Phase-modulated single-photon router

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The single-photon routing properties in the system composed of two waveguides and two coupled cavities interacting with a two-level system are investigated theoretically. The phases of the coupling strengths are considered. The analytical expressions for the single-photon routing probabilities are obtained. The study shows that single-photon routing in a single waveguide or between two waveguides can be switched on or off by modulating the phase difference between the coupling constants. The analytical results also exhibit that the coupling between the two nanocavities plays an important role in the phase-dependent single-photon routing. We also show numerically the influence of the dissipation of cavities and atoms on routing properties. Our results may be useful in controlling and manipulating light-matter interactions based on waveguides at the single-photon level.

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I. INTRODUCTION

Controllable photon scattering in a one-dimensional waveguide coupled to quantum emitters, namely, waveguide quantum electrodynamics (QED), has been widely investigated in recent years [1–42]. The platform of waveguide QED can be used in quantum information processing [43–50] and designing quantum devices at the single-photon level [51–57] since a waveguide can guide photons naturally and the strong coupling between the quantum emitters and waveguide has been realized. It can also be used to investigate many interesting quantum and nonlinear optical phenomena, such as superradiance [58–60], subradiance [61,62], two-photon blockade [63,64], anisotropic vacuum-induced interference [65], and enhancement of nonlinearity [66]. Many methods, such as real-space Hamiltonian [1], input-output formalism [23,67], path integral formalism-based scattering matrix [68], and dynamical time-dependent theory [16,27], have been developed to investigate waveguide quantum electrodynamics. There are also several proposals such as electromagnetical induced transparency [10,19] and designing different coupling schemes [54] to coherently control photon scattering properties. A review of waveguide QED can be found in Refs. [69–71]. However, in most of these proposals, only the amplitudes of coupling strengths are considered.

Recently, phase-dependent photon statistics in cavity QED have been reported. It has been shown that the phase difference between the coupling strengths plays an important role in the generation of single and multiple photons. For example, Pleinert *et al.* studied the phase controlled collective behavior of two atoms in a cavity [72]. They showed that the light field can be tuned from antibunched to (super-) bunched by modifying the phase difference between the coupling strengths. Wang *et al.* investigated the generation of nonclassical light fields from two coupled cavities interacting with a two-level system [73]. They found that photon antibunching can be affected by the phase differences of coupling constants. For the waveguide QED, the single-photon switch and nonreciprocal photon scattering properties have been reported by controlling the phase difference between two classical fields [74]. Many researchers have also investigated the influence of phase, which is induced by the photon propagation between two separated emitters, on the single-photon scattering spectra [11,33,43]. And the physical mechanisms of the phase controlled Fano-type transmission and reflection spectra have also been discussed [33,75].

Motivated by these advances, in this paper, we investigate how phases of the coupling constants affect single-photon routing properties. We consider the system composed of a pair of waveguides coupling to two nanocavities interacting with an atom. The single-photon scattering amplitudes are given analytically. We show that the single-photon scattering spectra depend strongly on the phases, and the routing properties can be switched on or off by modulating the phase difference between the coupling strengths.

The structure of this paper is organized as follows. In Sec. II, the model and the Hamiltonian of the system are introduced. In Sec. III, we consider phase-modulation single-photon routing in a single waveguide. In Sec. IV, the results for phase-dependent single-photon routing between two waveguides are presented. In Sec. V, we show the influence of dissipation on the routing properties. The possible experimental realization and conclusions are given in Sec. VI.

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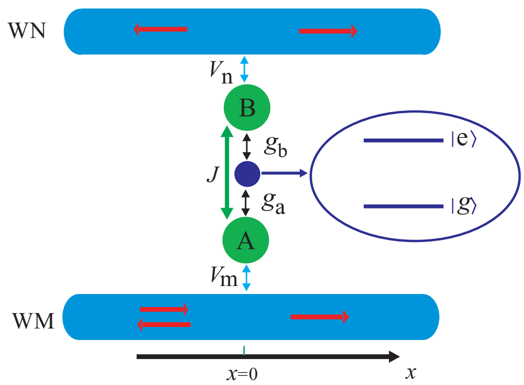

FIG. 1. Phase-controlled single-photon routing configuration considered in this paper. Waveguide M and waveguide N couple, respectively, to cavity A and cavity B. The two cavities are linked by a two-level system. There is a direct interaction between the two cavities.

II. MODEL AND HAMILTONIAN

The system under consideration is shown in Fig. 1. Waveguide M (WM) and waveguide N (WN) interact respectively with cavity A and cavity B. The interaction strengths are denoted by $V_m = |V_m|e^{i\varphi_m}$ and $V_n = |V_n|e^{i\varphi_n}$, respectively. An atom with two energy levels couples to the two cavities, and the coupling strengths are represented by $g_a = |g_a|e^{i\theta_a}$ and $g_b = |g_b|e^{i\theta_b}$, respectively. Here, to show the effects of phases of interaction strengths on the routing properties, we take the interaction strengths as complex numbers. We also suppose that there is direct interaction between the two cavities and the interaction strength is denoted by J.

The Hamiltonian in the real space is given by

$$H = H_w + H_c + H_a + H_{wc} + H_{ca} + H_{cc}, \tag{1}$$

where H_w, H_c, and H_a denote the freely propagating photonic, the free cavities, and the free atomic Hamiltonians, respectively. H_{wc} describes the interactions between the waveguides and cavity modes. H_{ca} refers to the interaction between the atomic and cavity modes. Finally, H_{cc} represents the interaction between the two cavity modes. The free photonic Hamiltonian H_w in the real space is given by [1]

$$H_w = \sum_{p=m,n} \int dx(-i v_{gp}) c_{Rp}^{\dagger} \frac{\partial}{\partial x} c_{Rp}(x) + \sum_{p=m,n} \int dx(-i v_{gp}) c_{Lp}^{\dagger} \frac{\partial}{\partial x} c_{Lp}(x). \tag{2}$$

Here $c_{Rp}^{\dagger}(x)$ [$c_{Lp}^{\dagger}(x)$] denotes creating a right (left) propagation photon at x in WM ($p = m$) or WN ($p = n$). v_{gp} is the group velocity of a photon in waveguide p. H_c for the two cavity modes reads

$$H_c = \left(\omega_a - i\frac{\gamma_a}{2}\right) a^{\dagger} a + \left(\omega_b - i\frac{\gamma_b}{2}\right) b^{\dagger} b, \tag{3}$$

where ω_a (ω_b) is the eigenfrequency of the cavity mode. $a^{\dagger}$ ($b^{\dagger}$) denotes the creating a photon in a (b) mode of the cavity A (B). γ_a (γ_b) is the dissipation rate of mode a (b). The Hamiltonian for the free atom is

$$H_a = \left(\omega_e - i\frac{\gamma_e}{2}\right)\sigma_{ee}, \tag{4}$$

where ω_e is the transition frequency between the excited state $|e\rangle$ and the ground state $|g\rangle$ of the atom. γ_e denotes the energy loss rate from the atom to the free space. σ_{ee} is the dipole operator. The interaction Hamiltonian H_{wc} between the waveguides and the two modes of the cavities reads

$$H_{wc} = V_m \sum_{d=R,L} \int dx\, \delta(x)\, c_{dm}^{\dagger} a + V_n \sum_{d=R,L} \int dx\, \delta(x)\, c_{dn}^{\dagger} b + \text{H.c.}, \tag{5}$$

where H.c. denotes Hermitian conjugate. H_{ca} takes the form

$$H_{ca} = g_a a^{\dagger} \sigma_{ge} + g_b b^{\dagger} \sigma_{ge} + \text{H.c.} \tag{6}$$

Finally, the interaction Hamiltonian H_{cc} between the two modes of the cavities reads

$$H_{cc} = J a^{\dagger} b + J b^{\dagger} a. \tag{7}$$

We concentrate on single-photon scattering in this system. At the initial time, the atom is supposed at ground state $|g\rangle$ and the cavity modes are not excited. Thus, the wave function of the system can be expressed as

$$|\Psi\rangle = \int dx\, \phi_{rm}(x) c_{Rm}^{\dagger}(x)|v\rangle + \int dx\, \phi_{rn}(x) c_{Rn}^{\dagger}(x)|v\rangle + \int dx\, \phi_{lm}(x) c_{Lm}^{\dagger}(x)|v\rangle + \int dx\, \phi_{ln}(x) c_{Ln}^{\dagger}(x)|v\rangle + \mu_a a^{\dagger}|v\rangle + \mu_b b^{\dagger}|v\rangle + \mu_e \sigma_{eg}|v\rangle, \tag{8}$$

where $\phi_{rm}(x)$, $\phi_{rn}(x)$, $\phi_{lm}(x)$, and $\phi_{ln}(x)$ denote the probability amplitudes of the right or left propagating photon in WM or WN. μ_a, μ_b, and μ_e are the excitation amplitudes of mode a, mode b, and the atom, respectively. $|v\rangle$ is the vacuum state, denoting no photon in the waveguides or cavity and the atom in ground state $|g\rangle$.

From the eigenvalue equation $H|\Psi\rangle = \omega|\Psi\rangle$, we obtain

$$-i v_{gm} \frac{\partial}{\partial x}\phi_{rm}(x) + V_m \mu_a \delta(x) = \omega \phi_{rm}(x), \tag{9a}$$

$$-i v_{gn} \frac{\partial}{\partial x}\phi_{rn}(x) + V_n \mu_b \delta(x) = \omega \phi_{rn}(x), \tag{9b}$$

$$i v_{gm} \frac{\partial}{\partial x}\phi_{lm}(x) + V_m \mu_a \delta(x) = \omega \phi_{lm}(x), \tag{9c}$$

$$i v_{gn} \frac{\partial}{\partial x}\phi_{ln}(x) + V_n \mu_b \delta(x) = \omega \phi_{ln}(x), \tag{9d}$$

$$V_m^* \phi_{rm}(0) + V_m^* \phi_{lm}(0) + J\mu_b + g_a \mu_e = \left(\omega - \omega_a + i\frac{\gamma_a}{2}\right)\mu_a, \tag{9e}$$

$$V_n^* \phi_{rn}(0) + V_n^* \phi_{ln}(0) + J\mu_a + g_b \mu_e = \left(\omega - \omega_b + i\frac{\gamma_b}{2}\right)\mu_a, \tag{9f}$$

$$g_a^* \mu_a + g_b^* \mu_b = \left(\omega - \omega_e + i\frac{\gamma_e}{2}\right)\mu_e. \tag{9g}$$

Suppose that the single photon is injected from the left of WM, then $\phi_{rm}(x)$, $\phi_{rn}(x)$, $\phi_{lm}(x)$, and $\phi_{ln}(x)$ can be expressed as

$$\phi_{rm}(x) = e^{ik_m x}[s(-x) + t_m s(x)], \tag{10a}$$

$$\phi_{rn}(x) = e^{ik_n x} t_n s(x), \tag{10b}$$

$$\phi_{lm}(x) = e^{-ik_m x} r_m s(-x), \tag{10c}$$

$$\phi_{ln}(x) = e^{-ik_n x} r_n s(-x), \tag{10d}$$

where $k_p = \omega/v_{gp}$. t_m (t_n) denotes the single-photon transmission amplitude in WM (WN). r_m (r_m) represents the single-photon reflection amplitude in WM (WN). $s(x)$ is the Heaviside step function with $s(0) = 1/2$. Substituting Eqs. (10) into Eqs. (9), one can get

$$t_m = \frac{(g_a g_b^* + g_a^* g_b)J + P + Q}{(g_a g_b^* + g_a^* g_b)J + P + Q + S}, \tag{11a}$$

$$r_m = \frac{-S}{(g_a g_b^* + g_a^* g_b)J + P + Q + S}, \tag{11b}$$

$$t_n = r_n = \frac{i\sqrt{G_m^* G_n}(Jk_e + g_a^* g_b)}{(g_a g_b^* + g_a^* g_b)J + P + Q + S}, \tag{11c}$$

where $P \equiv k_a|g_b|^2 + k_b|g_a|^2 + k_e J^2 - k_a k_b k_e$, $Q \equiv i(|g_a|^2 G_n - k_a k_e G_n)$, and $S \equiv i|g_b|^2 G_m + G_m G_n k_e - ik_b k_e G_m$, $k_{i=a,b,e} \equiv \omega - \omega_i + i\gamma_i/2$, $G_p \equiv |V_p|^2/v_{gp}$. Based on Eqs. (11a)–(11c), we can discuss the phase-controlled single-photon routing in this system.

III. ROUTING IN A SINGLE WAVEGUIDE

In this section, we show how to modulate single-photon routing in a single waveguide by controlling the phase difference between the coupling strengths. When $V_n = 0$, which means that the WN decouples from cavity B and the system degenerates into single waveguide case, then t_m and r_m become

$$t_m = \frac{(g_a g_b^* + g_a^* g_b)J + k_a|g_b|^2 + k_b|g_a|^2 + k_e J^2 - k_a k_b k_e}{(g_a g_b^* + g_a^* g_b)J + k_a|g_b|^2 + k_b|g_a|^2 + k_e J^2 - k_a k_b k_e + i|g_b|^2 G_m - ik_b k_e G_m}, \tag{12a}$$

$$r_m = \frac{-i|g_b|^2 G_m + ik_b k_e G_m}{(g_a g_b^* + g_a^* g_b)J + k_a|g_b|^2 + k_b|g_a|^2 + k_e J^2 - k_a k_b k_e + i|g_b|^2 G_m - ik_b k_e G_m}, \tag{12b}$$

and $t_n = r_n = 0$. It exhibits clearly that t_m and r_m depend strongly on the phase difference $\Delta\theta = \theta_a - \theta_b$. To show the phase-dependent photon routing more clearly, we set $\omega_a = \omega_b = \omega_e$ and neglect dissipations temporarily. When a resonant photon with $\omega = \omega_e$ incidents from the left of WM, t_m and r_m degenerate into

$$t_m^r = \frac{2J|g_a||g_b|\cos\Delta\theta}{2J|g_a||g_b|\cos\Delta\theta + i|g_b|^2 G_m}, \tag{13a}$$

$$r_m^r = \frac{-i|g_b|^2 G_m}{2J|g_a||g_b|\cos\Delta\theta + i|g_b|^2 G_m}, \tag{13b}$$

respectively. Here, the superscript r denotes that the incident photon is resonant with the two-level system. Equations (13a) and (13b) show that when $\Delta\theta = (2n+1)\pi/2$ (n is an integer), $T_m^r \equiv |t_m^r|^2 = 0$ and $R_m^r \equiv |r_m^r|^2 = 1$. The incident resonant photon is reflected. However, when $\Delta\theta = k\pi$ (k is an integer), $T_m^r \approx 1$ and $R_m^r \approx 0$ if $|g_b|^2 G_m \ll 2J|g_a||g_b|$. The incident resonant photon is almost completely transmitted. Thus, the single-photon routing properties can be modulated by the phase difference $\Delta\theta$.

To show this property more clearly, we plot Fig. 2. When the condition $|g_b|^2 G_m \ll 2J|g_a||g_b|$ is satisfied, as shown by the red solid lines in Fig. 2, T_m^r reaches the minimum value of 0 and R_m^r is the maximum value of 1 at $\Delta\theta = 0.5\pi$ and 1.5π. However, T_m^r is about 0.998 and R_m^r reaches 0.002 at $\Delta\theta = \pi$. The results are consistent with previous analysis. But when G_m becomes large, it deviates from the condition $|g_b|^2 G_m \ll 2J|g_a||g_b|$. The influence of $\Delta\theta$ becomes weak, as shown by the dashed-dotted lines in Fig. 2.

Figure 3 exhibits the single-photon scattering spectra. Here, $T_m \equiv |t_m|^2$, $R_m \equiv |r_m|^2$, and detuning $\Delta \equiv \omega - \omega_e$. It exhibits that $T_m(R_m)$ approaches 1 (0) when G_m becomes small for the case of the phase difference $\Delta\theta = \pi/2$ if $\Delta = 0$. We also note that there are several dips in the transmission spectra. The locations of dips in the transmission spectra can be found by letting the absolute value of the numerator of t_m given in Eq. (12a) be zero. The number of dips is determined by J, $|g_a|$, $|g_b|$, and $\Delta\theta$. There are three dips in the transmission spectra for most cases. However, when

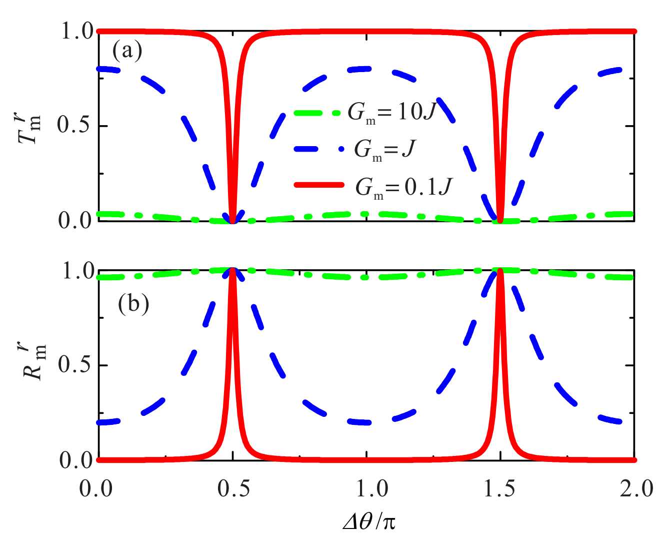

FIG. 2. Phase-controlled resonant single-photon scattering probabilities for the case of a single waveguide without decays. (a) Transmission probability T_m^r. (b) Reflection probability R_m^r. In the calculations, $\omega_a = \omega_b = \omega_e$, $|g_a| = |g_b| = J = 10^{-5}\omega_e$.

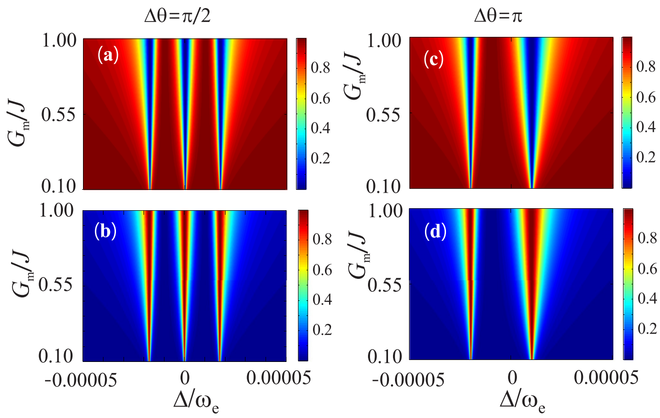

FIG. 3. Single-photon scattering properties as a function of G_m and detuning Δ for the case of a single waveguide without decays. The left column shows (a) T_m and (b) R_m when $\Delta\theta = \pi/2$, while the right column exhibits (c) T_m and (d) R_m when $\Delta\theta = \pi$. In the calculations, $\omega_a = \omega_b = \omega_e$, $J = |g_a| = |g_b| = 10^{-5}\omega_e$.

$J = |g_a| = |g_b|$ and $\Delta\theta = \pi$, there are just two dips which are located in $\omega = \omega_e + |g_a|$ and $\omega = \omega_e - 2|g_a|$.

The physical mechanism of the phase-dependent photon scattering property can be explained by the interference picture given in Refs. [73,76]. The interaction between the two cavities plays an important role. When $J = 0$, cavity A interacts with the two-level system with coupling strength g_a. And then, the two-level system couples to cavity B with coupling constant g_b. Thus, cavity A interacts indirectly with cavity B. When $J \neq 0$, cavities A and B are coupled directly. The quantum interference between the two paths leads to phase-dependent photon routing.

One can find that the phase difference $\Delta\varphi \equiv \varphi_m - \varphi_n$ does not affect the single-photon routing properties from Eq. (11). The physical mechanism is given as follows. The two cavities and the two-level system can be taken as a whole. Then the two waveguides interact indirectly, which is just one path. So the single-photon routing properties are independent of the phase difference $\Delta\varphi$.

The reason why phase-controlled single-photon routing depends on G_m can be explained as follows. As we stated before, one can take the two cavities and the two-level system as a whole. The interactions between them can be considered as internal interactions. While the coupling between cavity A and WM is external. When G_m is small, which means the external coupling is weak, the internal interactions can play significant roles. Thus, the single-photon routing properties strongly depend on $\Delta\theta$. However, when G_m is large, which means that the external coupling is strong, the effects of internal interactions become weak. Thus, T_m^r and R_m^r are insensitive to $\Delta\theta$.

IV. ROUTING BETWEEN TWO WAVEGUIDES

Now we turn our attention to the case of phase-dependent single-photon routing between two waveguides. When a resonant photon incidents from the left of the WM, t_m, r_m, t_n,

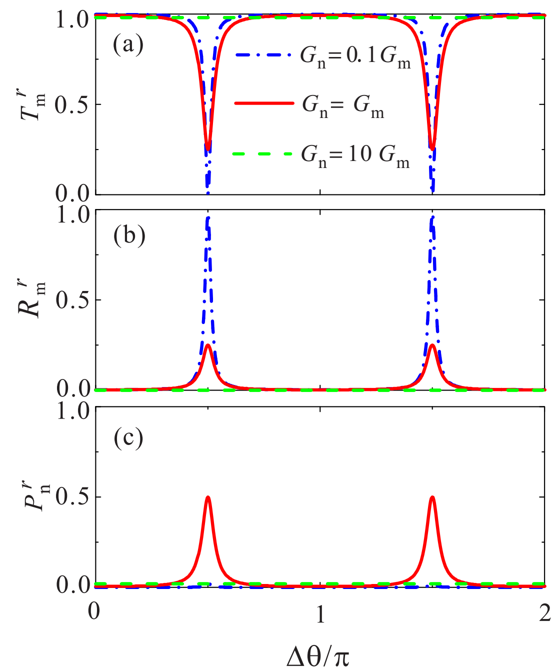

FIG. 4. Phase-dependent resonant single-photon scattering probability for the case of two waveguides without decays. (a) Transmission probability T_m^r. (b) Reflection probability R_m^r. (c) Transfer probability P_n^r. In the calculations, $\omega_a = \omega_b = \omega_e$, $G_m = |g_a| = |g_b| = 0.1J = 10^{-5}\omega_e$.

and r_n degenerate into

$$t_m^r = \frac{2J|g_a||g_b|\cos\Delta\theta + i|g_a|^2 G_n}{2J|g_a||g_b|\cos\Delta\theta + i|g_a|^2 G_n + i|g_b|^2 G_m}, \tag{14a}$$

$$r_m^r = \frac{-i|g_b|^2 G_m}{2J|g_a||g_b|\cos\Delta\theta + i|g_a|^2 G_n + i|g_b|^2 G_m}, \tag{14b}$$

$$t_n^r = \frac{i\sqrt{G_m^* G_n} g_a^* g_b}{2J|g_a||g_b|\cos\Delta\theta + i|g_a|^2 G_n + i|g_b|^2 G_m}, \tag{14c}$$

$$r_n^r = t_n^r, \tag{14d}$$

respectively. We introduce $P_n = T_n + R_n$ to denote the transfer probability of the single photon from WM to WN. When $\Delta\theta = (2n+1)\pi/2$, the real part of the denominator in Eq. (14c) is zero, thus both $T_n^r \equiv |t_n^r|^2$ and $R_n^r \equiv |r_n^r|^2$ reach the maximum value. The maximum value is determined by $|g_a|$, $|g_b|$, G_m, and G_n. When $|g_a|^2 G_n = |g_b|^2 G_m$, both T_n^r and R_n^r reach the maximum value of 0.25. Thus, $P_n^r \equiv T_n^r + R_n^r$ reaches the maximum value of 0.5. If $J|g_a||g_b| \gg |g_a|^2 G_n, |g_b|^2 G_m$, then P_n^r approaches zero at $\Delta\theta = \pi$. Figure 4 shows T_m^r, R_m^r, and P_n^r as functions of $\Delta\theta$ for the resonant photon with different G_n. It exhibits clearly that T_m^r, R_m^r, and P_n^r depend strongly on G_n and $\Delta\theta$. The red solid line in Fig. 4(c) shows P_n^r as a function of $\Delta\theta$ when the conditions $J|g_a||g_b| \gg |g_a|^2 G_n, |g_b|^2 G_m$ are satisfied. It indicates that one can switch on or off the single-photon routing between the two waveguides by modulating the phase difference $\Delta\theta$.

We also note that when $G_n \ll G_m$, the probability of the single photon being routed to the WN is very small. The

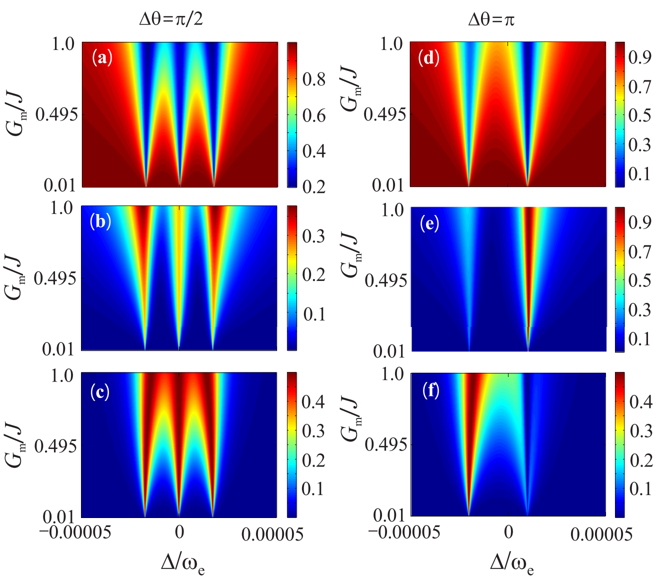

FIG. 5. Single-photon scattering properties as a function of G_m and detuning Δ without decays. The left column shows (a) T_m, (b) R_m, and (c) $P_n(c)$ when $\Delta\theta = \pi/2$, while the right column exhibits (d) T_m, (e) R_m, and (f) P_n when $\Delta\theta = \pi$. In the calculations, $\omega_a = \omega_b = \omega_e$, $|g_a| = |g_b| = 10^{-5}\omega_e$. $G_m = G_n$, $J = |g_a| = |g_b| = 10^{-5}\omega_e$.

single-photon scattering spectra are similar to those for a single waveguide due to the weak interaction between cavity B and WN. When $G_n \gg G_m$, a single photon which is incident from the left of WM can be transmitted from the right of WM since the interaction between cavity A and WM is very weak.

Figure 5 shows the single-photon scattering spectra with G_m and detuning Δ. The left column shows clearly that when $\Delta\theta = \pi/2$, P_n for the resonant incident photon reaches the maximum value of 0.5. The right column exhibits that $P_n \approx 0$ at $\Delta\theta = \pi$ for the resonant photon if the conditions $J \gg |g_a|^2 G_n, |g_b|^2 G_m$ are satisfied. Thus one can control the resonant single-photon routing between the two waveguides by manipulating $\Delta\theta$.

V. INFLUENCE OF DISSIPATION

In this part, we show how dissipation affects the photon routing probability. For the case of single waveguide, we have shown that the resonant photon can be transmitted with near unit probability when $\Delta\theta = \pi$, while it will be reflected with unit probability in the ideal case if $\Delta\theta = \pi/2$. Since dissipation can only reduce the peak values of the transmission and reflection spectra, we plot T_m^r with $\Delta\theta = \pi$ as a function of γ_a, γ_e in Fig. 6. R_m^r with $\Delta\theta = \pi/2$ as a function of decay is also shown. It exhibits clearly that both T_m^r and R_m^r decrease with increasing γ_a and γ_e. However, R_m^r is more sensitive to dissipation than T_m^r. This phenomenon can be explained as follows. When the decays are considered, Eq. (13) should be rewritten as

$$t_m^r = \frac{2J|g_a||g_b|\cos\Delta\theta + iC}{2J|g_a||g_b|\cos\Delta\theta + iD}, \tag{15a}$$

$$r_m^r = \frac{-i|g_b|^2 G_m + i\gamma_b\gamma_e G_m/2}{2J|g_a||g_b|\cos\Delta\theta + iD}. \tag{15b}$$

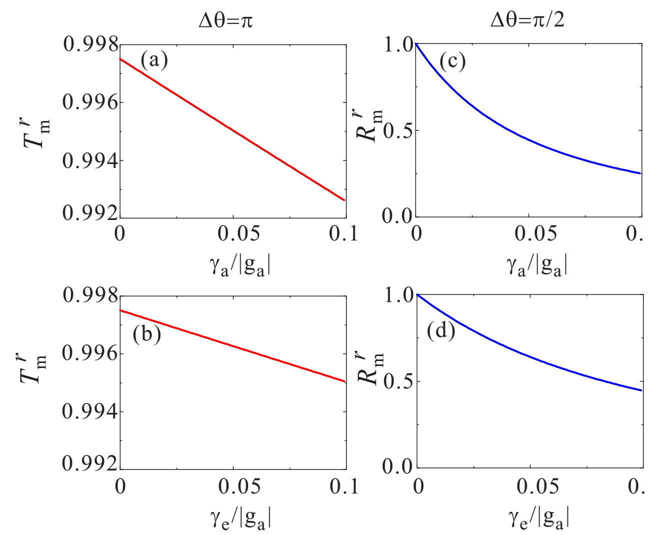

FIG. 6. Resonant photon transmission and reflection probabilities as a function of dissipation for the case of a single waveguide. The left column shows resonant photon transmission probabilities as a function of (a) γ_a and (b) γ_e with $\Delta\theta = \pi$. The right column exhibits resonant photon reflection probabilities as a function of (c) γ_a and (d) γ_e with $\Delta\theta = \pi/2$. In the calculations, $\omega_a = \omega_b = \omega_e$, $\gamma_a = \gamma_b$, $J = |g_a| = |g_b| = 10^{-5}\omega_e$. $G_m = 0.1|g_a|$.

Here, $C = (|g_b|^2\gamma_a + |g_a|^2\gamma_b)/2 + \gamma_a\gamma_b\gamma_e/8$, and $D = |g_b|^2 G_m + (|g_b|^2\gamma_a + |g_a|^2\gamma_b)/2 + \gamma_a\gamma_b\gamma_e/8 - G_m\gamma_b\gamma_e/4$. When $\Delta\theta = \pi$, $\cos\Delta\theta = -1$. The absolute value of $J|g_a||g_b|\cos\Delta\theta$ is much larger than other terms in t_m^r because we have set the conditions $|g_b|^2 G_m \ll 2J|g_a||g_b|$ and $\gamma_j \ll |g_a|$ ($|g_b|$). So, t_m^r is insensitive to decays in the parameter interval. However, when $\theta = \pi/2$,

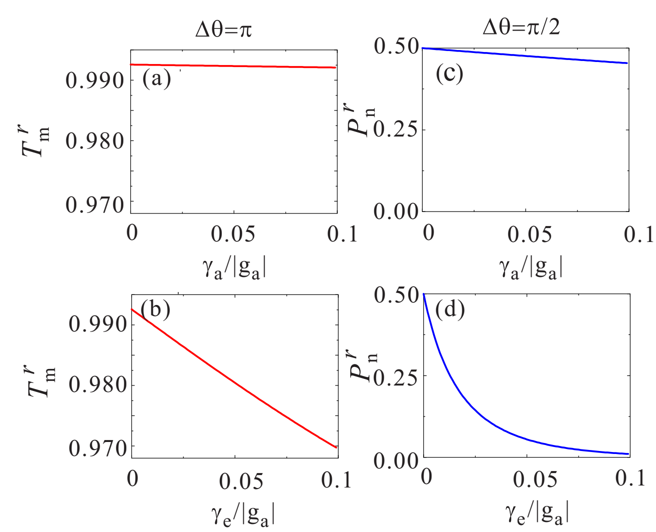

FIG. 7. Resonant photon routing probabilities as a function of dissipations for the case of two waveguides. The left column shows resonant photon transmission probabilities T_m^r as a function of (a) γ_a and (b) γ_e with $\Delta\theta = \pi$. While the right column exhibits probabilities of resonant photon P_n^r routed to the WN as a function of (c) γ_a and (d) γ_e with $\Delta\theta = \pi/2$. In the calculations, $\omega_a = \omega_b = \omega_e$, $\gamma_a = \gamma_b$, $G_m = G_n = 0.1J = |g_a| = |g_b| = 10^{-5}\omega_e$.

$J|g_a||g_b|\cos\Delta\theta = 0$. r_m^r depends strongly on G_m and γ_j. Especially when γ_j can be comparable with G_m, the influence of γ_j on r_m^r can be very large, as exhibited in Fig. 6. One can also find that the effect of γ_a is greater than that of γ_e on T_m^r and R_m^r. Luckily, cavity dissipation can be very small with modern technology.

Figure 7 shows T_m^r at $\Delta\theta = \pi$ and P_n^r at $\Delta\theta = \pi/2$ as functions of γ_a and γ_e for the case of two waveguides. It shows that P_n^r is sensitive to γ_e. This is because we choose the condition that J is larger than G_p, $|g_a|$, and $|g_b|$ in the numerical calculations to obtain a high value of P_n. One can see from Eq. (11c) that Jk_e can play a significant role in the expression of t_n and r_n. γ_e can affect $t_n(r_n)$ strongly due to the large J. Thus P_n is sensitive to γ_e.

VI. DISCUSSION AND CONCLUSIONS

This model may be realized by using a quantum dot coupled to a pair of coupled photonic crystal cavities. Majumdar *et al.* reported the cavity quantum electrodynamics with a single quantum dot coupled to a photonic molecule [77]. In their experiments, the strong coupling between the quantum dot and the two coupled cavities was realized and the cavity-cavity interaction between the two cavities was achieved. The configuration studied in this paper will be realized if the two cavities in their model respectively couple to two waveguides.

In summary, we have shown that the single-photon routing probabilities in a single waveguide or between two waveguides can be modulated by the phase difference $\Delta\theta$. For a single-waveguide system, the resonant photon is reflected with unit probability when $\Delta\theta = (2n+1)\pi/2$ while it will be transmitted with near unit when $\Delta\theta = k\pi$ in the ideal case. For the two-waveguide system, it is shown that the resonant single-photon routing probabilities between the two waveguides can be switched on or off by modulating $\Delta\theta$. When $\Delta\theta = (2n+1)\pi/2$, the single-photon routing probability P_n from WM to WN can reach the maximum value of 0.5. While when $\Delta\theta = p\pi$, P_n can be nearly zero. Our results are useful in designing all-optical quantum devices at the single-photon level.

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