Preserving quantum entanglement from parametric amplifications with a correlation modulation scheme

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Amplification of an entangled state is a matter of great significance in quantum communication. However, any parametric amplifier (PA) will introduce added noise into the system which unavoidably degrades the entanglement or even makes it disappear. Recently, it has been experimentally demonstrated that amplification of one half of a two-mode squeezed state is possible while preserving entanglement [Phys. Rev. Lett. **103**, 010501 (2009)]. However, such entanglement cannot be maintained when the strength of the amplification is large. To solve this problem, we propose a correlation modulation scheme (CMS), which fully exploits the quantum correlation between the two output states from the PA process, to suppress the added noise from the parametric amplifications. For amplifying an entangled state using a PA, we demonstrate that the CMS not only better maintains but also always preserves the entanglement whatever the strength of the PA. Such a CMS may pave the way to low-noise amplification of an entangled state.

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I. INTRODUCTION

Quantum entanglement [1], which plays an essential role in quantum cryptography [2], quantum repeaters [3], and quantum information [4,5], reveals the intrinsic statistical relations between two or more parties in a compound quantum system. For two quantum states sharing entanglement, each one of them cannot be independently described without the assistance of the other one. Based on bipartite entanglement, many pioneering ideas have been proposed theoretically and some of them have been experimentally realized, such as quantum key distribution (QKD) [6–17], quantum dense coding [18–23], and quantum teleportation [24–33]. However, quantum entanglement is sensitive to losses and can be easily degraded in the quantum transmission [34]. One solution to this problem is quantum phase-insensitive amplification which makes the entangled states amplified while preserving the intrinsic entanglement. Recently, a low-noise parametric amplifier (PA) has been experimentally realized [35]. By using cascaded four-wave mixing (FWM) processes, it has been demonstrated that, for two entangled beams, it is possible to amplify one of them while their entanglement can still exist. However, any optical amplification process is unavoidably limited by the added noise introduced from the PA. It is this added noise that makes it hard for the aforementioned PA to maintain the entanglement when the gain of the PA is larger than 2.8. In this sense, it is natural to ask that, for two entangled states, whether their entanglement can always be preserved from the parametric amplifications.

It is well known that for a PA based on the FWM process, although added noise will be introduced from the amplification processing, its two output states are quantum correlated with each other. It is this quantum correlation that gives the possibility of many interesting experiments such as the generation of two-mode squeezed states [36–43], the tunable delay of Einstein–Podolsky–Rosen entangled state [44], and the realization of an SU(1,1) interferometer [45–51]. Along this line, in this paper we propose a correlation modulation scheme (CMS) which fully exploits the quantum correlation produced from the FWM process to suppress the added noise introduced from the parametric amplification. For amplifying two entangled states, two kinds of amplification structure are considered. The first (second) one is asymmetrical (symmetrical) amplification which amplifies one (both) of the two entangled states. For both kinds of amplification structure using a PA based on the FWM process, we demonstrate that the CMS cannot only better maintain but also always preserve the entanglement whatever the strength of the parametric amplifier.

This paper is organized as follows: In Sec. II, we first study in detail how the CMS can enhance the entanglement for the asymmetrical amplification structure. The effect of losses on the CMS is also considered. Second, we expand our CMS to the symmetrical amplification structure. We conclude in Sec. III with a discussion.

II. ENTANGLEMENT ENHANCEMENT FROM THE CORRELATION MODULATION SCHEME

For simplification, we call a PA with CMS a "PACMS" in the following discussions. The schemes of amplifying one

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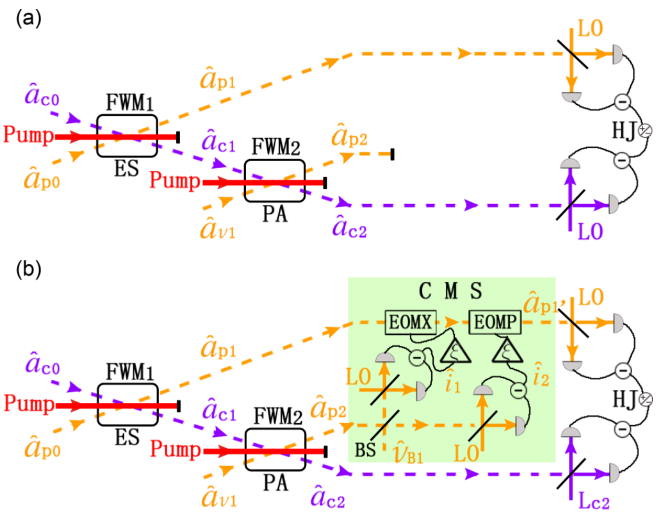

FIG. 1. Comparison between the schemes of amplifying one of two entangle states using (a) PA and (b) PACMS.

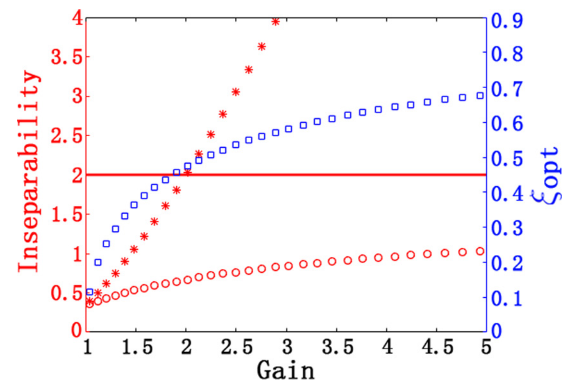

FIG. 2. Entanglement enhancement from the CMS in the ideal case. For two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$, the red asterisks (circles) show the inseparability I_{p1c2} ($I^{\min}_{p1c2'}$) between them versus the gain of the FWM$_2$ when the field $\hat{a}_{c1}$ is amplified by the PA (PACMS) to become $\hat{a}_{c2}$ ($\hat{a}_{c2'}$). The blue squares show the $\xi_{\rm opt}$ versus the gain of the FWM$_2$ minimizing $I^{\min}_{p1c2'}$. Here it is assumed that the gain of the FWM$_1$ is two.

of two entangled states (symmetrical amplification structure) using a PA and PACMS are shown in Figs. 1(a) and 1(b), respectively. For both schemes, the first FWM process (FWM$_1$) is an entanglement source (ES) for generating two entangled fields ($\hat{a}_{p1}$ and $\hat{a}_{c1}$), and the second FWM process (FWM$_2$) is an optical PA which amplifies the field $\hat{a}_{p1}$ to become $\hat{a}_{p2}$. The difference between these two schemes is that, for the scheme using PA, one ($\hat{a}_{p2}$) of the two output fields of the PA is discarded [35]. For the scheme using PACMS, the aforementioned discarded field $\hat{a}_{p2}$ is exploited by a Bell-state measurement and its output photocurrents are used to modulate the optical field $\hat{a}_{p1}$ (see below). For two vacuum fields ($\hat{a}_{p0}$ and $\hat{a}_{c0}$) entering the FWM$_1$ pumped by a strong beam (pump), two entangled fields called "probe" ($\hat{a}_{p1}$) and "conjugate" ($\hat{a}_{c1}$) are generated [38]. This FWM process can be expressed as

$$\hat{a}_{p1} = G_1\hat{a}_{p0} + g_1\hat{a}^{\dagger}_{c0}, \quad \hat{a}_{c1} = G_1\hat{a}_{c0} + g_1\hat{a}^{\dagger}_{p0}, \qquad (1)$$

where G_1 is the amplitude gain of the FWM$_1$ and $g_1^2 = G_1^2 - 1$. To amplify one of the two entangled fields, $\hat{a}_{c1}$ is seeded into another FWM process (FWM$_2$) and amplified to become $\hat{a}_{c2}$. The input-output relation for the FWM$_2$ can be given by

$$\hat{a}_{p2} = G_2\hat{a}_{v1} + g_2\hat{a}^{\dagger}_{c1}, \quad \hat{a}_{c2} = G_2\hat{a}_{c1} + g_2\hat{a}^{\dagger}_{v1}, \qquad (2)$$

where $\hat{a}_{v1}$ is in vacuum and $\hat{a}_{p2}$ is a new probe field accompanied with the generation of the field $\hat{a}_{c2}$. So far, two entangled fields ($\hat{a}_{p1}$ and $\hat{a}_{c1}$) have been prepared by the FWM$_1$ and one of them ($\hat{a}_{c1}$) has been amplified to become $\hat{a}_{c2}$ by the FWM$_2$. To describe the quantum correlation between any two optical fields $\hat{a}_1$ and $\hat{a}_2$, joint quadrature operators $\hat{X}_{-,12} = (\hat{X}_1 - \hat{X}_2)/\sqrt{2}$ and $\hat{Y}_{+,12} = (\hat{Y}_1 + \hat{Y}_2)/\sqrt{2}$ are defined, where $\hat{X}_k = \hat{a}^{\dagger}_k + \hat{a}_k$ and $\hat{Y}_k = i\hat{a}^{\dagger}_k - i\hat{a}_k$ are the quadrature-phase amplitudes of the corresponding fields ($k = 1$ and 2). The degree of entanglement or inseparability can be quantified by $I_{12} = \langle\Delta\hat{X}^2_{-,12}\rangle + \langle\Delta\hat{Y}^2_{+,12}\rangle$. Optical fields $\hat{a}_1$ and $\hat{a}_2$ are entangled or inseparable on the condition that $I_{12} < 2$ [52,53]. Experimentally, the variances of the joint quadratures $\langle\Delta\hat{X}^2_{-,p1c2}\rangle$ and $\langle\Delta\hat{Y}^2_{+,p1c2}\rangle$ can be measured by joint homodyne detection as shown in Fig. 1(a), which combines two individual homodyne detections with a hybrid junction (HJ) [35,38,44]. It can be calculated that, for the asymmetrical amplification of an entangled state using PA,

$$\langle\Delta\hat{X}^2_{-,p1c2}\rangle = \tfrac{1}{2}(g_1G_2 - G_1)^2\langle\hat{a}_{p0}\hat{a}^{\dagger}_{p0}\rangle + \tfrac{1}{2}(G_1G_2 - g_1)^2\langle\hat{a}_{c0}\hat{a}^{\dagger}_{c0}\rangle + \tfrac{1}{2}g_2^2\langle\hat{a}_{\nu_1}\hat{a}^{\dagger}_{\nu_1}\rangle, \quad (3)$$

where $\langle\Delta\hat{X}^2_{-,p1c2}\rangle = \langle\Delta\hat{Y}^2_{+,p1c2}\rangle$ and $\langle\hat{a}_{p0}\hat{a}^{\dagger}_{p0}\rangle = \langle\hat{a}_{c0}\hat{a}^{\dagger}_{c0}\rangle = \langle\hat{a}_{\nu_1}\hat{a}^{\dagger}_{\nu_1}\rangle = 1$. For the scheme using PA, the separability I_{p1c2} versus the gain of the FWM$_2$ (G_2^2) is shown by the red asterisks in Fig. 2, where the gain of the FWM$_1$ is $G_1^2 = 2$. It can be found that, with the increasing of G_2^2, the degree of the inseparability I_{p1c2} between the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$ increases. When $G_2^2 > 2$, $I_{p1c2} > 2$. In other words, the amplification of the field a_{c1} using the PA degrades the intrinsic inseparability between the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$. The larger the strength of the PA is, the worse the inseparability I_{p1c2} becomes. Optical fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$ are not inseparable until $G_2^2 > 2$. This is because, for an optical parametric amplification process, added noise will be unavoidably introduced by the amplifier [54]. For the PA based on the FWM process as shown in Fig. 1(a), the added noise is caused from both the direct amplification of the field $\hat{a}_{c1}$ and the introduction of the vacuum field $\hat{a}_{v1}$. To explain this point and study the mechanism of the amplification of the field $\hat{a}_{c1}$, it is worth studying the entanglement between the fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$. By taking $G_2^2 = 1$ in Eq. (3), it can be easily obtained that $\langle\Delta\hat{X}^2_{-,p1c1}\rangle = \langle\Delta\hat{Y}^2_{+,p1c1}\rangle$ and

$$\langle\Delta\hat{X}^2_{-,p1c1}\rangle = \tfrac{1}{2}(g_1 - G_1)^2\langle\hat{a}_{p0}\hat{a}^{\dagger}_{p0}\rangle + \tfrac{1}{2}(G_1 - g_1)^2\langle\hat{a}_{c0}\hat{a}^{\dagger}_{c0}\rangle, \qquad (4)$$

from which the inseparability I_{p1c1} between the fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ can be obtained. When the field $\hat{a}_{c1}$ is amplified to become $\hat{a}_{c2}$, the inseparability I_{p1c2} can be derived from Eq. (3). By comparing Eq. (3) with Eq. (4), it can be found that amplification of the field $\hat{a}_{c1}$ varies both terms in Eq. (4)

and introduces a new vacuum term $\frac{1}{2}g_2^2\langle\hat{a}_{\nu_1}\hat{a}_{\nu_1}^\dagger\rangle$. In the first term of Eq. (4), the factor g_1 in $\frac{1}{2}(g_1-G_1)^2\langle\hat{a}_{p0}\hat{a}_{p0}^\dagger\rangle$ becomes g_1G_2 in Eq. (3). In the second term of Eq. (4), the factor G_1 in $\frac{1}{2}(G_1-g_1)^2\langle\hat{a}_{c0}\hat{a}_{c0}^\dagger\rangle$ becomes G_1G_2 in Eq. (3). These two variations originate from the direct amplification of the field $\hat{a}_{c1}$ [see $\hat{a}_{c1}$ becomes $G_2\hat{a}_{c1}$ in the right half of Eq. (2)]. With the increasing of the gain of the FWM$_2$, the first two terms in Eq. (3) become larger and therefore the inseparability I_{p1c2} becomes worse. The vacuum term $\frac{1}{2}g_2^2\langle\hat{a}_{\nu_1}\hat{a}_{\nu_1}^\dagger\rangle$ in Eq. (3) corresponds to the amplified vacuum noise generated from the amplification process. When the field $\hat{a}_{c1}$ is amplified to become $\hat{a}_{c2}$ by the PA, the vacuum field $\hat{a}_{\nu_1}$ gets into $\hat{a}_{c2}$ and is also amplified [see the right half in Eq. (2)]. Such vacuum noise degrades the inseparability I_{p1c2} and becomes larger with the increasing of the gain of the FWM$_2$. In summary, the added noise from the PA in our system comes from both the direct amplification of the field $\hat{a}_{c1}$ and the introduction of the vacuum field $\hat{a}_{v1}$. It is this added noise that makes the inseparability between the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$ degraded or even disappear.

To solve this problem, we propose a CMS as shown in Fig. 1(b) which fully exploits the quantum correlation produced from the PA to enhance the entanglement between the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$. For the amplification of the field $\hat{a}_{c1}$, although added noise will be introduced into the field $\hat{a}_{c1}$ as discussed above, another field $\hat{a}_{p2}$ is simultaneously produced. Optical fields $\hat{a}_{p2}$ and $\hat{a}_{c2}$ are inherently time-correlated and this quantum correlation can be used to suppress the added noise introduced from the PA. The CMS can be divided into two steps: a "Bell-state" measurement and quadrature-phase amplitude modulation. For the "Bell-state" measurement, the field $\hat{a}_{p2}$ is first seeded into a beam splitter (BS) where it is assumed that the other port of the BS is seeded by a vacuum field $\hat{\nu}_{B1}$. Then the two output fields from the BS are measured by using two balanced homodyne detections, which eliminate the classical noise of the system such as the background electric noise of the photodetector and the excess noise of the local oscillator [55], to make a "Bell-state" measurement as mentioned in Ref. [27]. Therefore, the amplitude-sum quadrature $\hat{X}_{p2}+\hat{X}_{\nu_{B1}}$ and phase-difference quadrature $\hat{Y}_{p2}-\hat{Y}_{\nu_{B1}}$ can be obtained, which corresponds to the output photocurrents of the homodyne detections $\hat{i}_1$ and $\hat{i}_2$, respectively. Then the photocurrents $\hat{i}_1$ and $\hat{i}_2$ are respectively sent to an amplitude modulator (EOMX) and a phase modulator (EOMP) to modulate the field $\hat{a}_{p1}$ to become $\hat{a}'_{p1}$:

$$\hat{a}'_{p1}=\hat{a}_{p1}+\xi_1\hat{i}_1+i\xi_2\hat{i}_2, \tag{5}$$

where ξ_1 and ξ_2 are respectively the gains of the photocurrents $\hat{i}_1$ and $\hat{i}_2$. For simplification, we have $\xi=\xi_1=\xi_2$ in the following discussions. It can be calculated that, for the scheme of amplifying one of two entangled fields using PACMS,

$$\begin{aligned}\langle\Delta\hat{X}^2_{-,p1'c2}\rangle &= \tfrac{1}{2}(G_1+\xi g_1g_2-g_1G_2)^2\langle\hat{a}_{p0}\hat{a}_{p0}^\dagger\rangle\\ &\quad+\tfrac{1}{2}(g_1+\xi G_1g_2-G_1G_2)^2\langle\hat{a}_{c0}\hat{a}_{c0}^\dagger\rangle\\ &\quad+\tfrac{1}{2}(\xi G_2-g_2)^2\langle\hat{a}_{\nu_1}\hat{a}_{\nu_1}^\dagger\rangle+\tfrac{1}{2}\xi^2\langle\hat{\nu}_{B1}\hat{\nu}_{B1}^\dagger\rangle,\end{aligned} \tag{6}$$

where $\langle\Delta\hat{X}^2_{-,p1'c2}\rangle=\langle\Delta\hat{Y}^2_{+,p1'c2}\rangle$ and $\langle\hat{a}_{p0}\hat{a}_{p0}^\dagger\rangle=\langle\hat{a}_{c0}\hat{a}_{c0}^\dagger\rangle=\langle\hat{a}_{\nu_1}\hat{a}_{\nu_1}^\dagger\rangle=\langle\hat{\nu}_{B1}\hat{\nu}_{B1}^\dagger\rangle=1$. The red circles in Fig. 2 show $I^{\min}_{p1'c2}$ versus the gain of the FWM$_2$ (G_2^2) where the gain of the FWM$_1$ is $G_1^2=2$ and the optimal ξ ($\xi_{\rm opt}$) is achieved to minimize $I^{\min}_{p1'c2}$. For each G_2^2, it can be easily calculated that

$$\xi_{\rm opt}=\frac{2G_1^2G_2g_2-2G_1g_1g_2}{g_1^2g_2^2+G_1^2g_2^2+G_2^2+1}, \tag{7}$$

which is shown by the blue circles in Fig. 2. It can be found that, for the same gain of the FWM$_2$ all the red circles are smaller than the red asterisks in Fig. 2. It means that, by using the PACMS, the inseparability between the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$ can be better maintained compared with the scheme using PA. More interestingly, it can be demonstrated that $I^{\min}_{p1'c2}(G_2\to\infty)\to 2$, i.e., the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$ can be inseparable for any $G_2>0$. Such entanglement enhancement is because the CMS suppresses the added noise introduced from the PA by fully exploiting the quantum correlation between the two output fields of the FWM$_2$. As has been discussed above, the added noise is cause by the direct amplification of the field $\hat{a}_{c1}$ [see the factor G_2 in the first two terms of Eq. (3)] and the introduction of the vacuum field $\hat{a}_{\nu1}$ [see the factor g_2 in the last term of Eq. (3)]. By comparing Eq. (3) with Eq. (6), it can be found that the CMS varies all the three terms in Eq. (3). In each term, there is a new factor related to ξ and all these new factors originate from the CMS. It is these new factors that make it possible to weaken the added noise from the amplification process of the FWM$_2$. Therefore, the inseparability between the two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ can be well maintained by the CMS when one of the $\hat{a}_{c1}$ is amplified.

It is well known that entanglement is fragile and can be easily degraded by the losses of the system. The effect of the losses in the FWM process has been discussed in our previous works [49,56]. Here we study how the losses can affect the performance of the PACMS. To theoretically interpret the effect the losses of the system, a BS with reflectivity η_i is placed after each output field $\hat{a}_i$ of the FWM process: $\hat{a}'_i=\sqrt{1-\eta_i}\hat{a}_i+\sqrt{\eta_i}\hat{\nu}_i$, where $\hat{a}'_i$ is the output field of the BS and $\hat{\nu}_i$ is in vacuum. For simplification, it is assumed that $\eta=\eta_{p1}=\eta_{c1}=\eta_{p2}=\eta_{c2}$. Then, for the scheme using PACMS, the variances of the joint quadratures $\langle\Delta\hat{X}^2_{-,p1'c2}\rangle=\langle\Delta\hat{Y}^2_{+,p1c2}\rangle$ and the $\xi_{\rm opt}$ respectively become

$$\begin{aligned}\langle\hat{X}^2_{-,p1'c2}\rangle &= \frac{1}{2}[\sqrt{1-\eta}G_1+(1-\eta)\xi g_1g_2-(1-\eta)g_1G_2]^2+\frac{1}{2}[\sqrt{1-\eta}g_1+(1-\eta)\xi G_1g_2-(1-\eta)G_1G_2]^2\\ &\quad+\frac{1}{2}\eta(1-\eta)(\xi g_2-G_2)^2+\frac{1}{2}(1-\eta)(\xi G_2-g_2)^2+\frac{1}{2}\xi^2(1+\eta)+\eta,\end{aligned} \tag{8}$$

$$\xi_{\rm opt}=\frac{(1-\eta)^2\left(G_1^2+g_1^2\right)G_2g_2+(1-\eta^2)G_2g_2-2(1-\eta)^{\frac{3}{2}}G_1g_1g_2}{(1-\eta)^2\left(G_1^2+g_1^2\right)g_2^2+\eta(1-\eta)g_2^2+(1-\eta)G_2^2+\eta+1}. \tag{9}$$

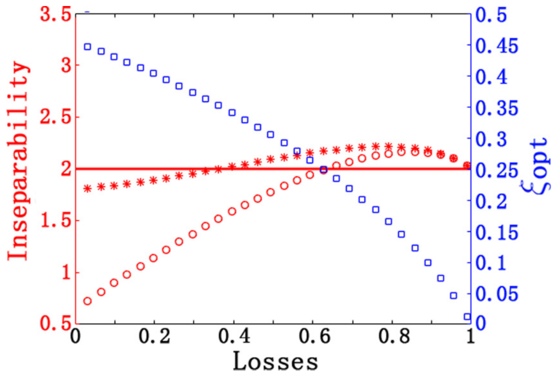

FIG. 3. The effect of the losses on the performance of the PA and PACMS. For two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$, the red asterisks (circles) show the inseparability I_{p1c2} ($I^{\min}_{p1'c2}$) between them versus the losses of the system when $\hat{a}_{c1}$ is amplified by the PA (PACMS). Here it is assumed that the gains of the FWM_1 and FWM_2 are respectively 2 and 1.9. The blue squares show the ξ_{opt} versus the losses for minimizing $I^{\min}_{p1c2'}$.

By taking $\xi = \xi_{\text{opt}}$ in Eq. (8), we can achieve the inseparability $I^{\min}_{p1'c2}$ versus the losses for the scheme using PACMS as shown by the red circles in Fig. 3. Here it is assumed that $G_1^2 = 2$ and $G_1^2 = 1.9$. For the scheme using PA, the inseparability I_{p1c2} can be obtained by taking $\xi = 0$ in Eq. (8) which is shown by the red asterisks in Fig. 3 for comparison. It can be found that, for both schemes using PA and PACMS, the inseparability (I_{p1c2} and $I^{\min}_{p1'c2}$) will be degraded or even disappear with the increasing of the losses. When the losses are close to 1, both I_{p1c2} and $I^{\min}_{p1'c2}$ approach the bound of the inseparability criterion. This is because, when $\eta = 1$, all the fields will be in vacuum and the inseparability between any two vacuum fields is two. By comparing the red asterisks with red circles in Fig. 3, it can be found that, for the scheme using PA, the inseparability disappears ($I_{p1c2} > 2$) when $\eta > 0.37$, while for the one using PACMS the inseparability $I^{\min}_{p1'c2}$ can be maintained until $\eta > 0.64$. Furthermore, it is clear that $I^{\min}_{p1'c2} < I_{p1c2}$ for all the losses. It means that for the same conditions of the system (i.e., the losses of the system, the gains of the FWM_1 and FWM_2), the PACMS has its unique superiority in maintaining the inseparability between the fields $\hat{a}_{p1}$ and $\hat{a}_{c2}$ compared with the PA. The reason why the CMS cannot maintain the inseparability between the fields $\hat{a}_{p1'}$ and $\hat{a}_{c2}$ for all the losses is that the vacuum field introduced from the losses is uncorrelated with any other fields in the system. Such vacuum noise cannot be weakened by the CMS and therefore the inseparability $I^{\min}_{p1'c2}$ disappears when the losses are large enough. In addition, it should be noted that the PACMS is not an optimal quantum amplifier which makes the signal amplified, meanwhile suppressing the added noise. Although the PACMS can suppress none of the noise of the two fields $\hat{a}_{p1}$, $\hat{a}_{c1}$, it maximally preserves the quantum entanglement from the parametric amplification of the bipartite entangled state $\hat{a}_{p1}$ and $\hat{a}_{c1}$.

We now expand our CMS to the asymmetrical amplification structure as shown in Fig. 4.

Different from Fig. 1(b) where for two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ only one of them is amplified, here both $\hat{a}_{p1}$ and $\hat{a}_{c1}$ are amplified by two PACMSs. First, two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ are respectively amplified by two FWM processes (FWM_2 and FWM_3). The input-output relation for the FWM_2 (FWM_3) can be expressed as

$$\hat{a}_{p2} = G_2\hat{a}_{v1} + g_2\hat{a}^\dagger_{c1}, \quad \hat{a}_{c2} = G_2\hat{a}_{c1} + g_2\hat{a}^\dagger_{v1},$$
$$(\hat{a}_{p3} = G_2\hat{a}_{p1} + g_2\hat{a}^\dagger_{v2}, \quad \hat{a}_{c3} = G_2\hat{a}_{v2} + g_2\hat{a}^\dagger_{p1}), \tag{10}$$

where $\hat{a}_{v1}$ and $\hat{a}_{v2}$ are in vacuum, $\hat{a}_{p2}$ and $\hat{a}_{c2}$ ($\hat{a}_{p3}$ and $\hat{a}_{c3}$) are the output fields from the FWM_2 (FWM_3). For simplification, it is assumed that the gains of both the FWM_2 and FWM_3 are equal to G_2^2. Then for the field $\hat{a}_{p2}$ ($\hat{a}_{c3}$), we make a "Bell-state" measurement of the amplitudes $\hat{X}_{p2} + \hat{X}_{v_{B1}}$ and $\hat{Y}_{p2} - \hat{Y}_{v_{B1}}$ ($\hat{X}_{c3} + \hat{X}_{v_{B2}}$ and $\hat{Y}_{c3} - \hat{Y}_{v_{B2}}$), which is used to modulate the field $\hat{a}_{p3}$ ($\hat{a}_{c2}$) to become $\hat{a}_{p3'}$ ($\hat{a}_{c2'}$). Then

$$\begin{aligned}\hat{a}'_{p3} &= \hat{a}_{p3} + \xi\big(\hat{X}_{p2} + \hat{X}_{v_{B1}}\big) + \xi\big(\hat{Y}_{p2} - \hat{Y}_{v_{B1}}\big),\\ \hat{a}'_{c2} &= \hat{a}_{c2} + \xi\big(\hat{X}_{c3} + \hat{X}_{v_{B2}}\big) + \xi\big(\hat{Y}_{c3} - \hat{Y}_{v_{B2}}\big),\end{aligned} \tag{11}$$

where $\hat{a}_{v_{B1}}$ and $\hat{a}_{v_{B2}}$ are the vacuum fields in the "Bell-state" measurements. By considering the losses of the system, it can be calculated that

$$\begin{aligned}\langle\Delta\hat{X}^2_{-,p3'c2'}\rangle = {}& (1-\eta)^2(G_1G_2 + \xi g_1g_2 - g_1G_2 - \xi G_1g_2)^2\\ &+ (1-\eta)(\xi G_2 - g_2)^2 + \eta(1-\eta)(G_2 - \xi g_2)^2\\ &+ \eta + \xi^2\eta + \xi^2,\end{aligned} \tag{12}$$

where $\langle\Delta\hat{X}^2_{-,p3'c2'}\rangle = \langle\Delta\hat{Y}^2_{+,p3'c2'}\rangle$ and the losses of each field after the FWM process are η. Then the optimal ξ for minimizing the inseparability $I_{p3'c2'}$ can be derived, which is

$$\xi_{\text{opt}} = \frac{(1-\eta^2)G_2g_2 - (1-\eta)(G_1G_2 - g_1G_2)(g_1g_2 - G_1g_2)}{(1-\eta)^2(g_1g_2 - G_1g_2)^2 + \eta(1-\eta)g_2^2 + (1-\eta)G_2^2 + \eta + 1}. \tag{13}$$

We first study the performance of the PACMS in the symmetrical amplification structure for the ideal case. By using Eqs. (12) and (13) and taking $\eta = 0$, we plot the minimal inseparability $I^{\min}_{p3'c2'}$ versus the gain of the FWM_2 as shown by the red circles in Fig. 5(a), where it is assumed that $G_1^2 = 2$. By taking $\eta = 0$ and $\xi = 0$ in Eq. (12), we also plot the inseparability I_{p3c2} without using CMS for comparison as shown by the red asterisks in Fig. 5(a). By comparing the symmetrical amplification [Fig. 5(a)] with the asymmetrical one (Fig. 2), it can be found that all the red asterisks in Fig. 5(a) are larger than the corresponding ones in Fig. 2. In other words, for two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$, their entanglement are degraded more easily in the symmetrical amplification than in the asymmetrical one. This is because,

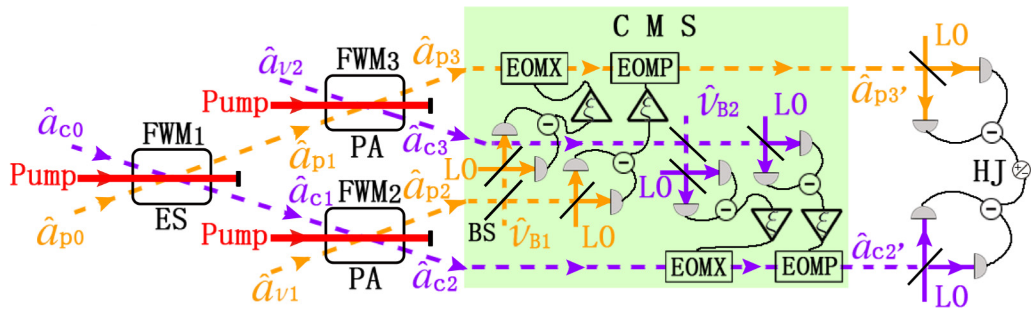

FIG. 4. The schemes of amplifying both two entangled states using two PACMSs.

in the symmetrical amplification, the fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ are individually amplified by the FWM_2 and FWM_3. While in the asymmetrical amplification, only the field $\hat{a}_{c1}$ is amplified by the FWM_2. The FWM_3 in the symmetrical amplification introduces more added noise into the system and therefore

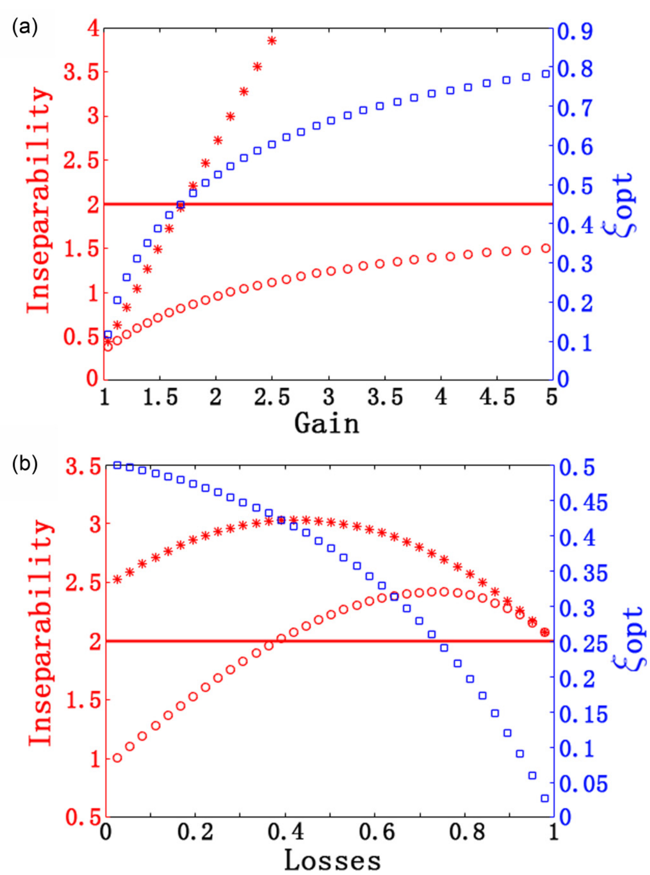

FIG. 5. The entanglement enhancement from the CMS in the symmetrical amplification for the (a) ideal case and (b) lossy case. For two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ in the ideal case, the red asterisks (circles) in panel (a) show the inseparability I_{p3c2} ($I^{\min}_{p3'c2'}$) between them versus G_2^2 when both of them are amplified by the PAs (PACMSs) to become $\hat{a}_{p3}$ and $\hat{a}_{c2}$ ($\hat{a}_{p3'}$ and $\hat{a}_{c2'}$). The blue squares in panel (a) show the ξ_{opt} versus the G_2^2 minimizing $I^{\min}_{p3'c2'}$. Here it is assumed that $G_1^2 = 2$. For the lossy case, the red asterisks (circles) in panel (b) show the inseparability I_{p3c2} ($I^{\min}_{p3'c2'}$) versus the losses, where it is assumed that $G_1^2 = 2$ and $G_2^2 = 1.9$. The blue squares in panel (b) show the ξ_{opt} versus the losses for minimizing $I^{\min}_{p3'c2'}$.

the inseparability between the fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ degrades more quickly than the asymmetrical one. However, the added noise introduced from both the FWM_2 and FWM_3 in the symmetrical amplification can also be suppressed by the CMS. By comparing the red circles with the red asterisks in Fig. 5 (a), it is clear that the former ones are all lower than the latter ones for the same G_2^2. Furthermore, it can be demonstrated that, for the ideal case $I^{\min}_{p3'c2'}(G_2 \to \infty) \to 2$, i.e., the fields $\hat{a}_{p3'}$ and $\hat{a}_{c2'}$ can be inseparable for any $G_2 > 0$. Therefore, our CMS can also be efficiently applied into the symmetrical amplification structure for the ideal case. For the lossy case, by using Eqs. (12) and (13) we plot the minimal inseparability $I^{\min}_{p3'c2'}$ versus the losses of the system as shown by the red circles in Fig. 5 (b), where it is assumed that $G_1^2 = 2$ and $G_2^2 = 1.9$. Similarly, by taking $\xi = 0$ in Eq. (12), we also plot the inseparability I_{p3c2} without using CMS versus the losses of the system as shown by the red asterisks in Fig. 5(b). It is clear that the red circles ($I^{\min}_{p3'p2'}$) are all lower than the red asterisks (I_{p3c2}) for all the losses. Therefore, for the lossy case in the symmetrical amplification the PACMS is able to improve the inseparability I_{p3c2} and has its advantage in maintaining the entanglement comparing with the PA. By comparing Fig. 5(b) with Fig. 3, it can be found that, for the same losses, all the red traces in Fig. 5(b) are larger than the corresponding ones in Fig. 3. It means that for two entangled fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$, their entanglement is more sensitive to the losses in the symmetrical amplification than in the asymmetrical one. This is because the symmetrical amplification has another FWM process (FWM_3) compared with the asymmetrical one. The FWM_3 will introduce more vacuum noise into the system which cannot be suppressed by the CMS. Therefore, the inseparability between the fields $\hat{a}_{p1}$ and $\hat{a}_{c1}$ is degraded more quickly in the symmetrical amplification than in the asymmetrical one.

III. CONCLUSION

Amplification of an entangled state is vital for the implementations of quantum communication. However, any parametric amplification process will introduce added noise into the system which unavoidably makes the entanglement degraded or even disappear. In spite of the unpreferred extra noise brought by the PA process, it also produces a field which is strongly correlated with the amplified field. Along this line, we have proposed a CMS in this paper, which fully exploits the quantum correlation between the two output fields from the FWM process, to suppress the added noise from amplification of an entangled state. By using the CMS, we have demonstrated that the entanglement of the system can be better maintained for both the ideal and lossy cases. The most interesting thing is that, for the ideal case, the CMS can always preserve the entanglement whatever the strength of the amplification is. So far, although we have only focused on the study of preservation of bipartite entanglement from amplification, it is also straightforward to apply our proposed CMS to preserve multipartite entanglement from amplification. Except for the FWM process, some other systems can also be treated as a good entanglement source such as the nondegenerate optical parametric oscillator (NOPO). For a NOPO with reasonable signal input into one of its ports, a high degree of bipartite entanglement can be possibly be generated [57,58]. Therefore, our CMS can be hopefully applied into the NOPO system to preserve the quantum entanglement from the parametric amplification when another NOPO is used to amplify a bipartite entangled state. In this sense, our proposed CMS could become a useful tool for achieving high fidelity quantum manipulation of entanglement. In addition, it should be noted that since the CMS presented here is a phase insensitive process focusing on preserving the quantum entanglement from the parametric amplification, our current work is different from some previous works, which study either the phase sensitive schemes [56,59,60] or intensity-difference squeezing [61] from the cascaded FWM processes.

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