Mixing of atomic levels by blackbody radiation and its consequences in an astrophysical context

T. Zalialiutdinov,[1,*] D. Solovyev,[1] L. Labzowsky,[1,2] and G. Plunien[3]

[1]*Department of Physics, St. Petersburg State University, Petrodvorets, Ulianovskaya 1, 198504, St. Petersburg, Russia*
[2]*Petersburg Nuclear Physics Institute, 188300, Gatchina, St. Petersburg, Russia*
[3]*Institut für Theoretische Physik, Technische Universität Dresden, Mommsenstraße 13, D-10162, Dresden, Germany*

(Received 26 March 2018; published 3 January 2019)

The mixing of atomic levels with opposite parity in the electric field induced by the blackbody radiation is studied. These studies are applied to the case when the source of blackbody radiation is the Cosmic Microwave Background. Our interest is focused on the $2s$ level of hydrogen, which played an important role during the epoch of cosmological recombination. Our studies show that the broadening of the $2s$ level due to the mixing effect induced by the blackbody radiation leads to a correction to the properties of Cosmic Microwave Background. This correction is of the same order as the other important corrections found during the last decades and is at the level of accuracy of the modern experimental data obtained from space probes.

DOI: 10.1103/PhysRevA.99.012502

I. INTRODUCTION

The influence of the blackbody radiation (BBR) on the atomic levels was an object of many investigations during the last 50 years. Theoretical calculations of the Stark shifts and broadening of atomic levels in the electric field produced by the BBR as well as the corresponding experimental studies are widely discussed in the literature [1–6]. The main goal of these investigations in the last years was the creation of atomic clocks and improvement of frequency standards. The current definition of a second is based on the microwave transition between the hyperfine levels of the ^{133}Cs ground state [6]. The operation of atomic clocks is generally carried out at room temperature, whereas the definition of the second refers to the clock transition in an atom at absolute zero. This implies that the clock transition frequency should be corrected for effects of finite temperature. The theoretical calculations of BBR shift contribution were based on the simple quantum mechanical model (QM) when the atomic level broadening was described by the emission and absorption of thermal (BBR) photons. The corresponding expression for the broadening of atomic level a then is [2] (in one-electron approximation)

$$\Gamma_a^{\mathrm{BBR-QM}} = \frac{4}{3}e^2\sum_n |\langle a|\mathbf{r}|n\rangle|^2\omega_{na}^3 n_\beta(\omega_{na}), \tag{1}$$

where $|n\rangle$ denote the one-electron wave functions (e.g., solutions of Hartree-Fock equations) corresponding to the eigenvalues E_n, $\omega_{na} = |E_n - E_a|$, E_n are the energy levels, $n_\beta(\omega) = (e^{\beta\omega} - 1)^{-1}$, $\beta = k_B T$, k_B is the Boltzmann constant, and T is the radiation temperature in kelvin. Summation over n in Eq. (1) is extended over all atomic spectrum. Note that only the states $|n\rangle$ with the parity opposite to the parity of the $|a\rangle$ state contribute to the sum in Eq. (1). This means that only $E1$ transitions are taken into account. In Eq. (1) relativistic units $\hbar = c = m = 1$ are used ($\hbar$ is the Planck constant, c is the speed of the light, m is the mass of electron). For the description of the thermal effects the quantum electrodynamical (QED) methods were also successfully employed [7]. A QED approach was applied recently for investigation of atomic level broadening in the presence of BBR [8–10]. The formula derived in [8] for the thermal line broadening is

$$\Gamma_a^{\mathrm{BBR-QED}} = \frac{2e^2}{3\pi}\sum_n |\langle a|\mathbf{r}|n\rangle|^2\int_0^\infty d\omega n_\beta(\omega)\omega^3 \times\left[\frac{\Gamma_{na}}{(\tilde{\omega}_{na}+\omega)^2+\frac{1}{4}\Gamma_{na}^2}+\frac{\Gamma_{na}}{(\tilde{\omega}_{na}-\omega)^2+\frac{1}{4}\Gamma_{na}^2}\right], \tag{2}$$

Here $\tilde{\omega}_{na} \equiv E_n - E_a + \Delta E_{na}^L$, $\Delta E_{na}^L = \Delta E_n^L - \Delta E_a^L$, ΔE_a^L is the corresponding radiative shift, $\Gamma_{na} = \Gamma_n + \Gamma_a$, Γ_n is the natural width of the state $|n\rangle$. The other notations are the same as in Eq. (1). This formula was derived in [8] as a contribution of the Feynman graph (Fig. 1) within thermal QED. The imaginary part of this graph corresponds to the broadening $\Gamma_a^{\mathrm{BBR-QED}}$ given by Eq. (2). A summation of the infinite chain of self-energy insertions into the internal electron line in Fig. 1 leads to geometric progression and to the corresponding regularization of the singular energy denominators [11] (see also [12]). After this regularization the expression in square brackets in the integrand in Eq. (2) arises. This summation is performed at the "resonant" values of the variable $\omega = \omega_{na}$. It can be easily seen that the contribution of the "resonant" terms to the integral in Eq. (3) gives immediately the result Eq. (1). Indeed, the "width" of the resonance is about Γ_{na} and the "height" of this area at the point of the resonance is Γ_{na}^{-1}. Then, when evaluating the contribution of this area to the integral is approximately equal to $\omega_{na}^3 n_\beta(\omega_{na})$ as in Eq. (1). Therefore we find

$$\Gamma_a^{\mathrm{BBR-QED-res}} = \Gamma_a^{\mathrm{BBR-QM}}, \tag{3}$$

where $\Gamma_a^{\mathrm{BBR-QED-res}}$ is the "resonant" part of $\Gamma_a^{\mathrm{BBR-QED}}$. The nonresonant part $\Gamma_a^{\mathrm{BBR-QED-nr}}$ of $\Gamma_a^{\mathrm{BBR-QED}}$ given also by

*t.zalialiutdinov@gmail.com

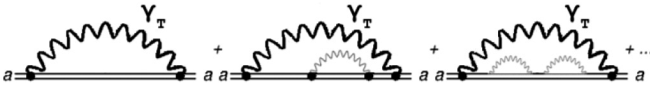

FIG. 1. Insertions of "ordinary" one-loop self-energy correction into the thermal one-loop self-energy correction leading to a "dressed" electron propagator. The internal wavy line denotes the virtual photon (ordinary photon propagator), while the photon line together with the index γ_T corresponds to the thermal photon loop (including the BBR photons).

Eq. (2) was interpreted as a "dynamic" mixing of the atomic levels with opposite parities by the BBR field.

Apart from the atomic clocks and frequency standards one of the most interesting applications of the theory of BBR interaction with atoms (in particular with the hydrogen atom) is the history of cosmological recombination. The main goal of the investigations in [8–10] and [13] was to demonstrate the importance of QED methods in atomic theory for description of the BBR interaction with atoms in the astrophysical context. We may add that the similar effects can be of importance for the description of the BBR influence on the 21-cm line profile [14,15]. In the course of the present work we have reconsidered our previous numerical results presented in [8–10] for the hydrogen atom in the BBR environment, as well as the results for the helium atom [13]. We found that the numbers presented for hydrogen in Table 3 in [8] and for helium in Tables 6 and 7 in [13] are strongly overestimated. They should be corrected by multiplication with a missing factor α^3.

More accurate calculations given in the present paper result as follows. These results concern the behavior of the hydrogen atom in the field of Cosmic Microwave Background (CMB) at the epoch of cosmological recombination. The field of CMB can be well approximated by the BBR field. We concentrate on the broadening of the $2s$ level of hydrogen, which plays a crucial role in the history of cosmological recombination. The cosmological recombination started in the early universe when the primordial plasma cooled down due to the expansion of the universe to the temperature defined by equality $k_B T \approx I$, where I is the ionization potential for the ground state of the hydrogen atom. This corresponds to the temperature $T = 15\,000$ K. The recombination occurs when the equilibrium between the process of the absorption and emission of photons is violated and the radiation escapes from the matter. There are two main channels for this escape. For the hydrogen atom in the $2p$ state the first channel is the emission of the $\mathrm{Ly}_\alpha(2p-1s)$ photon in the red wing of the Lorentz profile. Due to the expansion of the universe the frequency of this photon will be redshifted below the absorption edge before it will reach the neighboring atom [16]. The other channel first considered in [16,17] consists of the population of the $2s$ state. The $2s$ state dominantly decays with emission of two photons with nonresonant frequencies. These photons cannot be absorbed by other atoms and the corresponding radiation escapes the interaction with the matter.

Our calculation using Eq. (2) (see Sec. II) shows that for all $n > 2$ the contribution of $\Gamma_{2s,np}^{\mathrm{BBR-QED-res}}$ strongly dominates over $\Gamma_{2s,np}^{\mathrm{BBR-QED-nr}}$ at high radiation temperatures, but for the low temperatures $T < 300$ K the contribution $\Gamma_{2s,np}^{\mathrm{BBR-QED-nr}}$ dominates over the $\Gamma_{2s,np}^{\mathrm{BBR-QED-res}}$. The situation is different with the contribution $\Gamma_{2s,2p}^{\mathrm{BBR-QED-nr}}$ which dominates over $\Gamma_{2s,2p}^{\mathrm{BBR-QED-res}}$ at high temperatures and becomes equal to $\Gamma_{2s,2p}^{\mathrm{BBR-QM}}$ at low temperatures; see Fig. 2 and Tables I and II.

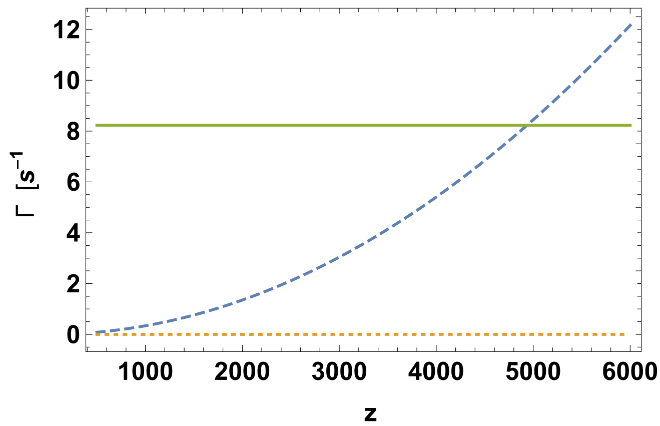

FIG. 2. The spontaneous decay rate $W_{2s,1s}^{(2\gamma)} = 8.229$ s^{-1} (bold line) and partial width $\Gamma_{2s,2p}^{\mathrm{BBR-QED}}$ (dashed line) as a function of redshift. With increasing temperature $\Gamma_{2s,2p}^{\mathrm{BBR-QED}}$ becomes comparable with $W_{2s,1s}^{(2\gamma)}$ due to the nonresonant contribution $\Gamma_{2s,2p}^{\mathrm{BBR-QED-nr}}$.

From the physical point of view $\Gamma_{2s}^{\mathrm{BBR-QED-res}}$ represents the broadening of the $2s$ level via absorption of the resonant thermal photons and transitions to the np levels. The $\Gamma_{2s,2p}^{\mathrm{BBR-QED-nr}}$ contribution can be understood as the broadening due to the influence of nonresonant thermal photons. Since the terms $\Gamma_{2s,np}^{\mathrm{BBR-QED-nr}}$ depend on Γ_{np} they can be considered as "dynamic mixing" of the $2s$ state and np levels by the alternating external BBR field. The connection of this

TABLE I. BBR-induced level width $\Gamma_{2s}^{\mathrm{BBR-QED}}$ [see Eq. (2)] and partial decay width $\Gamma_{2s,2p}^{\mathrm{BBR-QED}}$ [see Eq. (2) with only one term $n = 2p$ retained in the sum] in s^{-1} for the hydrogen atom at different temperatures. The number in square brackets indicates the power of ten. The full widths $\Gamma_{2s}^{\mathrm{BBR-QED}}$ become comparable with $\Gamma_{2s}^{\mathrm{BBR-QM}}$ (see Table II) at high temperatures, when the resonant contribution $\Gamma_{2s}^{\mathrm{BBR-QM-res}}$ dominates over the nonresonant one $\Gamma_{2s}^{\mathrm{BBR-QM-nr}}$. The partial width $\Gamma_{2s,2p}^{\mathrm{BBR-QED}}$ dominates over $\Gamma_{2s,2p}^{\mathrm{BBR-QM}}$ due to the nonresonant contribution $\Gamma_{2s,2p}^{\mathrm{BBR-QED-nr}}$ at high temperatures and becomes comparable to it at low temperatures; see also Fig. 3. One has to note that the accuracy of numerical evaluation of Eq. (2) is less then for Eq. (1) due to the presence of integration over frequency ω which is performed in a numerical way. However, this accuracy is sufficient to trace the behavior of $\Gamma_{2s}^{\mathrm{BBR-QED}}$.

T (K)	$\Gamma_{2s}^{\mathrm{BBR-QED}}$	$\Gamma_{2s}^{\mathrm{BBR-QED-nr}}$	$\Gamma_{2s,2p}^{\mathrm{BBR-QED}}$	$\Gamma_{2s,2p}^{\mathrm{BBR-QED-nr}}$
300	0.0041	0.0041	0.0041	0.0041
500	0.0115	0.0115	0.0110	0.0115
3000	4.6986[4]	0.4181	0.4136	0.4134
10 000	1.1216[7]	4.3002	4.5952	4.5948
20 000	5.0153[7]	18.362	18.381	18.380
30 000	9.6548[7]	43.064	41.357	41.356

TABLE II. BBR-induced level width $\Gamma_{2s}^{\rm BBR-QM}$ [see Eq. (1)] and partial decay width $\Gamma_{2s,2p}^{\rm BBR-QM}$ in s^{-1} for the hydrogen atom at different temperatures. The numbers in square brackets indicate the power of ten.

T (K)	$\Gamma_{2s}^{\rm BBR-QM}$	$\Gamma_{2s,2p}^{\rm BBR-QM}$
300	0.0000142	0.000014203
500	0.0000237	0.0000237
3000	4.69855[4]	0.000142041
10 000	1.12163[7]	0.000473474
20 000	5.01533[7]	0.00094695
30 000	9.65475[7]	0.00142043

"dynamic mixing" with the atomic level mixing in the static electric field is considered in more detail in Sec. II.

Naturally, this mixing becomes most significant in case $n = 2p$. The frequency of radiation associated with $\Gamma_{2s,2p}^{\rm BBR-QED-nr}$ coincides neither with $2s - 1s$ nor with $2p - 1s$ frequency but is distributed according to the Planck distribution. This becomes evident if we take into account the dependence on ω not only in the denominators but also in the numerators in Eq. (2). The corresponding corrections to $\Gamma_{2s,2p}^{\rm BBR-QED-res}$ are small but they demonstrate that the frequency of the radiation related to the $\Gamma_{2s,2p}^{\rm BBR-QED-nr}$ part is shifted from the $2s - 1s$ frequency by the small shift equal to the frequency of the thermal photons.

In the resonant case, for $\Gamma_{2s,2p}^{\rm BBR-QED-res}$ the frequency of radiation coincides with the $2p - 1s$ frequency. The $\Gamma_{2s,2p}^{\rm BBR-QED-res}$ broadening can be interpreted as the $2s - 2p - 1s$ cascade contribution. Unlike $\Gamma_{2s,2p}^{\rm BBR-QED-res} = \Gamma_{2s,2p}^{\rm BBR-QM}$ the contribution of $\Gamma_{2s,2p}^{\rm BBR-QED-nr}$ remained unnoticed within the QM approach but becomes evident with the application of QED methods. Similar studies, also with employment of QED methods were performed in [18,19] for the influence of BBR radiation on the noncontact and Van der Waals friction forces.

II. CONNECTION WITH THE LEVEL MIXING IN THE PRESENCE OF STATIC ELECTRIC FIELD

In the low-temperature limit Eq. (2) could be compared with the effect of quadratic level mixing in the presence of static electric field. External electric field mixes the opposite parity states, for instance, the $2s$ and $2p$ states of hydrogen. The mixed states can be denoted as $\overline{2s}$, $\overline{2p}$ states. Then the differential one-photon decay rate of the mixed $\overline{2s}$ state can be expressed as [20]

$$W_{\overline{2s}\,1s}^{(1\gamma)} = W_{2s\,1s}^{M1} + \frac{e^2 a_0^2 E_0^2}{\left(\Delta E_{2p2s}^L\right)^2 + \frac{1}{4}\Gamma_{2p}^2} W_{2p1s}^{E1}, \qquad (4)$$

where E_0 is the electric field strength and $W_{2s\,1s}^{M1}$ is the decay rate for the magnetic dipole transition $2s \to 1s + \gamma(M1)$, where $\gamma(M1)$ denotes the magnetic dipole photon. In Eq. (4) the Bohr radius in a.u., $a_0 = \alpha$ (α is the fine structure constant), $a_0^2 = |\langle 2s|\mathbf{r}|2p\rangle|^2$, and ΔE_{2p2s}^L is the Lamb shift. This expression shows that the additional one-photon electric dipole emission channel is allowed for the metastable $2s$ state in the presence of an external electric field. The term linear in the field vanishes after the integration over photon emission directions and does not contribute to the total transition probability. Contrary to this the term quadratic in the field leads to a significant increasing of the $2s$ level width [21,22].

In the low-temperature limit $k_B T \ll \Delta E_{2p2s}^L$ when the maximum of Planckian distribution is located at small frequencies the following approximation for the partial width $\Gamma_{2s,2p}^{\rm BBR-QED}$ is valid:

$$\Gamma_{2s,2p}^{\rm BBR-QED} \approx \frac{4e^2}{3\pi}|\langle 2s|\mathbf{r}|2p\rangle|^2 \int_0^\infty d\omega n_\beta(\omega)\omega^3 \times \left[\frac{\Gamma_{2p}}{\left(\Delta E_{2s\,2p}^L\right)^2 + \frac{1}{4}\Gamma_{2p}^2}\right]. \qquad (5)$$

The integration over frequency ω in the expression (5), with the use of the definition for the BBR-induced root mean squared electric field (in a.u.) [2],

$$\langle E^2\rangle = \frac{1}{2}\int_0^\infty \frac{8\alpha^3}{\pi}\omega^3 n_\beta(\omega) d\omega = \frac{4\pi^3}{15}\alpha^3(k_B T)^4 = (8.319\ \mathrm{V/cm})^2[T(\mathrm{K})/300]^4, \qquad (6)$$

leads to the second term in the expression (4), in which the square of the electric field strength E_0^2 is replaced by the root mean squared value of $\langle E^2\rangle$. Consequently, the radiation process takes place at the resonant frequency ω_{2s1s}, i.e., at the frequency shifted by the Lamb shift relative to the Ly $_\alpha$ frequency, as in the case of a static electric field.

The values of $\Gamma_{2s}^{\rm BBR-QED}$ and $\Gamma_{2s}^{\rm BBR-QM}$ calculated with the use of Eqs. (2) and (1), respectively, are presented in Tables I and II. Formula (2) contains both contributions: resonant $\Gamma_{2s}^{\rm BBR-QED-res}$ and nonresonant $\Gamma_{2s}^{\rm BBR-QED-nr}$. One can single out the nonresonant contribution from Eq. (2) with the use of the following equality:

$$\Gamma_{2s}^{\rm BBR-QED-nr} = \Gamma_{2s}^{\rm BBR-QED} - \Gamma_{2s}^{\rm BBR-QED-res}, \qquad (7)$$

where we assume $\Gamma_{2s}^{\rm BBR-QED-res} = \Gamma_{2s}^{\rm BBR-QM}$. The value $\Gamma_{2s}^{\rm BBR-QED-nr}$ is also presented in Table I.

III. APPLICATION TO THE COSMOLOGICAL RECOMBINATION PROBLEM

In this section we apply the QED method and in particular Eq. (2) to the study of the influence of the BBR-like CMB radiation on the cosmological recombination. The $\Gamma_{2s,2p}^{\rm BBR-QED-res}$ part of atomic level broadening is well known and commonly used in the astrophysical equations for the balance between the absorption (level population) and emission (level depopulation) of photons by atomic electrons. However, the $\Gamma_{2s,2p}^{\rm BBR-QED-nr}$ part was never yet considered in this respect. An important characteristic of the cosmological recombination status at the given redshift is the ionization fraction of the primordial plasma, i.e., the ratio of free electron density to the total density of hydrogen atoms and ions. Ionization fraction is defined by the system of kinetic balance equations for the level populations which accounts for all possible radiative decay channels which allow one to reach the ground state. Within the three-level approach (ground state, first excited state, and continuum) this system can be reduced to one ordinary differential equation [23]. In the standard three-level recombination model only two channels are taken into account which lead to the ground state (i.e., to recombine): Ly_α transition $2p \to 1s + \gamma(E1)$ and two-photon decay $2s \to 1s + 2\gamma(E1)$. To estimate the contribution $\Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p}$ to the ionization history we start from the differential equation for the ionization fraction $x_e = n_e/n_{\mathrm{H}}$, where n_e is the free electron number density and n_{H} is the total number density of hydrogen atoms and ions. The latter depend on redshift z as

$$n_{\mathrm{H}} = n^0_{\mathrm{H}}(1+z)^3, \tag{8}$$

where n^0_{H} is the value of hydrogen concentration at the present epoch. The dependence of radiation temperature on redshift z is given by

$$T = T_0(1+z), \tag{9}$$

where $T_0 = 2.725$ K is the present temperature of CMB radiation. The time-dependent behavior of the hydrogen ionization fraction in the isotropic homogeneous expanding universe within the three-level model of the atom is described by the kinetic equation [16,23],

$$\frac{dx_e}{dz} = C_{\mathrm{H}}\frac{\left[\alpha_{\mathrm{H}}x_e^2 n_{\mathrm{H}} - \beta_{\mathrm{H}}\exp\left(-\frac{\Delta E_{21}}{k_B T}\right)(1-x_e)\right]}{H(z)(1+z)}, \tag{10}$$

$$C_{\mathrm{H}} = \frac{1 + K_{\mathrm{H}}W^{(2\gamma)}_{2s,1s}n_{\mathrm{H}}(1-x_e)}{1 + K_{\mathrm{H}}\left(W^{(2\gamma)}_{2s,1s} + \beta_{\mathrm{H}}\right)n_{\mathrm{H}}(1-x_e)}, \tag{11}$$

$$K_{\mathrm{H}} = \frac{\lambda^3_\alpha}{8\pi H(z)}. \tag{12}$$

Here α_{H} is the total recombination coefficient for the excited states of hydrogen, β_{H} is the total ionization coefficient, λ_α is the wavelength of Ly_α transition, $W^{(2\gamma)}_{2s,1s} = 8.229\ \mathrm{s}^{-1}$ is the transition rate of spontaneous two-photon emission, $\Delta E_{21} = E_{2s} - E_{1s}$, and $H(z)$ is the Hubble factor describing the expansion of universe within the considered cosmological model.

Following [16] one can compare the rate by which hydrogen is allowed to recombine because photons are being redshifted out of the Ly_α line with the rate of recombinations via the two-photon emission process. This ratio is given by [16]

$$\frac{\mathrm{Ly}_\alpha\ \text{rate}}{\text{Two-photon rate}} = \frac{1}{K_{\mathrm{H}}W^{(2\gamma)}_{2s,1s}}. \tag{13}$$

If this the ratio (13) is small, recombination via the Ly_α channel is unimportant, and the factor C_{H} in Eq. (10) reduces to [16]

$$C_{\mathrm{H}} \approx \frac{W^{(2\gamma)}_{2s,1s}}{W^{(2\gamma)}_{2s,1s} + \beta_{\mathrm{H}}}, \tag{14}$$

which is just the branching ratio for two-photon decay of the $2s$ state of the hydrogen atom, i.e., the probability that the hydrogen atom in the $2s$ state will decay via two-photon emission before it is photodissociated. From Eq. (14) it is evident that the broadening of the $2s$ state due to the resonant contribution $\Gamma^{\mathrm{BBR-QED-res}}_{2s,2p} = \Gamma^{\mathrm{BBR-QM}}_{2s,2p}$ is negligible. Throughout the entire period of hydrogen recombination ($800 < z < 1600$) $\Gamma^{\mathrm{BBR-QM}}_{2s,2p}$ is of the order $10^{-4}\ \mathrm{s}^{-1}$ (see Table II and Fig. 3), and therefore should not be taken into account. However, as it was shown in Sec. II, nonresonant contribution $\Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p}$ is about three orders of magnitude larger than resonant one $\Gamma^{\mathrm{BBR-QED-res}}_{2s,2p}$ for the same recombination period (see Fig. 3). Then factor C_{H} should be modified to account for the contribution of $\Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p}$. This gives

$$\tilde{C}_{\mathrm{H}} = \frac{1 + K_{\mathrm{H}}W^{(2\gamma)}_{2s,1s}n_{\mathrm{H}}(1-x_e)}{1 + K_{\mathrm{H}}\left(W^{(2\gamma)}_{2s,1s} + \Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p} + \beta_{\mathrm{H}}\right)n_{\mathrm{H}}(1-x_e)}. \tag{15}$$

In the case when the ratio (13) is small we have

$$\tilde{C}_{\mathrm{H}} \approx \frac{W^{(2\gamma)}_{2s,1s}}{W^{(2\gamma)}_{2s,1s} + \Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p} + \beta_{\mathrm{H}}}. \tag{16}$$

Now $\tilde{C}_{\mathrm{H}}$ is the branching ratio for decay of the $2s$ state of the hydrogen atom with account for the $\Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p}$

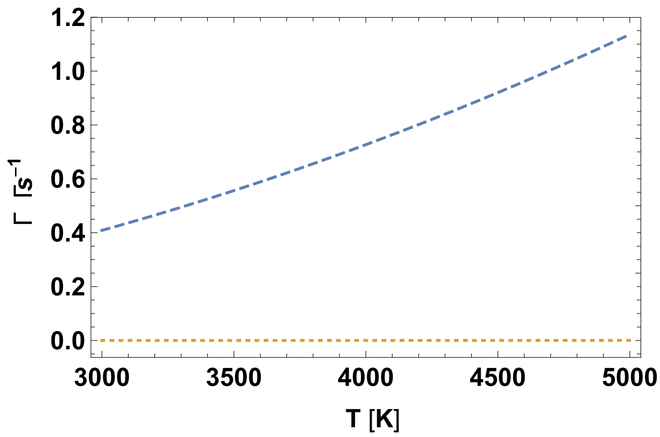

FIG. 3. The partial widths $\Gamma^{\mathrm{BBR-QED}}_{2s,2p}$ (dashed line) and $\Gamma^{\mathrm{BBR-QM}}_{2s,2p}$ (dotted line) as a function of temperature. The value $\Gamma^{\mathrm{BBR-QED}}_{2s,2p}$ dominates over $\Gamma^{\mathrm{BBR-QM}}_{2s,2p}$ at temperatures of cosmological hydrogen recombination due to the nonresonant contribution $\Gamma^{\mathrm{BBR-QED-nr}}_{2s,2p}$.

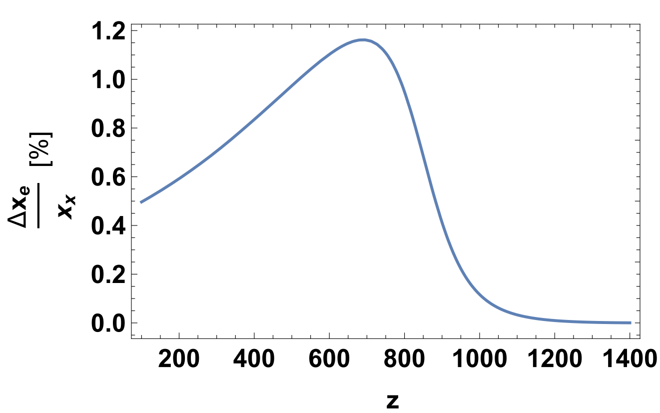

FIG. 4. Relative difference $\Delta x_e/x_e$ in percentages as a function of redshift z. The corresponding radiation temperature is given by equality $T = T_0(1+z)$, where $T_0 = 2.725$ K is the present CMB temperature.

TABLE III. Recent improvements for $\Delta x_e/x_e$ [24,26]. Here z is the redshift corresponding to the maximum of $\Delta x_e/x_e$.

Effect	$\Delta x_e/x_e$, %	z
Partial frequency redistribution of the Ly_α escape rate	−1.2	900
Time-dependent corrections to the Ly_α Sobolev escape probability	+1.2	1000
Two-photon transitions from higher levels	−0.4	1100
Stimulated two-photon transition $2s-1s$	+0.6	900
Thermodynamic asymmetry in the Ly_α profile	−1.9	1100
Direct recombination from continuum for HI	−0.0006	1280
Accounting for $\Gamma_{2s,2p}^{\text{BBR-QED-nr}}$ (this work)	+1.1	650

contribution to the broadening of the $2s$ state. The photoionization coefficient β_{H} in the denominators of Eqs. (14) and (16) actually plays a role of $n>2$ terms in the sum in Eq. (2). The relative difference between ionization fractions calculated from Eq. (10) with ordinary factor Eq. (11) and with the modified one $\tilde{C}_{\text{H}}$ given by Eq. (15), i.e., $\Delta x_e/x_e \equiv (\tilde{x}_e - x_e)/x_e$, is presented in Fig. 4. Rate equations for the ionization fraction were solved numerically with the standard cosmological parameters [25].

IV. RESULTS AND DISCUSSION

The broadening of atomic levels due to the influence of BBR was analyzed with the use of QED methods. A special attention was paid to the behavior of the $2s$ level of the hydrogen atom. The peculiarities of this behavior are connected with the metastable character of the $2s$ level. A standard QM description of BBR broadening from the QED point of view corresponds to the resonant scattering of thermal photons on atomic levels. Nonresonant scattering is commonly neglected. However, for the $2s$ level in hydrogen nonresonant BBR effects can dominate over the resonant one within some temperature intervals. This effect is similar to the atomic level mixing in an external electric field.

We found that accounting for the nonresonant contribution $\Gamma_{2s,2p}^{\text{BBR-QED-nr}}$ leads to a noticeable correction Δx_e to the ionization fraction x_e (i.e., to the ratio of free electron density to the total density of hydrogen atoms and ions). The maximum of ratio $\Delta x_e/x_e$ is about 1.1% at $z=650$. This correction is of the order magnitude of the other important corrections found during the last decades (see Table III) and is at the level of accuracy of the modern experimental methods for measuring the CMB properties [25]. We also would stress that the application of QED methods in the theory of thermal broadening of the atomic levels allows one to trace effects, which remained unnoticed by the simple QM approach. The correction appears to be important in the history of cosmological recombination.

ACKNOWLEDGMENTS

Section III of the present work was supported by Russian Science Foundation (Grant No. 17-12-01035). Section II was supported by TU Dresden (DAAD Programm Ostpartnerschaften) and by the foundation for the advancement of theoretical physics “BASIS.”

[1] T. F. Gallagher and W. E. Cooke, Phys. Rev. Lett. **42**, 835 (1979).
[2] J. W. Farley and W. H. Wing, Phys. Rev. A **23**, 2397 (1981).
[3] S. G. Porsev and A. Derevianko, Phys. Rev. A **74**, 020502(R) (2006).
[4] M. S. Safronova, M. G. Kozlov, and Charles W. Clark, Phys. Rev. Lett. **107**, 143006 (2011).
[5] E. J. Angstmann, V. A. Dzuba, and V. V. Flambaum, Phys. Rev. Lett. **97**, 040802 (2006).
[6] M. S. Safronova *et al.*, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **57**, 94 (2010).
[7] J. F. Donoghue and B. R. Holstein, Phys. Rev. D **28**, 340 (1983).
[8] D. Solovyev, L. Labzowsky, and G. Plunien, Phys. Rev. A **92**, 022508 (2015).
[9] T. Zalialiutdinov, D. Solovyev, L. Labzowsky, and G. Plunien, Phys. Rev. A **96**, 012512 (2017).
[10] T. Zalialiutdinov, D. Solovyev, and L. Labzowsky, JETP **126**, 8 (2018).
[11] F. Low, Phys. Rev. **88**, 53 (1952).
[12] O. Yu. Andreev, L. N. Labzowsky, G. Plunien, and D. A. Solovyev, Phys. Rep. **455**, 135 (2008).
[13] T. Zalialiutdinov, D. Solovyev, and L. Labzowsky, J. Phys. B **51**, 015003 (2018).
[14] J. R. Pritchard and A. Loeb, Rep. Prog. Phys. **75**, 086901 (2012).
[15] J. D. Bowman, Alan E. E. Rogers, R. A. Monsalve, T. J. Mozdzen, and N. Mahesh, Nature (London) **555**, 67 (2018).
[16] P. J. E. Peebles, Astrophys. J. **153**, 1 (1968).
[17] Ya. B. Zel’dovich, V. G. Kurt, and R. A. Sunyaev, Zh. Eksp. Teor. Fiz. **55**, 278 (1968) [Sov. Phys. JETP **28**, 146 (1969)].
[18] G. Lach, M. De Kieviet, and U. D. Jentschura, Phys. Rev. Lett. **108**, 043005 (2012).
[19] U. D. Jentschura, G. Łach, M. De Kieviet, and K. Pachucki, Phys. Rev. Lett. **114**, 043001 (2015).
[20] Ya. I. Azimov, A. A. Ansel’m, A. N. Moskalev, and R. M. Ryndin, Zh. Eksp. Teor. Fiz. **67**, 17 (1974) [Sov. Phys. JETP **40**, 8 (1975)].

[21] D. Solovyev, V. Sharipov, L. Labzowsky, and G. Plunien, J. Phys. B **43**, 074005 (2010).
[22] D. Solovyev and E. Solovyeva, Phys. Rev. A **91**, 042506 (2015).
[23] S. Seager, D. D. Sasselov, and D. Scott, Astrophys. J. **523**, L1 (1999).
[24] W. Y. Wong, Ph.D thesis, University of British Columbia, 2008.
[25] Planck Collaboration, Astron. Astrophys. A **13**, 594 (2016).
[26] J. A. Rubiño-Martín, J. Chluba, W. A. Fendt, and B. D. Wandelt, MNRAS **43**, 439 (2010).