Standard quantum limit of sensitivity of an optical gyroscope

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We find that the measurement sensitivity of an optical integrating gyroscope is fundamentally limited due to ponderomotive action of the light leading to the standard quantum limit of the rotation angle detection. The uncorrelated quantum fluctuations of power of clockwise and counterclockwise electromagnetic waves result in optical power-dependent uncertainty of the angular gyroscope position. We also show that, on the other hand, a quantum backaction evading measurement of the angular momentum of a gyroscope becomes feasible if the proper measurement strategy is selected. The angle is perturbed in this case. This observation hints at the fundamental inequivalence of integrating and rate gyroscopes.

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I. INTRODUCTION

There are two types of optical gyroscopes with respect to measurement observables: rate and integrating ones. Rate gyroscopes, for instance, passive fiber optic gyroscopes, measure rotation speed Ω. Integrating gyroscopes, for instance, laser gyroscopes, allow a direct observation of the rotation phase ϕ. Both devices are based on the optical Sagnac effect and seem to be identical from a classical physics perspective. From a quantum physics perspective, these measurements are not equivalent because angular momentum and rotation phase operators do not commute. In this paper we study fundamental quantum limitations of rotation measurement accuracy using a passive resonant optical resonant gyroscope as a model. We also study quantum limitations of the torque measurement using the system.

Passive resonant optical gyroscopes utilize the rotation-dependent change of the distance traveled by light in the clockwise and counterclockwise directions with respect to the rotation axis to measure the rotation rate and rotation angle. The nonreciprocity of the rotating system results in the removal of the frequency degeneracy between clockwise and counterclockwise modes of the ring cavity. The frequencies of the modes can be measured in various ways, for instance, by observation of the phase shift of the clockwise and counterclockwise light interacting with them. The resonant nature of the device is important since the signal is proportional to the finesse of the cavity.

Usual quantum analysis of a gyroscope involves study of the optical quantum noise of the device [1,2]. The noise results in sensitivity limitation that can be lifted with an increase of the optical pump power. It was noticed that the sensitivity of a gyroscope can also be limited due to the ponderomotive effect [3]. It was envisioned that, in case the device has movable parts, such as mirrors, the optical power fluctuations at the mirrors introduce an additional noise term, called quantum backaction [4], that eventually limits the overall sensitivity of the gyroscope for any optical power, similarly to the limitation of sensitivity of any interferometric measurement of this kind. Miniaturization of a gyroscope made out of a nonlinear material also results in the backaction [5].

Our analysis complements the studies performed in the realm of modern optomechanics [6,7]. Optomechanical structures utilize light to manipulate mechanical systems. For instance, radiation pressure induced by light modifies the dynamics of harmonically bound micromirrors and introduces additional mechanical rigidity and attenuation. The attenuation can be replaced with amplification if the optical detuning between the pump light frequency and the optical mode frequency is properly selected in resonant optomechanical structures. The attenuation can be used to cool the mechanical degree of freedom. At the quantum limit, the optomechanical interaction results in limitation of the continuous position detection of the mechanical system leading to the standard quantum limit (SQL) of the coordinate measurement. We expand the observations of the optomechanics to the rotational degrees of freedom and apply the developed technique to find sensitivity limitations of the rotation measurements. In our case the angular momentum and rotation phase replace the momentum and coordinate of a standard optomechanical system. We also expect that the study reported here will lead to a wealth of efforts directed towards application of the optomechanical techniques to rotating mechanical systems.

In this paper we study the fundamental limitations of the sensitivity of a macroscopic gyroscope occurring due to the uncertainty principle for the angular momentum and phase of the gyroscope. In Sec. II we show that one can obtain the SQL for the rotation measurements using this uncertainty principle. In Sec. III we analyze the dynamics of a quantum gyroscope using the Hamiltonian approach, derive the SQL within the framework of this approach, and discuss the possibility of a backaction-evading measurement and the SQL surpassing a variational readout. In Sec. IV we discuss an analogy between a quantum gyroscope and a quantum translational speed meter. Section V summarizes the paper.

II. STANDARD QUANTUM LIMIT OF ROTATION ANGLE

Let us consider rotation of a rigid thin circular thread around an axis Z. The motion can be described by angular momentum and rotation phase operators in analogy with the momentum and coordinate of a pointlike particle. The measurement of the rotation speed corresponds to the measurement of angular momentum $p_\phi = mr^2\Omega$, where m is the mass and r is the radius of the circle. The quantum operators corresponding to angular momentum and the phase, $\hat{p}_\phi = -i\hbar\partial/\partial\hat{\phi}$ and $\hat{\phi}$, respectively, do not commute,

$$[\hat{p}_\phi, \hat{\phi}] = -i\hbar, \tag{2.1}$$

which means that both observables cannot be measured accurately at the same time and hence the rate and integrating gyroscopes are not equivalent. Heisenberg's uncertainty principle leads to the requirement $\Delta_p\Delta_\phi \geqslant \hbar/2$, which also can be rewritten in the form $\Delta_\Omega\Delta_\phi \geqslant \hbar/2mr^2$. Here Δ_p, Δ_Ω, and $\Delta\phi$ are uncertainties of the momentum, frequency, and phase, respectively. The expressions can be generalized for a three-dimensional object, however we omit this exercise herein for the sake of simplicity.

It is this inequality that results in the appearance of the effect of quantum backaction in the rotation measurements. We have two instantaneous measurements of the angle ϕ separated by time t_m in order to control variation of the angle $\Delta\phi$. Then backaction from the first measurement leads to the appearance of the SQL written as

$$\Delta\phi_{\rm SQL} = \sqrt{\frac{\hbar t_m}{mr^2}}, \tag{2.2}$$

$$\Delta\Omega_{\rm SQL} = \sqrt{\frac{\hbar}{mr^2 t_m}}. \tag{2.3}$$

The SQL for the rotation angle is completely analogous to the corresponding SQL of the coordinate and momentum of a free mass [4,8–11].

The SQL value is small in practical devices, but can become measurable for a micro- or nanodevice. For instance, $\Delta\phi_{\rm SQL} = 1.8 \times 10^{-5}$ deg for $m = 10^{-5}$ g, $r = 10^{-2}$ cm, and $t_m = 10^5$ s. The bias drift of a good gyroscope is two orders of magnitude larger for the same time of observation. On the other hand, assuming that the trend of technology development of smart nanostructures continues, we can see that there is a certain limit of miniaturization when the fundamental quantum limitations become important.

Equations (2.2) and (2.3) also show that the measurements of the rotation phase and frequency are fundamentally inequivalent. It also means that a true integrating gyroscope and a rate gyroscope are also fundamentally different. The difference can be visible only on a quantum level, which is too low for the vast majority of practical applications.

III. STANDARD QUANTUM LIMIT OF ROTATION ANGLE FOR AN OPTICAL GYROSCOPE

A. Model of the gyroscope

Let us explain the origin of the SQL of the rotation angle using an example of a resonant optical gyroscope. An resonant optical gyroscope measures either angular speed or rotation angle, depending on the configuration. The measurements are based on the Sagnac effect, which results in the frequency shift of a ring cavity mode as a function of the rotation frequency. In the case of clockwise rotation of the cavity, the clockwise and counterclockwise frequencies of the cavity modes (ω_+ and ω_-, respectively) shift as

$$\Delta\omega_\pm = \mp\frac{r\Omega}{cn_0}, \tag{3.1}$$

where c is the speed of light in the vacuum and n_0 is the refractive index of the material. A gyroscope can detect the Sagnac-effect-mediated phase shift of the light passing through the rotating cavity, the frequency shift of light generated in the resonator filled with a lasing medium, or a fringe shift resulting from the interference of clockwise and counterclockwise light emitted by the gyroscope cavity. In the first two cases the gyroscope measures rotation speed. In the third case it measures the rotation angle.

To derive Eq. (3.1) we use standard formalism describing modes of a rotating optical cavity [12,13]. At this point we neglect the mechanical degree of freedom and consider only the optical part of the system. We assume that the phase velocity of light in the motionless cavity is $V_{\rm ph} = c/n_0$. For the rotating cavity the optical path increases in the clockwise direction (denoted by the subscript $+$) and decreases in the counterclockwise direction (denoted by the subscript $-$) in accordance with kinematic formulas

$$L_\pm = 2\pi r \pm r\Omega t_\pm, \tag{3.2}$$

$$V_\pm = \frac{V_{\rm ph} \pm r\Omega}{1 \pm V_{\rm ph} r\Omega/c^2}, \tag{3.3}$$

where $t_\pm$ is the cavity round-trip travel time and $V_\pm$ is the phase velocity of the clockwise and counterclockwise waves in the laboratory frame of reference. Using the definition of $t_\pm$, we find an expression for the optical path length $L_\pm$ for the clockwise and counterclockwise waves in the laboratory reference frame using a special relativity formalism

$$t_\pm = \frac{L_\pm}{V_\pm}, \tag{3.4}$$

which implies that

$$L_\pm = 2\pi r \pm r\Omega\frac{L_\pm(1 \pm V_{\rm ph} r\Omega/c^2)}{V_{\rm ph} \pm r\Omega}, \tag{3.5}$$

$$L_\pm = 2\pi r\frac{V_{\rm ph} \pm r\Omega}{V_{\rm ph}(1 - [r\Omega/c]^2)}. \tag{3.6}$$

We can see from this expression that the optical length increases in the clockwise direction and decreases in the counterclockwise direction when the resonator rotates in the clockwise direction.

Let us consider a motionless cavity made out of material with dielectric permittivity ϵ and magnetic permeability $\mu = 1$. The refractive index of the cavity material is related to the dielectric permittivity $n_0 = \sqrt{\epsilon}$. Our goal is to find equations describing generalized canonical amplitudes $q_\pm(t)$ for clockwise and counterclockwise modes of the cavity.

We assume that the radius of the cavity and the effective cross section of the modes are large enough: $r \gg S^{1/2} \gg \lambda$ (λ is the mean optical wavelength in the vacuum). In this case electric ($E_\pm$) and magnetic ($H_\pm$) field amplitudes of the modes can be expressed as (see [14])

$$E_\pm(t) = \mp\sqrt{\frac{2\pi}{nS_\pm L_\pm}} f_\pm e^{\pm ik_\pm r\phi}\partial_t q_\pm(t), \tag{3.7a}$$

$$H_\pm(t) = \sqrt{\frac{2\pi n}{S_\pm L_\pm}} f_\pm e^{\pm ik_\pm r\phi}\omega_\pm q_\pm(t), \tag{3.7b}$$

$$f_\pm = f_\pm(\vec{r}_\perp), \quad S_\pm = \int |f_\pm|^2 d\vec{r}_\perp, \tag{3.7c}$$

$$k_\pm = \frac{\omega_\pm}{V_{\rm ph}} = \frac{n\omega_\pm}{c}, \quad V_{\rm ph} = \frac{c}{n}, \tag{3.7d}$$

where ϕ is azimuthal angle and $r\phi$ is the coordinate along the rim of the ring cavity.

The translation condition $E_\pm(t) = E_\pm(t + t_\pm)$ defines normal frequencies of the ring cavity. For the case of the cavity at rest ($\Omega = 0$)

$$k_\pm L_\pm = 2\pi\ell, \quad L_\pm = 2\pi r, \tag{3.8}$$

which implies that

$$\omega_\pm = \ell\frac{V_{\rm ph}}{r} \equiv \omega_0, \tag{3.9}$$

where ℓ is an integer. Normal frequencies of the clockwise and counterclockwise modes are degenerate $\omega_+ = \omega_-$ and are equal to ω_0.

Kinetic ($T_\pm$) and potential ($U_\pm$) energies for the optical fields and associated Lagrangians $\mathcal{L}_\pm$ can be expressed in terms of the canonical amplitudes as

$$T_\pm = \int \frac{L_\pm\epsilon\langle|E_\pm|\rangle^2}{8\pi} dr_\perp = \frac{\partial_t q_\pm^2}{2}, \tag{3.10}$$

$$U_\pm = \int \frac{L_\pm\langle|H_\pm|\rangle^2}{8\pi} dr_\perp = \frac{\omega_\pm^2 q_\pm^2}{2}, \tag{3.11}$$

$$\mathcal{L}_\pm = T_\pm - U_\pm = \frac{\partial_t q_\pm^2}{2} - \frac{\omega_\pm^2 q_\pm^2}{2}. \tag{3.12}$$

We can find generalized momenta $p_\pm$ for the light confined in the modes and write down the corresponding Hamiltonians $H_\pm$ for the optical fields propagating in the clockwise and counterclockwise directions

$$p_\pm = \frac{\partial\mathcal{L}_\pm}{\partial\dot{q}_\pm} = \partial_t q_\pm, \tag{3.13}$$

$$H_\pm = p_\pm\partial_t q_\pm - \mathcal{L}_\pm = \frac{p_\pm^2}{2} + \frac{\omega_\pm^2 q_\pm^2}{2}. \tag{3.14}$$

For a gyroscope rotating with frequency $\dot{\phi} = \Omega$, the formulas (3.7) are valid. Using the translation conditions, we obtain

$$k_\pm L_\pm = 2\pi\ell \quad [\text{Eq. (3.6)}], \tag{3.15}$$

which implies that

$$\omega_\pm = 2\pi\ell\frac{V_\pm}{L_\pm}, \tag{3.16}$$

where ω_0 is defined in (3.8). Linearizing the expression with respect to $r\phi/V_{\rm ph}$, we derive

$$\omega_\pm \simeq \omega_0\left(1 \mp \frac{r\dot{\phi}}{n_0 c}\right), \tag{3.17}$$

which results in Eq. (3.1).

B. Hamilton formalism for the optomechanical system

We have characterized the field amplitudes of the optical modes of the ring cavity used in an optical gyroscope in a classical canonical way. To describe the measurement of the rotation using the gyroscope in the quantum picture we need to describe the interaction of light and the mechanical degree of freedom in a classical canonical way and then quantize it. This can be done in two ways. We can either use Eq. (3.1) and derive the complete Hamiltonian of the optomechanical system through the Lagrangian formalism or utilize the Lagrangian of the electromagnetic modes given by Eq. (3.12) directly, along with the Lagrangian of the mechanical system. The first method is similar to the standard optomechanical approach stating that the mechanical motion modifies the frequency of light and does not change the photon number. The second method is more general as it does not require photon-number conservation in the modes but rather shows that the number is conserved. Both methods result in the same final expression of the optomechanical Hamiltonian of a rotating system, and we present both derivations in what follows.

1. *Derivation of the Hamiltonian using Eq. (3.1)*

Let us consider a thin planar lossless fiber loop cavity of mass m that can rotate. The loop confines monochromatic light in one of its clockwise-counterclockwise mode pairs. The kinetic and potential energies of the loop are

$$T = \tfrac{1}{2}mr^2\dot{\phi}^2, \tag{3.18}$$

$$V = \mathcal{E}_+\left[1 - \frac{r\dot{\phi}}{cn_0}\right] + \mathcal{E}_-\left[1 + \frac{r\dot{\phi}}{cn_0}\right], \tag{3.19}$$

$$\mathcal{E}_\pm = \frac{p_\pm^2}{2} + \frac{\omega_0^2 q_\pm^2}{2}. \tag{3.20}$$

where we expressed optical energy stored in the clockwise and counterclockwise modes as $\mathcal{E}_\pm = T_\pm + U_\pm$ when the cavity is at rest. The canonical angular momentum of the system is

$$p_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = mr^2\dot{\phi} + \frac{r}{cn_0}(\mathcal{E}_+ - \mathcal{E}_-), \tag{3.21}$$

where $\mathcal{L} = T - V$ is the Lagrangian. The Hamiltonian of the optomechanical system is defined as

$$\begin{aligned} H &= p_\phi\dot{\phi} - \mathcal{L} \\ &= \frac{1}{2I}\left(p_\phi - \frac{r}{cn_0}(\mathcal{E}_+ - \mathcal{E}_-)\right)^2 + \mathcal{E}_+ + \mathcal{E}_-, \end{aligned} \tag{3.22}$$

where $I = mr^2$ is the moment of inertia.

2. *Derivation of the Hamiltonian using Eq. (3.12)*

Usage of Eq. (3.12) allows for a more generalized derivation of the Hamiltonian. We add a Lagrangian for a mechanical degree of freedom to the Lagrangian of light confined in the resonator modes to find a Lagrangian of the rotating optomechanical system

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_+ + \mathcal{L}_- + \mathcal{L}_{\rm int}, \tag{3.23}$$

$$\mathcal{L}_m = \frac{I\dot{\phi}^2}{2}, \quad \mathcal{L}_\pm = \frac{\partial_t q_\pm^2}{2} - \frac{\omega_0^2 q_\pm^2}{2}, \tag{3.24}$$

$$\mathcal{L}_{\rm int} = \omega_0^2 \frac{r\dot{\phi}}{n_0 c}(q_+^2 - q_-^2). \tag{3.25}$$

To derive the expression for $\mathcal{L}_{\rm int} \equiv (r\dot{\phi}/n_0c)(\mathcal{E}_+ - \mathcal{E}_-)$ we took into account that, for the cavity at rest, $p_\pm^2 = \omega_0^2 q_\pm^2$.

The canonical angular momentum p_ϕ and the canonical optical momenta p_+ and p_- of the system can be found from

$$p_\phi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = I\dot{\phi} + \omega_0^2 \frac{r}{n_0 c}(q_+^2 - q_-^2), \tag{3.26}$$

$$p_\pm \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}_\pm} = \dot{q}_\pm, \tag{3.27}$$

$$\dot{\phi} = \frac{1}{I}\left(p_\phi - \omega_0^2 \frac{r}{n_0 c}(q_+^2 - q_-^2)\right). \tag{3.28}$$

The Hamiltonian of the system is defined as

$$H = p_\phi \dot{\phi} + \dot{q}_+ p_+ + \dot{q}_- p_- - \mathcal{L} \tag{3.29a}$$

$$= H_+ + H_- + H_m, \tag{3.29b}$$

$$H_\pm = \frac{p_\pm^2}{2} + \frac{\omega_0^2 q_\pm^2}{2}, \tag{3.29c}$$

$$H_m = \frac{1}{2I}\left(p_\phi - \omega_0^2 \frac{r}{n_0 c}(q_+^2 - q_-^2)\right)^2. \tag{3.29d}$$

3. Equations of motion

Now using (3.22) or (3.29), one can write down equations of motion

$$\partial_t q_\pm = p_\pm, \tag{3.30a}$$

$$\partial_t p_\pm = -\omega_0^2 q_\pm \pm \pm \frac{1}{I}\left(p_\phi - \omega_0^2 \frac{r}{n_0 c}(q_+^2 - q_-^2)\right) 2\omega_0^2 \frac{R}{nc} q_\pm, \tag{3.30b}$$

$$\partial_t p_\phi = 0, \tag{3.30c}$$

$$\partial_t \phi = \frac{1}{I}\left(p_\phi - \omega_0^2 \frac{r}{n_0 c}(q_+^2 - q_-^2)\right). \tag{3.30d}$$

These equations can be reduced to

$$\partial_t^2 q_+ + \omega_0^2\left(1 - 2\frac{r}{n_0 c}\partial_t \phi\right) q_+ = 0, \tag{3.31a}$$

$$\partial_t^2 q_- + \omega_0^2\left(1 + 2\frac{r}{n_0 c}\partial_t \phi\right) q_- = 0, \tag{3.31b}$$

$$I\partial_t^2 \phi + \omega_0^2 \frac{r}{n_0 c}\partial_t (q_+^2 - q_-^2) = 0. \tag{3.31c}$$

C. Quantization

At this point we are ready to quantize the rotating optomechanical system. Introducing annihilation and creation operators for the optical modes ($\hat{a}_\pm$ and $\hat{a}_\pm^\dagger$, respectively), we obtain

$$\hat{q}_\pm = \sqrt{\frac{\hbar}{2\omega_0}}(\hat{a}_\pm + \hat{a}_\pm^\dagger), \quad \hat{p}_\pm = \sqrt{\frac{\hbar\omega_0}{2}}\left(\frac{\hat{a}_\pm + \hat{a}_\pm^\dagger}{i}\right),$$

$$\hat{p}_\phi = mr^2\dot{\phi} + \frac{\hbar\omega_0 r}{cn_0}(\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-); \tag{3.32}$$

$$\hat{H} = \hat{H}_m + \hat{H}_+ + \hat{H}_-, \quad \hat{H}_\pm = \hbar\omega_0\left(\hat{a}_\pm^\dagger \hat{a}_\pm + \tfrac{1}{2}\right),$$

$$\hat{H}_m = \frac{1}{2I}\left(\hat{p}_\phi - \hbar\omega_0 \frac{r}{n_0 c}(\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-)\right)^2. \tag{3.33}$$

Here we used the rotating-wave approximation and dropped fast oscillating terms $\sim \hat{a}_\pm^2$ and $(\hat{a}_\pm^\dagger)^2$. To complete the picture we need to take into account the commutator given by Eq. (2.1).

Equation (3.33) immediately shows that the canonical angular momentum $\hat{p}_\phi$ is not perturbed by the interaction since $[\hat{H}, \hat{p}_\phi] = 0$. Therefore, $\hat{p}_\phi$ is conserved in the measurement, and similarly for the photon numbers. On the other hand, the rotation phase as well as optical phase is perturbed due to quantum backaction. To find an expression for the quantum backaction we write the Hamiltonian equations

$$\dot{\hat{p}}_\phi = \partial_\phi H = 0, \quad \partial_t(\hat{a}_\pm^\dagger \hat{a}_\pm) = 0, \tag{3.34a}$$

$$\dot{\hat{\phi}} = \frac{\hat{p}_\phi}{I} - \frac{\hbar\omega_0 r}{Icn_0}(\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-), \tag{3.34b}$$

$$\dot{\hat{a}}_\pm = -i\omega_0\left(1 \mp \frac{r\dot{\hat{\phi}}}{cn_0}\right)\hat{a}_\pm, \tag{3.34c}$$

which can be solved exactly, taking into account that $\hat{a}_\pm^\dagger \hat{a}_\pm$ and $\hat{p}_\phi$ are conserved,

$$\hat{\phi} = \hat{\phi}_0 + \left(\frac{\hat{p}_\phi}{I} - \frac{\hbar\omega_0 r}{Icn_0}(\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-)\right)t, \tag{3.35}$$

$$\hat{a}_\pm = e^{-i\omega_0 t}\exp\left[\pm i\frac{r}{cn_0}\left(\frac{\hat{p}_\phi}{I} - \frac{\hbar\omega_0 r}{Icn_0} \times (\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-)\right)t\right]\hat{a}_\pm(0). \tag{3.36}$$

D. Origin of the SQL and possibility of backaction evading measurement in the rotating system

There are several possible problems that can be considered with respect to the system to explain the SQL introduced in Eqs. (2.2) and (2.3). In what follows we will consider two of them. One is related to the measurement of the initial angular velocity of the rotating gyroscope, while the other is related to the measurement of the velocity change occurring during the measurement procedure. In what follows we argue that in the first case the accuracy of the measurement can be infinite, in accordance with Eq. (2.3), while in the second case it is limited by the standard quantum limit of phase detection [Eq. (2.2)]. Interestingly, by selecting a proper measurement procedure one can remove the quantum backaction in the second case.

Let us consider an empty open cavity rotating with angular velocity Ω. The cavity is adiabatically interrogated with two, clockwise and counterclockwise, optical pulses. Since the operator $\hat{p}_\phi$ is conserved during the interaction, the angular momentum (and angular velocity) will be the same at the end of the measurement as the angular velocity at the beginning of the measurement. In other words, $\hat{p}_\phi$ is an integral of motion, hence it fulfills the condition of being a quantum nondemolition (QND) variable [9]. The accuracy of our measurement procedure is limited by the SQL (2.3) though.

The information about the initial angular momentum $\hat{p}_\phi(t=0)$ is contained in the phases $\varphi_\pm$ of the pulses exiting the cavity (3.34c):

$$\varphi_\pm = \mp\frac{r\Omega\omega_0\tau}{cn_0}. \tag{3.37}$$

Here τ is the time duration of each pulse. The phase shift $(\varphi_- - \varphi_+)/2$ can be measured with certain accuracy. For the simplest case when optical pulses are in the coherent state with phase uncertainties $\Delta\varphi_\pm \simeq 1/2\sqrt{n_\pm}$ and equal mean photon numbers $n_\pm = n$, we estimate the measurement error to be

$$\Delta\Omega_{\text{meas}} = \frac{cn_0}{2\omega_0 r\tau}\sqrt{\Delta\varphi_+^2 + \Delta\varphi_-^2} \simeq \frac{cn_0}{2\omega_0 r\tau\sqrt{2n}}. \tag{3.38}$$

On the other hand, during the measurement we get information on the *perturbed* angular velocity, as it follows from (3.34b),

$$\Delta\Omega_{\text{ba}} = \frac{\hbar\omega_0 r}{Icn_0}(\Delta n_+ + \Delta n_-) \simeq \frac{\hbar\omega_0 r}{Icn_0}2\sqrt{n}. \tag{3.39}$$

Here we again assume that optical pulses are in the coherent state with uncertainties of phonon numbers $\Delta n_\pm \simeq \sqrt{n}$.

The minimal error of the measurement can be derived from (3.38) and (3.39) by minimizing $\Delta\Omega_{\text{meas}} + \Delta\Omega_{\text{ba}}$ by selecting the optimal value of the photon number n,

$$\Delta\Omega_{\min} = \sqrt{\frac{2\hbar}{I\tau}}, \quad \Delta p_\phi = \sqrt{\frac{2\hbar}{\tau}}. \tag{3.40}$$

This differs from the SQL (2.3) by a numeric multiplier only.

Obviously, even within the SQL boundaries we can improve the accuracy of the measurement by increasing the measurement time τ or by utilizing the consequence of repeating measuring pulses. The latter technique simply allows involving multiple optical pulses to increase the measurement accuracy. Let us assume that during the procedure the measurement accuracy of the angular momentum is Δ_p. The measurement can be repeated N times. Since each measurement does not disturb the initial angular momentum and the errors of the measurements are not correlated, the accuracy of the set of measurements becomes $\Delta_p/\sqrt{N}$. The overall accuracy of the set of measurements increases with an increase in N.

While the backaction in operator $\hat{p}_\phi$ is removed after the measurement, the perturbation of the phase cannot be removed. This perturbation occurs in accordance with (3.35) [compare with (3.39)] and leads to

$$\Delta\phi_{\text{ba}} \simeq \frac{\hbar\omega_0 r\tau}{Icn_0}\sqrt{\Delta n_+^2 + \Delta n_-^2} \simeq \frac{\hbar\omega_0 r\tau\sqrt{2n}}{Icn_0}. \tag{3.41}$$

It is possible to derive the standard uncertainty relationship for the measurements using (3.38) and (3.41),

$$I\Delta\Omega_{\text{meas}}\Delta\phi_{\text{ba}} \simeq \frac{\hbar}{2}. \tag{3.42}$$

If we use the phase of the output light waves to detect a change of the angular velocity that happens due to the action of an external torque, the measurement sensitivity will be limited by the SQL related to the phase, not the angular velocity. We consider this case in the next section.

Depending on the measurement procedure, the SQL can be lifted. The SQL appears in the QND measurement of the angular velocity because the backaction (3.39) is erased *after* the measurement took place and the initial $\hat{p}_\phi$ is not disturbed (it is a QND variable) at the end. However, *during* the measurement it restricts the accuracy. In order to realize a QND measurement along with a backaction-evading measurement we have to use not the semiclassical coherent state, but a specifically prepared quantum state, the preparation of which is not discussed herein. We also can surpass the SQL by applying the procedure of variational measurement [15–17] [see also the discussion and formula (3.61) in what follows].

E. Continuous measurement of torque

In the preceding section we briefly mentioned a continuous measurement of a change of angular velocity of the system performed by detection of the phase of the light interacting with the rotating cavity. Let us consider this problem in more detail and study the accuracy of detection of classical torque acting on the ring cavity in the optomechanical system and consider an open lossless optical configuration by introducing the coupling rate γ and the associated Langevin terms into the optic subsystem-related equations. We consider the continuous measurement of the torque acting on the cavity and assume that (i) the probe's clockwise light and counterclockwise light are resonantly tuned and (ii) the mechanical system stays without dissipation. The equations of motion describing the behavior of the *open* system become

$$\dot{\hat{\phi}} = \frac{\hat{p}_\phi}{I} - \frac{\hbar\omega_0 r}{Icn_0}(\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-) + \int\frac{T_s}{I}dt, \tag{3.43a}$$

$$\dot{\hat{a}}_\pm + (i\omega_0 + \gamma)\hat{a}_\pm = \pm i\frac{r\dot{\hat{\phi}}}{cn_0}\hat{a}_\pm + \sqrt{\frac{2\gamma}{\tau}}\hat{b}_\pm, \tag{3.43b}$$

$$\hat{d}_\pm = -\hat{b}_\pm + \sqrt{2\gamma\tau}\hat{a}_\pm. \tag{3.43c}$$

Here T_s is a time-dependent signal torque, $\hat{b}_\pm$ are the amplitudes of the clockwise and counterclockwise pump fields including fluctuation (Langevin) terms, $\tau = 2\pi r/c$ is the round-trip time for the ring cavity, and $\hat{d}_\pm$ are output amplitudes of the clockwise and counterclockwise waves to be analyzed.

We remove the fast oscillating terms and present the optical field operators as sums of classical and quantum terms

$$\hat{a}_\pm e^{i\omega_0 t} = A_\pm + a_\pm, \tag{3.44}$$

$$\hat{b}_\pm e^{i\omega_0 t} = B_\pm + b_\pm, \tag{3.45}$$

$$\hat{d}_\pm e^{i\omega_0 t} = D_\pm + d_\pm, \tag{3.46}$$

where $A_\pm$ are the field amplitudes inside the cavity and $B_\pm$ and $D_\pm$ are the input and output field mean amplitudes. For the sake of simplicity, we assume that the mean amplitudes are real numbers and write

$$A_\pm = \sqrt{\frac{2}{\gamma\tau}} B_\pm, \tag{3.47}$$

$$D_\pm = B_\pm - \sqrt{2\gamma\tau} A_\pm = -B_\pm. \tag{3.48}$$

We derive a set of equations in the linear approximation for fluctuation amplitudes

$$\dot{a}_+ + \gamma a_+ = i\omega_0 \frac{r\dot{\phi}}{n_0 c} A_+ + \sqrt{\frac{2\gamma}{\tau}} b_+, \tag{3.49a}$$

$$\dot{a}_- + \gamma a_- = -i\omega_0 \frac{r\dot{\phi}}{n_0 c} A_- + \sqrt{\frac{2\gamma}{\tau}} b_-, \tag{3.49b}$$

$$I\ddot{\phi} + \frac{\sqrt{2}\hbar\omega_0 r}{n_0 c} \partial_t (A_+ a_{a+} - A_- a_{a-}) = T_s, \tag{3.49c}$$

$$d_\pm = b_\pm - \sqrt{2\gamma\tau} a_\pm. \tag{3.49d}$$

The amplitude noise components a_{a+} and a_{a-} are defined in Eq. (3.53a).

The equations for the fluctuations can be solved in the frequency domain using the Fourier transform if we neglect the initial conditions of the optomechanical system:

$$a_\pm(t) = \int_{-\omega_0}^{\infty} \alpha_\pm(\omega) e^{-i\omega t} d\omega, \tag{3.50a}$$

$$b_\pm(t) = \int_{-\omega_0}^{\infty} \beta_\pm(\omega) e^{-i\omega t} d\omega, \tag{3.50b}$$

$$d_\pm(t) = \int_{-\omega_0}^{\infty} \delta_\pm(\omega) e^{-i\omega t} d\omega, \tag{3.50c}$$

$$\Omega(t) = \int_{-\infty}^{\infty} \Omega(\omega) e^{-i\omega t} d\omega. \tag{3.50d}$$

While omitting the initial conditions for the optical modes is substantiated for the optical amplitudes in the case of a continuous measurement, it is not straightforward for the case of the mechanical degree of freedom. We use the motion of the initial conditions which can be removed at the stage of the processing when the classical signal is taken during the measurements [18].

For the operators and their Fourier amplitudes the usual commutation relations are valid, for example,

$$[a_\pm(t), a_\pm^\dagger(t')] = \delta(t - t'), \tag{3.51}$$

$$[\alpha(\omega), \alpha^\dagger(\omega')] = 2\pi\delta(\omega - \omega'). \tag{3.52}$$

Similar relationships are valid for the other operators.

It is convenient to introduce amplitude and phase quadratures for the fields

$$a_{a\pm}(t) \equiv \frac{a_\pm(t) + a_\pm^\dagger(t)}{\sqrt{2}}, \tag{3.53a}$$

$$a_{\text{ph}\pm} \equiv \frac{a_\pm(t) - a_\pm^\dagger(t)}{i\sqrt{2}}, \tag{3.53b}$$

$$\alpha_{a\pm}(\omega) \equiv \frac{\alpha_\pm(\omega) + \alpha_\pm^\dagger(-\omega)}{\sqrt{2}}, \tag{3.53c}$$

$$\alpha_{\text{ph}\pm} \equiv \frac{\alpha_\pm(\omega) - \alpha_\pm^\dagger(-\omega)}{i\sqrt{2}} \tag{3.53d}$$

and rewrite Eqs. (3.49) for the quadratures in the frequency domain

$$\alpha_{a\pm} = \sqrt{\frac{2\gamma}{\tau}} \frac{\beta_{a\pm}}{\gamma - i\omega}, \tag{3.54a}$$

$$\alpha_{\text{ph}\pm} = \pm \frac{\sqrt{2}\omega_0 r\Omega}{n_0 c} \frac{A_\pm}{\gamma - i\omega} + \sqrt{\frac{2\gamma}{\tau}} \frac{\beta_{\text{ph}\pm}}{\gamma - i\omega}, \tag{3.54b}$$

$$-i\omega\left(I\Omega + \frac{\sqrt{2}\hbar\omega_0 r}{n_0 c}(A_+\alpha_{a+} - A_-\alpha_{a-})\right) = T_s. \tag{3.54c}$$

For the output waves we obtain

$$\delta_{a\pm} = -\beta\beta_{a\pm}, \quad \beta \equiv \frac{\gamma + i\omega}{\gamma - i\omega}, \tag{3.55a}$$

$$\delta_{\text{ph}\pm} = \mp \frac{2\sqrt{\gamma\tau}\omega_0 r\Omega}{n_0 c} \frac{A_\pm}{\gamma - i\omega} - \beta\beta_{\text{ph}\pm}, \tag{3.55b}$$

$$\Omega = \frac{iT_s}{\omega I} - \frac{\sqrt{2}\hbar\omega_0 r}{I n_0 c}(A_+\alpha_{a+} - A_-\alpha_{a-}). \tag{3.55c}$$

After substitution of (3.55c) into (3.55b) we obtain

$$\delta_{\text{ph}\pm} = \pm \frac{2\sqrt{2\gamma\tau}\hbar(\omega_0 r)^2}{(n_0 c)^2 I} \frac{A_\pm(A_+\alpha_{a+} - A_-\alpha_{a-})}{\gamma - i\omega} \mp \frac{2\sqrt{\gamma\tau}\omega_0 r A_\pm}{n_0 c(\gamma - i\omega)} \frac{iT_s}{\omega I} - \beta\beta_{\text{ph}\pm}. \tag{3.56}$$

These equations can be rewritten with respect to the input fields

$$\delta_{a\pm} = -\beta\beta_{a\pm}, \tag{3.57a}$$

$$\delta_{\text{ph}\pm} = \pm \frac{8\hbar(\omega_0 r)^2}{(n_0 c)^2 I\tau} \frac{B_\pm(B_+\beta_{a+} - B_-\beta_{a-})}{(\gamma - i\omega)^2} \mp \frac{2\sqrt{2}\omega_0 r B_\pm}{n_0 c(\gamma - i\omega)} \frac{iT_s}{\omega I} - \beta\beta_{\text{ph}\pm}. \tag{3.57b}$$

As expected, we see that the phase quadrature is contaminated with the optical power-dependent fluctuation term that comes along with the signal.

Let us consider the measurement procedure of T_s in which the sums and differences of outputs are detected:

$$\tilde{d}_+ = \frac{d_+ + d_-}{\sqrt{2}}, \quad \tilde{d}_- = \frac{d_+ - d_-}{\sqrt{2}}, \tag{3.58a}$$

$$\tilde{b}_+ = \frac{b_+ + b_-}{\sqrt{2}}, \quad \tilde{b}_- = \frac{b_+ - b_-}{\sqrt{2}}. \tag{3.58b}$$

For the sake of simplicity we also assume that the measurement is completely balanced $B_\pm = B$. In the frequency domain we obtain

$$\tilde{\delta}_{a\pm} = -\beta\tilde{\beta}_{a\pm}, \quad \tilde{\delta}_{\mathrm{ph}+} = -\beta\tilde{\beta}_{\mathrm{ph}+}, \tag{3.59a}$$

$$\begin{aligned}\tilde{\delta}_{\mathrm{ph}-} &= \beta\left(\frac{16\hbar(\omega_0 r)^2B^2}{I(n_0c)^2\tau(\gamma^2+\omega^2)}\tilde{\beta}_{a-} - \tilde{\beta}_{\mathrm{ph}-}\right)\\ &\quad - \frac{4\omega_0 rB}{n_0c(\gamma - i\omega)}\frac{iT_s}{\omega I}\\ &= \beta(\mathcal{K}\tilde{\beta}_{a-} - \tilde{\beta}_{\mathrm{ph}-}) - i\sqrt{2\beta\mathcal{K}}\frac{T_s}{T_{\mathrm{SQL}}},\end{aligned} \tag{3.59b}$$

$$\mathcal{K} \equiv \frac{16\hbar(\omega_0 r)^2B^2}{I(n_0c)^2\tau(\gamma^2+\omega^2)}, \tag{3.59c}$$

$$T_{\mathrm{SQL}}(\omega) \equiv \sqrt{2\hbar I\omega^2}. \tag{3.59d}$$

Here the quadrature $\tilde{\beta}_{\mathrm{ph}-}$ is responsible for measurement error while the quadrature $\tilde{\beta}_{a-}$ is responsible for backaction in the measurements. In accordance with Eq. (3.59b), the torque can be measured if

$$\frac{T_s}{T_{\mathrm{SQL}}(\omega)} > \sqrt{\frac{\mathcal{K}}{2} + \frac{1}{2\mathcal{K}}}. \tag{3.60}$$

The minimum detectable torque is $T_s = T_{\mathrm{SQL}}$ when $\mathcal{K} = 1$ if one detects the quadrature $\tilde{\delta}_{\mathrm{ph}-}$. We see that this straightforward measurement of the torque has limited maximum sensitivity even though an optimal strategy for the angular velocity measurement has unlimited sensitivity.

In order to surpass the SQL we can apply the procedure of a variational measurement [15–17] of quadrature $\xi = \tilde{\delta}_{a-}\cos\theta + \tilde{\delta}_{\mathrm{ph}-}\sin\theta$ defined as

$$\begin{aligned}\xi &= \beta([\mathcal{K}\sin\theta + \cos\theta]\tilde{\beta}_{a-} - \sin\theta\tilde{\beta}_{\mathrm{ph}-})\\ &\quad - i\sin\theta\sqrt{2\beta\mathcal{K}}\frac{T_s}{T_{\mathrm{SQL}}},\end{aligned} \tag{3.61}$$

where the homodyne angle θ is selected so that $\cot\theta = -1/\mathcal{K}$. Such a selection allows for compensation of the backaction. Due to the frequency dependence of $\mathcal{K}$, this compensation is possible in a limited frequency band only. A similar method is used in a conventional quantum speed meter to overcome the SQL restriction [19,20].

IV. DISCUSSION

In this paper we analyze fundamental restrictions of the quantum sensitivity of a gyroscope and show that the gyroscope should be extremely small in size to allow these quantum effects to emerge. Microring optical cavities can have high finesse and can be monolithically integrated. This gives us grounds to expect that the quantum effects can be observed with this type of device. On the other hand, the demonstrated measurement sensitivity of the existing microgyroscopes [21–29] is not very high. It was shown that an on-chip gyroscope having resolution on the order of 10°/h is feasible if an InP-based ring cavity with a Q factor of approximately 6×10^5 and footprint of 10 mm^2 is utilized [30]. An active on-chip gyroscope having similar resolution has also been demonstrated [31]. A resonant gyroscope [32,33] utilizing ultrahigh-Q crystalline whispering gallery mode resonators and having 2°/h resolution was also demonstrated. The sensitivity should be improved significantly to allow the quantum effects described in this paper to become visible.

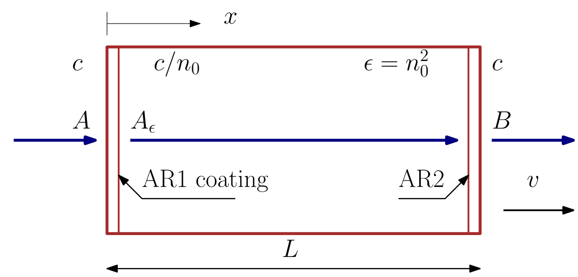

FIG. 1. Toy model of the speed meter. The velocity v of a dielectric (the refractive index is n_0) probe mass with antireflecting coatings (AR1 and AR2) is measured by light traveling through it via detection of the phase shift of the light proportional to velocity v.

The formalism developed in this paper also can be useful for measurements performed with straight moving bodies. The measurement of the angular velocity is similar to the measurement of velocity of a body. The analogy becomes obvious if one considers, for instance, a rotating gyroscope as a pointer mass that can move on a round string. We show here the analogy by introducing a toy semiclassical model of the speed meter shown in Fig. 1.

The light passes through a dielectric test mass without reflections (its surfaces are covered by antireflecting coatings) and its phase φ shift depends on the velocity v of the test mass (see the notation in Fig. 1)

$$\phi = \frac{\omega_0}{c}v\tau_0(n_0 - 1), \quad \tau_0 = \frac{n_0L}{c}. \tag{4.1}$$

Here ω_0 is the optical frequency and n_0 is the refractive index of the material. Let us assume that the light is in the coherent state. The error of the velocity measurement is

$$\delta\phi \simeq \frac{1}{2\sqrt{n}}, \quad \Delta v_{\mathrm{meas}} \simeq \frac{1}{2\sqrt{n}}\frac{c}{\omega_0\tau(n_0-1)}. \tag{4.2}$$

Here n is number of "used" (passed through) light quanta and τ is the duration of the measurement.

The momenta of the optical quanta outside and inside the test mass are equal to

$$p_{\mathrm{out}} = \frac{\hbar\omega_0}{c}, \quad p_{\mathrm{in}} = \frac{n_0\hbar\omega_0}{c}, \tag{4.3}$$

respectively. Hence, while the photon is traveling inside, the probe mass receives additional momentum $p_{\mathrm{out}} - p_{\mathrm{in}}$, which transforms into a position shift of the probe mass

$$x = \frac{p_{\mathrm{out}} - p_{\mathrm{in}}}{m}\tau. \tag{4.4}$$

The uncertainty of the quantum number $\sqrt{n}$ produces the backaction noise of the velocity of the probe mass m during the measurement

$$\Delta v_{\mathrm{ba}} \simeq \frac{\sqrt{n}\hbar\omega_0(n_0-1)}{cm}. \tag{4.5}$$

This perturbation is erased after the measurement because of the photon-number- and energy-conservation laws.

Combining (4.2) and (4.5), one obtains a minimal error of the measurement

$$\Delta v_{\rm min} = \sqrt{\frac{\hbar}{m\tau}}, \tag{4.6}$$

which coincides with the SQL for the velocity. We can surpass the SQL by applying the procedure of variational measurement [15–17].

The uncertainty of the quantum number $\sqrt{n}$ produces the uncertainty of position (backaction)

$$\Delta x_{\rm ba} \simeq \frac{\sqrt{N}\hbar\omega_0(n_0-1)}{cm}\tau, \tag{4.7}$$

leading to the uncertainty relation

$$m\Delta v_{\rm meas}\Delta x_{\rm ba} \simeq \frac{\hbar}{2}. \tag{4.8}$$

The formulas (4.2), (4.7), and (4.8) here directly relate to the formulas (3.38), (3.41), and (3.42), respectively, for the quantum gyroscope.

The backaction-evading QND measurement strategy introduced at the end of Sec. III D is also valid for the speed meter. Let us consider a light pulse passing through the test mass and then reflecting from a perfect mirror and passing the test mass in the opposite direction. During the second propagation the light-induced ponderomotive force is in the opposite direction and is completely erased. The accuracy of the test mass velocity is defined only by (4.2) and can be decreased just by an increase of the photon number n of the optical pulse.

V. CONCLUSION

We have shown that the sensitivity of a generalized gyroscope is restricted by the standard quantum limit in a way similar to the free mass coordinate measurement sensitivity limitations. Quantum theory indicates that the detection of the rotation rate and of the rotation phase are fundamentally different because the rotation phase and the canonical angular momentum operators do not commute. As the result, the observables cannot be measured simultaneously with high accuracy. Using an example of a resonant optical gyroscope, we have found the ultimate limit of the sensitivity of the device and discussed requirements for achievement of the sensitivity in an experiment. A backaction-evading measurement technique allowing surpassing the standard quantum limit was proposed. We also analyzed quantum restrictions of the sensitivity of measurements of the classical torque applied to the gyroscope.

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