# Creating, probing, and manipulating fractionally charged excitations of fractional Chern insulators in optical lattices

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We propose a set of schemes to create and probe fractionally charged excitations of a fractional Chern insulator state in an optical lattice. This includes the creation of localized quasiparticles and quasiholes using both static local defects and the dynamical local insertion of synthetic flux quanta. Simulations of repulsively interacting bosons on a finite square lattice with experimentally relevant open boundary conditions show that already a four-particle system exhibits signatures of charge fractionalization in the quantum-Hall-like state at the filling fraction of 1/2 particle per flux quantum. This result is favorable for the prospects of adiabatic preparation of fractional Chern insulators. Our work is inspired by recent experimental breakthroughs in atomic quantum gases: the realization of strong artificial magnetic fields in optical lattices, the ability of single-site addressing in quantum gas microscopes, and the preparation of low-entropy insulating states by engineering an entropy-absorbing metallic reservoir.

## I. INTRODUCTION

Topologically ordered states of matter that support anyonic excitations are a fascinating example for the emergence of intriguing properties from the interplay of many interacting degrees of freedom [1]. Moreover, it has been shown that such states can form a platform for robust (topologically protected) quantum information processing [2], provided one is able to create and manipulate the anyonic excitations in a coherent fashion. Well known examples of topologically ordered states are fractional quantum Hall (FQH) states [3] and the closely related fractional Chern insulators (FCIs) in lattice systems [4–6]. However, in solid-state devices even the direct observation of individual anyonic quasiparticles is extremely difficult (see Ref. [7] for a recent proposal), not to mention their coherent creation and manipulation.

Atomic quantum gases have been considered as an alternative environment for studying FQH physics already for more than a decade [8–11]. However, for a long time the achievable effective magnetic fields were not strong enough to observe quantum Hall physics and also the entropies (or temperatures) were rather high. This situation has now changed with several experimental breakthroughs. On the one hand, strong artificial magnetic fields and two-dimensional spin-orbit coupling were achieved in optical lattice systems [12–27] allowing for the observation of topologically nontrivial band structures [24,26], a (quantized) bulk Hall response [19,23], and chiral edge transport [20–22]. On the other hand, quantum gas microscopes were established as tools for

manipulating and imaging atoms on single lattice sites [28–39]. Using digital micromirror devices they allow for tailoring light-shift potentials with high spatiotemporal resolution. This technique was very recently employed to prepare low-entropy insulating states by engineering a potential that gives rise to an entropy-absorbing metallic shell at the boundary of a small system [37].

Inspired by these developments, in this paper we address two crucial questions regarding the possibility to realize and observe FQH physics in a quantum gas microscope. The first question is: Provided a FCI state has been realized, what are experimentally feasible probes that can reveal its characteristic signatures? Here our approach is to design schemes for creating and manipulating the elementary excitations of the system and to probe their fractional "charge" (i.e., particle number). This also paves the way towards studying their fractional (anyonic) statistics in the future. The second question is: Are characteristic features of a FCI state still accessible in small systems of just a few particles? This question is of eminent practical importance because, in contrast to solid-state systems, quantum gases are well isolated rather than kept at a given temperature by their environment. This implies that the state of the system has to be prepared adiabatically, starting from a topologically trivial ground state and passing through a continuous phase transition into the desired FCI state [40,41] while relying on a finite-size gap in the spectrum.

In order to address these questions, we propose three different probes that can be implemented in quantum gas microscopes, and simulate them exactly for small systems of four particles on about 50 sites. We consider the experimentally relevant scenario of repulsively interacting bosons on a square lattice subjected to a homogeneous flux and focus on regimes where one expects a Laughlin-like FCI state at the filling of $\nu = 1/2$ particles per flux quantum. Note that it is not our aim to study fundamental properties of FCI states as

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such, as they have been investigated already in a number of previous studies (see, e.g., Refs. [40–46] and also [11,47–50] for other types of optical lattices). We rather address how and whether it is possible to observe signatures of FCIs in small realistic systems. The first probe is the interaction energy (which can be obtained from measuring site occupations) as a function of the filling factor $\nu$. As expected for a FCI, we find a pronounced minimum in the interaction energy around $\nu = 1/2$. The second probe is the accumulation of quantized fractional charge $\nu$ near engineered local defects as a unique fingerprint revealing the localization of individual fractionally charged excitations. Finally, as a third probe, we study the adiabatic creation and pumping of fractionally charged excitations via the insertion of a flux quantum through a thin solenoid localized on single plaquettes (as they can be implemented using Floquet engineering [51]). Simulations of these probes indicate FQH physics already in the small systems studied here.

## II. MODEL

We consider interacting bosons moving in a two-dimensional square lattice subjected to an effective magnetic field. The system is described by a Bose-Hubbard Hamiltonian, written in terms of annihilation and density operators $a_\ell$ and $n_\ell = a_\ell^\dagger a_\ell$ for bosons on site $\ell$,

$$H = -t \sum_{\langle \ell' \ell \rangle} e^{i\theta_{\ell'\ell}} a_{\ell'}^\dagger a_\ell + \frac{U}{2} \sum_\ell \hat{n}_\ell(\hat{n}_\ell - 1) + \sum_\ell V_\ell \hat{n}_\ell. \quad (1)$$

Here $t$ describes nearest-neighbor tunneling. The Peierls phases $\theta_{\ell'\ell}$, which encode a uniform flux $\phi = 2\pi\alpha$ and possibly also local fluxes on certain plaquettes, can be implemented using Floquet engineering [14]. Whereas a homogeneous flux has already been achieved experimentally [18,19,25], the creation of solenoid-type single-plaquette fluxes is proposed in Refs. [51,52]. The Hubbard parameter $U$ describes on-site interactions. We also consider energy offsets $V_\ell$ on some sites, to be used for creating and/or trapping quasiparticle and quasihole excitations.

Motivated by a recent experiment [19], we focus on the regime $\alpha = 1/4$. In this situation, the band structure consists of four bands. The lowermost (which is adiabatically connected to the lowest Landau level in the low-flux regime) is protected by a gap much wider than its bandwidth. The formation of a topological band structure was observed by measuring the Chern number $\mathcal{C} = 1$ with convincing precision and fueled the hope for a possible many-body experiment. For strong interactions this system is expected to stabilize an incompressible FCI state at certain fractional fillings $\nu$, such as a $\nu = 1/2$-Laughlin-type state [40–46,53].

## III. FINITE-SIZE SYSTEM

In our numerical simulations we consider a square lattice of $N_x \times N_y$ sites with open boundary conditions, which contains $(N_x - 1)(N_y - 1)$ plaquettes. Such a system can be realized in current quantum gas microscope setups. To reach the strongly interacting regime $U \gg t$, we set $U/t = 7.5$. However, varying the ratio $U/t$ we find little qualitative de-
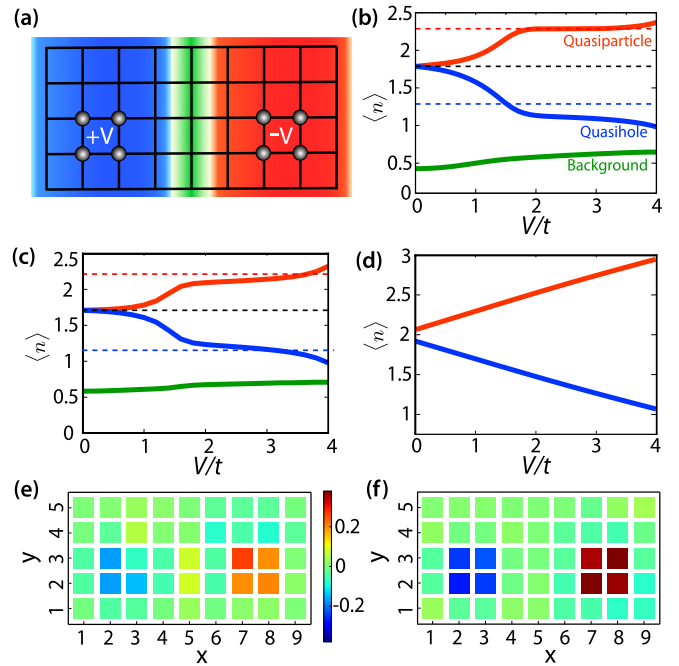


FIG. 1. Lattice geometry and charge redistribution due to additional potential offsets. (a) Typical geometry of $9 \times 5$ sites. Potential offsets $\pm V$ are each distributed uniformly over four sites. The narrow "control" region in the center (green shade) separates the left (blue) and right (red) regions where density change is calculated. Charge accumulation and depletion with quantized steps of magnitude one half is seen in lattices (b) 4@$9 \times 5$ and (c) 4@$7 \times 7$ operating in the topological regime; here $U/t = 7.5$ and $\nu = 1/2$. (d) A trivial band insulator leads to continuous charge flow. Density change induced by the impurity potentials: for (e) fractional quantum Hall state at $\nu = 1/2$, $V = 3$, $U = 7.5$ and (f) integer quantum Hall state at $\nu = 1$, $V = 9$, $U = 0$.

pendence on its actual value and most of the presented results are also reproduced in the hard-core limit. The samples are pierced by a uniform background flux, and further perturbed by either: (i) introducing localized attractive and repulsive potential offsets, each distributed over four lattice sites surrounding a single plaquette, or (ii) inserting two additional localized solenoid-type fluxes of equal magnitude and opposite signs, each penetrating a single plaquette [51]. Figure 1(a) shows a typical example with dimensions $N_x \times N_y = 9 \times 5$. Note that the two locations of the potential offsets or the solenoid fluxes (the affected plaquettes are marked by dots) are well separated from each other and avoid the boundary sites. In Fig. 1(a) we also mark three regions in which charge redistribution will be monitored. The broad region on the left (marked with blue shade) accommodates the quasihole, and its counterpart on the right (red shade) accommodates the quasiparticle. The two regions are separated by the narrow "neutral" stripe (green shade) that helps verify their localization.

In an infinite square lattice (or for periodic boundary conditions) the number of sites $N_{\text{sites}}$ (i.e., of single-particle states) matches the number of plaquettes $N_{\text{plaq}}$. Thus, the filling factor $\nu$ is uniquely defined as the ratio between the number of particles per plaquette $N/N_{\text{plaq}}$ and the number of
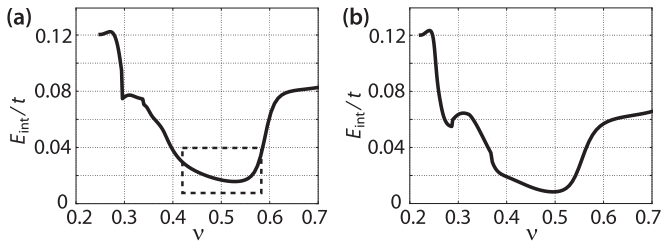
FIG. 2. Scaled interaction energy, as it can be measured in a quantum gas microscope, versus filling factor, for configurations $4@9 \times 5$ (a) and $4@7 \times 7$ (b) obtained for $U/t = 7.5$. The filling is tuned via plaquette flux $\phi$. Broad minima appear close to the expected filling factor $\nu = 1/2$.

flux quanta per plaquette $\alpha$,

$$\nu = \frac{N/N_{\text{plaq}}}{\alpha}. \qquad (2)$$

Any deviation from the nominal rational value of the filling factor that corresponds to a given FQH state results in creation of quasiparticle excitations. In a finite system, an ambiguity arises because of extra sites lying at the system boundary. Counting sites rather than plaquettes in Eq. (2) would produce an alternative filling factor: $\tilde{\nu} = \nu(N_x - 1)(N_y - 1)/N_x N_y$. This issue—being of eminent importance for the preparation of FQH states in quantum gas microscopes—poses a natural question: What is the optimal filling for the stabilization of a FQH droplet in a small system?

### A. Interaction energy

In order to address the posed question, we consider $N$ bosons on a square lattice of $N_x \times N_y$ sites (denoted $N@N_x \times N_y$ hereafter). We focus on a homogeneous system with uniform plaquette flux $\phi = 2\pi\alpha$ and in the absence of on-site potentials, i.e., $V_\ell = 0$, $\forall \ell$. In an experiment, a continuous control of the flux can be achieved, e.g., by implementing the moving superlattice used for the Floquet engineering of the plaquette fluxes by a digital micromirror device [25]. The contribution to the ground-state energy that comes from the interaction of particles should attain a minimum at fractional filling factors that correspond to the formation of the FCI state. This approach was introduced for interacting fermions [54] on finite lattices with periodic boundary conditions, and conspicuous minima at odd-denominator fractional fillings were demonstrated. We define the average interaction energy as

$$E_{\text{int}} = \langle \Psi_{\text{gs}} | \frac{U}{2} \sum_\ell \hat{n}_\ell(\hat{n}_\ell - 1) | \Psi_{\text{gs}} \rangle, \qquad (3)$$

where $|\Psi_{\text{gs}}\rangle$ denotes the ground state obtained from exact diagonalization.

We begin with our principal $4@9 \times 5$ configuration, which gives $\nu = 1/2$ for $\alpha = 1/4$ in Eq. (2), and vary the filling $\nu$ by tuning $\alpha$. Figure 2(a) shows that the broad minimum of $E_{\text{int}}$ has its deepest point at $\nu \approx 0.525$ which is just slightly off the nominal value $\nu = 1/2$. The alternative definition $\tilde{\nu} \approx 0.71 \nu$ would lead to an obvious deviation. In Fig. 2(b) we repeat the numerical experiment for another lattice geometry $4@7 \times 7$, which is of comparable area. The minimum of
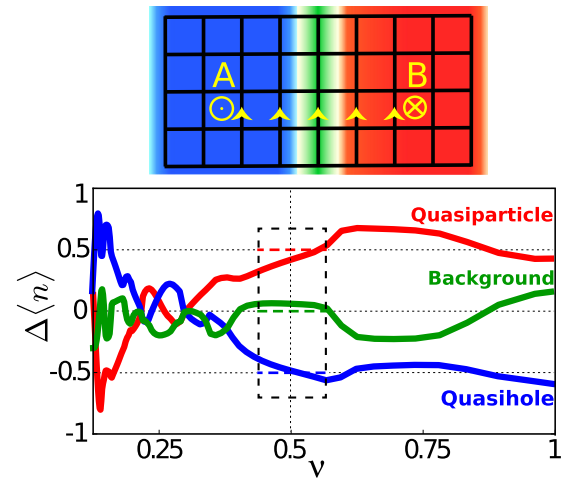


FIG. 3. The net charge transferred in a $4@9 \times 5$ lattice in response to the insertion of a flux quantum plotted as a function of the filling factor. The rectangle drawn in the black dashed line marks the vicinity of $\nu = 1/2$. The background flux is set to $\alpha = 1/4$, interaction $U/t = 7.5$, and additional potential $V = \pm 0.2$ is distributed over the A/B plaquettes.

the interaction energy is again centered around $\nu = 1/2$ but would shift when plotted against $\tilde{\nu} \approx 0.73 \nu$. Note that for the parameters and protocol of Fig. 2 we do not expect a FCI state at quarter filling. In this scenario, the filling is adjusted by tuning the plaquette flux and $\nu = 1/4$ corresponds to $\phi = \pi$ that neither breaks time-reversal symmetry nor gives rise to separate Bloch bands with nonzero Chern numbers.

In both Figs. 2(a) and 2(b) we observe rather broad minima of width $\sim 0.1$ with respect to the filling. Below (see Fig. 3) we will see that the corresponding filling factors coincide with those intervals of $\nu$ for which we also find the quantized fractional charge transport expected for a FCI state. This finding suggests a robustness of the FCI state with respect to filling, which should greatly facilitate its experimental preparation. We attribute this effect to the fact that in our small systems, a finite fraction of the total particle number can be accommodated in gapless edge modes without significantly changing the bulk properties of the system (which are protected by a gap). This effect is another advantage of considering small systems that complements their advantage for adiabatic state preparation.

We stress that besides serving as a numerical benchmark, the interaction energy can also be used as an experimental indicator for the formation of FQH states. The single-site resolution of quantum gas microscopes allows for extracting both the mean and the fluctuations of the on-site occupations via measuring their statistics in repeated experiments. This turns $E_{\text{int}}$ into an experimentally accessible quantity.

### B. Incompressibility and charge fractionalization

While the minimum of the interaction energy around half-filling is consistent with a FCI ground state, it does not reveal specific signatures, such as charge fractionalization, of this state and could also indicate, e.g., a gapped density wave at half-filling. Therefore, we propose another experimentally

feasible method to probe the fractional excitations of this system. The method is based on the idea to introduce localized impurity potentials [46] to pin quasiparticles or quasiholes. For that purpose we assume that the system is prepared in the presence of two spatially separated local defects in the bulk: a potential dip and a potential bump. Monitoring the integrated particle density ("charge") in the vicinity of each defect, we expect two signatures for a FCI state: First, the incompressibility associated with the bulk gap should make the system stiff against weak impurities. Second, ramping up the impurities further, we expect the dip and the bump to attract a quasiparticle and a quasihole, respectively, corresponding to a quantized charge accumulation of $+\nu$ and $-\nu$.

We observe that in our situation potentials situated on a single site (cf. Ref. [46]) do not constitute the optimal choice. In fact, due to the finite extent of the quasiparticles and quasiholes, it is preferable to work with potentials $|V_\ell| = V/4$ distributed over four neighboring sites, as depicted in Fig. 1(a) for the 4@9 × 5 lattice. We verified that results do not deteriorate when the impurity potentials are smeared out over more than four sites as modeled by Gaussians with a width of a lattice constant.

We calculate the ground state as a function of $V$ and plot the charge accumulated in the three control regions [indicated by the background colors in Fig. 1(a)] in Fig. 1(b). We see that as the potential strength $V$ grows from 0 to $2t$, the number of particles in the left (right) end of the elongated lattice gradually grows (decays) from the initial value of around 1.8 to a stable value of 2.3 (1.3) (depicted with dashed lines) and remains pinned to this value in a broad region of potential strengths up to $V \approx 4t$. We note that the initial region of the curves also displays insensitivity against weak potentials. Thus, we make two observations that point to the successful creation of fractionally charged excitations in our small system: not only the transferred charge equals $\pm 1/2$ with a good accuracy but also the initial and the final states display rigidity typical for incompressible FCI states. The small deviation from the expected quasihole charge of $-1/2$ can be attributed to a finite-size effect.

To further corroborate the observations, we show the results of analogous simulations in Figs. 1(c) and 1(d), respectively, for a sample of different shape (7 × 7) and for a topologically trivial system (with four Bloch bands created by a superlattice potential rather than by magnetic flux). In the former case the results reproduce the features seen for lattices 9 × 5 while in the latter case we see a featureless uniform growth of the transferred charge.

In Fig. 1(e) we plot the density change induced by local potentials of strength $V = 3$ [considering the configuration of Figs. 1(a) and (b)]. For completeness, we have also plotted the density change found for an integer quantum Hall state on the same lattice but with $\nu = 1$ and $U = 0$. Here a larger impurity $V = 9$ is required to create integer charged quasiparticles and holes, since the single-particle band gap is much larger that the many-body gap protecting the FCI.

### C. Creating excitations and fractional charge pumping

A further hallmark of FCI states is fractionally quantized adiabatic charge pumping in response to the insertion of a flux quantum [40,51,55]. In order to probe this effect, we consider the configuration shown in the inset of Fig. 3, where additional Peierls phases of $\delta\phi$ are added on the bonds marked by yellow arrows. This leads to local solenoid-type fluxes $\pm\delta\phi$ piercing the marked plaquettes A and B on top of the homogeneous background flux $\phi$. When $\delta\phi$ is ramped up slowly from 0 to $2\pi$, we expect that a quasihole of charge $-\nu$ is created at plaquette A and a quasiparticle of charge $\nu$ at plaquette B. This corresponds to the adiabatic pumping of fractional charge $\nu$ from A to B. In order to prevent the so-created excitations from dispersing, we also add small static local energy offsets of the same form as the ones considered previously [Fig. 1(a)]. We choose a very small value $V = \pm 0.2t$, which hardly alters the ground-state charge distribution [as can be seen from Fig. 1(b)], but is still large enough to pin the dynamically created local quasiparticles and quasiholes [46].

For this protocol we simulate the time evolution. Starting from the ground state of a small system (4@9 × 5), $\delta\phi$ is switched on linearly from 0 to $2\pi$ moderately slowly, with the ramp time $\tau = 5\hbar/t$. We note that the ramp time $\tau$ should not be too large as the pumped charge would have time to disperse during the slow ramp, whereas the opposite limit of too fast ramping would heat the system up. In Fig. 3 we depict the so-induced change in the particle numbers in the three regions indicated with different background colors in the inset. Here the filling factor $\nu$ is again controlled by changing the background flux $\phi$. The dashed lines correspond to the fractional charge transfer predicted for the FCI state at half-filling. We find that the pumped charge saturates approximately to the expected value only in the region marked by the rectangular box, where the particle density in the (green) control region also remains fixed. In addition to being centered around $\nu = 1/2$, this region matches the broad minimum of the interaction energy shown in Fig. 2(a) for the same system. This provides another indication that a system as small as the one considered here can support a FCI-type ground state.

### IV. CONCLUSIONS AND OUTLOOK

In summary, we have proposed and investigated two complementary schemes for probing charge fractionalization in optical lattice systems as a signature of FCI states. They rely on different strategies for creating and transporting quasiholes and quasiparticles: either by static impurities or by the local insertion of a flux quantum. As alternative probes are not easy to implement, the proposed density-based protocols will be crucial for measuring unique signatures of a FCI state at a particular filling $\nu$. For example, observing the fractionally quantized bulk Hall response to a homogeneous force is difficult because of the external confinement, and so far, there is also no straightforward method proposed for measuring the exchange phase $2\pi\nu$ characteristic to the anyonic quasiparticles. Also probing chiral edge modes via the dynamics of defects [45] or the (experimentally challenging) measurement of single-particle coherence [40] do not allow for a clear distinction between integer and fractional Chern insulator states. Additionally, we have pointed out that also the interaction energy is a measurable quantity that provides

further support regarding the realization of insulating states of matter at fractional filling. Simulating these probes for a bosonic system on a square lattice, we found that already small systems of just four particles feature signatures of FQH state at half-filling. This result is of immediate practical relevance, since the adiabatic preparation of such a state relies on the finite-size gap of the system.

In future work it will be interesting to look for signatures of charge fractionalization for FCI states at filling factors different from $\nu = 1/2$, either Laughlin states at lower filling or hierarchy states. Another fascinating perspective is to study whether the techniques introduced here can be extended also to measure signatures of anyonic statistics.

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