$ and $N_\pm$ are arbitrary constants. Since the dynamics governed by master equation (5) conserves the photon-number parity which is defined by the operator $P = e^{i\pi a^\dagger a}$, the parity of the steady state of the resonator is determined by the parity of its initial state. If the resonator is initialized with even parity (for example, an initial state is the vacuum state $|0\rangle$), $N_+ = N_-$ and the steady state of the resonator will be the even Schrödinger cat state

$$|\psi_s\rangle = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2(1 + e^{-2|\alpha|^2})}. \tag{6}$$

Similarly, if the resonator is initialized with odd parity (for example, an initial state is the Fock state $|1\rangle$), $N_+ = -N_-$ and the steady state of the resonator will be the odd Schrödinger cat state

$$|\psi_s\rangle = (|\alpha\rangle - |-\alpha\rangle)/\sqrt{2(1 - e^{-2|\alpha|^2})}. \tag{7}$$

Based on quantum reservoir engineering, the dissipation of the qubit as a resource is utilized to drive the resonator into the Schrödinger cat states. Moreover, since the steady state of the qubit is the ground state, this approach is robust against the dephasing of the qubit. The resulting nonclassical states are a valuable resource not only for understanding the role of decoherence in macroscopic systems, but also for implementing universal quantum computation [46–49].

To show the quantum character of the resulting Schrödinger cat state, we apply the Wigner function

$$W(\alpha) = \frac{2}{\pi}\langle D_\alpha P D_{-\alpha}\rangle, \tag{8}$$

which is a representation of a quantum state defined over the complex plane [35]. Here, $D_\alpha = e^{\alpha a^\dagger - \alpha^* a}$ is the state displaced operator, $P = e^{i\pi a^\dagger a}$ is the parity operator, and $\langle \cdots \rangle$ represents the expectation value with respect to a given quantum state. With the system initially prepared in the ground state $|g\rangle \otimes |0\rangle$, we numerically solve master equation (5). The Wigner function $W(\alpha)$ of the resulting steady state is depicted in Fig. 2. The negative quasiprobability distribution clearly displays the nonclassical quantum features of the steady state. We check that the Wigner function shown in Fig. 2 is exactly the same as that of the even cat state $|\psi_s\rangle = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2(1 + e^{-2|\alpha|^2})}$ [34] with $\alpha = 2i$. Thus, the resonator eventually evolves into the even cat state at steady state.

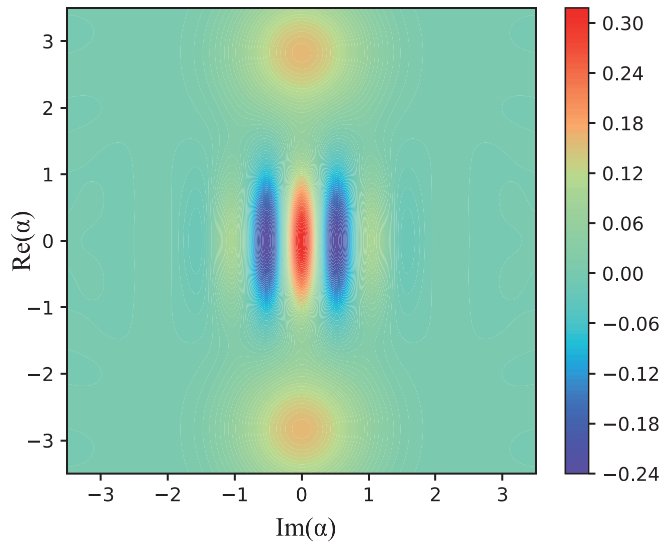

FIG. 2. The Wigner function of steady state by numerically solving master equation (5) with the initial state $|g\rangle \otimes |0\rangle$, where the relevant parameters are chosen to be $\delta/2\pi = 12$, $\omega/2\pi = 6$, $g/2\pi = 0.3$, $\Omega/2\pi = 0.06$, $\Gamma/2\pi = 0.015$ GHz, and $\theta = \pi/4$.

On the other hand, the resonator will inevitably interact with the environment in practice, and the single-photon loss channel is opened. The photon decay of the resonator will destroy the coherence, leading to a statistical mixture of $|\alpha\rangle$ and $|-\alpha\rangle$ [50]. However, if the photon pair exchange rate is much larger than the single-photon decay rate, the two-photon drive and dissipation process will dominate the dynamic evolution, and the coherence can be maintained. In recent experiments, the strong nonlinear two-photon coupling about hundreds of kilohertz has been realized between two resonator modes via the cross-Kerr interaction; i.e., the decay mode extracts only photons in pairs from the other one, and the transient quantum superposition is observed [35,36]. Compared with the above method, the nonlinear coupling in our scheme is the dispersive coupling $\frac{g^2 \sin 2\theta}{\omega}$. For the parameters of $g \approx 2\pi \times 300$ MHz, $\omega = 2\pi \times 6$ GHz, and $\theta = \frac{\pi}{4}(\delta_x = \delta_z)$, the nonlinear coupling strength is about $2\pi \times 15$ MHz. It is almost two orders of magnitude larger than that with respect to the cross-Kerr effect. In this sense, the generation of cat states in our scheme can be greatly speeded up, and the resonator with a high-fidelity cat state can be stabilized for long enough time against the single-photon decay. Moreover, only one dissipative qubit in our scheme is needed for generating the cat state, which greatly simplifies the experimental implementation with respect to the previous ones [35,36].

To investigate the effect of photon leakage of the resonator, we also perform the numerical simulation by including the single-photon decay rate κ into master equation (5), where $\kappa = \frac{Q}{\omega}$, and Q is the quality factor. Here, we define the fidelity $F = {\rm Tr}[\rho_s \rho]$ to quantify the overlap of the resulting state with the target one, where ρ is the density matrix of the system and $\rho_s = |\psi_s\rangle\langle\psi_s|$; i.e., $|\psi_s\rangle$ is the even cat state with $\alpha = 2i$. In Fig. 3, the time evolution of fidelity F is plotted. In the absence of single-photon decay, the target state is obtained at steady state with $F = 1$. For $Q = 10^6$, the

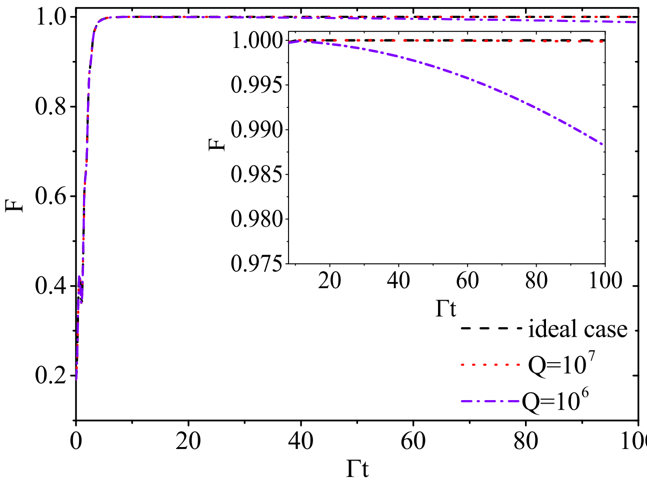

FIG. 3. Fidelity F versus the dimensionless function Γt by numerically solving master equation (5), in which the single-photon decay rate is included and the parameter is chosen the same as in Fig. 2. The corresponding target state is $|\psi_s\rangle = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2(1+e^{-2|\alpha|^2})}$ with $\alpha = 2i$.

fidelity is slightly decreased with the time evolution. But, it is clearly observed that the single-photon decay can hardly affect the quantum state with $Q = 10^7$, and the coherence can be kept for a long time. Experimentally, the transmission-line resonator with a quality factor $Q > 10^7$ has been fabricated [51], and the quality factor beyond 10^8 has been demonstrated in three-dimensional superconducting resonators [52]. So, the parameters in the numerical simulation are reachable in the current experiment. Moreover, it is also noted that the jump of photon-number parity resulting from single-photon loss can be continuously monitored and compensated with a measurement-based feedback in experiment [53,54].

Finally, we extend our idea for generating two-mode Schrödinger cat states of microwave fields in two superconducting resonators. As depicted in Fig. 4, the extended architecture under consideration consists of one flux qubit simultaneously coupled to two spatially separated resonators. Using the same method as above, we can work out the effective Hamiltonian

$$H_{\rm eff} = \frac{6g_x^2}{\omega}|e\rangle\langle e| + \frac{2g_x^2}{\omega}(a_1^\dagger + a_2^\dagger)(a_1 + a_2)|e\rangle\langle e| - \frac{2g_x g_z}{\omega}[\sigma_+(a_1+a_2)^2 + \sigma_-(a_1^\dagger + a_2^\dagger)^2] + \Omega(\sigma_+ + \sigma_-), \quad (9)$$

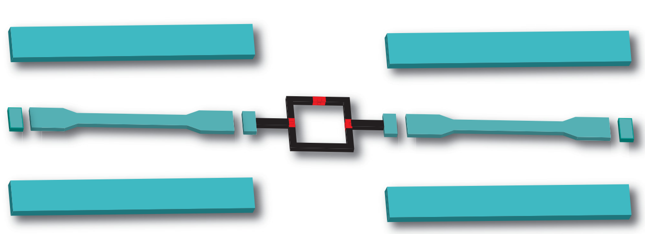

FIG. 4. The schematic of a superconducting flux qubit simultaneously coupled to two spatially separated superconducting transmission line resonators.

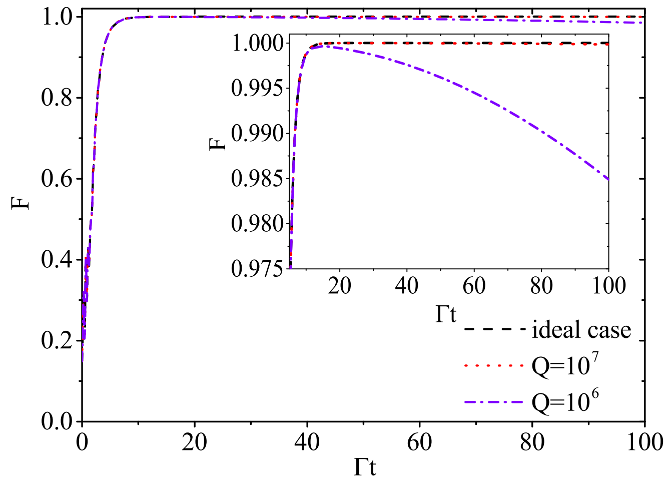

FIG. 5. Fidelity F versus the dimensionless function Γt by numerically solving master equation (11) with the initial state $|g\rangle \otimes |0,0\rangle$, in which the single-photon decay rate is included and the parameter is chosen the same as in Fig. 2 except for $\Omega/2\pi = 0.09$ GHz. The corresponding target state is $|\psi_s\rangle = (|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle)/\sqrt{2(1+e^{-4|\alpha|^2})}$ with $\alpha = 1.5i$.

where a_1 and a_2 are photon annihilation operators of the two resonator modes, respectively. The crucial part of the above Hamiltonian is the third term, which represents the pairwise exchange of photons between the qubit and the two resonators.

Including the decay of the qubit, the dynamics of the density matrix ρ of the system is governed by the master equation

$$\frac{d\rho}{dt} = -i[H_{\rm eff}, \rho] + \Gamma[2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-]. \quad (10)$$

Similarly, it also describes a two-photon drive and dissipation process, and conserves the photon-number parity. According to the quantum dissipative dynamical process, the whole system will be steered into the steady state $|\Psi_s\rangle = |g\rangle \otimes |\psi_s\rangle$, where $|\psi_s\rangle$ depends on the initial parity of the two resonators. With the system initially prepared in an even-parity state $|g\rangle \otimes |0,0\rangle$, the steady state of the two resonators will be

$$|\psi_s\rangle = (|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle)/\sqrt{2(1+e^{-4|\alpha|^2})}. \quad (11)$$

It is just the entangled coherent state with even parity [41]. This two-mode cat state exhibits a great manifestation of mesoscopic superposition and entanglement, and has very wide applications in the field of quantum information, such as quantum metrology [55], quantum networks, and teleportation [56].

To confirm the above result, we perform the numerical simulation by solving master equation (11). By setting the system with initial state $|g\rangle \otimes |0,0\rangle$, the time evolution of the fidelity is plotted in Fig. 5. Without single-photon decay, the entangled coherent state with $\alpha = 1.5i$ is obtained at steady state for $F = 1$. In the presence of photon decay, it will break the photon-number parity and the quantum superposition is gradually washed out. For $Q = 10^7$, the photon decay has negligible effect on the fidelity, and the entanglement can be sustained for a long time. Although the generation of a two-mode cat state has been proposed in several protocols [37–40] and achieved in experiment [41], all of them need to transfer the coherence from qubits to resonators. Here, the decay of the qubit as a resource is utilized for preparing the quantum entangled state. Thus, our approach is more feasible in experiment compared with the previous ones.

IV. CONCLUSION

We have proposed an approach for dissipative generation of Schrödinger cat states of microwave fields. By introducing a superconducting qubit with broken inversion symmetry, which is coupled to a resonator of the electromagnetic field, we show that the strong two-photon nonlinear coupling between qubit and resonator can be mediated via the transverse and longitudinal couplings. By means of a two-photon driving and dissipation process, the cat states of a resonator can be engineered with the assistance of energy relaxation of the qubit. Compared with the previous method [33–36], the nonlinear coupling strength in our scheme can be increased by almost two orders of magnitude. Therefore, the generation of cat states is greatly speeded up, and the coherence can be preserved for a long time against the single-photon decay. In addition, the scheme is also extended for producing the two-mode cat state of two separated resonators. Based on the remarkable progress in superconducting quantum circuits, we believe that the present scheme can be implemented in current experiments.

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