# Highly directional photon superbunching from a few-atom chain of emitters

Qurrat-ul-Ain Gulfam[1,*] and Zbigniew Ficek[2,3,†]
[1]Department of Physics, Faculty of Science, Jazan University, P.O. Box 114, Gizan 45142, Saudi Arabia
[2]The National Centre for Applied Physics, KACST, P.O. Box 6086, Riyadh 11442, Saudi Arabia
[3]Quantum Optics and Engineering Division, Institute of Physics, University of Zielona Góra, Szafrana 4a, Zielona Góra 65-516, Poland

(Received 9 August 2018; published 14 December 2018)

We examine angular distribution of the probability of correlated fluorescence photon emission from a linear chain of identical equidistant two-level atoms. We selectively excite one of the atoms by a resonant laser field. The atoms are coupled to each other via the dipole-dipole interaction and collective spontaneous emission. Our attention is focused on the simultaneous observation of correlated pairs of photons. It is found that the interference between the emitting atoms can result in a highly directional emission of photon pairs. These pairs of photons exhibit strong correlations and their emission is highly concentrated into specific directions. We demonstrate the crucial role of the selective coherent excitation in such a geometrical configuration. Shifting the driving field from an atom located at one end of the chain to the other causes the radiation pattern to flip to the opposite half of the detection plane. Furthermore, we find that atomic systems in which only an atom situated at a particular position within the linear chain is driven by a laser field can radiate correlated twin photons in directions along which the radiation of single photons is significantly reduced. Alternatively, superbunching in the emitted photon statistics preferentially occurs in directions of negligible or vanishing single-photon emission. The effect of superbunching strengthens as more emitters are added to the chain. Depending on the number of atoms and the position of the driven atom within the chain, the strongly correlated pairs of photons can be emitted into few well-defined directions.

DOI: 10.1103/PhysRevA.98.063824

## I. INTRODUCTION

The radiative properties of correlated systems is an active area of research study in quantum optics and quantum information science. The presence of correlations results in the collective behavior of the radiating systems which can significantly modify the properties of the emitted radiation, in particular, can lead to the phenomenon of superradiance [1–4]. A good deal of attention has been given to the problem of collective behavior of spatially separated atoms arranged into a linear chain [5–13] or two-dimensional arrays [14,15]. Such arrangements can be realized in practice with atoms trapped at small distances and interacting cooperatively with a common radiation field (reservoir) [16–21]. If the atoms are close enough, the coupling results in the dipole-dipole interaction between the atoms and the collective damping of the atomic transitions.

The radiative properties of the atoms are usually studied in terms of correlation functions of the amplitudes of the emitted field. For example, the one-time first-order correlation function is used to determine the intensity, whereas the two-time correlation function is used to determine spectral properties of the field [22]. Higher-order correlation functions, in particular, the second-order correlation functions, are widely applied to determine whether the radiated field is quantum or classical in nature or, in other words, if two emitted photons are correlated or anticorrelated [23].

It is well known that the collective behavior of the radiating atoms may lead to a strongly directional emission of photons and the directivity increases with an increasing number of atoms [10,14,24–28]. The directivity of the emission is determined by the angular distribution of the radiation intensity or, equivalently, by the angular distribution of the first-order correlations of the emitted photons.

High directivity can be observed in the higher-order correlations, in particular, in the second-order correlations. The correlations are usually studied by using the normalized second-order correlation function $g^{(2)}$, which can have values ranging from zero to infinity. Values of $g^{(2)} < 1$ refer to photon antibunching, $1 < g^{(2)} < 2$ to photon bunching, and $g^{(2)} > 2$ to photon superbunching or extrabunching. Photon antibunching, corresponding to the emission of single photons in a regular manner, is a nonclassical phenomenon and has been the subject of intensive theoretical and experimental studies [29–37]. Photon bunching is characteristic of thermal light, whereas superbunching, indicating strong correlations between photons, has been found characteristic of nonclassical states of light such as squeezed states [38] and entangled Gaussian states [39].

In an early study of the angular distribution of the second-order correlations, Wiegand [40] demonstrated that the occurrence of antibunching or superbunching of photons emitted by a system composed of two interacting and weakly driven two-level atoms strongly depends on the direction of observation and on the distance between the atoms. Richter [41,42] studied

*qgulfam@jazanu.edu.sa
†zficek@kacst.edu.sa

 

the dependence of the angular distribution of the second-order correlations of photons emitted from three atoms on the arrangement of the atoms and the pumping process. It was found that the angular distribution of the correlation function is different for the atoms arranged in a triangle or in an equidistant chain. It is also different for coherently and incoherently pumped atoms.

Correlated pairs of photons have been found useful in many applications in interferometric lithography [43–45], ghost imaging [46,47], probing interactions in Rydberg ensembles [48,49], to detect two-photon atom-cavity field entangled states [50–52], and also in heralding measurements which require double-photon coincidences [53–55]. Sources of correlated pairs of photons are also required for the realization of many quantum information proposals such as linear optical quantum computation [56], quantum memories [57,58], entanglement swapping [59], and teleportation [60].

The schemes for generating superbunched pairs of photons have generally been based on either nonlinear processes such as parametric down conversion [61–63] or multilevel atom systems driven by laser fields [64]. The Rydberg blockade mechanism, a well-known mechanism to generate single photons, has also been proposed to produce correlated photon pairs in multiatom systems [48,49]. Other interesting methods include measurement-induced superbunching of photons emitted from independent photon sources [65–69], and multiple two-photon path interference in which two-photon superbunching of a thermal light has been predicted and experimentally observed [70–72]. Here, we consider a scheme involving a multiatom system in which the generation of correlated pairs of photons is based on a nonsymmetric excitation of the atomic system and the interaction between the atoms mediated by the coupling to a common three-dimensional vacuum field. The coupling results in the collective damping of the atoms and the dipole-dipole interaction between them.

The atoms are geometrically arranged in an equidistant chain and the emitted photons are detected by a single photodetector located in the far-field zone of the system. We assume that only one, specially selected atom of the chain is driven on resonance with an external laser field. This nonsymmetric excitation of the atomic system allows us to manipulate and control the correlation properties of the emitted photons. The correlation properties of the emitted photons are determined by the first- and second-order correlation functions, and we concentrate on the properties of their angular distributions. We consider situations where either one of end atoms or the middle atom of the chain is selectively driven by the laser. With this we find by considering the normalized second-order correlation function that at small distances between the atoms strongly correlated (superbunched) pairs of photons can be emitted into two or four specific directions. Moreover, we find that depending on whether the number of atoms is even or odd, the emission of superbunched pairs of photons can occur into one or two specific directions. These directions are found to be always located in the half-plane opposite to the half-plane where the driven atom is present. We also see quite clear that superbunching is not associated with strong correlations between the emitted photons, but it rather results from a significant reduction of the probability of the emission of single photons relative to the probability of the simultaneous emission of two photons. Finally, we point out that the strongly directional emission of correlated pairs of photons emitted by a linear chain of atoms is predicted without the requirement of the presence of interactions mediated by a waveguide [73,74] or a cavity [75,76], or by the creation of a chiral-type coupling between the atoms [77].

The paper is organized as follows. In Sec. II we give detailed description of the model system which is composed of two-level atoms geometrically arranged in an equidistant chain. The atoms are selectively driven by a resonant laser field and the fluorescence photons are detected by a photodetector located in the far-field zone of the system. In Sec. III, the definitions of the first- and second-order correlation functions in terms of the atomic dipole operators are presented, and the master-equation approach for the evaluation of density matrix elements is outlined. Polar plots for the angular distributions of the correlation functions are presented in Sec. IV. The plots demonstrate the existence of specific directions in which strongly correlated (superbunched) pairs of photons are emitted. This is accompanied by the detailed discussion of the controlled switching of these directions, associated with the driving of specially selected atom. The variation of the angular distribution of the correlation functions with the number of atoms is also discussed. In Sec. V we comment about the physical meaning of superbunching by comparing the results for the normalized second-order correlation function with those for a different measure of two-photon correlations which is less sensitive to single-photon emissions. Our conclusions are summarized in Sec. VI.

## II. MODEL SYSTEM

The system under investigation consists of a set of $N$ identical two-level atoms located at positions $r_i$ and organized into a line such that they form a linear chain, as shown in Fig. 1. The atoms are close to each other such that they couple to a common three-dimensional vacuum field. The coupling results in the collective damping and the dipole-dipole interaction between the atoms. An external laser field of the Rabi frequency $\Omega$ and the angular frequency equal to the atomic transition frequency $\omega_0$ drives one of the atoms (black dot) in the chain. In particular, the leftmost, or the middle, or the rightmost atom is driven at a time. The laser field propagates in the direction perpendicular to the atomic line. Our aim is to record correlated pairs of simultaneously emitted photons. For this purpose, two photodetectors located at distances $R_1$ and $R_2$, and at polar angles $\theta_1$ and $\theta_2$, respectively, in the far-field region detect the scattered light. The pairs can be detected by a single or two photodetectors located in a plane orthogonal to the direction of alignment of atomic dipole moments. The left inset shows the internal energy-level structure corresponding to a laser-driven atom (black dot) in which the atomic transition is coupled with the laser field while right inset refers to any atom which is not being driven (white dot). $|e\rangle$ ($|g\rangle$) represents the excited (ground) state of a two-level atom.

## III. CORRELATION FUNCTIONS

We focus on correlation properties of the photons emitted by an atomic system which, in turn, can be used to determine

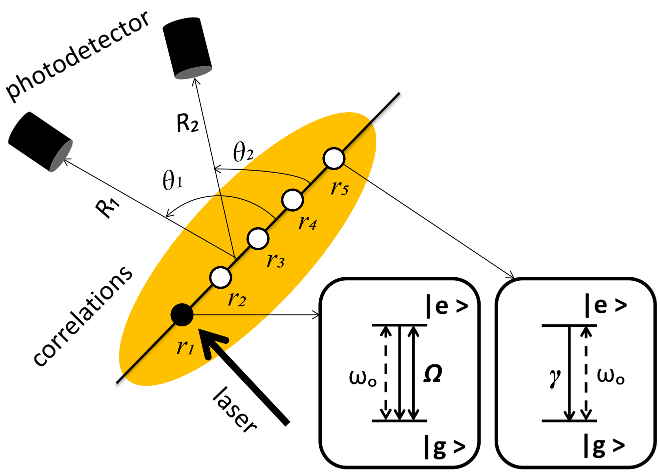


FIG. 1. A chain of equidistant two-level atoms. An external laser field of Rabi frequency $\Omega$ drives one of the atoms (black dot). The atoms are coupled to each other through the dipole-dipole interaction and the collective spontaneous emission resulting from the coupling of the atoms to a common vacuum field. The emitted fluorescence field is detected by a single or two photodetectors at distances $R_1$ and $R_2$ and angles $\theta_1$ and $\theta_2$ from the atomic line, located in the plane normal to the plane defined by the polarization of the atomic dipole moments $\vec{\mu}\cdot\hat{r}_{ij}=0$. Left (right) blob and inset shows a coherently driven (nondriven) atom.

the correlation properties of the atoms. In order to do this, we introduce first-order correlation function of the normally ordered electric field operators associated with the fluorescence field emitted by the atoms and detected in the far-field zone [22,78]

$$G^{(1)}(\vec{R},t)=\left(\frac{R^2}{2\pi k_0}\right)\langle\vec{E}^{(-)}(\vec{R},t)\cdot\vec{E}^{(+)}(\vec{R},t)\rangle, \quad (1)$$

and the normally ordered intensity correlation function

$$\begin{aligned}G^{(2)}(\vec{R}_1,t_1;\vec{R}_2,t_2)&=\langle :I(\vec{R}_1,t_1)I(\vec{R}_2,t_2):\rangle\\&=\left(\frac{R_1R_2}{2\pi k_0}\right)^2\langle\vec{E}^{(-)}(\vec{R}_1,t_1)\vec{E}^{(-)}(\vec{R}_2,t_2)\\&\quad\times\vec{E}^{(+)}(\vec{R}_2,t_2)\vec{E}^{(+)}(\vec{R}_1,t_1)\rangle, \quad (2)\end{aligned}$$

where $\vec{E}^{(+)}$ ($\vec{E}^{(-)}$) denotes the positive (negative) frequency part of the electric field. In the definition of the correlation function $G^{(1)}(\vec{R},t)$ [Eq. (1)], we have introduced the factor $(R^2/2\pi k_0)$ so that $G^{(1)}(\vec{R},t)d\Omega_R dt$ is the probability of finding a photon inside the solid angle $d\Omega_R$ around the direction $\vec{R}$ in the time interval $dt$.

Assume that the system of radiating atoms is composed of $N$-equidistant identical two-level atoms. An atom, say the $i$th one, is represented by its ground state $|g_i\rangle$, an excited state $|e_i\rangle$, the Bohr atomic transition frequency $\omega_0$, the transition dipole moment $\vec{\mu}$, and its position $\vec{r}_i$ along the atomic line. In the far-field zone of the radiating atoms, the contribution from the free field can be neglected, and the positive frequency part of the scattered electric field can be expressed in terms of the transition dipole moments of the atoms as

$$\vec{E}^{(+)}(\vec{R},t)=\frac{\omega_0^2\mu^2}{4\pi\epsilon_0c^2}\frac{[(\hat{R}\times\hat{\mu})\times\hat{R}]}{R}\sum_{i=1}^{N}S_i^-e^{-i(k\hat{R}\cdot\vec{r}_i-\omega_0t)}, \quad (3)$$

where $S_i^-=|g_i\rangle\langle e_i|$ is the atomic lowering operator for the $i$th atom, $\mu=|\langle g_i|\vec{\mu}|e_i\rangle|$ is the magnitude of the atomic dipole moment, $R=|\vec{R}|$, $\hat{\mu}$ is the unit vector in the direction of the atomic transition dipole moment, and $\hat{R}$ is the unit vector in the direction of observation $\vec{R}$.

After substituting Eq. (3) into Eqs. (1) and (2), we obtain

$$G^{(1)}(\vec{R},t)=u(\hat{R})\gamma\sum_{\{i,j\}=1}^{N}\langle S_i^+(t)S_j^-(t)\rangle e^{ik\vec{r}_{ij}\cdot\hat{R}}, \quad (4)$$

and

$$\begin{aligned}G^{(2)}(\vec{R}_1,t_1;\vec{R}_2,t_2)&=u(\hat{R}_1)u(\hat{R}_2)\gamma^2\sum_{i\neq j=1}^{N}\sum_{l\neq k=1}^{N}\langle S_i^+(t_1)\\&\quad\times S_j^+(t_2)S_k^-(t_2)S_l^-(t_1)\rangle\\&\quad\times\exp[ik(\vec{r}_{il}\cdot\hat{R}_1+\vec{r}_{jk}\cdot\hat{R}_2)], \quad (5)\end{aligned}$$

where $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$ is the spatial distance vector between the atoms $i$ and $j$, $u(\hat{R})=(3/8\pi)[1-(\hat{\mu}\cdot\hat{R})^2]$ is the radiation pattern of a single atomic dipole, and $\gamma$ is the decay rate of the atomic transition.

The evolution of the atomic correlation functions appearing in Eqs. (4) and (5) is governed by a master equation for the atomic density operator $\rho$. Within the Born-Markov and the rotating-wave approximations, the density operator satisfies the Lehmberg-Agarwal master equation [2–4,79]

$$\begin{aligned}\frac{\partial\rho}{\partial t}&=-\frac{i}{\hbar}[H,\rho]-i\sum_{i\neq j=1}^{N}\Omega_{ij}[S_i^+S_j^-,\rho]\\&\quad-\frac{1}{2}\sum_{i,j=1}^{N}\gamma_{ij}(S_i^+S_j^-\rho+\rho S_i^+S_j^--2S_j^-\rho S_i^+), \quad (6)\end{aligned}$$

where

$$H=\hbar\,\omega_0\sum_{i=1}^{N}S_i^+S_i^-+\frac{1}{2}\hbar\Omega(S_l^+e^{-i\omega_0t}+S_l^-e^{i\omega_0t}), \quad (7)$$

$$\begin{aligned}\gamma_{ij}&=\frac{3}{2}\gamma\left\{[1-(\hat{\mu}\cdot\hat{r}_{ij})^2]\frac{\sin\xi_{ij}}{\xi_{ij}}+[1-3(\hat{\mu}\cdot\hat{r}_{ij})^2]\right.\\&\quad\left.\times\left[\frac{\cos\xi_{ij}}{\xi_{ij}^2}-\frac{\sin\xi_{ij}}{\xi_{ij}^3}\right]\right\}, \quad (8)\end{aligned}$$

$$\begin{aligned}\Omega_{ij}&=\frac{3}{4}\gamma\left\{-[1-(\hat{\mu}\cdot\hat{r}_{ij})^2]\frac{\cos\xi_{ij}}{\xi_{ij}}+[1-3(\hat{\mu}\cdot\hat{r}_{ij})^2]\right.\\&\quad\left.\times\left[\frac{\sin\xi_{ij}}{\xi_{ij}^2}+\frac{\cos\xi_{ij}}{\xi_{ij}^3}\right]\right\}, \quad (9)\end{aligned}$$

in which $\gamma\equiv\gamma_{ii}$ is the spontaneous emission rate for each individual atom, and

$$\xi_{ij}=\frac{2\pi r_{ij}}{\lambda},\quad r_{ij}\equiv|\vec{r}_{ij}|=|\vec{r}_j-\vec{r}_i|. \quad (10)$$

The $\gamma_{ij}$ terms describe the collective damping which results from an incoherent exchange of photons between the atoms $i$ and $j$, and the $\Omega_{ij}$ terms describe the collective shift of the atomic energy levels. The shift results from a coherent exchange of photons, the dipole-dipole interaction between the atoms. The effect of $\Omega_{ij}$ on the atomic system is the shift of the energy of the collective states from the single-atom energy states. We have chosen the laser field with Rabi frequency $\Omega$ to drive only one, the $l$th atom of the chain, and to be exactly resonant with the atomic transition frequency. With this driving arrangement, an excitation can be transferred between the atoms through the dipole-dipole interaction and collective damping of the atoms.

We solve the master equation (6) numerically up to $N = 5$ atoms and calculate the steady-state values of the correlation functions. We investigate angular distributions of the correlation functions and analyze their dependence on the way an atom at a particular position in the chain is excited.

## IV. ANGULAR DISTRIBUTIONS OF THE CORRELATIONS

Let us now demonstrate how one could determine the correlation properties between two atoms of a chain composed of $N \geqslant 2$ atoms by monitoring the angular distribution of the correlation functions of the emitted photons. We illustrate this in detail with examples of atomic chains composed of two and three atoms and then consider the case of $N > 3$. We determine the general conditions for the angular distribution of the correlation functions and its dependence on the number of atoms and detectors.

### A. Atomic chain composed of two atoms

Consider first the simplest case, a chain composed of only two atoms and calculate the first-order correlation function of the steady-state radiation field $G^{(1)}(\vec{R}) \equiv \lim_{t\to\infty} G^{(1)}(\vec{R}, t)$. The correlation function gives us the information about the probability of emitting single photons in the direction of detection $\vec{R}$. It also describes the intensity of the radiation field emitted in the observation direction $\vec{R}$. The general properties of the first-order correlation function can be determined from Eq. (4), which can be written as

$$\begin{aligned} G^{(1)}(\vec{R}) = u(\hat{R})\gamma\{&\langle S_1^+ S_1^-\rangle + \langle S_2^+ S_2^-\rangle \\ &+ 2\,\mathrm{Re}[\langle S_1^+ S_2^-\rangle]\cos(k\,r_{12}\cos\theta) \\ &+ 2\,\mathrm{Im}[\langle S_1^+ S_2^-\rangle]\sin(k\,r_{12}\cos\theta)\}, \end{aligned} \tag{11}$$

where $\theta$ is the angle between the interatomic axis and the direction of observation. The expression (11) can be written in a compact form as

$$G^{(1)}(\vec{R}) = u(\hat{R})\gamma(I_1 + I_2)[1 + \upsilon_{12}\cos(k\,r_{12}\cos\theta - \psi_{12})], \tag{12}$$

where $I_i = \langle S_i^+ S_i^-\rangle$ is the intensity of light emitted by atom $i$,

$$\upsilon_{12} = \frac{2|\langle S_1^+ S_2^-\rangle|}{\langle S_1^+ S_1^-\rangle + \langle S_2^+ S_2^-\rangle} \tag{13}$$

is the first-order coherence between the atoms 1 and 2, and $\psi_{12} = \arg(\langle S_1^+ S_2^-\rangle)$. The angle $\psi_{12}$ depends on the sign of the real and imaginary parts of $\langle S_1^+ S_2^-\rangle$ such that

$$\psi_{12} = \tan^{-1}\left(\frac{\mathrm{Im}\langle S_1^+ S_2^-\rangle}{\mathrm{Re}\langle S_1^+ S_2^-\rangle}\right) \tag{14}$$

when $\mathrm{Re}\langle S_1^+ S_2^-\rangle > 0,\ \mathrm{Im}\langle S_1^+ S_2^-\rangle > 0$;

$$\psi_{12} = \pi - \tan^{-1}\left(\frac{\mathrm{Im}\langle S_1^+ S_2^-\rangle}{\mathrm{Re}\langle S_1^+ S_2^-\rangle}\right) \tag{15}$$

when $\mathrm{Re}\langle S_1^+ S_2^-\rangle < 0,\ \mathrm{Im}\langle S_1^+ S_2^-\rangle > 0$;

$$\psi_{12} = -\tan^{-1}\left(\frac{\mathrm{Im}\langle S_1^+ S_2^-\rangle}{\mathrm{Re}\langle S_1^+ S_2^-\rangle}\right) \tag{16}$$

when $\mathrm{Re}\langle S_1^+ S_2^-\rangle > 0,\ \mathrm{Im}\langle S_1^+ S_2^-\rangle < 0$; and

$$\psi_{12} = -\pi + \tan^{-1}\left(\frac{\mathrm{Im}\langle S_1^+ S_2^-\rangle}{\mathrm{Re}\langle S_1^+ S_2^-\rangle}\right) \tag{17}$$

when $\mathrm{Re}\langle S_1^+ S_2^-\rangle < 0,\ \mathrm{Im}\langle S_1^+ S_2^-\rangle < 0$.

Viewing as a function of the detection angle $\theta$, we see that in general the angular distribution of $G^{(1)}(\vec{R})$ is not spherically symmetric. The correlation function exhibits an interference pattern dependent not only on the separation between the atoms, but also on the populations of the atoms and the coherences between them. In other words, the radiation intensity pattern depends on the way the atoms are excited. When the atoms are prepared in a state or driven such that the correlation function $\langle S_1^+ S_2^-\rangle$ is real, the modulation of the angular distribution depends solely on $r_{12}$. However, when the correlation function is complex, then not only the distance between the atoms but also the interatomic correlation function plays a crucial role in the angular distribution of the emitted photons. Thus, the depth of the modulation, determined by the cosine term, not only depends on the geometry of the system, determined by $r_{12}$, but also on the way the atoms are correlated, which is determined by $\upsilon_{12}$ and $\psi_{12}$.

Let us find the values of the detection angle $\theta$ at which $G^{(1)}(\vec{R})$ can be maximal or minimal, corresponding to directions of a strong focusing or divergence of the emitted photons, and how these observation angles depend on $r_{12}$ and $\psi_{12}$. Note that directions in which $G^{(1)}(\vec{R})$ is zero or close to zero correspond to elimination of single-photon emission in those directions. To find these observation angles we evaluate $\partial G^{(1)}/\partial\theta$ and get

$$\frac{\partial G^{(1)}}{\partial\theta} = u(\hat{R})\gamma(I_1 + I_2)\upsilon_{12}\,k\,r_{12}\sin(k\,r_{12}\cos\theta - \psi_{12})\sin\theta. \tag{18}$$

Next,

$$\frac{\partial G^{(1)}}{\partial\theta} = 0 \Leftrightarrow \sin\theta = 0 \quad \text{or} \quad \sin(k\,r_{12}\cos\theta - \psi_{12}) = 0. \tag{19}$$

Hence, the values of $\theta$ at which $G^{(1)}(\vec{R})$ is maximal or minimal are

$$\theta = n\pi \quad \text{or} \quad \theta = \arccos\left(\frac{n\pi + \psi_{12}}{k\,r_{12}}\right), \quad n \in \{0, \pm1, \pm2\}. \tag{20}$$

From this it follows that there are two separate criteria for $\theta$ at which maxima and minima of $G^{(1)}(\vec{R})$ could occur. The criterion $\theta = n\pi$ is independent of the distance between the atoms and the angle $\psi_{12}$. Therefore, there always can be either maximum or minimum of $G^{(1)}(\vec{R})$ along the atomic axis. The second criterion shows that there can be maxima and minima where the angular distribution depends on both the distance between the atoms and $\psi_{12}$. Clearly, if $\langle S_1^+ S_2^- \rangle$ has zero imaginary part, then the angular distribution of $G^{(1)}(\vec{R})$ depends solely on $r_{12}$. Thus, the interference is not purely geometrical; it also depends on the manner the atoms are correlated.

From Eq. (20) it is also seen that maxima and minima may appear in directions other than the direction of the atomic axis if

$$r_{12}/\lambda \geqslant \tfrac{1}{2}|(n + \psi_{12}/\pi)|. \tag{21}$$

Note that for the pure geometrical case of $\psi_{12} = 0$ and atomic separations $r_{12} < \lambda/2$, the condition for the optimum negative value (that is $-1$) of the term $\cos(k r_{12}\cos\theta)$ necessary to achieve optimal reduction of $G^{(1)}(\vec{R})$ cannot be achieved for all values of $\theta$. In physical terms, at distances $r_{12} < \lambda/2$ the atomic dipole moments oscillate in phase resulting in an enhanced emission of photons. However, the term $\cos(kr_{12}\cos\theta - \psi_{12})$ can reach the optimum negative value for $r_{12} < \lambda/2$, i.e., a nonzero $\psi_{12}$ can lift the limit. For example, in the case of $\psi_{12} = -3\pi/4$ corresponding to $\mathrm{Re}\langle S_1^+ S_2^- \rangle = \mathrm{Im}\langle S_1^+ S_2^- \rangle < 0$, and $r_{12} = \lambda/4$, there are two directions at which $\cos(kr_{12}\cos\theta - \psi_{12}) = -1$, namely, $\theta = \pi/6$ and $5\pi/3$. At these two directions $G^{(1)}(\vec{R})$ can be optimally reduced. It is interesting that a change of the sign of $\mathrm{Im}\langle S_1^+ S_2^- \rangle$ from negative to positive results in a rotation of those two directions by $\pi$. In physical terms, a nonzero $\psi_{12}$ can shift the phase difference between the atomic dipole moments such that the dipoles could oscillate in an opposite phase resulting in a reduction or even inhibition of the emission of photons. The inhibition of the single-photon emission may occur only when $\upsilon_{12} = 1$, i.e., when the oscillations of the atomic dipole moments are perfectly coherent.

Consider now the second-order correlation function $G^{(2)}(\vec{R}_1, t_1; \vec{R}_2, t_2)$. It is not difficult to see from Eq. (20) that in the case of two atoms the correlation function can exhibit cosine modulation only if measured by two distinguishable detectors located at two different geometric points. A simple calculation gives

$$G^{(2)}(\vec{R}_1, \vec{R}_2) = u(\hat{R}_1)u(\hat{R}_2)\gamma^2\langle S_1^+ S_2^+ S_1^- S_2^- \rangle \times \{1 + \cos[k\,\vec{r}_{12}\cdot(\hat{R}_1 - \hat{R}_2)]\}. \tag{22}$$

It is noted that the second-order correlation function manifests an interference pattern dependent on the separation between the two detection positions. The visibility of the interference pattern is independent of the way the atoms are excited.

However, in the case when the measurement is made with a single detector or two detectors recording in sync, $G^{(2)}(\vec{R}_1, \vec{R}_2) = G^{(2)}(\vec{R}, \vec{R})$ becomes independent of the direction of detection such that there is no interference pattern in the second-order sense. Thus, simultaneous emission of two photons is spherically symmetric. In other words, photons emitted simultaneously in the same direction do not interfere.

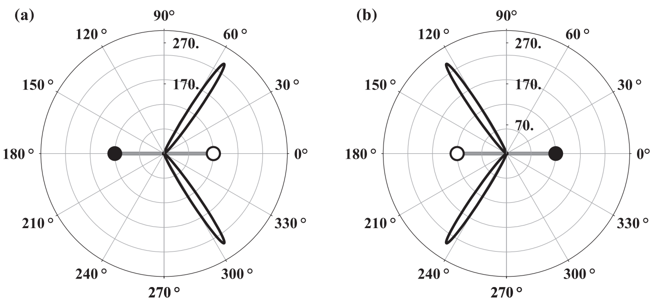

FIG. 2. Angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ for a chain composed of two atoms illustrated for $r_{12} = \lambda/2$ and $\Omega = 0.15\gamma$, the parameter values set according to the Rabi frequency $\Omega/2\pi = 4$ MHz and the damping rates of the atoms $\gamma/2\pi = 26$ MHz, used in the experiment of Loo *et al.* [80]. (a) Shows the angular distribution corresponding to the excitation of the left-end atom whereas (b) corresponds to the excitation of the right-end atom with the same Rabi frequency. The black dot indicates the laser-driven atom.

Comparing the properties of $G^{(2)}(\vec{R}, \vec{R})$ with those of $G^{(1)}(\vec{R})$ we see that two photons can be detected anywhere despite the fact that one photon can never be detected in certain directions. This fact that one photon can never be detected or can be detected but only with a very small probability in certain directions may result in the *superbunching* effect such that the normalized second-order correlation function given by

$$g^{(2)}(\vec{R}, \vec{R}) = \frac{G^{(2)}(\vec{R}, \vec{R})}{G^{(1)}(\vec{R})G^{(1)}(\vec{R})} \tag{23}$$

could have very large values in directions at which $G^{(1)}(\vec{R})$ is very small.

To illustrate this, we consider the angular distribution of $g^{(2)}(\vec{R}, \vec{R})$, which, with the result (12) takes the form

$$g^{(2)}(\vec{R}, \vec{R}) = \frac{\eta_{1212}}{[1 + \upsilon_{12}\cos(k\,r_{12}\cos\theta - \psi_{12})]^2}, \tag{24}$$

where

$$\eta_{1212} = \frac{4\langle S_1^+ S_2^+ S_1^- S_2^- \rangle}{(\langle S_1^+ S_1^- \rangle + \langle S_2^+ S_2^- \rangle)^2} \tag{25}$$

is the second-order coherence between the atoms 1 and 2. It is vivid that in the case of a large degree of the first-order coherence between the atoms ($\upsilon_{12} \approx 1$), the correlation function $g^{(2)}(\vec{R}, \vec{R})$ can be very large or even infinite for some directions $\theta$. This indicates that in these directions two photons are simultaneously emitted with the absence of single-photon emission. Although $g^{(2)}(\vec{R}, \vec{R})$ is mostly regarded as a measure of photon-photon correlations, it is evident from Eq. (24) that $g^{(2)}(\vec{R}, \vec{R})$ provides much more information about single-photon emissions, determined by $[G^{(1)}(\vec{R})]^2$, than about two-photon correlations, determined by $G^{(2)}(\vec{R}, \vec{R})$. We will return to this issue later in Sec. V.

We display first, in Fig. 2, the the angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ for two different excitation configurations at separation $r_{12} = \lambda/2$ and we set $\Omega = 0.15\gamma$ according to the Rabi frequency $\Omega/2\pi = 4$ MHz and the damping rates of the atoms $\gamma/2\pi = 26$ MHz, used in a relevant experiment [80]. In the experiment, strong photon-mediated interactions were created between artificial atoms located at fixed separations corresponding to $r_{12} \leqslant \lambda$. It is seen that the shape of the radiation pattern is very simple. Under excitation of the left-end atom, the pattern of $g^{(2)}(\vec{R}, \vec{R})$ shows two pronounced correlation peaks (superbunching) spatially concentrated in the right half of the pattern, the half in which the undriven atom is located. The directions of these peaks point precisely where $G^{(1)}(\vec{R})$ has optimal minima corresponding to an extremely small probability of emission of single photons. When the laser excitation is turned on the right-end atom, the correlation pattern flips over the axis vertical to the interatomic axis or equivalently rotates by $\pi$ radians. Therefore, it is the way the atoms are excited which accounts for the qualitative change of the pattern. In other words, there are preferred directions of suppressed single-photon emission imposed by the excitation field. The superbunching effect results from a nonzero phase shift $\psi_{12}$ and thus from the creation of minima of $G^{(1)}(\vec{R}, t)$ which is a proof that the single-photon emission is significantly suppressed. With the parameter values of Fig. 2(a), $r_{12} = \lambda/2$ and the laser field of the Rabi frequency $\Omega = 0.15\gamma$ turned on the left-side atom, $\mathrm{Re}[\langle S_1^+ S_2^- \rangle] \approx -0.000134$ and $\mathrm{Im}[\langle S_1^+ S_2^- \rangle] = -0.000297$, so that $\psi_{12} \approx -0.64\pi$. When the excitation is turned on the right-sided atom, the case illustrated in Fig. 2(b), $\mathrm{Im}[\langle S_1^+ S_2^- \rangle]$ reverses sign and thus $\psi_{12}$ turns out to be $0.64\pi$.

The magnitude of the superbunched peaks is very sensitive to the separation between the atoms and the Rabi frequency of the driving field. This is illustrated in Fig. 3 which shows the angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ for two different separations $r_{12} = \lambda/4$ and $\lambda/2$, and for a very weak driving field with $\Omega = 0.02\gamma$. Clearly, the magnitude and sharpness of the peaks depend on the distance between the atoms. For $r_{12} = \lambda/2$ the peaks are more narrowed, needle shaped, and have magnitudes much larger than ones for $r_{12} = \lambda/4$. Moreover, the magnitude of the superbunched peaks increases with a decreasing $\Omega$. Comparing the results for $\Omega = 0.02\gamma$, Figs. 3(c) and 3(d), with those for $\Omega = 0.15\gamma$, Fig. 2, one can see that at lower Rabi frequencies of the driving field the magnitude of the superbunched peaks is few orders larger. A further decrease of the Rabi frequency leads to an increase of the magnitude of the superbunched peaks, and their magnitudes may attend infinity in the limit of $\Omega \to 0$. This result is as expected from the fact that in the limit of $\Omega \to 0$, the first-order correlation function $G^{(1)}(\vec{R}) \to 0$.

One might argue that a larger number of correlation peaks could be witnessed in the angular distribution of two-photon correlated emission probability pattern when the two atoms are well separated, that is, when $r_{12} > \lambda/2$. However, for $r_{12} > \lambda/2$, the degree of the first-order coherence $\upsilon_{12}$ is considerably reduced so that there is no significant reduction of $G^{(1)}(\vec{R})$ present and no subsequent superbunching is possible. Figure 4 illustrates the variation of $\upsilon_{12}$ with $r_{12}/\lambda$. It is apparent that $\upsilon_{12} \approx 1$ for atomic separations $1/4 < r_{12}/\lambda < 1/2$. Thus, in the case of two atoms forming the chain and the laser field driving only one of the atoms in the chain, an almost perfect coherence between the atomic dipole moments is possible to be achieved for interatomic separations $r_{12} \leqslant \lambda/2$.

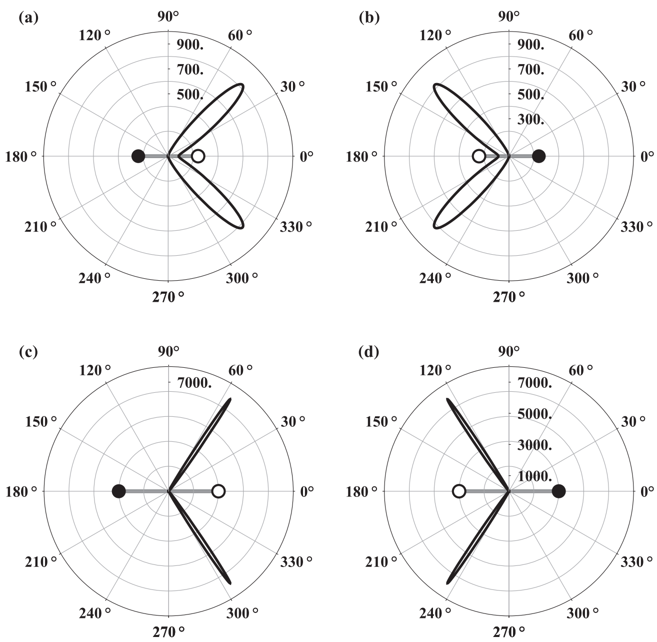


FIG. 3. Angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ for a chain composed of two atoms illustrated for two different separations between the atoms and two different excitation configurations. In (a) and (b), $r_{12} = \lambda/4$, and in (c) and (d), $r_{12} = \lambda/2$. (a), (c) Show angular distributions corresponding to the excitation of the left-end atom with a laser field of the Rabi frequency $\Omega = 0.02\gamma$. (b), (d) Correspond to the excitation of the right-end atom with the same Rabi frequency. The black dot indicates the laser-driven atom.

### B. Atomic chain composed of three atoms

When the chain is composed of three atoms, both the first- and second-order correlation functions depend on the direction of detection such that there is an interference pattern not only in the first-order but also in the second-order sense.

In the case of three atoms, the angular distribution of the first-order correlation function is of the form

$$\begin{aligned}\frac{G^{(1)}(\vec{R})}{u(\hat{R})} &= (I_1 + I_2)\left[\frac{1}{2} + \upsilon_{12}\cos(k\, r_{12}\cos\theta - \psi_{12})\right] \\ &\quad + (I_2 + I_3)\left[\frac{1}{2} + \upsilon_{23}\cos(k\, r_{23}\cos\theta - \psi_{23})\right] \\ &\quad + (I_3 + I_1)\left[\frac{1}{2} + \upsilon_{31}\cos(k\, r_{31}\cos\theta - \psi_{31})\right], \quad (26)\end{aligned}$$

where $I_i = \langle S_i^+ S_i^- \rangle$, $\upsilon_{ij}$ is the first-order coherence between atoms $i$ and $j$, and $\psi_{ij} = \arg(\langle S_i^+ S_j^- \rangle)$. The correlation function is composed of three terms resulting from the three possible pairs of atoms forming the three-atom chain.

The angular distribution of the second-order correlation function has the form

$$\begin{aligned}\frac{G^{(2)}(\vec{R}, \vec{R})}{4u^2(\hat{R})} &= G_{1212} + G_{2323} + G_{3131} \\ &\quad + 2|G_{1312}|\cos(k\, r_{12}\cos\theta - \phi_{12}) \\ &\quad + 2|G_{2313}|\cos(k\, r_{23}\cos\theta - \phi_{23}) \\ &\quad + 2|G_{3221}|\cos(k\, r_{31}\cos\theta - \phi_{31}), \quad (27)\end{aligned}$$

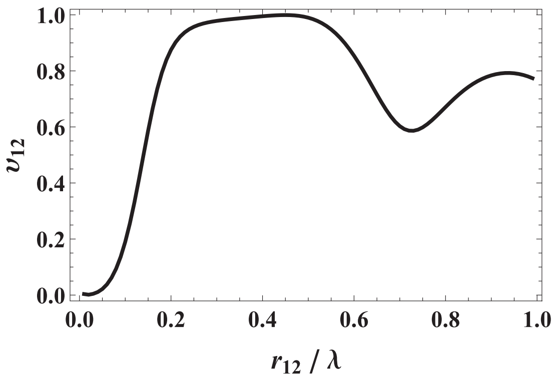


FIG. 4. Variation of the degree of the first-order coherence $\upsilon_{12}$ with the scaled interatomic separation $r_{12}/\lambda$ for the case of the left-side atom driven by a laser field of Rabi frequency $\Omega = 0.02\gamma$.

where $G_{ijkl} = |\langle S_i^+ S_j^+ S_k^- S_l^-\rangle|$ and $\phi_{il} = \arg(G_{ijkl})$. In writing the above expression we have used the fact that $G_{1213} = G_{1312}^*$, $G_{2312} = G_{1223}^*$, and $G_{1323} = G_{2313}^*$.

If we introduce the abbreviations

$$G_1 \equiv G_{1212}, \quad G_2 \equiv G_{2323}, \quad G_3 \equiv G_{3131},$$
$$\sigma_{12} = \frac{2|G_{1312}|}{G_1 + G_2}, \quad \sigma_{23} = \frac{2|G_{2313}|}{G_2 + G_3}, \quad \sigma_{31} = \frac{2|G_{3221}|}{G_3 + G_1}, \tag{28}$$

then we can write the second-order correlation function divided by the prefactor (27) as

$$\begin{aligned}\frac{G^{(2)}(\vec{R},\vec{R})}{4u^2(\hat{R})} &= (G_1+G_2)\left[\frac{1}{2}+\sigma_{12}\cos(k\,r_{12}\cos\theta-\phi_{12})\right]\\ &\quad+(G_2+G_3)\left[\frac{1}{2}+\sigma_{23}\cos(k\,r_{23}\cos\theta-\phi_{23})\right]\\ &\quad+(G_3+G_1)\left[\frac{1}{2}+\sigma_{31}\cos(k\,r_{31}\cos\theta-\phi_{31})\right].\end{aligned} \tag{29}$$

Note that $\sigma_{ij}$ do not represent the correlation coefficients in the same sense as the first-order coherence $\upsilon_{ij}$. They represent some kind of correlations but do not necessarily obey $\sigma_{ij} \leqslant 1$.

It is interesting that $G^{(1)}(\vec{R})$ [Eq. (26)] and $G^{(2)}(\vec{R},\vec{R})$ [Eq. (29)] are so similar in form. Thus, the angular distribution of $G^{(2)}(\vec{R},\vec{R})$ is expected to be similar in form to that of $G^{(1)}(\vec{R})$, except that the magnitudes and the directions of their maxima and minima might be different.

To illustrate the behaviors of the first- and second-order correlation functions, we show in Fig. 5 the angular distributions of $G^{(2)}(\vec{R},\vec{R})/[u(\hat{R})]^2$ and $G^{(1)}(\vec{R})/u(\hat{R})$ for several different values of the Rabi frequency of the laser field driving the left-end atom of the chain. We observe that indeed the angular distributions of the correlation functions are similar in form. Both the first- and second-order correlation functions are maximal in the direction $\theta = \pi$, the backward direction relative to the direction of the chain. However, in these directions, $G^{(1)}(\vec{R})$ is larger than $G^{(2)}(\vec{R},\vec{R})$ indicating antibunching of the emitted photons. Correlated pairs of photons with $G^{(2)}(\vec{R},\vec{R})$ comparable and even larger than $G^{(1)}(\vec{R})$ are

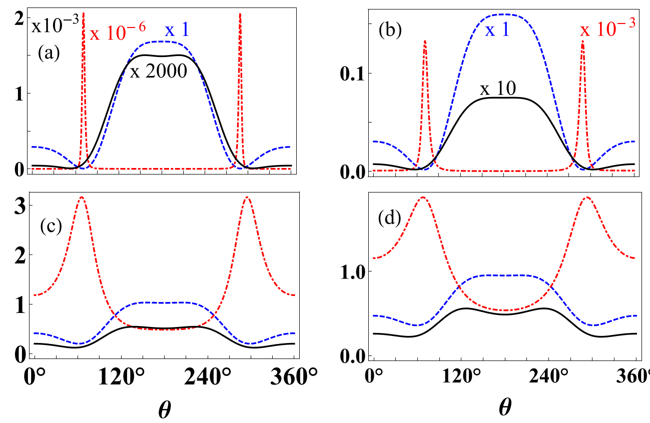


FIG. 5. Angular distribution of $G^{(2)}(\vec{R},\vec{R})/[u(\hat{R})]^2$ (solid black line) and $G^{(1)}(\vec{R})/u(\hat{R})$ (dashed blue line) for $N = 3$ with $r_{12} = r_{23} = \lambda/4$ plotted against detection angle $\theta$. Different Rabi frequencies of the laser field drive the left-end atom, (a) $\Omega = 0.02\gamma$, (b) $\Omega = 0.2\gamma$, (c) $\Omega = \gamma$, and (d) $\Omega = 10\gamma$. Also shown is the normalized second-order correlation function $g^{(2)}(\vec{R},\vec{R})$ (dashed-dotted red line). The curves in (a) and (b) have been scaled with constant factors.

emitted in directions located on that side of the pattern where the undriven atoms are located.

In Fig. 6 we show polar diagrams of the $g^{(2)}(\vec{R},\vec{R})$ function for a chain composed of three atoms with $r_{12} = r_{23} = \lambda/4$ and for different driving configurations. It is seen that the direction and the number of correlation (superbunching) peaks depend on the driving configuration. When one of the side atoms is driven [Figs. 6(a) and 6(c)], the pattern exhibits two pronounced peaks spatially concentrated in that half of the

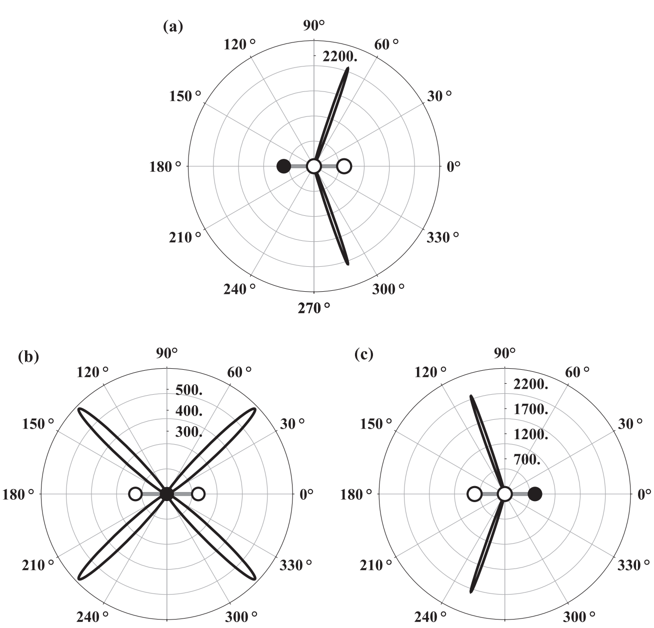


FIG. 6. Angular distribution of $g^{(2)}(\vec{R},\vec{R})$ for a chain composed of three atoms with $r_{12} = r_{23} = \lambda/4$ and for different excitation configurations, (a) left-end atom, (b) middle atom, and (c) right-sided atom driven by a laser field of the Rabi frequency $\Omega = 0.02\gamma$. The black dot indicates the laser-driven atom.

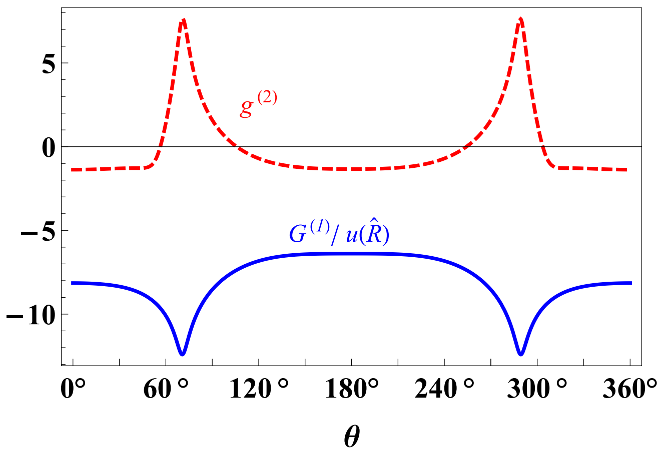


FIG. 7. Comparison of the angular distributions of the correlation function $G^{(1)}(\vec{R})/u(\hat{R})$ (solid blue line) with that of $g^{(2)}(\vec{R},\vec{R})$ (dashed red line) plotted using logarithmic scale against the detection angle $\theta$. The parameters are same as in Fig. 6(a).

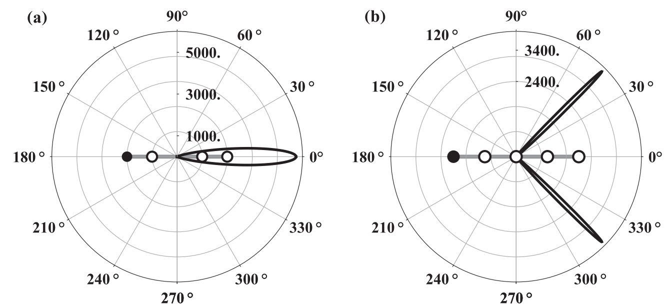


FIG. 8. Angular distribution of $g^{(2)}(\vec{R},\vec{R})$ for a chain composed of (a) $N = 4$, (b) $N = 5$ equidistant atoms with $r_{ij} = \lambda/4$. The left-end atom is driven by a laser field of the Rabi frequency $\Omega = 0.02\gamma$. The black dot indicates the laser-driven atom.

pattern where the undriven atoms are located. Comparing the results for the chain composed of three atoms with those for two atoms (Fig. 3), we see that the angular distribution is qualitatively the same except the magnitude of the correlation peaks is larger and the peaks are significantly narrower. Thus, a longer chain not only generates a stronger superbunched light, but also leads to a better localization of the strongly correlated pairs of photons. When the middle atom is driven, the pattern is composed of four pronounced peaks. It is easy to understand why four instead of two peaks appear in the pattern. When the middle atom is driven, the total system is equivalent to the case of two atomic subchains each composed of two atoms. In the subchain composed of atoms 1 and 2, the right-sided atom is driven, whereas in the subchain composed of atoms 2 and 3 the left-end atom is driven. These constitute two sources of correlated pairs of photons, each radiating into two directions located in opposite half of the correlation pattern.

We may summarize the results for a chain composed of three atoms that, as in the case of two atoms, the superbunched peaks also occur in directions in which $G^{(1)}(\vec{R})$ reaches its minimum value. This conclusion is supported by plots of the angular distributions of $g^{(2)}(\vec{R},\vec{R})$ and $G^{(1)}(\vec{R})$ shown in Fig. 7. It is evident that superbunched values of $g^{(2)}(\vec{R},\vec{R})$ occur in the directions in which $G^{(1)}(\vec{R})$, the probability of the emission of single photons, is minimal.

### C. Atomic chains composed of $N > 3$ atoms

We now turn to extend the analysis to the case where the atomic chain is composed of more than three atoms. Figure 8 shows the angular distribution of $g^{(2)}(\vec{R},\vec{R})$ for chains composed of $N = 4$ and 5 atoms separated at distance $r_{ij} = \lambda/4$. One can immediately notice that the angular distribution of the correlation function for $N = 4$ is significantly different than for $N = 3$ and 5. While the distributions for $N = 3$ and 5 atoms exhibit two superbunched peaks, only a single peak is exhibited for $N = 4$. The directions of the peaks are also different. It is seen that in the case of $N = 4$ atoms the preferred direction of the emission of strongly correlated pairs of photons is in the "forward" direction, along the interatomic axis. The directions of the correlated peaks for $N = 5$ are forming smaller angles with the interatomic axis than in the case of $N = 3$ atoms. This could be regarded as the formation of superradiance that the collective behavior of a large number of atoms turns the emission towards the interatomic axis.

Hence, we may conclude that the effect of increasing the number of atoms in the chain is to enhance the superbunching effects and to turn the corresponding directions of emission toward the interatomic axis. The collective interaction between a large number of atoms turns the emission in the direction along the interatomic axis. When the left- or right-sided atom of the chain is excited, the excitation is then transferred to other atoms through the collective interaction between them. The interaction establishes a constant phase relation between the atoms which yields to superradiance and directionality in the emitted radiation [25–27].

### D. Case of two atoms simultaneously driven

In the previous subsections, the analysis of the angular distribution of the two-photon correlations was carried out under the assumptions that only one of the atoms composing the chain is driven by an external laser field. In this case, a fixed phase difference between the emitters required for the interference to exist is solely established by the interaction between the atoms, in particular, by the dipole-dipole interaction. In this section, we extend this analysis to the case where at least two atoms are simultaneously driven by the laser field. In particular, we will consider a chain composed of three atoms simultaneously driven by a laser field. In this case, a constant phase difference between the atoms can be established not only by the interatomic interactions, but also by the driving field. We will also illustrate the case where apart from the field driving one of the end atoms there is an additional probe field interacting with all atoms and propagating along the interatomic axis.

To illustrate the directional properties of the emission of correlated pairs of photons in the case where at least two atoms of the chain are simultaneously driven by a laser field, we show in Fig. 9(a) the angular distribution of $g^{(2)}(\vec{R},\vec{R})$ obtained using the same parameters as in Fig. 6, except that instead of assuming that only one atom is driven, we have

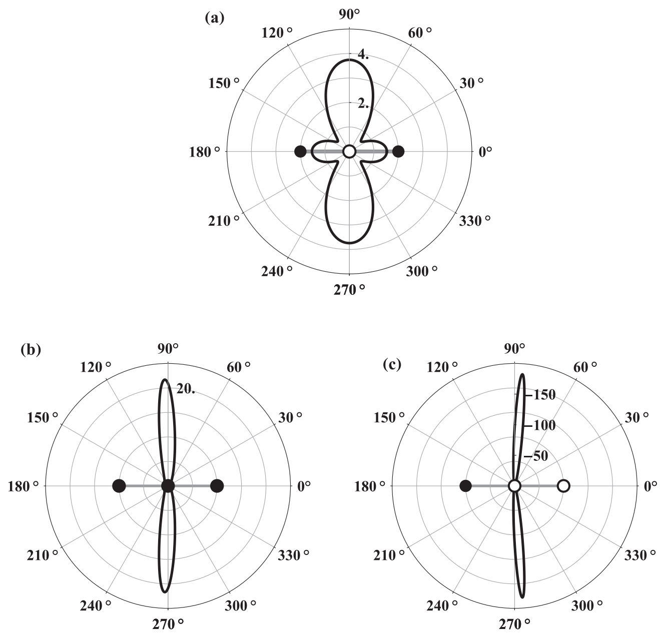


FIG. 9. Angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ for a chain composed of three atoms with $r_{12} = r_{23} = \lambda/4$ when (a) both end atoms are driven by a laser field of the Rabi frequency $\Omega = 0.02\gamma$, (b) all atoms are driven by a weak field of the Rabi frequency $\Omega_p = 0.02\gamma$ propagating along the interatomic axis, (c) in addition to the weak probe field interacting with all atoms the left-side atom is driven by another field of the same Rabi frequency. The black dots indicate the laser-driven atoms.

both end atoms driven. In Fig. 9(b) we show the angular distribution when all atoms interact with a weak field propagating along the interatomic axis, and in Fig. 9(c) we show the angular distribution when, in addition to a weak probe field interacting with all atoms, the left-end atom is driven by another field. Since the probe field propagates along the interatomic axis, the atoms "see" different phases of the field that the Rabi frequency at the left-end atom is $\Omega_p$, whereas at the middle atom is $\Omega_p \exp(ikr_{12})$, and at the right-end atom is $\Omega_p \exp(ikr_{13})$.

When both end atoms are simultaneously driven by the field, the angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ is seen to be qualitatively different from the case when only one atom is driven [see Figs. 6(a) and 6(c)]. The distribution exhibits four broad peaks pointing in directions perpendicular and along the interatomic axis.

This could be interpreted as resulting from interference between the directions in which the correlated pairs of photons are emitted when only one atom is driven. When only the left-end atom is driven, correlated pairs of photons are emitted in directions $\theta = 71^\circ$ and $289^\circ$. On the other hand, when the driving field is turned on the right-end atom, the correlated pairs of photons are emitted in directions $\theta = 109^\circ$ and $251^\circ$. Note that the directions are symmetrical about the interatomic axis and about the direction perpendicular to the interatomic axis. When both atoms are driven, the correlated pairs of photons are simultaneously emitted into these four directions. Since it is not known into which direction photons are emitted there is an interference. In other words, the probability amplitudes of the emission into these directions interfere with each other. resulting in emission peaks pointing in directions perpendicular and along the interatomic axis. Note that the angle between peaks pointing into the directions $\theta = 71^\circ$ and $109^\circ$ is smaller than between $\theta = 71^\circ$ and $289^\circ$. Similarly, the angle between $\theta = 251^\circ$ and $289^\circ$ is smaller than between $\theta = 109^\circ$ and $251^\circ$. Therefore, the magnitudes of the peaks in the $\theta = 90^\circ$ and $270^\circ$ directions are much greater than those in the directions $\theta = 0^\circ$ and $180^\circ$.

The significantly different angular distribution of the correlated pairs of photons that occurs in the case of both atoms driven may also be qualitatively understood in terms of quantum eraser [81]. When only one atom is driven, the emission of the correlated pairs of photons into different modes can be attributed to the fact that it is known which atom of the chain is driven. There is no interference between the directions because there is now known which atom generates photons. The situation differs when both end atoms are simultaneously driven. In this case, the information which atom generates photons is erased. The lack of information which atom generates photons leads to the interference between the different directions. As the result of the interference, a different angular distribution of the correlations would be found.

Figure 9(b) shows the angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ under the assumption that all atoms interact with a weak field propagating along the interatomic axis, and Fig. 9(c) shows the distribution when in addition to the weak probe field the left-end atom is driven by another field. If the atoms interact only with the propagating field, the angular distribution of $g^{(2)}(\vec{R}, \vec{R})$ exhibits two broad peaks in directions perpendicular to the interatomic axis. By adding an extra field driving the left-end atom the directions of these two peaks turn towards the interatomic axis, as it is seen from Fig. 9(c). Moreover, the magnitude of the peaks is significantly enhanced. It is easy to understand, when in addition to the propagating field the left-end atom is driven by another field, it is possible, in principle, to determine which atom produces photons. In other words, the information of which atom is producing photons is partly restored, resulting in a partial cancellation of the interference.

## V. REMARK ON THE MEANING OF SUPERBUNCHING

The term superbunching is used in general for $g^{(2)}(\vec{R}, \vec{R}) \gg 1$ and is interpreted as a signature of strong photon-photon correlations. The considerations of Sec. IV show that the question of whether superbunching means strong photon-photon correlations may be irrelevant to the problem of obtaining large values of $g^{(2)}(\vec{R}, \vec{R})$ at directions where single photons are not emitted. Let us illustrate this point more clearly.

To see whether the feature of superbunching is solely due to the strong two-photon correlations, we consider angular behaviors of $G^{(2)}(\vec{R}, \vec{R})/(u(\hat{R}))^2$ and $[G^{(1)}(\hat{R})/u(\hat{R})]^2$ separately. In Fig. 10 we show the angular distribution of the correlation functions $G^{(1)}(\vec{R})$, $G^{(2)}(\vec{R}, \vec{R})$, and $g^{(2)}(\vec{R}, \vec{R})$ for the case of three atoms separated by $r_{12} = r_{23} = \lambda/4$. Note that the curves have been renormalized in magnitude in order to visualize better their variations at $\theta = 71^\circ$. It is seen that at the angles where superbunching occurs, $G^{(1)}(\vec{R})$ reaches its

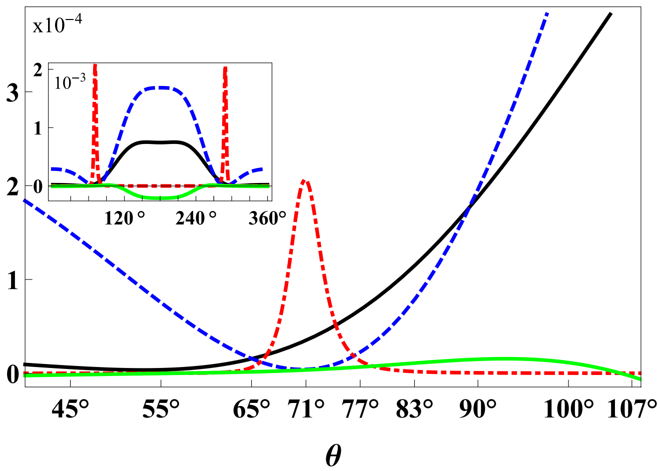


FIG. 10. Variation of the correlation functions $[G^{(2)}(\vec{R},\vec{R})/(u(\hat{R}))^2]\times 10^3$ (solid black line), $G^{(1)}(\hat{R})/u(\hat{R})$ (dashed blue line) and $g^{(2)}(\vec{R},\vec{R})\times 10^{-7}$ (dashed-dotted red line) in the vicinity of $\theta = 71^\circ$ at which $g^{(2)}(\vec{R},\vec{R})$ exhibits a pronounced peak. Also shown is a function $C^{(2)}(\vec{R},\vec{R})\times 10^2$ (solid green line). Inset shows the angular distribution of the correlation functions over the entire angle $\theta$. The plots are for a chain composed of three atoms separated by $r_{12} = r_{23} = \lambda/4$ and the left-end atom driven by a laser field of the Rabi frequency $\Omega = 0.02\gamma$.

minimum value and also $G^{(2)}(\vec{R},\vec{R})$ is significantly reduced. Extracting the values of the correlation functions we find that at the angles where the maximum superbunching occurs, i.e., at $\theta = 71^\circ$ and $289^\circ$, the second-order correlation function $G^{(2)}(\vec{R},\vec{R})$ is very small, $G^{(2)}(\vec{R},\vec{R})/(u(\hat{R}))^2 \sim 3.5\times 10^{-8}$. It turns out that at these angles $[G^{(1)}(\vec{R})]^2$ is much smaller than $G^{(2)}(\vec{R},\vec{R})$, $[G^{(1)}(\vec{R})/u(\hat{R})]^2 \sim 1.7\times 10^{-11}$. Correspondingly, the ratio $G^{(2)}(\vec{R},\vec{R})/[G^{(1)}(\vec{R})]^2$ is very large. This means that $g^{(2)}(\vec{R},\vec{R})$ varies much more rapidly with $[G^{(1)}(\vec{R})]^2$ rather than with $G^{(2)}(\vec{R},\vec{R})$. For this reason $g^{(2)}(\vec{R},\vec{R})$ could be regarded as a better measure of $[G^{(1)}(\vec{R})]^2$ rather than $G^{(2)}(\vec{R},\vec{R})$. In other words, $g^{(2)}(\vec{R},\vec{R})$ provides much more information about $[G^{(1)}(\vec{R})]^2$ than about $G^{(2)}(\vec{R},\vec{R})$.

One can also see from Fig. 10 that at the angles $\theta$ at which $g^{(2)}(\vec{R},\vec{R})$ is maximal, the difference between $G^{(2)}(\vec{R},\vec{R})$ and $G^{(1)}(\vec{R})$ is also maximal. Therefore, instead of $g^{(2)}(\vec{R},\vec{R})$, we may consider a correlation measure defined in [82]

$$C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2 = G^{(2)}(\vec{R},\vec{R}) - [G^{(1)}(\vec{R})]^2. \quad (30)$$

The correlation function $C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2$ is less sensitive to single-photon emissions than $g^{(2)}(\vec{R},\vec{R})$ and provides a clearer measure of $G^{(2)}(\vec{R},\vec{R})$, the probability of the simultaneous emission of two photons. Positive values of $C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2$ indicate that simultaneous emission of two photons dominates over the single-photon emissions, $C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2 = 0$ corresponds to a coherent emission and $C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2 < 0$ indicates emission of single photons, with the minimum negative value $C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2 = -G^{(1)}(\vec{R})G^{(1)}(\vec{R})$ corresponding to the emission of a single photon. It is evident from Fig. 10 that $C^{(2)}(\vec{R},\vec{R})(u(\hat{R}))^2$ is less sensitive to $[G^{(1)}(\vec{R})]^2$ than $g^{(2)}(\vec{R},\vec{R})$ and provides information about values of $G^{(2)}(\vec{R},\vec{R})$ even if $G^{(1)}(\vec{R}) = 0$.

Following the above analysis, we thus conclude that the feature of superbunching as determined by $g^{(2)}(\vec{R},\vec{R}) \gg 1$ is not necessary resulting from very strong two-photon correlations, but rather is a consequence of a significant reduction of one photon emission. In other words, $g^{(2)}(\vec{R},\vec{R}) \gg 1$ indicates that the probability of the emission of two single photons is much smaller than the probability of the simultaneous emission of two photons.

## VI. SUMMARY

In this paper, we have studied the correlation properties of fluorescence photons emitted from a linear chain of identical two-level atoms. It has been assumed that only one of the atoms composing the chain is selectively driven by a coherent laser field. The atoms interact with each other through the dipole-dipole interaction and the collective spontaneous emission resulting from the coupling of the atoms to a common vacuum field. In such a system, the interference pattern of the radiation field and the correlations between the atoms depend not only on the physical geometry of the system (distance between the atoms), but also on the arrangement of driven atom. We have found that the effect of selective driving of only a single atom results in a shift of the phase difference between neighboring atoms. The shift leads to a destructive interference of the emitted radiation that significantly reduces the probability of emission of single photons. The immediate effect of the reduced single-photon emission is to produce pronounced peaks in the angular distribution of the normalized second-order correlation function. The maximum value of these peaks can be made huge, values of the order of hundreds or thousands, which is termed as superbunching. When one of the end atoms is driven by the laser and the separation between atoms is kept less than or equal to half of the resonant atomic transition wavelength, the normalized second-order correlation function exhibits single or two superbunched peaks. When the driving field is turned on the other end atom, the directions of the superbunched peaks flip by $\pi$ radians. Turning the driving field on the middle atom of the chain results in two or four superbunched peaks in the angular distribution of the normalized second-order correlation function. The effect of increasing the number of atoms in the chain is to produce more prominent superbunched peaks which tend to turn toward the atomic line. The meaning of superbunching has also been discussed and we have argued that the normalized second-order correlation function, which is regarded as a measure of photon-photon correlations, provides much more information about single-photon emission than about simultaneous two-photon emission.

[1] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[2] R. H. Lehmberg, Phys. Rev. A **2**, 883 (1970).
[3] G. S. Agarwal, *Springer Tracts in Modern Physics: Quantum Optics* (Springer, Berlin, 1974).

[4] Z. Ficek and S. Swain, *Quantum Interference and Coherence: Theory and Experiments*, Vol. 100 (Springer, New York, 2005).
[5] Z. Ficek, R. Tanaś, and S. Kielich, Phys. A (Amsterdam) **146**, 452 (1987).
[6] S. Das, G. S. Agarwal, and M. O. Scully, Phys. Rev. Lett. **101**, 153601 (2008).
[7] J. Eschner *et al.*, Nature (London) **413**, 495 (2001); S. Rist, J. Eschner, M. Hennrich, and G. Morigi, Phys. Rev. A **78**, 013808 (2008).
[8] C. Hettich *et al.*, Science **298**, 385 (2002).
[9] J.-T. Chang, J. Evers, M. O. Scully, and M. S. Zubairy, Phys. Rev. A **73**, 031803(R) (2006); J.-T. Chang, J. Evers, and M. S. Zubairy, *ibid.* **74**, 043820 (2006); Q. Gulfam and J. Evers, J. Phys. B: At., Mol. Opt. Phys. **43**, 045501 (2010).
[10] J. P. Clemens, L. Horvath, B. C. Sanders, and H. J. Carmichael, Phys. Rev. A **68**, 023809 (2003).
[11] C. J. Mewton and Z. Ficek, J. Phys. B: At., Mol. Opt. Phys. **40**, S181 (2007).
[12] D. Porras and J. I. Cirac, Phys. Rev. A **78**, 053816 (2008).
[13] Z. Liao and M. S. Zubairy, Phys. Rev. A **90**, 053805 (2014).
[14] H. Blank, M. Blank, K. Blum, and A. Faridani, Phys. Lett. A **105**, 39 (1984).
[15] H. S. Freedhoff, J. Chem. Phys. **85**, 6110 (1986); J. Phys. B: At. Mol. Phys. **19**, 3035 (1986); Phys. Rev. A **69**, 013814 (2004).
[16] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[17] M. Saffman, T. G. Walker, and K. Molmer, Rev. Mod. Phys. **82**, 2313 (2010).
[18] D. Comparat and P. Pillet, J. Opt. Soc. Am. B **27**, A208 (2010).
[19] K. P. Nayak, P. N. Melentiev, M. Morinaga, F. Le Kien, V. I. Balykin, and K. Hakuta, Opt. Express **15**, 5431 (2007).
[20] E. Vetsch, D. Reitz, G. Sagué, R. Schmidt, S. T. Dawkins, and A. Rauschenbeutel, Phys. Rev. Lett. **104**, 203603 (2010).
[21] C. Weitenberg *et al.*, Phys. Rev. Lett. **106**, 215301 (2011).
[22] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
[23] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).
[24] N. E. Rehler and J. H. Eberly, Phys. Rev. A **3**, 1735 (1971).
[25] M. O. Scully, E. S. Fry, C. H. Raymond Ooi, and K. Wódkiewicz, Phys. Rev. Lett. **96**, 010501 (2006).
[26] A. A. Svidzinsky, J.-T. Chang, and M. O. Scully, Phys. Rev. Lett. **100**, 160504 (2008).
[27] M. O. Scully and A. A. Svidzinsky, Science **325**, 1510 (2009).
[28] Q. Gulfam and Z. Ficek, Phys. Rev. A **94**, 053831 (2016); AIP Conf. Proc. **1976**, 020003 (2018).
[29] H. J. Carmichael and D. F. Walls, J. Phys. B: At. Mol. Phys. **9**, 1199 (1976).
[30] H. J. Kimble and L. Mandel, Phys. Rev. A **13**, 2123 (1976).
[31] H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).
[32] A. Kuhn, M. Hennrich, and G. Rempe, Phys. Rev. Lett. **89**, 067901 (2002).
[33] J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, and H. J. Kimble, Science **303**, 1992 (2004).
[34] M. Keller, B. Lange, K. Hayasaka, W. Lange, and H. Walther, Nature (London) **431**, 1075 (2004).
[35] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Nature (London) **436**, 87 (2005).
[36] M. Hijlkema, B. Weber, H. P. Specht, S. C. Webster, A. Kuhn, and G. Rempe, Nat. Phys. **3**, 253 (2007).
[37] B. Dayan, A. S. Parkins, T. Aoki, E. P. Ostby, K. J. Vahala, and H. J. Kimble, Science **319**, 1062 (2008).
[38] *Quantum Squeezing*, edited by P. D. Drummond and Z. Ficek (Springer, New York, 2004).
[39] M. Stobińska and K. Wódkiewicz, Phys. Rev. A **71**, 032304 (2005).
[40] M. Wiegand, Opt. Commun. **36**, 297 (1981).
[41] T. Richter, J. Phys. B: At. Mol. Phys. **15**, 1293 (1982).
[42] T. Richter, Ann. Phys. (NY) **495**, 234 (1983).
[43] M. De Angelo, Y.-H. Kim, S. P. Kulik, and Y. Shih, Phys. Rev. Lett. **92**, 233601 (2004).
[44] H. J. Chang, H. Shin, M. N. O'Sullivan-Hale, and R. W. Boyd, J. Mod. Opt. **53**, 2271 (2006).
[45] Z. Ficek and R. Tanaś, *Quantum-Limit Spectroscopy* (Springer, New York, 2017), Chap. 9.
[46] M. I. Kolobov, *Quantum Imaging* (Springer, Berlin, 2007).
[47] J. Liu, J. Wang, H. Chen, H. Zheng, Y. Liu, Y. Zhou, F.-L. Li, and Z. Xu, Opt. Commun. **410**, 824 (2018).
[48] J. D. Pritchard, C. S. Adams, and K. Molmer, Phys. Rev. Lett. **108**, 043601 (2012).
[49] F. Bariani, Y. O. Dudin, T. A. B. Kennedy, and A. Kuzmich, Phys. Rev. Lett. **108**, 030501 (2012).
[50] H. J. Carmichael, P. Kochan, and B. C. Sanders, Phys. Rev. Lett. **77**, 631 (1996).
[51] I. Schuster, A. Kubanek, A. Fuhrmanek, T. Puppe, P. W. H. Pinkse, K. Murr, and G. Rempe, Nat. Phys. **4**, 382 (2008).
[52] L. Schneebeli, M. Kira, and S. W. Koch, Phys. Rev. Lett. **101**, 097401 (2008).
[53] C. Wagenknecht, Nat. Photonics **4**, 549 (2010).
[54] C. A. Husko, A. S. Clark, M. J. Collins, A. De Rossi, S. Combrié, G. Lehoucq, I. H. Rey, T. F. Krauss, Ch. Xiong, and B. J. Eggleton, Sci. Rep. **3**, 3087 (2013).
[55] D. Rielander, A. Lenhard, M. Mazzera, and H. de Riedmatten, New J. Phys. **18**, 123013 (2016).
[56] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London) **409**, 46 (2001).
[57] A. I. Lvovsky, B. C. Sanders, and W. Tittel, Nat. Photonics **3**, 706 (2009).
[58] R. Zhao, Y. O. Dudin, S. D. Jenkins, C. J. Campbell, D. N. Matsukevich, T. A. B. Kennedy, and A. Kuzmich, Nat. Phys. **5**, 100 (2009).
[59] H. de Riedmatten, I. Marcikic, J. A. W. van Houwelingen, W. Tittel, H. Zbinden, and N. Gisin, Phys. Rev. A **71**, 050302 (2005).
[60] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997).
[61] F. Boitier, A. Godard, N. Dubreuil, P. Delaye, C. Fabre, and E. Rosencher, Nat. Commun. **2**, 425 (2011); Phys. Rev. A **87**, 013844 (2013).
[62] T. Sh. Iskhakov, A. M. Pérez, K. Yu. Spasibko, M. V. Chekhova, and G. Leuchs, Opt. Lett. **37**, 1919 (2012).
[63] A. Allevi and M. Bondani, Opt. Lett. **40**, 3089 (2015).
[64] S. Swain, P. Zhou, and Z. Ficek, Phys. Rev. A **61**, 043410 (2000).
[65] R. Weigner, J. von Zanthier, and G. S. Agarwal, Phys. Rev. A **84**, 023805 (2011).
[66] S. Oppel, R. Wiegner, G. S. Agarwal, and J. von Zanthier, Phys. Rev. Lett. **113**, 263606 (2014).

[67] R. Wiegner, S. Oppel, D. Bhatti, J. von Zanthier, and G. S. Agarwal, Phys. Rev. A **92**, 033832 (2015).
[68] D. Bhatti, J. von Zanthier, and G. S. Agarwal, Sci. Rep. **5**, 17335 (2015).
[69] D. Bhatti, S. Oppel, R. Wiegner, G. S. Agarwal, and J. von Zanthier, Phys. Rev. A **94**, 013810 (2016).
[70] P. L. Hong, J. B. Liu, and G. Q. Zhang, Phys. Rev. A **86**, 013807 (2012).
[71] B. Bai, J. Liu, Y. Zhou, H. Zheng, H. Chen, S. Zhang, Y. He, F. Li, and Z. Xu, J. Opt. Soc. Am. B **34**, 2081 (2017).
[72] Y. Zhou, F. L. Li, B. Bai, H. Chen, J. B. Liu, Z. Xu, and H. B. Zheng, Phys. Rev. A **95**, 053809 (2017).
[73] D. E. Chang, L. Jiang, A. V. Gorshkov, and H. J. Kimble, New J. Phys. **14**, 063003 (2012).
[74] D. E. Chang, J. I. Cirac, and H. J. Kimble, Phys. Rev. Lett. **110**, 113606 (2013).
[75] P. Kochan, H. J. Carmichael, P. R. Morrow, and M. G. Raizen, Phys. Rev. Lett. **75**, 45 (1995).
[76] E. V. Goldstein and P. Meystre, Phys. Rev. A **56**, 5135 (1997).
[77] B. Vermersch, T. Ramos, P. Hauke, and P. Zoller, Phys. Rev. A **93**, 063830 (2016).
[78] R. J. Glauber, Phys. Rev. **130**, 2529 (1963).
[79] M. Kiffner, M. Macovei, J. Evers, and C. H. Keitel, Vacuum-induced processes in multi-level atoms, in *Progress in Optics*, Vol. 55, edited by E. Wolf (Elsevier, Amsterdam, 2010).
[80] A. F. van Loo, A. Fedorov, K. Lalumiére, B. C. Sanders, A. Blais, and A. Wallraff, Science **342**, 1494 (2013).
[81] M. O. Scully and K. Drühl, Phys. Rev. A **25**, 2208 (1982).
[82] A. Kubanek, A. Ourjoumtsev, I. Schuster, M. Koch, P. W. H. Pinkse, K. Murr, and G. Rempe, Phys. Rev. Lett. **101**, 203602 (2008).