Decoherence of quantum systems sequentially interacting with a common environment

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A composite quantum channel is derived for two independent quantum systems that interact sequentially with a common environment. One of the two quantum systems first interacts with the environment for a finite time, and after that the other one interacts with the same environment. In the process, the environment does not simultaneously interact with the two quantum systems. It is important to note that the second quantum system interacts with the environment that has been disturbed by the first one. As a result, the correlation between the two quantum systems, not directly interacting with each other, is created through the environment. An approximation to the composite quantum channel is also provided, which is applicable if a correlation time of the environment is not so long. When two independent qubits interact sequentially with a common bosonic environment via a dephasing coupling, it is explicitly shown that the time evolution of the second qubit can be non-Markovian due to the disturbance effect caused by the first qubit, even if the time evolution of the first qubit is Markovian. Entanglement, total, classical, and quantum correlations are calculated to find how the disturbance affects bipartite correlations. Furthermore, in state transmission through the composite quantum channel, it is found that the fidelity of quantum states can be enhanced by the disturbance effect.

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I. INTRODUCTION

A quantum system in the real world is not isolated from its surrounding environment, which includes a thermal reservoir, a measurement apparatus, and a controller. Such a quantum system is referred to as an open system [1–8]. An interaction with an environment leads to irreversible time evolution of an open quantum system, during which quantumness of the system is degraded and disappears eventually. Quantumness means coherence, quantum correlation, entanglement, nonlocality, and so on [9–14]. To study how quantumness of an open system is destructed during irreversible time evolution is of great importance not only on the basis of quantum mechanics [15] but also in quantum information processing [16]. Hence open quantum systems have been studied extensively by many authors [17–38]. To investigate their properties, several methods have been developed, including the phenomenological method [39–41], the stochastic Schrödinger equation [6,7], the quantum master equation [2,4], the classical stochastic method [1,2], and the path integral method [42,43]. These methods provide a reduced density operator of a relevant quantum system, which is derived from whole dynamics by eliminating environmental degrees of freedom. A reduced density operator can explain all the statistical properties of single-time events that occurred in an open quantum system. Recently, several methods for calculating two- and multitime correlations functions of open quantum systems have been formulated [44–48]. A two- or multitime correlation function of an open quantum system plays an important role in the linear-response theory [2,49–52], the weak values of postselected quantum systems [53–58], and the temporal nonlocality (the Leggett-Garg inequalities) [59–63].

An environmental system is usually assumed to be in a thermal equilibrium state. Thus it may be interesting to consider the degradation of quantumness of an open system if an environmental state is not in a thermal equilibrium state. For instance, let us consider that two quantum systems interact sequentially with a common environment. One of the two quantum system is transmitted from a sender to a receiver through a noisy environment. After its arrival at the receiver, the other quantum system is sent through the same environment. In this case, although the first quantum system interacts with the environment in a thermal equilibrium state, the second one may interact with the environment in a nonstationary state that has been created by the interaction with the first quantum system. Such state transmission has been phenomenologically treated in previous works to show that correlation between two input states can enhance the channel capacity of classical information [64–66]. In these works, a disturbance effect has been taken into account as a memory effect of the Kraus operators describing the quantum channel. However, it should be noted that the whole quantum channel itself is Markovian in the sense that the quantum channel is described in terms of the time-independent Kraus operators. Thus in the present paper we formulate the problem microscopically and derive a general expression of a composite quantum channel describing time evolution of two quantum systems that interact sequentially with a common environment. Applying the result to a two-qubit system that interacts with a bosonic environment via a dephasing coupling, we investigate how the disturbance caused by the first qubit affects the non-Markovianity of the reduced time evolution of the second qubit [67–72]. The degradation of entanglement, total, classical, and quantum correlation between the two qubits

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are also investigated [73–76]. Furthermore, state transmission through the composite quantum channel is examined. We will find that the disturbance effect decreases an individual fidelity while it can enhance a collective fidelity.

This paper is organized as follows. In Sec. II, we derive a general formula for a composite quantum channel that describes the time evolution of two quantum systems, each of which interacts sequentially with a common environment. It will be determined how the quantum system is influenced by the environment that is disturbed by the other system. An approximation to the quantum composite channel is provided, which is valid if a correlation time of the environment is not so long. In Sec. III, we investigate the non-Markovianity when the two quantum systems are qubits and have a dephasing coupling with a bosonic environment. It is shown that the disturbance effect of the environment caused by the first qubit enhances the non-Markovianity of the second one. In Sec. IV, we clarify the disturbance effect on bipartite correlation of the qubits, including entanglement, total, classical, and quantum correlations. In Sec. V, we consider state transmission of two qubits through the composite quantum channel. It will be shown that the disturbance by the qubit has a negative effect when the transmission is treated individually for the two qubits while it has a positive effect when the transmission is treated collectively. In Sec. VI, we give a brief summary of this paper.

II. QUANTUM CHANNELS FOR QUANTUM SYSTEMS

We suppose that two quantum systems A and B are sequentially transmitted from a sender to a receiver through a noisy environment R. First the quantum system A is propagated from time t_0 to time t_1. After the quantum system A is received, the other system B is propagated from time t_2 to time t_3. Here it is important to note that the environment R does not interact simultaneously with the two quantum systems during the transmission, that is, the inequality $t_0 < t_1 \leqslant t_2 < t_3$ is satisfied. This assumption is essential for our consideration. The transmission times $t_1 - t_0$ and $t_3 - t_2$ of the quantum systems A and B may be different. The characteristic feature is that the quantum system B is influenced by the environment R that has been disturbed by the quantum system A. It may be reasonable to consider that if the time difference $t_2 - t_1$ is much larger than the correlation time of the environment R, the disturbance effect is negligible. The whole process that we consider in this paper is depicted in Fig. 1.

To derive a quantum channel that describes how the two quantum systems evolve in time, we denote Hamiltonians of the quantum systems A and B and the environment R, respectively, as H_A, H_B, and H_R. The interaction Hamiltonians are H_{AR} and H_{BR}. An initial state prepared at the time $t = t_0$ is assumed to be described by a density operator $\rho^{\text{in}}_{ABR} = \rho^{\text{in}}_{AB} \otimes \rho^{\text{eq}}_R$. The two quantum systems A and B may be initially correlated with each other while the environment R is initially in a thermal equilibrium state and does not have any correlation with the quantum A or B. For the sake of simplicity, we assume that the two quantum systems A and B remain unchanged after and before the interaction with the environment R (outside the environment R). Since the whole system evolves by $H_A + H_R + H_{AR}$ from t_0 to t_1, by H_R from t_1 to t_2, and by $H_B + H_R + H_{BR}$ from t_2 to t_3, the output state ρ^{out}_{ABR} is given by

$$\rho^{\text{out}}_{ABR} = e^{(L_B+L_R+L_{BR})(t_3-t_2)} e^{L_R(t_2-t_1)} e^{(L_A+L_R+L_{AR})(t_1-t_0)} \rho^{\text{in}}_{ABR}, \tag{1}$$

where the Liouvillian superoperator L_X is defined by $L_X\bullet = -(i/\hbar)[H_X, \bullet]$ with $X = A, B, R, AR, BR$ [2,4]. The

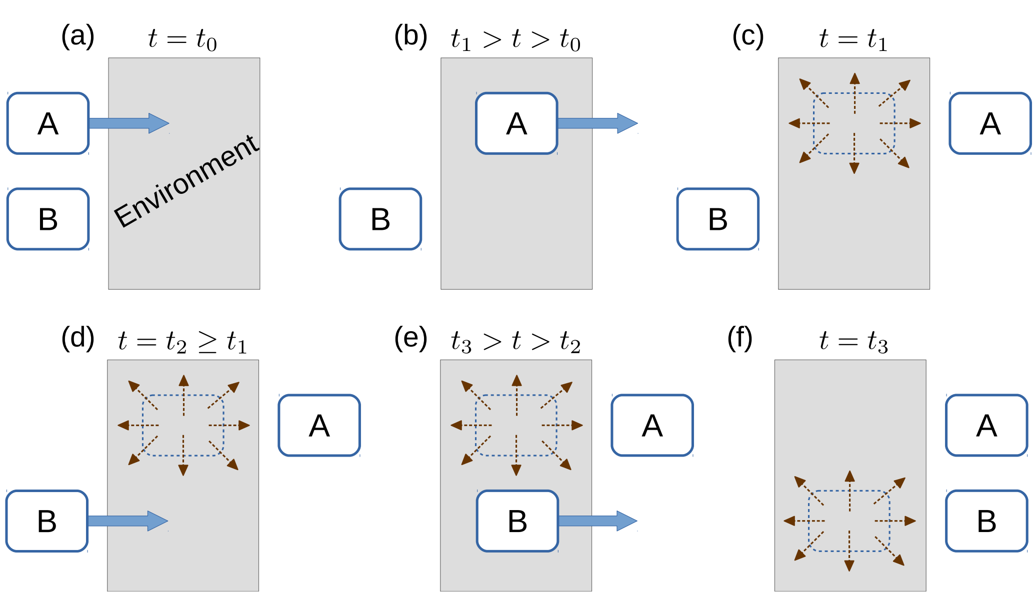

FIG. 1. A schematic representation of the whole process. (a) The quantum system A begins interacting with the environment R at time t_0 and (b) continues to interact until time t_1. (c) The disturbance caused by the quantum system A is left in the environment after the interaction. (d) The quantum system B starts interacting with the environment R at time t_2 and (e) continues to interact until time t_3. (f) All the processes are finished at time t_3. The environmental system does not simultaneously interact with the quantum systems A and B.

reduced output state of the quantum systems A and B is obtained by tracing out the environmental degrees of freedom,

$$\rho_{AB}^{\text{out}} = V_{AB}(t_3, t_2; t_1, t_0)\rho_{AB}^{\text{in}}, \tag{2}$$

where the composite quantum channel $V_{AB}(t_3, t_2; t_1, t_0)$ for the quantum systems A and B is given by

$$V_{AB}(t_3, t_2; t_1, t_0) = \langle e^{(L_B+L_R+L_{BR})(t_3-t_2)} e^{L_R(t_2-t_1)} e^{(L_A+L_R+L_{AR})(t_1-t_0)}\rangle_R, \tag{3}$$

with $\langle\cdots\rangle_R = \text{Tr}_R[\cdots\rho_R^{\text{eq}}]$. If there is no initial correlation between the two quantum states, we can obtain the reduced quantum states $\rho_A^{\text{out}} = \text{Tr}_B\rho_{AB}^{\text{out}}$ and $\rho_B^{\text{out}} = \text{Tr}_A\rho_{AB}^{\text{out}}$ by substituting $\rho_{AB}^{\text{in}} = \rho_A^{\text{in}} \otimes \rho_B^{\text{in}}$ into Eq. (3),

$$\rho_A^{\text{out}} = V_A(t_1, t_0)\rho_A^{\text{in}}, \tag{4}$$

$$\rho_B^{\text{out}} = V_B(t_3, t_2|t_1, t_0)\rho_B^{\text{in}}. \tag{5}$$

The reduced quantum channel $V_A(t_1, t_0)$ for the system A is given by

$$V_A(t_1, t_0) = \langle e^{(L_A+L_R+L_{AR})(t_1-t_0)}\rangle_R. \tag{6}$$

On the other hand, the reduced quantum channel $V_B(t_3, t_2|t_1, t_0)$ for the system B includes the disturbance effect caused by the system A, which has already interacted with the environment R,

$$V_B(t_3, t_2|t_1, t_0) = \text{Tr}_R[e^{(L_B+L_R+L_{BR})(t_3-t_2)}\rho_R(t_2|t_1, t_0)], \tag{7}$$

where $\rho_R(t_2|t_1, t_0)$ is the environmental state disturbed by the quantum system A,

$$\rho_R(t_2|t_1, t_0) = e^{L_R(t_2-t_1)}\langle e^{(L_A+L_R+L_{AR})(t_1-t_0)}\rangle_A \rho_R^{\text{eq}}, \tag{8}$$

with $\langle\cdots\rangle_A = \text{Tr}_A[\cdots\rho_A^{\text{in}}]$. If a correlation time of the environment is sufficiently short in comparison with the time difference $t_2 - t_1$, we expect that the environment returns to the thermal equilibrium state before interacting with the quantum system B, that is, $\rho_R(t_2|t_1, t_0) = \rho_R^{\text{eq}}$. In this case, the composite quantum channel is factorized into the individual quantum channels, that is, $V_{AB}(t_3, t_2; t_1, t_0) = V_B(t_3, t_2) \otimes V_A(t_1, t_0)$ with $V_B(t_3, t_2) = \langle e^{(L_B+L_R+L_{BR})(t_3-t_2)}\rangle_R$.

Our task is to find the composite quantum channel $V_{AB}(t_3, t_2; t_1, t_0)$ by calculating the partial average with respect to the environment R. This can be done by making use of the method that has been developed for calculating a reduced density operator [77] and a two-time correlation function [48] of an open quantum system in contact with a Gaussian environment. First we rewrite Eq. (3) into the interaction picture. To do this, we use the two identities

$$e^{(L_B+L_R+L_{BR})(t_3-t_2)} = e^{(L_B+L_R)(t_3-t_0)}V_{BR}(t_3, t_2|t_0)e^{-(L_B+L_R)(t_2-t_0)}, \tag{9}$$

$$e^{(L_A+L_R+L_{AR})(t_1-t_0)} = e^{(L_A+L_R)(t_1-t_0)}V_{AR}(t_1, t_0|t_0), \tag{10}$$

with

$$V_{BR}(t_3, t_2|t_0) = \text{T}\exp\left(\int_{t_2}^{t_3} d\tau\, L_{BR}(\tau|t_0)\right), \tag{11}$$

$$V_{AR}(t_1, t_0|t_0) = \text{T}\exp\left(\int_{t_0}^{t_1} d\tau\, L_{AR}(\tau|t_0)\right), \tag{12}$$

where T stands for the time-ordering operation that the superoperators $L_{AR}(\tau|t_0)$ and $L_{BR}(\tau|t_0)$ are placed from the right to the left of in chronological order, and the Liouvillian superoperators $L_{BR}(t|t_0)$ and $L_{BR}(t|t_0)$ are given by

$$L_{BR}(\tau|t_0) = e^{-(L_B+L_R)(\tau-t_0)}L_{BR}e^{(L_B+L_R)(\tau-t_0)}, \tag{13}$$

$$L_{AR}(\tau|t_0) = e^{-(L_A+L_R)(\tau-t_0)}L_{AR}e^{(L_A+L_R)(\tau-t_0)}. \tag{14}$$

Substituting Eqs. (9) and (10) into Eq. (3), we obtain

$$V_{AB}(t_3, t_2; t_1, t_0) = e^{L_B(t_3-t_0)+L_A(t_1-t_0)}\text{Tr}_R\big[V_{BA}(t_3, t_2|t_0) \times V_{AR}(t_1, t_0|t_0)\rho_R^{\text{eq}}\big]e^{-L_B(t_2-t_0)}, \tag{15}$$

where we have used the commutativity $[V_{AR}(t_1, t_0|t_0), L_B] = [V_{BR}(t_2, t_1|t_0), L_A] = 0$. Here it is convenient to introduce the superoperators $L_{ABR}(\tau|t_0)$ and $\hat{V}_{AB}(t_3, t_2; t_1, t_0)$ by

$$L_{ABR}(\tau|t_0) = \theta(\tau - t_2)L_{BR}(\tau|t_0) + \theta(t_1 - \tau)L_{AR}(\tau|t_0), \tag{16}$$

$$\hat{V}_{AB}(t_3, t_2; t_1, t_0) = \text{Tr}_R\left[\text{T}\exp\left(\int_{t_0}^{t_3} d\tau\, L_{ABR}(\tau|t_0)\right)\rho_R^{\text{eq}}\right], \tag{17}$$

where $\theta(t)$ is the usual step function [$\theta(t) = 1$ for $t \geqslant 0$ and otherwise zero]. Then we can express the quantum channel $V_{AB}(t_3, t_2; t_1, t_0)$ as

$$V_{AB}(t_3, t_2; t_1, t_0) = e^{L_B(t_3-t_0)+L_A(t_1-t_0)}\hat{V}_{AB}(t_3, t_2; t_1, t_0)e^{-L_B(t_2-t_0)}. \tag{18}$$

To proceed further, we assume that the interaction Hamiltonians are given by $H_{AR} = \hbar A \otimes R$ and $H_{BR} = \hbar B \otimes R$, where A and B are observables of the quantum systems and R is an environmental operator. The generalization to $H_{AR} = \sum_j A_j \otimes R_j$ and $H_{BR} = \sum_j B_j \otimes R_j$ is straightforward. In the interaction picture, we have $H_{AR}(t|t_0) = \hbar A(t|t_0) \otimes R(t|t_0)$ and $H_{BR}(t|t_0) = \hbar B(t|t_0) \otimes R(t|t_0)$ with $X(t|t_0) = e^{-L_X(t-t_0)}Xe^{L_X(t-t_0)} = e^{iH_X(t-t_0)/\hbar}Xe^{-iH_X(t-t_0)/\hbar}$ $(X = A, B)$. Then the quantum channel $\hat{V}_{AB}(t_3, t_2; t_1, t_0)$ becomes

$$\hat{V}_{AB}(t_3, t_2; t_1, t_0) = \text{Tr}_R\left[\text{T}\exp\left(-i\int_0^{t_3} d\tau\, [S(\tau|0) \otimes R(\tau|t_0)]^{\times}\right)\rho_R^{\text{eq}}\right], \tag{19}$$

with $X^{\times}\bullet = [X, \bullet]$ and

$$S(\tau|t_0) = \theta(\tau - t_2)B(\tau|t_0) + \theta(t_1 - \tau)A(\tau|t_0). \tag{20}$$

Furthermore we assume that the environment is Gaussian. This means that all the time-ordered cumulants of $R(t|t_0)$ higher than the second order vanish in the thermal equilibrium state $\rho_R^{\rm eq}$. Using the method for calculating reduced density operators [77–79], we can derive from Eq. (17)

$$\hat{V}_{AB}(t_3, t_2; t_1, t_0) = \mathrm{T}\exp\bigg(-\int_{t_0}^{t_3} d\tau \int_{t_0}^{\tau} d\tau'\, S^{\times}(\tau|t_0)[C_R(\tau-\tau')S^{\times}(\tau'|t_0) + iC_I(\tau-\tau')S^{\circ}(\tau'|t_0)]\bigg), \tag{21}$$

where $S^{\circ}(\tau'|t_0)\bullet = \{S(\tau'|t_0), \bullet\}$ (anticommutator), and $C_R(\tau-\tau')$ and $C_I(\tau-\tau')$ are the real and imaginary parts of the two-time correlation function $C(\tau-\tau')$ of the environmental operator R,

$$C(\tau-\tau') = C_R(\tau-\tau') + iC_I(\tau-\tau') = \langle R(\tau|t_0)R(\tau'|t_0)\rangle_R. \tag{22}$$

Note that the correlation function is a function of time difference since the environment is initially in the thermal equilibrium state $\rho_R^{\rm eq}$. If the environment is not Gaussian, Eq. (21) is equivalent to the second-order approximation with respect to the system-environment interaction in the time-local quantum master equation derived by the projection operator technique [1,2,4].

Finally, substituting Eq. (20) into Eq. (21), we obtain the quantum channel $\hat{V}_{AB}(t_3, t_2; t_1, t_0)$ in the interaction picture,

$$\begin{aligned}\hat{V}_{AB}(t_3, t_2; t_1, t_0) = \mathrm{T}\exp\bigg(&-\int_{t_2}^{t_3} d\tau \int_{t_2}^{\tau} d\tau'\, B^{\times}(\tau|t_0)[C_R(\tau-\tau')B^{\times}(\tau'|t_0) + iC_I(\tau-\tau')B^{\circ}(\tau'|t_0)]\\ &-\int_{t_2}^{t_3} d\tau \int_{t_0}^{t_1} d\tau'\, B^{\times}(\tau|t_0)[C_R(\tau-\tau')A^{\times}(\tau'|t_0) + iC_I(\tau-\tau')A^{\circ}(\tau'|t_0)]\\ &-\int_{t_0}^{t_1} d\tau \int_{t_0}^{\tau} d\tau'\, A^{\times}(\tau|t_0)[C_R(\tau-\tau')A^{\times}(\tau'|t_0) + iC_I(\tau-\tau')A^{\circ}(\tau'|t_0)]\bigg).\end{aligned} \tag{23}$$

Here we introduce vectors of the system superoperators and a matrix of the environmental correlation function [48],

$$\mathsf{C}(\tau) = \begin{pmatrix} C_R(\tau) & iC_I(\tau) \\ 0 & 0 \end{pmatrix}, \quad \mathsf{A}(\tau|t_0) = \begin{pmatrix} A^{\times}(\tau|t_0) \\ A^{\circ}(\tau|t_0) \end{pmatrix}, \quad \mathsf{B}(\tau|t_0) = \begin{pmatrix} B^{\times}(\tau|t_0) \\ B^{\circ}(\tau|t_0) \end{pmatrix}, \tag{24}$$

in terms of which we can rewrite Eq. (23) into

$$\begin{aligned}\hat{V}_{AB}(t_3, t_2; t_1, t_0) = \mathrm{T}\exp\bigg(&-\sum_{\mu,\nu}\int_{t_2}^{t_3} d\tau \int_{t_2}^{\tau} d\tau'\, \mathsf{B}_\mu(\tau|t_0)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{B}_\nu(\tau'|t_0)\\ &-\sum_{\mu,\nu}\int_{t_2}^{t_3} d\tau \int_{t_0}^{t_1} d\tau'\, \mathsf{B}_\mu(\tau|t_0)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{A}_\nu(\tau'|t_0) - \sum_{\mu,\nu}\int_{t_0}^{t_1} d\tau \int_{t_0}^{\tau} d\tau'\, \mathsf{A}_\mu(\tau|t_0)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{A}_\nu(\tau'|t_0)\bigg).\end{aligned} \tag{25}$$

Therefore, using the fact that the equality $\mathsf{B}_\mu(\tau|t_2) = e^{L_B(t_2-t_0)}\mathsf{B}_\mu(\tau|t_0)e^{-L_B(t_2-t_0)}$ is established since $\mathsf{B}_\mu(\tau|t_0) = e^{-L_B(\tau-t_0)}\mathsf{B}_\mu e^{L_B(\tau-t_0)}$ and $\mathsf{B}_\mu(\tau|t_2) = e^{-L_B(\tau-t_2)}\mathsf{B}_\mu e^{L_B(\tau-t_2)}$, we can obtain the quantum channel $V_{BA}(t_3, t_2; t_1, t_0)$ in the Schrödinger picture,

$$\begin{aligned}V_{BA}(t_3, t_2; t_1, t_0) = e^{L_B(t_3-t_2)+L_A(t_1-t_0)}\mathrm{T}\exp\bigg(&-\sum_{\mu,\nu}\int_{t_2}^{t_3} d\tau \int_{t_2}^{\tau} d\tau'\, \mathsf{B}_\mu(\tau|t_2)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{B}_\nu(\tau'|t_2)\\ &-\sum_{\mu,\nu}\int_{t_2}^{t_3} d\tau \int_{t_0}^{t_1} d\tau'\, \mathsf{B}_\mu(\tau|t_2)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{A}_\nu(\tau'|t_0) - \sum_{\mu,\nu}\int_{t_0}^{t_1} d\tau \int_{t_0}^{\tau} d\tau'\, \mathsf{A}_\mu(\tau|t_0)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{A}_\nu(\tau'|t_0)\bigg).\end{aligned} \tag{26}$$

This is one of the main results of this paper. If the time difference $t_2 - t_1$ is much larger than the correlation time of the environment, the second double integral on the right-hand side is negligible since there is no overlap between the two integrations. In this case, the equality $V_{BA}(t_3, t_2; t_1, t_0) = V_B(t_3, t_2) \otimes V_A(t_1, t_0)$ is established.

Although the quantum channel $V_{BA}(t_3, t_2; t_1, t_0)$ given by Eq. (26) is a general formula, it is difficult to explicitly calculate the quantum channel due to the time-ordering operation. So we derive an approximated formula by assuming that the time difference $t_2 - t_1$ is not so small in comparison with the correlation time τ_R of the environment. In this case, $\mathsf{C}_{\mu\nu}(\tau-\tau')$ is considered a small parameter if $\tau > t_2$ and $\tau' < t_1$ [48]. In the following, we derive an approximation to the quantum channel $V_{BA}(t_3, t_2; t_1, t_0)$ up to the lowest-order correction. First we expand the second exponential on the right-hand side of Eq. (26)

under the time ordering, and we obtain up to the first order

$$\begin{aligned} V_{BA}(t_3,t_2;t_1,t_0) = e^{L_B(t_3-t_2)+L_A(t_1-t_0)}\mathrm{T}\Bigg[&\exp\left(-\sum_{\mu',\nu'}\int_{t_2}^{t_3} d\tau \int_{t_2}^{\tau} d\tau'\, \mathsf{B}_{\mu'}(\tau|t_2)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{B}_{\nu'}(\tau'|t_2)\right) \\ &\times \left(1-\sum_{\mu,\nu}\int_{t_2}^{t_3} d\tau \int_{t_0}^{t_1} d\tau'\, \mathsf{B}_{\mu}(\tau|t_2)\mathsf{C}_{\mu\nu}(\tau-\tau')\mathsf{A}_{\nu}(\tau'|t_0)\right) \\ &\times \exp\left(-\sum_{\mu',\nu'}\int_{t_0}^{t_1} d\tau \int_{t_0}^{\tau} d\tau'\, \mathsf{A}_{\mu'}(\tau|t_0)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{A}_{\nu'}(\tau'|t_0)\right)\Bigg] \equiv V_{AB}^{(0)}(t_3,t_2;t_1,t_0) + V_{AB}^{(1)}(t_3,t_2;t_1,t_0). \end{aligned} \tag{27}$$

If the disturbance effect is ignored and the quantum systems A and B interact with the environment R in the thermal equilibrium state, we have the reduced quantum channels $V_A(t_j,t_k)$ and $V_B(t_j,t_k)$,

$$V_A(t_j,t_k) = \langle e^{(L_A+L_{AR}+L_R)(t_j-t_k)}\rangle_R = e^{L_A(t_j-t_k)}\mathrm{T}\exp\left(-\sum_{\mu',\nu'}\int_{t_k}^{t_j} d\tau \int_{t_0}^{\tau} d\tau'\, \mathsf{A}_{\mu'}(\tau|t_k)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{A}_{\nu'}(\tau'|t_j)\right), \tag{28}$$

$$V_B(t_j,t_k) = \langle^{(L_B+L_{BR}+L_R)(t_j-t_k)}\rangle_R = e^{L_B(t_j-t_k)}\mathrm{T}\exp\left(-\sum_{\mu',\nu'}\int_{t_k}^{t_j} d\tau \int_{t_2}^{\tau} d\tau'\, \mathsf{B}_{\mu'}(\tau|t_k)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{B}_{\nu'}(\tau'|t_j)\right). \tag{29}$$

Then the lowest-order term of the quantum channel is given by

$$V_{AB}^{(0)}(t_3,t_2;t_1,t_0) = V_A(t_1,t_0)\otimes V_B(t_3,t_2). \tag{30}$$

The next task is to calculate the first-order correction term $V_{BA}^{(1)}(t_3,t_2;t_1,t_0)$ to the composite quantum channel, which can be rewritten as

$$\begin{aligned} V_{BA}^{(1)}(t_3,t_2;t_1,t_0) = &-\sum_{\mu,\nu}\int_{t_2}^{t_3} dt \int_{t_0}^{t_1} dt'\, \mathsf{C}_{\mu\nu}(t-t')e^{L_B(t_3-t_2)+L_A(t_1-t_0)} \\ &\times \mathrm{T}\left[\mathsf{B}_{\mu}(t|t_2)\exp\left(-\sum_{\mu',\nu'}\int_{t_2}^{t_3} d\tau \int_{t_2}^{\tau} d\tau'\, \mathsf{B}_{\mu'}(\tau|t_2)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{B}_{\nu'}(\tau'|t_2)\right)\right] \\ &\times \mathrm{T}\left[\mathsf{A}_{\nu}(t'|t_0)\exp\left(-\sum_{\mu',\nu'}\int_{t_0}^{t_1} d\tau \int_{t_0}^{\tau} d\tau'\, \mathsf{A}_{\mu'}(\tau|t_0)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{A}_{\nu'}(\tau'|t_0)\right)\right], \end{aligned} \tag{31}$$

where we have used the property of the time-ordered product. Since the first-order term with respect to the small parameter $\mathsf{C}_{\mu\nu}(\tau-\tau')$ with $\tau > t_2$ and $\tau' < t_1$ is already present in front of the exponentials in Eq. (31), we can ignore such small parameters in the first and second exponentials. Then Eq. (31) can be further approximated as

$$\begin{aligned} V_{BA}^{(1)}(t_3,t_2;t_1,t_0) = &-\sum_{\mu,\nu}\int_{t_2}^{t_3} dt \int_{t_0}^{t_1} dt'\, \mathsf{C}_{\mu\nu}(t-t')e^{L_B(t_3-t_2)+L_A(t_1-t_0)} \\ &\times \mathrm{T}\exp\left(-\sum_{\mu',\nu'}\int_{t}^{t_3} d\tau \int_{t}^{\tau} d\tau'\, \mathsf{B}_{\mu'}(\tau|t_2)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{B}_{\nu'}(\tau'|t_2)\right) \\ &\times \mathsf{B}_{\mu}(t|t_2)\mathrm{T}\exp\left(-\sum_{\mu',\nu'}\int_{t_2}^{t} d\tau \int_{t_2}^{\tau} d\tau'\, \mathsf{B}_{\mu'}(\tau|t_2)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{B}_{\nu'}(\tau'|t_2)\right) \\ &\times \mathrm{T}\exp\left(-\sum_{\mu',\nu'}\int_{t'}^{t_1} d\tau \int_{t'}^{\tau} d\tau'\, \mathsf{A}_{\mu'}(\tau|t_0)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{A}_{\nu'}(\tau'|t_0)\right) \\ &\times \mathsf{A}_{\nu}(t'|t_0)\mathrm{T}\exp\left(-\sum_{\mu',\nu'}\int_{t_0}^{t'} d\tau \int_{t_0}^{\tau} d\tau'\, \mathsf{A}_{\mu'}(\tau|t_0)\mathsf{C}_{\mu'\nu'}(\tau-\tau')\mathsf{A}_{\nu'}(\tau'|t_0)\right). \end{aligned} \tag{32}$$

After some calculation, we can finally obtain the first-order correction term of the composite quantum channel $V_{BA}(t_3, t_2; t_1, t_0)$,

$$V^{(1)}_{BA}(t_3, t_2; t_1, t_0) = -\sum_{\mu,\nu}\int_{t_2}^{t_3} dt \int_{t_0}^{t_1} dt'\, \mathsf{C}_{\mu\nu}(t-t')V_B(t_3, t)\mathsf{B}_\mu V_B(t, t_2) \otimes V_A(t_1, t')\mathsf{A}_\nu V_A(t', t_0), \tag{33}$$

where $V_A(t_j, t_k)$ and $V_B(t_j, t_k)$ are given by Eqs. (28) and (29). The method to derive this result from Eq. (31) is similar to that for calculating two-time correlation functions of an open quantum system [48]. The difference is that the composite quantum channel in this paper treats two quantum systems while the two-time correlation function in Ref. [48] treats a single quantum system.

We can simplify the composite quantum channel given by Eq. (27) when the interaction between the quantum system and the environment is a dephasing coupling. In this case, the operators $A(t|t_0)$ and $B(t|t_0)$ become independent of time due to the commutativity $[H_A, H_{AR}] = [H_B, H_{BR}] = 0$. Then the time ordering in Eq. (26) is removed and thus the quantum channel $V_{BA}(t_3, t_2; t_1, t_0)$ is given by

$$V_{BA}(t_3, t_2; t_1, t_0) = e^{L_B(t_3-t_2)+L_A(t_1-t_0)} \exp\left[-\sum_{\mu,\nu}(\mathsf{B}_\mu \mathsf{G}_{\mu\nu}(t_3, t_2)\mathsf{B}_\nu + \mathsf{B}_\mu \mathsf{G}_{\mu\nu}(t_3, t_2; t_1, t_0)\mathsf{A}_\nu + \mathsf{A}_\mu \mathsf{G}_{\mu\nu}(t_1, t_0)\mathsf{A}_\nu)\right], \tag{34}$$

with

$$\mathsf{G}_{\mu\nu}(t_j, t_k) = \int_{t_k}^{t_k} d\tau \int_{t_k}^{\tau} d\tau'\, \mathsf{C}_{\mu\nu}(\tau-\tau'), \tag{35}$$

$$\mathsf{G}_{\mu\nu}(t_j, t_k; t_l, t_m) = \int_{t_k}^{t_k} d\tau \int_{t_m}^{t_l} d\tau'\, \mathsf{C}_{\mu\nu}(\tau-\tau'). \tag{36}$$

If the two quantum systems are initially uncorrelated, the reduced output states $\rho^{\text{out}}_A = \text{Tr}_B \rho^{\text{out}}_{AB}$ and $\rho^{\text{out}}_B = \text{Tr}_A \rho^{\text{out}}_{AB}$ of the two quantum systems become

$$\rho^{\text{out}}_A = V_A(t_1, t_0)\rho^{\text{in}}_A, \tag{37}$$

$$\rho^{\text{out}}_B = V_B(t_3, t_2)V_B(t_3, t_2|t_1, t_0)\rho^{\text{in}}_B, \tag{38}$$

where $V_A(t_1, t_0)$, $V_B(t_3, t_2)$, and $V_B(t_3, t_2|t_1, t_0)$ are given by

$$V_A(t_1, t_0) = e^{L_A(t_1-t_0)} \exp\left(-\sum_{\mu,\nu}\mathsf{A}_\mu \mathsf{G}_{\mu\nu}(t_1, t_0)\mathsf{A}_\nu\right), \tag{39}$$

$$V_B(t_3, t_2) = e^{L_B(t_3-t_2)} \exp\left(-\sum_{\mu,\nu}\mathsf{B}_\mu \mathsf{G}_{\mu\nu}(t_3, t_2)\mathsf{B}_\nu\right), \tag{40}$$

$$V_B(t_3, t_2|t_1, t_0) = \left\langle \exp\left(-\sum_{\mu,\nu}\mathsf{B}_\mu \mathsf{G}_{\mu\nu}(t_3, t_2; t_1, t_0)\mathsf{A}_\nu\right)\right\rangle_A. \tag{41}$$

The disturbance effect caused by the quantum system A is described by the conditional quantum channel $V_B(t_3, t_2|t_1, t_0)$. If the time difference $t_2 - t_1$ is much larger than the correlation time of the environment, we obtain $V_B(t_3, t_2|t_1, t_0) = 1$. Hence the disturbance effect vanishes in this case.

III. REDUCED TIME EVOLUTION AND ITS NON-MARKOVIANITY

In this section, when the quantum systems A and B undergo a pure dephasing due to the interaction with the environment R, we investigate how the disturbance in the environment R that is caused by the quantum system A affects the reduced output state of the quantum system B. To make the problem analytically tractable, we assume that the two quantum system are qubits (two-level systems), the Hamiltonians of which are given by $H_A = (1/2)\hbar\sigma^z_A$ and $H_A = (1/2)\hbar\sigma^z_A$, where $\sigma^z_{A,B}$ is the Pauli operator of the z component of spin-1/2. The interaction Hamiltonians are assumed to be $H_{AR} = (1/2)\hbar\sigma^z_A \otimes R$ and $H_{BR} = (1/2)\hbar\sigma^z_B \otimes R$. Substituting $A = (1/2)\sigma^z_A$ and $B = (1/2)\sigma^z_B$ into Eq. (34), we obtain the output state ρ^{out}_{AB} of the two qubits,

$$\begin{aligned}
\rho^{\text{out}}_{AB} = {} & \rho_{ee,ee}\overbrace{|e\rangle\langle e|}^{A}\otimes\overbrace{|e\rangle\langle e|}^{B} + \rho_{ee,gg}|e\rangle\langle e| \otimes |g\rangle\langle g| + \rho_{gg,ee}|g\rangle\langle g| \otimes |e\rangle\langle e| + \rho_{gg,gg}|g\rangle\langle g| \otimes |g\rangle\langle g| \\
& + \rho_{ee,eg}e^{-i\omega_B(t_3-t_2)-iG_I(t_3,t_2;t_1,t_0)-G_R(t_3,t_2)}|e\rangle\langle e| \otimes |e\rangle\langle g| + \rho_{ee,ge}e^{i\omega_B(t_3-t_2)+iG_I(t_3,t_2;t_1,t_0)-G_R(t_3,t_2)}|e\rangle\langle e| \otimes |g\rangle\langle e| \\
& + \rho_{gg,eg}e^{-i\omega_B(t_3-t_2)+iG_I(t_3,t_2;t_1,t_0)-G_R(t_3,t_2)}|g\rangle\langle g| \otimes |e\rangle\langle g| + \rho_{gg,ge}e^{i\omega_B(t_3-t_2)-iG_I(t_3,t_2;t_1,t_0)-G_R(t_3,t_2)}|g\rangle\langle g| \otimes |g\rangle\langle e| \\
& + \rho_{eg,ee}e^{-i\omega_A(t_1-t_0)-G_R(t_1,t_0)}|e\rangle\langle g| \otimes |e\rangle\langle e| + \rho_{ge,ee}e^{i\omega_A(t_1-t_0)-G_R(t_1,t_0)}|g\rangle\langle e| \otimes |e\rangle\langle e| \\
& + \rho_{eg,gg}e^{-i\omega_A(t_1-t_0)-G_R(t_1,t_0)}|e\rangle\langle g| \otimes |g\rangle\langle g| + \rho_{ge,gg}e^{i\omega_A(t_1-t_0)-G_R(t_1,t_0)}|g\rangle\langle e| \otimes |g\rangle\langle g| \\
& + \rho_{eg,eg}e^{-i\omega_A(t_1-t_0)-i\omega_B(t_3-t_2)-G_R(t_1,t_0)-G_R(t_3,t_2)-G_R(t_3,t_2;t_1,t_0)}|e\rangle\langle g| \otimes |e\rangle\langle g| \\
& + \rho_{eg,ge}e^{-i\omega_A(t_1-t_0)+i\omega_B(t_3-t_2)-G_R(t_1,t_0)-G_R(t_3,t_2)+G_R(t_3,t_2;t_1,t_0)}|e\rangle\langle g| \otimes |g\rangle\langle e| \\
& + \rho_{ge,eg}e^{i\omega_A(t_1-t_0)-i\omega_B(t_3-t_2)-G_R(t_1,t_0)-G_R(t_3,t_2)+G_R(t_3,t_2;t_1,t_0)}|g\rangle\langle e| \otimes |e\rangle\langle g| \\
& + \rho_{ge,ge}e^{i\omega_A(t_1-t_0)+i\omega_B(t_3-t_2)-G_R(t_1,t_0)-G_R(t_3,t_2)-G_R(t_3,t_2;t_1,t_0)}|g\rangle\langle e| \otimes |g\rangle\langle e|,
\end{aligned} \tag{42}$$

where $|g\rangle$ and $|e\rangle$ are the eigenstates of the Pauli operator σ^z such that $\sigma^z|g\rangle = -|g\rangle$ and $\sigma^z|e\rangle = |e\rangle$. In this equation, we set $\rho_{jk,mn} = \mathrm{Tr}[(|k\rangle\langle j| \otimes |n\rangle\langle m|)\rho_{AB}^{\mathrm{in}}]$ and

$$G_{R,I}(t_j, t_k) = \int_{t_k}^{t_k} d\tau \int_{t_k}^{\tau} d\tau' \, C_{R,I}(\tau - \tau'), \tag{43}$$

$$G_{R,I}(t_j, t_k; t_l, t_m) = \int_{t_k}^{t_k} d\tau \int_{t_m}^{t_l} d\tau' \, C_{R,I}(\tau - \tau'). \tag{44}$$

In this paper, when we denote $|j\rangle\langle k| \otimes |m\rangle\langle n|$, the left operator $|j\rangle\langle k|$ is for qubit A and the right operator $|m\rangle\langle n|$ is for qubit B. Furthermore, we assume that there is no initial correlation between the two qubits. Substituting the input state $\rho_{AB}^{\mathrm{in}} = \rho_A^{\mathrm{in}} \otimes \rho_B^{\mathrm{in}}$ into Eq. (42), we obtain the reduced output states,

$$\begin{aligned}\rho_A^{\mathrm{out}} &= \rho_{ee}^a|e\rangle\langle e| + \rho_{gg}^a|g\rangle\langle g| + \rho_{eg}^a e^{-i\omega_A(t_1-t_0)-G_R(t_1,t_0)}|e\rangle\langle g| \\ &\quad + \rho_{ge}^a e^{i\omega_A(t_1-t_0)-G_R(t_1,t_0)}|g\rangle\langle e|,\end{aligned} \tag{45}$$

$$\begin{aligned}\rho_B^{\mathrm{out}} &= \rho_{ee}^b|e\rangle\langle e| + \rho_{gg}^b|g\rangle\langle g| + \rho_{eg}^b[\cos G_I(t_3, t_2; t_1, t_0) \\ &\quad - i\langle\sigma_A^z\rangle \sin G_I(t_3, t_2; t_1, t_0)]e^{-i\omega_B(t_3-t_2)-G_R(t_3,t_2)}|e\rangle\langle g| \\ &\quad + \rho_{ge}^b\big[\cos G_I(t_3, t_2; t_1, t_0) + i\langle\sigma_A^z\rangle \sin G_I(t_3, t_2; t_1, t_0)\big] \\ &\quad \times e^{i\omega_B(t_3-t_2)-G_R(t_3,t_2)}|g\rangle\langle e|,\end{aligned} \tag{46}$$

with $\rho_{jk}^{a,b} = \langle j|\rho_{A,B}^{\mathrm{in}}|k\rangle$. In deriving Eq. (46), we have used the relations $\rho_{ee}^A = (1 + \langle\sigma_A^z\rangle)/2$ and $\rho_{gg}^A = (1 - \langle\sigma_A^z\rangle)/2$ with the initial average $\langle\sigma_A^z\rangle = \mathrm{Tr}[\sigma_A^z\rho_A^{\mathrm{in}}]$. It is obvious from Eq. (46) that the disturbance effect vanishes if the equality $G_I(t_3, t_2; t_1, t_0) = 0$ holds. This means that the imaginary part of the two-time correlation function $\langle R(t|t_0)R(t'|t_0)\rangle_R$ is indispensable for creating the disturbance effect. In other words, the existence of the disturbance effect requires noncommutativity of $R(t|t_0)$ and $R(t'|t_0)$ $(t = t')$. This also implies that there is no disturbance effect in a classical environment. On the other hand, the reduced state of qubit A is independent of qubit B since the future event does not affect the past.

Next we investigate the property of the output state ρ_B^{out} of qubit B. For a pure dephasing process, in general, a density operator of a qubit can be expressed as $\rho(t) = \rho_{ee}|e\rangle\langle e| + \rho_{gg}|g\rangle\langle g| + \rho_{eg}f(t)|e\rangle\langle g| + \rho_{ge}f^*(t)|g\rangle\langle e|$, which is a solution of the time-local quantum master equation,

$$\frac{\partial}{\partial t}\rho(t) = -(i/2)\omega(t)[\sigma_z, \rho(t)] + \gamma(t)[\sigma_z\rho(t)\sigma_z - \rho(t)], \tag{47}$$

where the time-dependent frequency $\omega(t)$ and the relaxation parameter $\gamma(t)$ are related to the parameter $f(t)$ by

$$\omega(t) = -\mathrm{Im}\left(\frac{\dot{f}(t)}{f(t)}\right), \quad \gamma(t) = -\frac{1}{2}\mathrm{Re}\left(\frac{\dot{f}(t)}{f(t)}\right). \tag{48}$$

The time evolution of the qubit is non-Markovian if the relaxation parameter $\gamma(t)$ can take a negative value [70]. Now we define the density operator $\rho(t)$ of qubit B with $t_2 \leqslant t \leqslant t_3$ by

$$\begin{aligned}\rho(t) &= \rho_{ee}^b|e\rangle\langle e| + \rho_{gg}^b|g\rangle\langle g| + \rho_{eg}^b f(t)|e\rangle\langle g| \\ &\quad + \rho_{ge}^b f^*(t)|g\rangle\langle e|,\end{aligned} \tag{49}$$

where the time-dependent parameter $f(t)$ is

$$\begin{aligned}f(t) &= \big[\cos G_I(t, t_2; t_1, t_0) - i\langle\sigma_z^A\rangle \sin G_I(t, t_2; t_1, t_0)\big] \\ &\quad \times e^{-i\omega_B(t-t_2)-G_R(t,t_2)}.\end{aligned} \tag{50}$$

We have the reduced output state $\rho_B^{\mathrm{out}} = \rho(t_3)$. The time-dependent frequency and relaxation parameters are given, respectively, by

$$\omega(t) = \omega_B + \frac{\langle\sigma_z^A\rangle\dot{G}_I(t, t_2; t_1, t_0)}{\cos^2 G_I(t, t_2; t_1, t_0) + \langle\sigma_z^A\rangle^2 \sin^2 G_I(t, t_2; t_1, t_0)}, \tag{51}$$

$$\gamma(t) = \frac{1}{2}\left[\dot{G}_R(t, t_2) + \frac{\left(1 - \langle\sigma_z^A\rangle^2\right)\cos G_I(t, t_2; t_1, t_0)\sin G_I(t, t_2; t_1, t_0)}{\cos^2 G_I(t, t_2; t_1, t_0) + \langle\sigma_z^A\rangle^2 \sin^2 G_I(t, t_2; t_1, t_0)}\dot{G}_I(t, t_2; t_1, t_0)\right], \tag{52}$$

where we set $\dot{X}(t) = dX(t)/dt$. If there is no disturbance effect, these parameters become

$$\omega^{(0)}(t) = \omega_B, \quad \gamma^{(0)}(t) = \tfrac{1}{2}\dot{G}_R(t, t_2). \tag{53}$$

Comparing Eq. (52) with Eq. (53), we find that the relaxation parameter $\gamma(t)$ can possibly take a negative value even if the inequality $\gamma^{(0)}(t) > 0$ is always satisfied. This implies that the disturbance in the environment R caused by qubit A induces non-Markovianity of the reduced time evolution of qubit B. For instance, setting $\langle\sigma_A^z\rangle = 0$ in Eq. (52), we have

$$\gamma(t) = \tfrac{1}{2}[\dot{G}_R(t, t_2) + \dot{G}_I(t, t_2; t_1, t_0)\tan G_I(t, t_2; t_1, t_0)]. \tag{54}$$

In this case, if $G_I(t, t_2; t_1, t_0)$ changes around $\pi/2$, the relaxation parameter $\gamma(t)$ can take negative values unless $\dot{G}_I(t, t_2; t_1, t_0) = 0$ at this point.

To explicitly derive the relaxation parameter $\gamma(t)$ of qubit B, we assume that the environment R consists of independent harmonic oscillators. Then we have the Hamiltonian $H_R = \sum_k \hbar\omega_k a_k^\dagger a_k$ and the environmental operator $R = \sum_k \hbar g_k(a_k + a_k^\dagger)$, where a_k is an annihilation operator of the kth environmental oscillator with angular frequency ω_k, and g_k represents a coupling strength between the qubit and the kth oscillator. The two-time correlation function of the environment is $\langle R(t|t_0)R(t'|t_0)\rangle_R = \sum_k g_k^2[(\bar{n}_k + 1)e^{-i\omega_k(t-t')} + \bar{n}_k e^{i\omega_k(t-t')}]$ with $\bar{n}_k = (e^{\hbar\omega_k/k_BT} + 1)^{-1}$. In the following, we further assume that the environment is initially in the ground state $(T = 0)$ and it has Lorentzian spectral density [4,28], $J(\omega) = (\gamma/2\pi)\lambda^2/[(\omega - \Delta)^2 + \lambda^2]$. Then we obtain

$$\begin{aligned}G(t_j, t_k) &= G_R(t_j, t_k) + iG_I(t_j, t_k) \\ &= \frac{\gamma\lambda}{2(\lambda + i\Delta)^2}\big[(\lambda + i\Delta)(t_j - t_k) - 1 + e^{-(\lambda+i\Delta)(t_j-t_k)}\big]\end{aligned} \tag{55}$$

and

$$G(t_3,t_2;t_1,t_0) = G_R(t_3,t_2;t_1,t_0) + iG_I(t_3,t_2;t_1,t_0)$$
$$= \frac{\gamma\lambda}{2(\lambda+i\Delta)^2}[1-e^{-(\lambda+i\Delta)(t_3-t_2)}] \times e^{-(\lambda+i\Delta)(t_2-t_1)}[1-e^{-(\lambda+i\Delta)(t_1-t_0)}]. \quad (56)$$

It is obvious from this equation that $G(t_3,t_2;t_1,t_0)\approx 0$ if the time separation between the output of qubit A and the input of the qubit system B is sufficiently large, that is, $\lambda(t_2-t_1)\gg 1$.

We investigate the reduced time evolution of qubit B, the density operator of which is obtained by substituting Eq. (50) with Eqs. (55) and (56) into Eq. (49). To make our discussion clear and focus on the disturbance effect caused by qubit A, we assume that $t_1=t_2$ and $\lambda(t_1-t_0)\gg 1$ in the rest of this section. Then we have the reduced density operator of qubit B,

$$\rho(t)=\rho^b_{ee}|e\rangle\langle e|+\rho^b_{gg}|g\rangle\langle g|+\rho^b_{eg}f(t)|e\rangle\langle g|+\rho^b_{ge}f^*(t)|g\rangle\langle e|, \quad (57)$$

with

$$f(t)=\left[\cos g_i(t)-i\langle\sigma^A_z\rangle\sin g_i(t)\right]e^{-i\omega_B t-g_r(t)}, \quad (58)$$

where $g_r(t)$ and $g_i(t)$ are given by

$$g_r(t)=\left(\frac{\gamma}{2\lambda}\right)\frac{1}{1+(\Delta/\lambda)^2}\left[\lambda t-\frac{1-(\Delta/\lambda)^2}{1+(\Delta/\lambda)^2}(1-e^{-\lambda t}\cos\Delta t) - \frac{2\Delta/\lambda}{1+(\Delta/\lambda)^2}e^{-\lambda t}\sin\Delta t\right], \quad (59)$$

$$g_i(t)=\left(\frac{\gamma}{2\lambda}\right)\frac{1}{1+(\Delta/\lambda)^2}\left[\frac{1-(\Delta/\lambda)^2}{1+(\Delta/\lambda)^2}e^{-\lambda t}\sin\Delta t - \frac{2(\Delta/\lambda)}{1+(\Delta/\lambda)^2}(1-e^{-\lambda t}\cos\Delta t)\right]. \quad (60)$$

In these equations, time t stands for an elapsed time from time t_2 at which qubit B is input. The relaxation parameter $\gamma(t)$ is given by

$$\gamma(t)=\frac{1}{2}\left[\dot g_r(t)+\frac{\left(1-\langle\sigma^A_z\rangle^2\right)\cos g_i(t)\sin g_i(t)}{\cos^2 g_I(t)+\langle\sigma^A_z\rangle^2\sin^2 g_i(t)}\dot g_i(t)\right] \quad (61)$$

with

$$\dot g_r(t)=\left(\frac{\gamma}{2}\right)\frac{1}{1+(\Delta/\lambda)^2}\left[1-e^{-\lambda}\cos\Delta t + \left(\frac{\Delta}{\lambda}\right)e^{-\lambda t}\sin\Delta t\right], \quad (62)$$

$$\dot g_i(t)=-\left(\frac{\gamma}{2}\right)\frac{1}{1+(\Delta/\lambda)^2}\left[\sin\Delta t+\left(\frac{\Delta}{\lambda}\right)\cos\Delta t\right]e^{-\lambda t}. \quad (63)$$

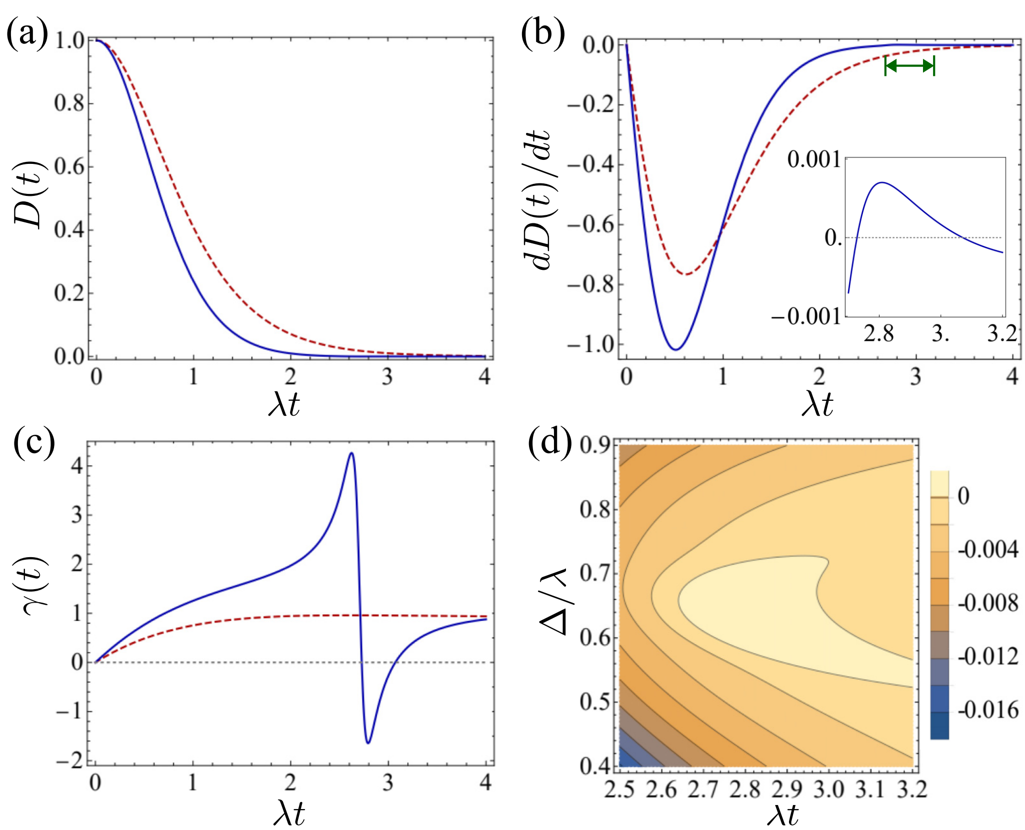

FIG. 2. The time evolution of (a) the trace distance $D(t)$, (b) its time derivative $dD(t)/dt$, (c) the relaxation parameter $\gamma(t)$, and (d) the contour plot of $dD(t)/dt$. In panels (a)–(c) we set $\Delta/\lambda = 0.6$, and the solid blue (dashed red) line stands for the time dependence with (without) the disturbance effect. In all of the panels, we set $\gamma/\lambda = 6.0$ and $\langle\sigma^A_z\rangle = 0.01$. The inset in panel (b) is an enlarged view of the part indicated by the green arrow.

The non-Markovianity of the reduced time evolution of qubit B is investigated in terms of the trace distance $D(t)$ between two density operators $\rho(t)$ and $\rho'(t)$ [16,67,68],

$$D(t)=\frac{1}{2}\mathrm{Tr}|\rho(t)-\rho'(t)|=\sqrt{\left(\Delta\rho^b_{ee}\right)^2+|f(t)|^2\left|\Delta\rho^b_{eg}\right|^2}, \tag{64}$$

the time derivative of which is given by

$$\frac{dD(t)}{dt}=\frac{[d|f(t)|^2/dt]\left|\Delta\rho^b_{eg}\right|^2}{2\sqrt{\left(\Delta\rho^b_{ee}\right)^2+|f(t)|^2\left|\Delta\rho^b_{eg}\right|^2}}. \tag{65}$$

In these equations, $\Delta\rho^b_{jk}$ is the difference between the matrix elements of the two different input states, ρ^{in}_B and $\rho^{\mathrm{in}\prime}_B$. In the dephasing process, the time derivative $dD(t)/dt$ becomes most significant when $\rho^{\mathrm{in}}_B=|\psi_+\rangle\langle\psi_+|$ and $\rho^{\mathrm{in}\prime}_B=|\psi_-\rangle\langle\psi_-|$ with $|\psi_\pm\rangle=(|e\rangle\pm|g\rangle)/\sqrt{2}$. In this case, we have $D(t)=|f(t)|^2$ and $dD(t)/dt=-4\gamma(t)|f(t)|^2$. So it is obvious that $dD(t)/dt>0$ is equivalent to $\gamma(t)<0$. If the time derivative $dD(t)/dt$ can take a negative value, the reduced time evolution of qubit B is non-Markovian [70]. When we ignore the disturbance effect, the parameters $f(t)$ and $\gamma(t)$ given by Eqs. (58) and (61) are replaced by $f^{(0)}(t)=e^{-i\omega_B t-g_r(t)}$ and $\gamma^{(0)}(t)=(1/2)\dot{g}_r(t)$. The time evolution of the trace distances with and without the disturbance effect is depicted in Fig. 2(a). It is found from the figure that the disturbance effect enhances the decay of the trace distance. The time derivative of the trace distance is plotted in Fig. 2(b). For given values of the parameters, the figure shows that although $dD(t)/dt$ is always negative if the disturbance effect is ignored, it becomes negative around $\lambda t=2.9$ due to the disturbance effect. Hence we have found that the disturbance effect caused by qubit A induces the non-Markovianity of the reduced time evolution of qubit B. In Fig. 2(c), we plot the relaxation parameter $\gamma(t)$, which clearly shows the non-Markovianity of the reduced time evolution. We can observe the characteristic behavior from the expression

$$\gamma(t)=\frac{1}{2}\left[\dot{g}_r(t)+\dot{g}_i(t)\frac{\left(1-\langle\sigma^z_A\rangle^2\right)\tan g_i(t)}{1+\langle\sigma^z_A\rangle^2\tan g_i^2(t)}\right]. \tag{66}$$

From Eq. (60), we find that $g_i(t)\approx\pi/2$ at $\lambda t\approx 2.71$ when $\Delta/\lambda=0.6$, $\gamma/\lambda=6.0$, and $\langle\sigma^A_z\rangle=0.01$. Hence the relaxation parameter $\gamma(t)$ changes from positive to negative around $\lambda t\approx 2.71$. The contour plot of $dD(t)/dt$ is shown in Fig. 2(d). The brightest area is the region where the reduced time evolution is non-Markovian. The time dependence of the relaxation parameter $\gamma(t)$ is plotted for several values of the parameter Δ/λ in Fig. 3. It is found from the figure that the non-Markovianity appears in the two different parameter regions [see Figs. 3(b) and 3(d)]. One is around $\Delta/\lambda=0.5$ and the non-Markovianity is caused by the disturbance effect. The other is the region of large values of Δ/λ. In this case, the environment is intrinsically non-Markovian since $\dot{g}_r(t)$ takes negative values and the disturbance effect becomes negligible.

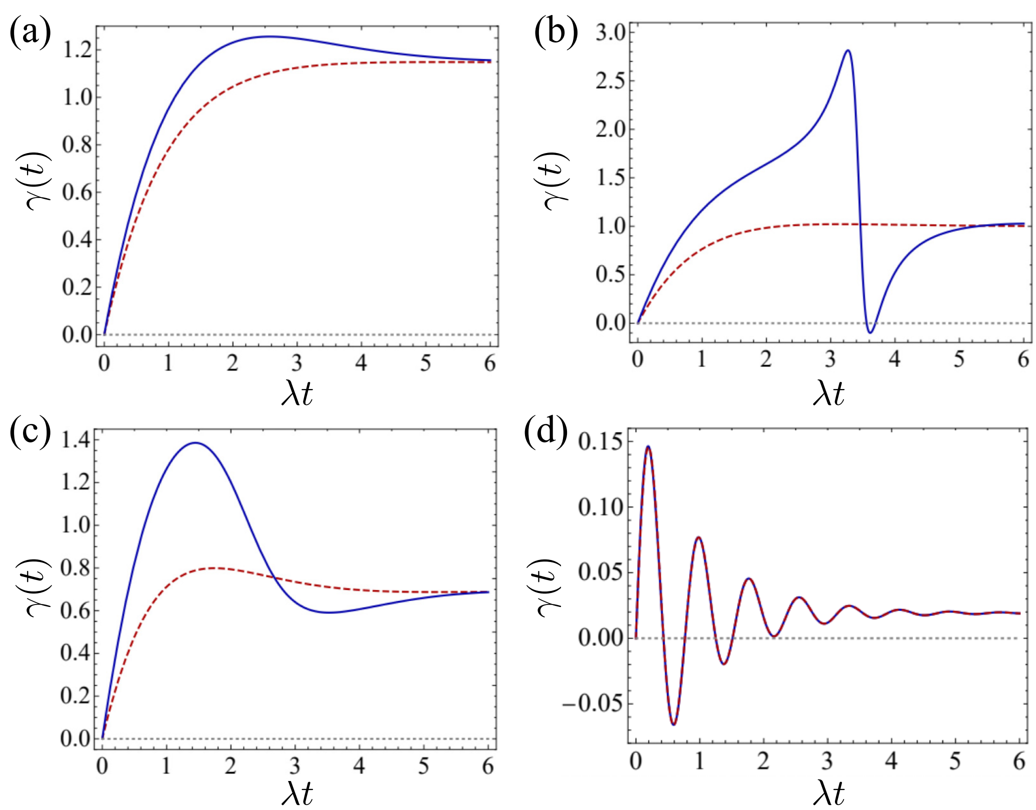

FIG. 3. The time dependence of the relaxation parameter $\gamma(t)$ with (a) $\Delta/\lambda=0.3$, (b) $\Delta/\lambda=0.5$, (c) $\Delta/\lambda=0.9$, and (d) $\Delta/\lambda=8.0$, where we set $\gamma/\lambda=5.0$ and $\langle\sigma^A_z\rangle=0.01$. In each panel, the solid blue (dashed red) line stands for the time dependence with (without) the disturbance effect.

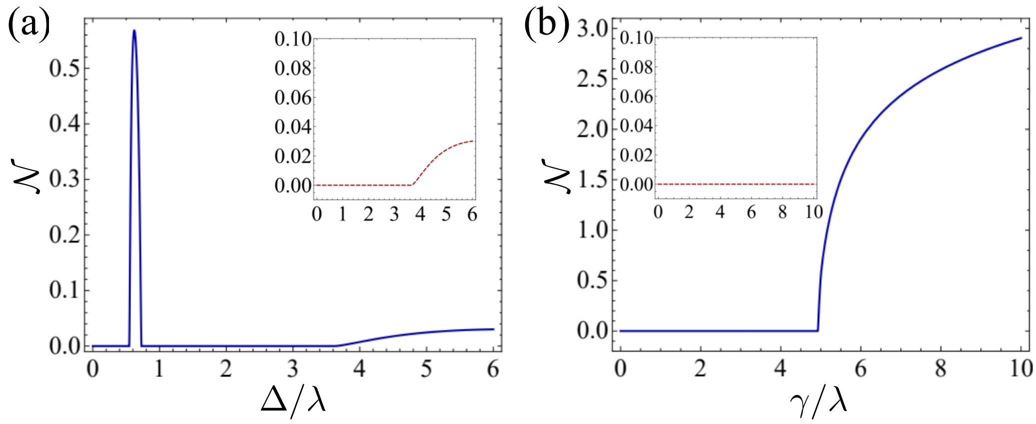

FIG. 4. The non-Markovianity $\mathcal{N}$ of the reduced time evolution of the qubit B, where we set (a) $\gamma/\lambda = 5.0$, $\langle\sigma_z^A\rangle = 0.01$ and (b) $\Delta/\lambda = 0.6$, $\langle\sigma_z^A\rangle = 0.01$. The inset of each panel stands for the non-Markovianity when the disturbance effect is ignored.

Finally, we quantify the non-Markovianity [70] by

$$\mathcal{N} = -2\int_{\gamma(t)<0} \gamma(t)\,dt = \int_0^\infty [|\gamma(t)| - \gamma(t)]dt, \quad (67)$$

which is depicted in Fig. 4. It is found from Fig. 4(a) that the non-Markovianity around $\Delta/\lambda \approx 0.6$ is caused by the disturbance effect while the non-Markovianity for $\Delta/\lambda > 3.6$ is due purely to the environmental property, where the disturbance effect is negligible. From Fig. 4(b), the reduced time evolution is non-Markovian for $\gamma/\lambda \gtrapprox 0.5$, while it is always Markovian in the absence of the disturbance effect.

IV. DECAY OF TWO-QUBIT CORRELATION

In this section, the decay of two-qubit correlation, including entanglement and quantum discord, is investigated when the time evolution of two qubits A and B is described by the composite quantum channel $V_{AB}(t_3, t_2; t_1, t_0)$. For this purpose, we suppose that the two qubits to be sent through the quantum channel are initially prepared in a statistical mixture of Bell states [21],

$$\rho_{AB}^{\mathrm{in}} = \frac{1+c}{2}|\Phi_+\rangle\langle\Phi_+| + \frac{1-c}{2}|\Psi_+\rangle\langle\Psi_+|, \quad (68)$$

with $|\Phi_+\rangle = (|ee\rangle + |gg\rangle)/\sqrt{2}$ and $|\Psi_+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}$. The positivity of the density operator ρ_{AB}^{in} requires the inequality $|c| \leqslant 1$. Then the output state ρ_{AB}^{out} is derived from Eq. (42),

$$\rho_{AB}^{\mathrm{out}} = \frac{1}{4}\begin{pmatrix} 1+c & 0 & 0 & F_+(\vec{t})(1+c) \\ 0 & 1-c & F_-(\vec{t})(1-c) & 0 \\ 0 & F_-^*(\vec{t})(1-c) & 1-c & 0 \\ F_+^*(\vec{t})(1+c) & 0 & 0 & 1+c \end{pmatrix}, \quad (69)$$

with the basis vectors $|1\rangle = |ee\rangle$, $|2\rangle = |eg\rangle$, $|3\rangle = |ge\rangle$, and $|4\rangle = |gg\rangle$. In this equation, we set $F_\pm(\vec{t}) = e^{-i\omega_A(t_1-t_0)\mp i\omega_B(t_3-t_1)}G_\pm(\vec{t})$ with

$$G_\pm(\vec{t}) = e^{-G_R(t_1,t_0)-G_R(t_3,t_2)\mp G_R(t_3,t_2;t_1,t_0)}. \quad (70)$$

If the disturbance effect is ignored, the equality $G_+(\vec{t}) = G_-(\vec{t}) = e^{-G_R(t_1,t_0)-G_R(t_3,t_2)}$ is established. In the rest of this section, we assume that the transmission times of qubits A and B are the same and we set $\tau = t_3 - t_2 = t_1 - t_0$ and $\delta\tau = t_2 - t_1$. We quantify the entanglement of the output state ρ_{AB}^{out} with the concurrence [73], which is calculated to be

$$C(\tau) = \max\left[0, \frac{1+c}{2}G_+(\vec{t}) - \frac{1-c}{2}, \frac{1-c}{2}G_-(\vec{t}) - \frac{1+c}{2}\right]. \quad (71)$$

If the disturbance effect is negligible, the concurrence is invariant under the replacement of c by $-c$. The dependence of the concurrence on the transmission time τ and the parameter Δ/λ is plotted in Fig. 5. It is found from Figs. 5(a) and 5(b) that for a positive value of c, the concurrence decays with the transmission time, and the entanglement sudden death (ESD) [17,18] takes place at a finite time. The ESD time becomes larger as the value of the parameter Δ/λ is larger since the relaxation parameter $\gamma(t)$ is small for a large value of Δ/λ. Furthermore, the EDS time becomes smaller as the value of γ/λ is larger since the parameter γ stands for the strength of the interaction between the qubit and the environment. Although the concurrence decays monotonously with time when Δ/λ is small, the oscillatory behavior of the concurrence is observed when the value of Δ/λ becomes larger. The oscillation is enhanced when γ/λ becomes large. On the

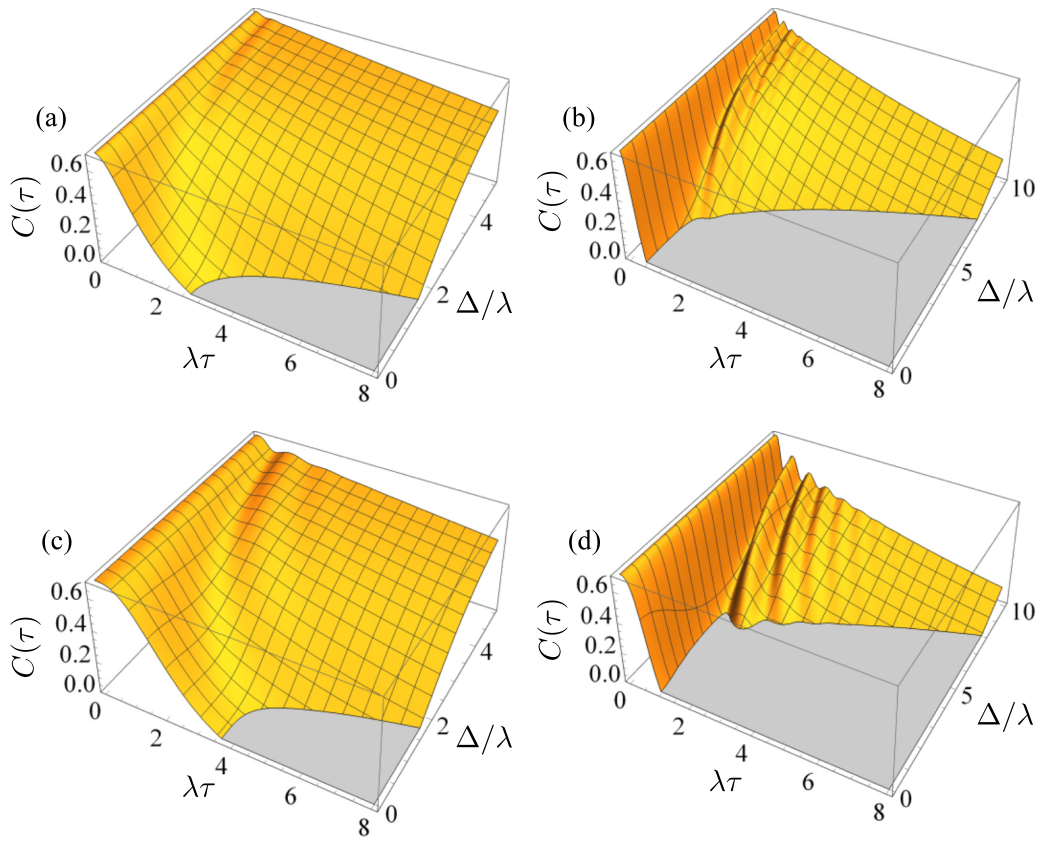

FIG. 5. The dependence of the concurrence $C(\tau)$ on the transmission time τ and the parameter Δ/τ with (a) $(\gamma/\lambda, c) = (0.8, 0.7)$, (b) $(\gamma/\lambda, c) = (12.0, 0.7)$, (c) $(\gamma/\lambda, c) = (0.8, -0.7)$, and (d) $(\gamma/\lambda, c) = (12.0, -0.7)$. In this figure, we set $\delta\tau = 0$.

other hand, in the case of $c < 0$, it is found from Fig. 5(d) that the oscillatory behavior of the concurrence becomes more significant. In particular, for larger values of γ/λ, we can observe not only the ESD but also the entanglement sudden birth (ESB) [19,20] around $\Delta/\lambda \approx 0.4$. The difference between the concurrences with positive and negative values of c is caused by the disturbance effect, which leads to $G_+(\vec{t}) \neq G_-(\vec{t})$. The difference disappears if the value of $\lambda\delta\tau$ is large since the disturbance effect caused by qubit A vanishes before qubit B is input.

Next we investigate total, classical, and quantum correlation of the output state ρ_{AB}^{out} [80]. For this purpose, it is convenient to rewrite Eq. (69) into

$$\rho_{AB}^{\text{out}} = \frac{1}{4}\begin{pmatrix} 1+c_z & 0 & 0 & c_x - c_y \\ 0 & 1-c_z & c_x+c_y & 0 \\ 0 & c_x+c_y & 1-c_z & 0 \\ c_x - c_y & 0 & 0 & 1+c_z \end{pmatrix}, \tag{72}$$

with

$$c_x = \tfrac{1}{2}[G_-(\vec{t})(1-c) + G_+(\vec{t})(1+c)], \tag{73}$$

$$c_y = \tfrac{1}{2}[G_-(\vec{t})(1-c) - G_+(\vec{t})(1+c)], \tag{74}$$

and $c_z = c$. In Eq. (72), we have neglected the unimportant phase factors, which can be removed by a local unitary transformation. The total correlation $\mathcal{C}_T$ is quantified with the von Neumann mutual information [80]. For the two-qubit state ρ_{AB}^{out}, we obtain

$$\begin{aligned}\mathcal{C}_T =& S(\rho_A^{\text{out}}) + S(\rho_B^{\text{out}}) - S(\rho_{AB}^{\text{out}}) = \tfrac{1}{2}(1-c)\log_2(1-c) + \tfrac{1}{2}(1+c)\log_2(1+c) \\ &+ \tfrac{1}{4}(1-c)([1-G_-(\vec{t})]\log_2[1-G_-(\vec{t})] + [1+G_-(\vec{t})]\log_2[1+G_-(\vec{t})]) \\ &+ \tfrac{1}{4}(1+c)([1-G_+(\vec{t})]\log_2[1-G_+(\vec{t})] + [1+G_+(\vec{t})]\log_2[1+G_+(\vec{t})]),\end{aligned} \tag{75}$$

where $S(\rho)$ is the von Neumann entropy of ρ. The classical correlation $\mathcal{C}_C$ is given by [80]

$$\mathcal{C}_C = \tfrac{1}{2}(1-c_{\max})\log_2(1-c_{\max}) + \tfrac{1}{2}(1+c_{\max})\log_2(1+c_{\max}) \tag{76}$$

with $c_{\max} = \max[|c_x|, |c_y|, |c_z|]$. The quantum correlation $\mathcal{C}_Q$ is quantified by means of the quantum discord [80], which is the difference between total and classical correlations. Then we obtain from Eqs. (75) and (76)

$$\begin{aligned}\mathcal{C}_Q = {}& \tfrac{1}{2}(1-c)\log_2(1-c) + \tfrac{1}{2}(1+c)\log_2(1+c) + \tfrac{1}{4}(1-c)([1-G_-(\vec{t})]\log_2[1-G_-(\vec{t})] + [1+G_-(\vec{t})]\log_2[1+G_-(\vec{t})]) \\ & + \tfrac{1}{4}(1+c)([1-G_+(\vec{t})]\log_2[1-G_+(\vec{t})] + [1+G_+(\vec{t})]\log_2[1+G_+(\vec{t})]) \\ & - \tfrac{1}{2}(1-c_{\max})\log_2(1-c_{\max}) - \tfrac{1}{2}(1+c_{\max})\log_2(1+c_{\max}).\end{aligned} \tag{77}$$

If the disturbance effect is ignored, the classical and quantum correlations become

$$\mathcal{C}_C = \tfrac{1}{2}(1-\tilde{c}_{\max})\log_2(1-\tilde{c}_{\max}) + \tfrac{1}{2}(1+\tilde{c}_{\max})\log_2(1+\tilde{c}_{\max}), \tag{78}$$

$$\mathcal{C}_Q = \tfrac{1}{2}(1-\tilde{c}_{\min})\log_2(1-\tilde{c}_{\min}) + \tfrac{1}{2}(1+\tilde{c}_{\min})\log_2(1+\tilde{c}_{\min}), \tag{79}$$

with $\tilde{c}_{\max} = \max[|c|, G(\vec{t})]$ and $\tilde{c}_{\min} = \min[|c|, G(\vec{t})]$. In this case, the inequality $\mathcal{C}_C \geqslant \mathcal{C}_Q$ is always satisfied and either $\mathcal{C}_C$ or $\mathcal{C}_Q$ remains unchanged in time. The correlations for the initial state with $c = -0.6$ are plotted in Fig. 6. We note that when $\lambda\delta\tau = 5.0$, the plots given in Figs. 6(b), 6(d) 6(f), and 6(h) are equal to those obtained for the initial state with $c = 0.6$ since the disturbance effect on qubit B is negligible and so $G_+(\vec{t}) \approx G_-(\vec{t})$ is established. The oscillatory behavior observed in the total and classical correlations in Figs. 6(c), 6(d) 6(g), and 6(h) is due to the intrinsic non-Markovianity of the environment. On the other hand, the increase of quantum discord observed in Figs. 6(a), 6(e) and 6(g) is caused by the disturbance effect. Since this effect is small, it is not apparent in the total and classical correlations. In Figs. 6(e) and 6(g), there is a time at which the inequality $\mathcal{C}_Q > \mathcal{C}_C$ is established.

V. FIDELITY OF STATE TRANSMISSION

Finally, we investigate how faithfully the composite quantum channel $V_{AB}(t_3, t_2; t_1, t_0)$ can transmit quantum states of qubits A and B [16], where the transmission times are the same, namely, $\tau = t_3 - t_2 = t_1 - t_0$. We set the time separation $\delta\tau = t_2 - t_1$ between the two transmissions. We suppose that the input states of the two qubits are, respectively, $|\psi_A^{\text{in}}\rangle = \alpha_A|e\rangle + \beta_A|g\rangle$ and $|\psi_B^{\text{in}}\rangle = \alpha_B|e\rangle + \beta_B|g\rangle$ with $|\alpha_A|^2 + |\beta_A|^2 = 1$ and $|\alpha_B|^2 + |\beta_B|^2 = 1$. Then the output state ρ_{AB}^{out} is given by

$$\begin{aligned}\rho_{AB}^{\text{out}} = {}& |\alpha_A|^2|\alpha_B|^2|e\rangle\langle e|\otimes|e\rangle\langle e| + |\alpha_A|^2|\beta_B|^2|e\rangle\langle e|\otimes|g\rangle\langle g| + |\beta_A|^2|\alpha_B|^2|g\rangle\langle g|\otimes|e\rangle\langle e| \\ & + |\beta_A|^2|\beta_B|^2|g\rangle\langle g|\otimes|g\rangle\langle g| + |\alpha_A|^2\alpha_B\beta_B^* e^{-i\omega_B\tau - i\Omega(\tau) - \Gamma(\tau)}|e\rangle\langle e|\otimes|e\rangle\langle g| \\ & + |\alpha_A|^2\alpha_B^*\beta_B e^{i\omega_B\tau + i\Omega(\tau) - \Gamma(\tau)}|e\rangle\langle e|\otimes|g\rangle\langle e| + |\beta_A|^2\alpha_B\beta_B^* e^{-i\omega_B\tau + i\Omega(\tau) - \Gamma(\tau)}|g\rangle\langle g|\otimes|e\rangle\langle g| \\ & + |\beta_A|^2\alpha_B^*\beta_B e^{i\omega_B\tau - i\Omega(\tau) - \Gamma(\tau)}|g\rangle\langle g|\otimes|g\rangle\langle e| + \alpha_A\beta_A^*|\alpha_B|^2 e^{-i\omega_A\tau - \Gamma(\tau)}|e\rangle\langle g|\otimes|e\rangle\langle e| \\ & + \alpha_A^*\beta_A|\alpha_B|^2 e^{i\omega_A\tau - \Gamma(\tau)}|g\rangle\langle e|\otimes|e\rangle\langle e| + \alpha_A\beta_A^*|\beta_B|^2 e^{-i\omega_A\tau - \Gamma(\tau)}|e\rangle\langle g|\otimes|g\rangle\langle g| \\ & + \alpha_A^*\beta_A|\beta_B|^2 e^{i\omega_A\tau - \Gamma(\tau)}|g\rangle\langle e|\otimes|g\rangle\langle g| + \alpha_A\beta_A^*\alpha_B\beta_B^* e^{-i\omega_A\tau - i\omega_B\tau - 2\Gamma(\tau) - \Upsilon(\tau)}|e\rangle\langle g|\otimes|e\rangle\langle g| \\ & + \alpha_A\beta_A^*\alpha_B^*\beta_B e^{-i\omega_A\tau + i\omega_B\tau - 2\Gamma(\tau) + \Upsilon(\tau)}|e\rangle\langle g|\otimes|g\rangle\langle e| + \alpha_A^*\beta_A\alpha_B\beta_B^* e^{i\omega_A\tau - i\omega_B\tau - 2\Gamma(\tau) + \Upsilon(\tau)}|g\rangle\langle e|\otimes|e\rangle\langle g| \\ & + \alpha_A^*\beta_A\alpha_B^*\beta_B e^{i\omega_A\tau + i\omega_B\tau - 2\Gamma(\tau) - \Upsilon(\tau)}|g\rangle\langle e|\otimes|g\rangle\langle e|,\end{aligned} \tag{80}$$

where the time-dependent parameters are $\Gamma(\tau) = G_R(t_3, t_2) = G_R(t_1, t_0)$, $\Upsilon(\tau) = G_R(t_3, t_2; t_1, t_0)$, and $\Omega(\tau) = G_I(t_3, t_2; t_1, t_0)$. First we evaluate the state transmission of the two qubits individually. The reduced output states of qubits A and B are given by

$$\rho_A^{\text{out}} = |\alpha_A|^2|e\rangle\langle e| + |\beta_A|^2|g\rangle\langle g| + \alpha_A\beta_A^* e^{-i\omega_A\tau - \Gamma(\tau)}|e\rangle\langle g| + \alpha_A^*\beta_A e^{i\omega_A\tau - \Gamma(\tau)}|g\rangle\langle e|, \tag{81}$$

$$\begin{aligned}\rho_B^{\text{out}} = {}& |\alpha_B|^2|e\rangle\langle e| + |\beta_B|^2|g\rangle\langle g| + \alpha_B\beta_B^* e^{-i\omega_B\tau - \Gamma(\tau)}(|\alpha_A|^2 e^{-i\Omega(\tau)} + |\beta_A|^2 e^{i\Omega(\tau)})|e\rangle\langle g| \\ & + \alpha_B^*\beta_B e^{i\omega_B\tau - \Gamma(\tau)}(|\alpha_A|^2 e^{i\Omega(\tau)} + |\beta_A|^2 e^{-i\Omega(\tau)})|g\rangle\langle e|,\end{aligned} \tag{82}$$

both of which are independent of the parameter $\Upsilon(\tau)$. To evaluate the state transmission through the composite quantum channel $V_{AB}(t_3, t_2; t_1, t_0)$, we calculate the average fidelity [16] between the reduced output state $\rho_{A,B}^{\text{out}}$ and the time-evolved input state $|\psi_{A,B}^{\text{in}}(\tau)\rangle = e^{-i\omega_{A,B}\tau}\alpha_{A,B}|e\rangle + e^{i\omega_{A,B}\tau}\beta_{A,B}|g\rangle$ generated by the free Hamiltonian $H_{A,B}$. The average is taken over the Bloch sphere of the input state. Then we obtain the average fidelities from Eqs. (81) and (82),

$$F_A = \overline{\langle\psi_{\text{in}}^A(\tau)|\rho_A^{\text{out}}|\psi_{\text{in}}^A(\tau)\rangle} = \frac{2 + e^{-\Gamma(\tau)}}{3}, \tag{83}$$

$$F_B = \overline{\langle\psi_{\text{in}}^B(\tau)|\rho_B^{\text{out}}|\psi_{\text{in}}^B(\tau)\rangle} = \frac{2 + e^{-\Gamma(\tau)}\cos\Omega(\tau)}{3}. \tag{84}$$

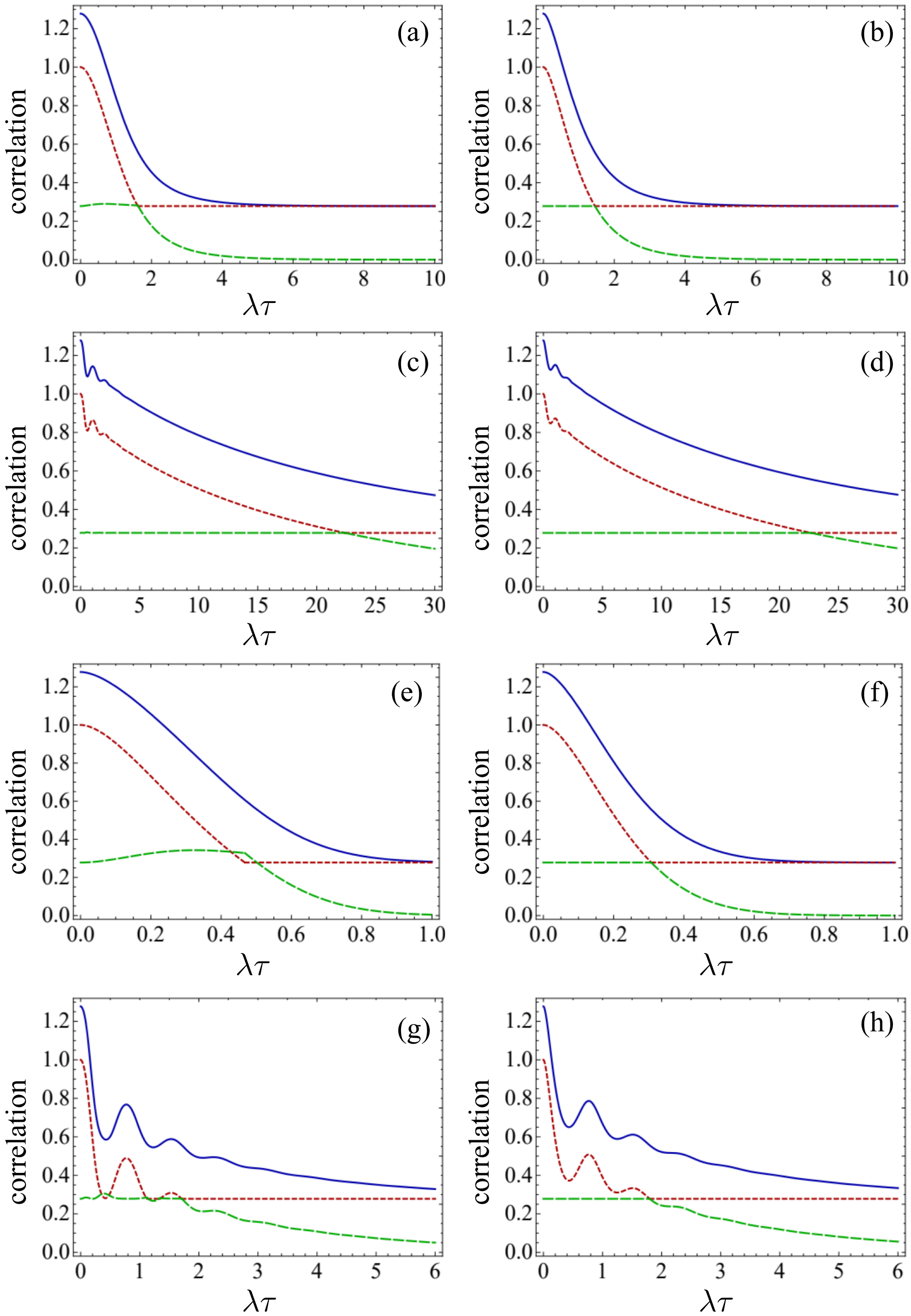

FIG. 6. The dependence of the total, classical, and quantum correlations of the output state ρ_{AB}^{out} with $c = -0.6$ on the transmission time τ, where the solid line (blue) stands for the total correlation $\mathcal{C}_T$, the dotted line (red) for the classical correlation $\mathcal{C}_C$, and the dashed line (green) for the quantum correlation $\mathcal{C}_Q$. In this figure, we set $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (0.0, 0.8, 0.8)$ in panel (a), $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (5.0, 0.8, 0.8)$ in panel (b), $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (0.0, 6.0, 0.8)$ in panel (c), $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (5.0, 6.0, 0.8)$ in panel (d), $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (0.0, 0.8.12.0)$ in panel (e), $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (5.0, 0.8, 12.0)$ in panel (f), $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (0.0, 8.0, 12.0)$ in panel (h), and $(\lambda\delta\tau, \Delta/\lambda, \gamma/\tau) = (5.0, 8.0, 12.0)$ in panel (g). In panels (b), (d), (f), and (h), $G_+(\vec{t}) \approx G_-(\vec{t})$ is satisfied.

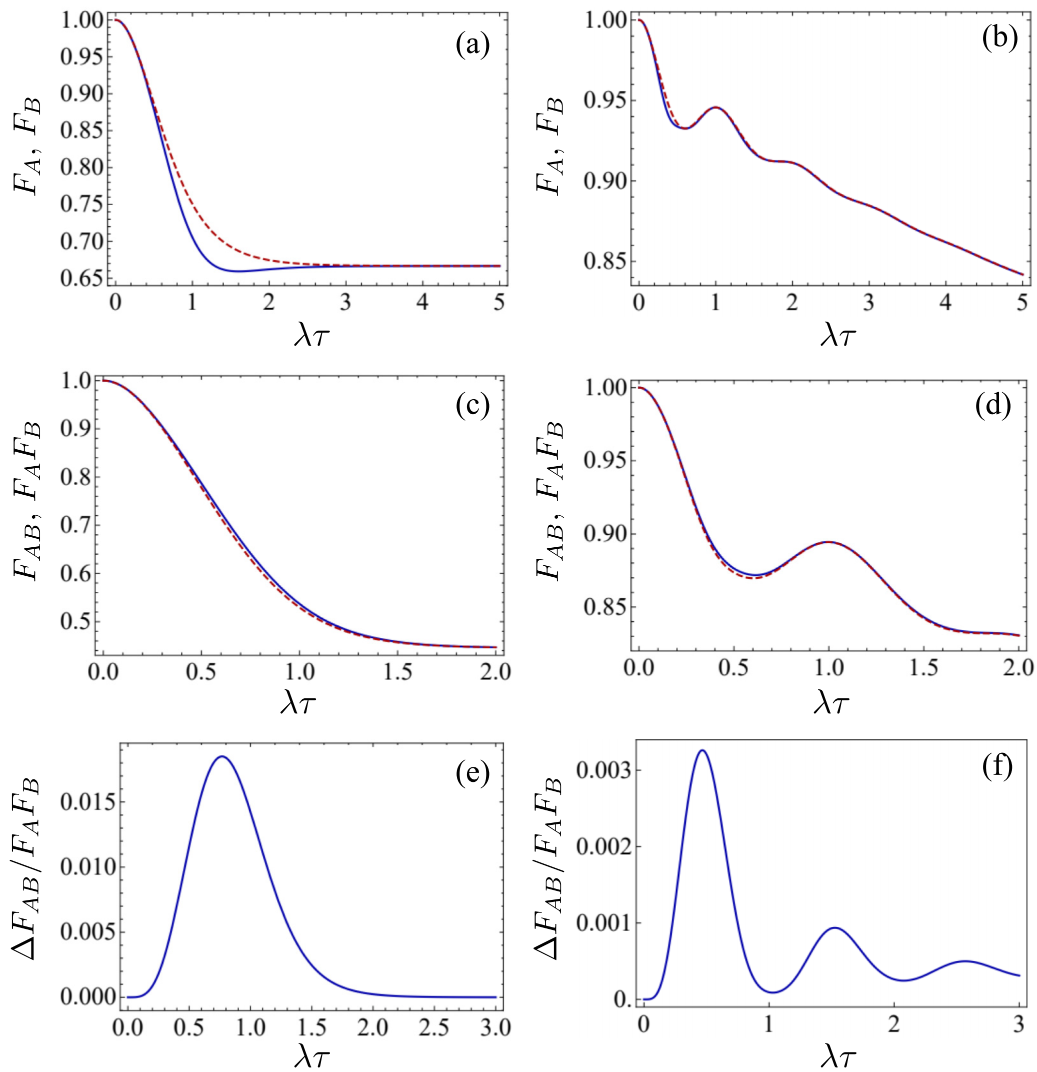

FIG. 7. The average fidelity of the state transmission through the composite quantum channel. In panels (a) and (b), the dashed line (red) stands for F_A and the solid line (blue) for F_B. In panels (c) and (d), the dashed line (red) stands for $F_A F_B$ and the solid line (blue) for F_{AB}. In panels (e) and (f), the ratio $\Delta F_{AB}/F_A F_B$ is plotted. We set $\Delta/\lambda = 1.0$ in panels (a), (c), and (e), and $\Delta/\lambda = 6.0$, $\gamma/\lambda = 8.0$ in panels (b), (d), and (f). In all of the panels, we set $\gamma/\lambda = 8.0$ and $\lambda\delta\tau = 0.0$.

If the disturbance effect is negligible [$\Omega(\tau) \approx 0$], the equality $F_A = F_B$ holds. It is obvious from these equations that the disturbance effect decreases the average fidelity of qubit B. On the other hand, when the state transmission is treated collectively, the average fidelity between the output two-qubit state ρ_{AB}^{in} and the compound input state $|\psi_A^{\text{in}}(\tau)\rangle \otimes |\psi_B^{\text{in}}(\tau)\rangle$ is given by

$$F_{AB} = \tfrac{1}{9}[4 + 2e^{-\Gamma(\tau)} + 2e^{-\Gamma(\tau)} \cos \Omega(\tau) + e^{-2\Gamma(\tau)} \cosh \Upsilon(\tau)]. \tag{85}$$

The fidelity difference between the individual and collective state transmissions is obtained,

$$\Delta F_{AB} = F_{AB} - F_A F_B = \tfrac{1}{9} e^{-2\Gamma(\tau)} [\cosh \Upsilon(\tau) - \cos \Omega(\tau)], \tag{86}$$

which clearly satisfies the inequality $\Delta F_{AB} \geqslant 0$. Thus the correlation between the two qubits that is created by the environmental disturbance has a positive effect on the collective state transmission, though it has a negative effect on the individual state transmission. The dependence of the fidelities on the transmission time τ is depicted in Fig. 7. As pointed out above, the fidelity of qubit B is smaller than that of qubit A. When Δ/λ is small, the fidelity F_A decays monotonously with the transmission time while the fidelity F_B first decreases to the minimum value and then increases slightly up to the stationary value [Fig. 7(a)]. On the other hand, when Δ/λ is large, F_A and F_B are nearly equal and demonstrate oscillatory behavior [Fig. 7(b)]. The disturbance effect is small in this case. In Figs. 7(c) and 7(d), the collective fidelity F_{AB} is compared with the individual fidelity $F_A F_B$, and it is found that F_{AB} is slightly larger than $F_A F_B$. To make this point clear, the ratio $\Delta F_{AB}/F_A F_B$ is plotted in Figs. 7(e) and 7(f). The result means that the disturbance effect enhances collective fidelity but reduces individual fidelity.

VI. SUMMARY

In this paper, when two independent quantum systems A and B interact sequentially with a common environment, we have derived a general expression of the composite quantum channel describing the time evolution of the two quantum systems. In the process, the quantum system A first interacts with the environment in the thermal equilibrium state, and after that the quantum system B interacts with the same environment, which has been disturbed by the quantum system A. Hence the quantum system B, which does not directly interact with the quantum system A, is affected indirectly by the quantum system A through the environment. The quantum channel derived in this paper is exact if the environment is Gaussian, otherwise it is the second-order approximation with respect to the system-environment interaction. We have also provided the approximation to the quantum channel that is valid if the correlation time of the environment is not so long in comparison with the time difference $t_2 - t_1$, where the quantum system A finishes the interaction at time t_1 and the quantum system B begins the interaction at time t_2. If the two quantum systems interact with the environment via a dephasing coupling, the general formula is analytically tractable since the time-ordering operation is removed.

Assuming that the two quantum states are qubits interacting with the environment via a dephasing coupling, we have investigated the non-Markovianity of the reduced time evolution, the degradation of the two-qubit correlation, and the state transmission through the composite quantum channel. Qubit B is influenced by the environment disturbed by qubit A. For qubit A, the reduced time evolution is non-Markovian only if the correlation time of the environment is long in comparison with the characteristic time of the qubit A. On the other hand, for qubit B, the non-Markovianity is caused by two different origins. One is the same as that for qubit A. The other is the disturbance effect of the environment caused by qubit A. In fact, it has been found that the non-Markovianity appears in the different parameter regions. The result implies that the condition for non-Markovianity is weakened when the environment is out of thermal equilibrium.

For a statistical mixture of Bell states, we have shown that the entanglement of the two-qubit state coming out of the composite quantum channel exhibits not only sudden death but also sudden birth in the case of strong coupling with the environment. Although classical correlation is always greater than quantum correlation without the disturbance effect, this is not true in the presence of the disturbance effect. The increase of quantum correlation has been observed due to the disturbance effect. Finally, we have investigated the state transfer of two qubits through the composite quantum channel. When the transmissions of qubits A and B are treated individually, the disturbance effect decreases the average fidelity of qubit B. However, when the state transmissions are treated collectively, the average fidelity of qubits A and B is increased in the presence of the disturbance effect. In this paper, we have found that the disturbance effect in the environment has a nontrivial influence on the non-Markovianity, the bipartite correlation, and the state transmission. In Secs. III–V, we have assumed that the environment has Lorentzian spectral density. So it may be important to investigate the disturbance effect when it has a different spectral density, such as Ohmic spectral density.

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