# Classical analog of quantum contextuality in spin-orbit laser modes

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We experimentally observed the violation of Kujula–Dzhafarov noncontextuality inequalities by nonseparable spin-orbit laser modes. Qubits are encoded on polarization and transverse modes of an intense laser beam. A spin-orbit nonseparable mode was produced by means of a radial polarization converter ($S$ plate). Our results show that the quantum contextuality can be emulated by nonseparable spin-orbit modes in an intense laser beam. Additionally, an improvement in the nonseparable mode preparation allowed us to observe a greater violation of the Clauser–Horne–Shimony–Holt inequality for such system. The results are in very good agreement with the theoretical predictions of quantum mechanics.

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## I. INTRODUCTION

In a famous paper, Kochen and Specker (KS) [1] challenged certain realistic models for quantum mechanics [2]. In it, they showed that a realistic theory that reproduced the outcomes of sets of observable properties had to be contextual, independently of the state of the quantum system. To do this, they produced a set of operators such that the observed properties of certain subsets of commuting operators (defining different experimental contexts) was inconsistent with the properties of the whole set, under the assumption that quantum properties do not change with context. For a three-level system, Klyachko, Can, Binicioglu, and Shumovsky proposed a state-dependent contextuality inequality [3]. Experimental studies of quantum contextuality were performed by using different systems, such as neutrons [4], ion traps [5], nuclear magnetic resonance [6], and photons [7–9].

Of course, entanglement and contextuality are related. Cabello discussed the link between Bell-type inequalities and noncontextual theories [10]. Entanglement and contextual quantum behavior of spin and orbital angular momentum was observed for twin photons [11].

In quantum computation, contextuality is an important ingredient in computation using states with only real amplitudes (rebits) [12] and in computation based in measurements [13]. For computation based on correlation, contextual correlations are an important ingredient for the deterministic evaluation of nonlinear Boolean functions [14]. The reliability of this computation was investigated and it was showed that, for bipartite systems, Clauser–Horne–Shimony–Holt (CHSH) correlation [15] is a sufficient condition for reliable computation [16]. Recent work presented the noncontextual wirings, a component that provide a class of contextuality-free operations [17], a very important feature for quantum computation.

Contextuality, according to Kochen and Specker [1], is the impossibility of consistently assigning values to physical properties associated with observables in a way that is independent of the experimental setup. As KS demonstrated, for any quantum system with dimension three or greater, it is possible to find a set of observables (in their example, 117 observables) corresponding to 0 or 1 eigenvalues (yes-no questions) that must be contextual. Cabello later provided a simpler example of a contextual set of 18 observables in a Hilbert space of dimension four [18]. The idea of the KS proof is the following: Imagine we have an observable $P_1$ that is simultaneously measured with $P_2$, $P_3$, and $P_4$ in one experimental setup (context 1), but is also measured in another experiment with observables $P_5$, $P_6$, and $P_7$ (context 2). The assumption that all such quantum observables $P_i$ have the same truth value in all contexts leads to a logical contradiction. Therefore, it is not possible to consistently assign to a property $P_i$ in one context the same truth value in another context for all properties in the set. In other words, the algebra of quantum observables leads to contextuality.

For systems with deterministic inputs and random outputs, Kujala, Dzhafarov, and Larsson proposed a definition and measurement for contextuality [19]. A study on two binary systems, including entangled half-spin particles, based on Ref. [19], provided a set of four bound inequalities, called here Kajula–Dzhafarov (KD) inequalities, that are necessary and sufficient to verify contextuality [20]. Violation of at least one of them constitute a sufficient condition to attest that the system is contextual. Here, we use KD inequalities to verify the contextuality in spin-orbit laser modes.

Linear optical systems have been largely used to explore quantum features by encoding qubits in degrees of freedom of the electromagnetic field. Two-particle entanglement was simulated by two polarized laser beams at different frequencies [21]. The Stokes parameter and Poincaré sphere can be directly related to the Bloch sphere of a single qubit [22]. The Jones matrix formalism [23] allow us to manipulate polarization in the same way as quantum operators on qubits. An analogy between classical polarization optics and two-level quantum systems was established [24]. First-order transverse modes present the same structure of polarization transformations and an analog of the Poincaré sphere can be constructed [25]. Polarization and transverse modes of a laser beam (spin-orbit modes) can be used to produce the so-called nonseparable modes [26], which present the same mathematical structure of two-qubit entanglement. It was showed that such modes violated the Clauser–Horne–Shimony–Holt (CHSH) inequality [27–29]. By encoding qubits in amplitude and phase modulation in a laser beam, a Bell-like inequality was also violated [30]. Other classical laser beam systems were also used to explore the CHSH inequality [31,32] and the Wright inequality [33]. Classical vector beams were used to simulate the behavior of entangled states, showing the strong analogy between classical nonseparable modes and quantum entangled states [34]. Classical hypercorrelation in wave optics was used to construct an analogy of quantum superdense coding [35]. A tripartite system was investigated by adding a propagation path in spin-orbit-mode degrees of freedom, also violating the quantum-like Mermin's inequality [36]. The entanglement generated by system-environment interaction was recently emulated by spin-orbit modes [37]. Several other tasks such as quantum cryptography [38–40], quantum gates and algorithms [41–43], quantum games [44,45], and teleportation [46–48] were investigated by using intense laser beams and linear optical circuits. The results of such studies hold strong mathematical analogies with the predictions of quantum mechanics and can bring interesting perspectives to optical coherence and polarization theory [49–58]. Of course, such intriguing results awaken discussions about its physical means. For instance, Ref. [59] states that the boundary between classical and quantum physics is not under conflict once the analogy between nonseparable modes and entangled states is due to mathematical formalities. Here, we are interested in the significance of nonseparable modes in the optical field and how it can be used. As a practical consequence of this established field of investigation, the well succeeded application of nonseparable classical states on polarization metrology was reported [60]. On the other hand, an improvement in the BB84 quantum cryptography protocol was proposed by using the transverse mode as an additional degree of freedom to eliminate the need for a shared reference frame between Alice and Bob's laboratory [38]. This proposal was experimentally investigated by using a classical laser beam and the results showed a clear accordance with that predicted by quantum mechanics [39].

Contextuality was also investigated in a classical system by exploring a linear optical setup [61,62]. The polarization and path propagation of a laser beam were used to encode the analog of a tripartite system, called classical trit, to investigate the KCBS inequality. As a result, this system provided classical states that violated the KCBS inequality, showing an analogy with contextual quantum systems.

In this work we explored spin-orbit laser modes to investigate the existence of contextuality in bipartite systems by means of the Kujala–Dzahafarov (KD) inequality. Our results show that the maximally nonseparable spin-orbit mode violates the KD inequality, a necessary condition to characterize such a system as presenting contextuality. We also revisited the experiment of violation of the CHSH inequality for spin-orbit modes [27] by improving mode preparation in order to achieve a higher violation of CHSH inequality. Despite nonseparable modes, which emulate entangled states, do not present spatial separation, this classical optical analogy also allows us to investigate some mathematical properties of entanglement with simple and inexpensive experiments and can be a useful tool for testing some quantum information protocols. Then, contextuality can be studied in a straightforward and simple way. The article is organized as follows: Section II presents the CHSH and Kujala–Dzhafarov inequalities for spin-orbit modes. The experimental setup and procedures are presented in Sec. III. Section IV contain results and discussions. In Sec. V we summarize our work.

## II. SPIN-ORBIT LASER MODES AND QUANTUM-LIKE INEQUALITIES

In this section we explore the analogy between Bell's inequalities for quantum mechanics and their classical counterpart, known as spin-orbit inequalities [27,35,36]. Following Ref. [63], the vector structure of electromagnetic field is common to quantum mechanics and classical optics, since the degrees of freedom of the electromagnetic field in a laser beam can be described by a vector product. In this way, we explore the vector nature of polarization and spatial modes (Hermitian–Gaussian mode of first order) to combine them and build the so-called spin-orbit mode.

The linear polarization vector can be described by the unit vectors $\hat{e}_V$ for vertical polarization and $\hat{e}_H$ for horizontal polarization. The first-order of Hermitian–Gaussian modes, $HG_{01}(\vec{r}\,)$ and $HG_{10}(\vec{r}\,)$, can be described by direction of transverse mode, vertical and horizontal. Therefore, $\{\hat{e}_H, \hat{e}_V\}$ and $\{HG_{10}(\vec{r}\,), HG_{01}(\vec{r}\,)\}$ form a basis to describe the polarization and transverse mode states, respectively.

When the electromagnetic field can be written as a product between the transverse structure and a polarization vector $[\vec{E}(\vec{r}\,) = \psi(\vec{r}\,)\hat{e}]$, we have the so-called separable mode, which is analogous to the product state in quantum mechanics.

The most general vector of the spin-orbit laser mode can be written as

$$\vec{E}(\vec{r}\,) = c_1 HG_{01}(\vec{r}\,)\hat{e}_V + c_2 HG_{01}(\vec{r}\,)\hat{e}_H + c_3 HG_{10}(\vec{r}\,)\hat{e}_V + c_4 HG_{10}(\vec{r}\,)\hat{e}_H, \tag{1}$$

where $c_i$, with $i = 1$, 2, 3, and 4, are the complex numbers satisfying the normalization condition. Here, normalization means that the sum of the intensity of each term divided by the total intensity is equal to one. The values of complex coefficients $c_i$ can be chosen to obtain a nonseparable mode, which is nonfactorable. We can write the spin-orbit laser modes as the radial ($\Psi_\pm$) and azimuthal ($\Phi_\pm$) polarization beams:

$$\Psi_+(\vec{r}\,) = \frac{1}{\sqrt{2}}[HG_{10}(\vec{r}\,)\hat{e}_H + HG_{01}(\vec{r}\,)\hat{e}_V], \tag{2}$$

$$\Psi_-(\vec{r}\,) = \frac{1}{\sqrt{2}}[HG_{10}(\vec{r}\,)\hat{e}_H - HG_{01}(\vec{r}\,)\hat{e}_V], \tag{3}$$

$$\Phi_+(\vec{r}\,) = \frac{1}{\sqrt{2}}[HG_{10}(\vec{r}\,)\hat{e}_V + HG_{01}(\vec{r}\,)\hat{e}_H], \tag{4}$$

$$\Phi_-(\vec{r}\,) = \frac{1}{\sqrt{2}}[HG_{10}(\vec{r}\,)\hat{e}_V - HG_{01}(\vec{r}\,)\hat{e}_H]. \tag{5}$$

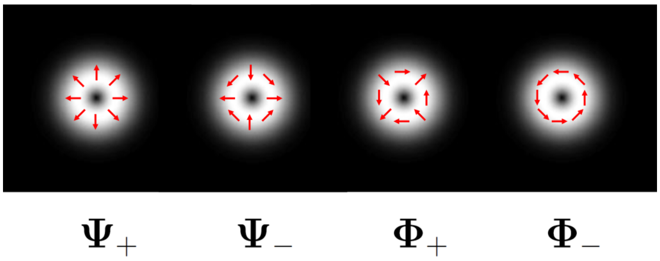


FIG. 1. Transverse structure of maximally nonseparable laser modes. The intensity has a donut shape. The arrows represent the polarization. $\Psi_\pm$ are the laser modes known as possessing radial polarization and $\Phi_\pm$ the azimuthal polarization.

As can be seen, this state cannot be written as the product of polarization and a first-order Hermitian–Gaussian mode. Therefore, the maximally nonseparable mode in Eqs. (2)–(5) present a vector structure that is similar to entangled states and can be associated with Bell's basis. Indeed, $\{\Psi_\pm, \Phi_\pm\}$ form an orthonormal mode basis. Figure 1 presents the transverse structures of the maximally nonseparable modes. The arrows distributed in the donut transverse structure illustrate radial and azimuthal polarization.

This analogy enables us to use the definition of the analog to quantum concurrence C to quantify the nonseparability [26]. For a general mode, described by Eq. (1), we can define

$$C = 2|c_1c_4 - c_2c_3|. \tag{6}$$

Note that, for a maximally nonseparable mode, Eqs. (2)–(5), $C = 1$, which is equivalent to the concurrence for the maximally entangled states of the Bell basis. In analogy to product states, $C = 0$ for any separable mode. Consequently, due to such an analogy, quantum-like inequalities for spin-orbit laser modes can be written and experimentally tested.

### A. Clauser–Horne–Shimony–Holt inequality

Following Ref. [27], we use the same argument of Clauser, Horne, Shimony, and Holt in quantum mechanics [15] to demonstrate the spin-orbit inequality. For instance, we define the rotated basis of polarization and transverse modes, so that

$$\begin{aligned}
\hat{e}_{\alpha+} &= (\cos\alpha)\hat{e}_V + (\sin\alpha)\hat{e}_H,\\
\hat{e}_{\alpha-} &= (\sin\alpha)\hat{e}_V + (\cos\alpha)\hat{e}_H,\\
HG_+(\vec{r}\,) &= (\cos\beta)HG_{01} + (\sin\beta)HG_{10}(\vec{r}\,),\\
HG_-(\vec{r}\,) &= (\sin\beta)HG_{01} - (\cos\beta)HG_{10}(\vec{r}\,).
\end{aligned} \tag{7}$$

Equation (2) can be rewritten in the rotated basis as

$$\begin{aligned}
\Psi_+(\vec{r}\,) = \frac{1}{\sqrt{2}}&\cos(\beta-\alpha)[HG_+(\vec{r}\,)\hat{e}_{\alpha+} + HG_-(\vec{r}\,)\hat{e}_{\alpha-}]\\
&+ \sin(\beta-\alpha)[HG_-(\vec{r}\,)\hat{e}_{\alpha+} - HG_+(\vec{r}\,)\hat{e}_{\alpha-}].
\end{aligned} \tag{8}$$

Defining $I_{\pm\pm}$ as the normalized squared amplitude of each term of Eq. (8), the normalized intensity of each component plays the role of probabilities in a quantum mechanics scenario. As Eq. (8) is normalized and the base vectors are unitary, we can shown that [27]

$$I_{++}(\alpha,\beta) + I_{+-}(\alpha,\beta) + I_{-+}(\alpha,\beta) + I_{--}(\alpha,\beta) = 1. \tag{9}$$

In addition, the quantity $M(\alpha,\beta)$ can be defined as [27]

$$M(\alpha,\beta) = I_{++}(\alpha,\beta) + I_{--}(\alpha,\beta) - I_{+-}(\alpha,\beta) - I_{-+}(\alpha,\beta), \tag{10}$$

and it is easy to show that

$$M(\alpha,\beta) = \cos[2(\beta-\alpha)], \tag{11}$$

which has the same result as the quantum-mechanical correlations for a two-level system. The next step is to derive a Bell-type inequality for spin-orbit modes, given by

$$S = M(\alpha_1,\beta_1) + M(\alpha_1,\beta_2) - M(\alpha_2,\beta_1) + M(\alpha_2,\beta_2), \tag{12}$$

where $\alpha_k$ and $\beta_k$, with $k = 1, 2$, are two distinct angles. Again, as in quantum mechanics, for separable modes, $-2 \leqslant S \leqslant 2$, but for nonseparable modes the Bell-type inequalities can be violated. For a set of angles, $\alpha_1 = \pi/8$, $\alpha_2 = 3\pi/8$, $\beta_1 = 0$, and $\beta_2 = \pi/4$, we obtain the maximum violation predicted by the inequality, $S = 2\sqrt{2}$. This inequality can be maximally violated by all modes presented in Eqs. (2)–(5).

### B. Kujala–Dzhafarov inequality

Satisfaction of the CHSH inequalities is a necessary and sufficient condition for the existence of a joint probability distribution for all experimental outcomes. However, as it is well known, they rely on a major assumption; namely, that the nonsignaling condition is valid. Yet, due to many experimental factors, photon-correlation experiments often exhibit some form of violation of the nonsignaling condition [64], with this condition being satisfied recently only in loophole-free Bell-type experiments [65,66].

Let us consider the standard Bell–Einstein–Podolsky–Rosen (Bell-EPR) setup, where we have two settings for Alice and Bob's detectors, and four random variables representing the outcomes of measurements, often labeled $A_1$, $A_2$, $B_1$, and $B_2$, for Alice and Bob, respectively. It is a consequence of probability theory that $\langle A_1\rangle$ and $\langle A_2\rangle$ are independent of whether they were measured in conjunction with $B_1$ or $B_2$. However, if the nonsignaling condition does not hold, this means that the intensity at the detector is not independent of the choices of the other detectors, and there is no possibility of having random variables (and therefore an underlying joint probability distribution) for $A_1$, $A_2$, $B_1$, and $B_2$, . Consequently, the use of the CHSH inequalities to determine contextuality (through the nonexistence of a joint probability distribution) is inadequate, because a joint probability already does not exist. Therefore, to examine contextuality, we need a different set of inequalities from CHSH.

Dzhafarov and Kujala [20] provided a consistent framework within classical probability theory to describe general systems that are contextual. Here we focus on the Bell-EPR setup to present their approach and inequalities. Since, because of violations of nonsignaling, the statistical distribution of property $A_1$ when measured with $B_1$ is different from when it is measured with $B_2$, we label its corresponding random variable as $A_{11}$ for the latter and $A_{12}$ for the former, with the first index indicating that we are measuring $A_1$ and the second index indicating the context $B_1$ or $B_2$, respectively. With such indexing, we have eight random variables, instead of the original four; namely, $A_{11}$, $A_{12}$, $A_{21}$, $A_{22}$, $B_{11}$, $B_{12}$, $B_{21}$, and $B_{22}$. The nonsignaling condition would be represented, in this notation, by the requirement that $p(A_{ij} = A_{ij'}) = 1$ and $p(B_{ij} = B_{ij'}) = 1$ for $i, j, j' = 1, 2$ and $j \neq j'$.

Of course, because they are measured together, the random variables $A_{ij}$ and $B_{jk}$ are jointly distributed. Furthermore, under no additional assumptions, $A_{ij}$ and $B_{kl}$ are not jointly distributed, but it is always possible to create a new set of jointly distributed random variables $A'_{ij}$ and $B'_{kl}$ such that all the stochastic properties of $A_{ij}$ and $B_{kl}$ are reproduced. Such a new set of random variables is called in probability theory a "coupling," since it connects variables from two different contexts as a single joint probability distribution.

Because such couplings always exist (in fact an infinite number of them), providing a coupling is not sufficient to tell us whether the physical system is contextual. To address the question of contextuality, we need to look at all possible couplings under some additional condition. Let us, for example, consider $A_{11}$ and $A_{12}$, which are two random variables measured in different contexts and therefore not jointly distributed. When constructing a coupling, we can make whatever assumptions we want about the connection between those two variables. But if we want to think of them as the same property being measured in different contexts, we should try to construct a coupling such that the probability that they are the same is maximal—being one is not a possibility, since their distributions are different due to artifacts that may cause a violation of the nonsignaling violations. We can also take the same approach when trying to find a coupling that involves all variables. This constraint, i.e., the requirement that the coupling be compatible with the maximal probability that a random variable representing a property in one context is equal to the same property in another context, is called "multimaximal coupling" [67]. Now, even though a coupling always exist that reproduces the marginals, a multimaximal coupling does not always exist, which means that a joint probability distribution with a multimaximal coupling does not exist. When this is the case, the system is said to be contextual.[1]

For the standard Bell-EPR case, with the notation shown above, we are now in a position to write a set of inequalities that, if satisfied, guarantee that the system is noncontextual. Given the random variables $A_{ij}$, $B_{kl}$, this system is noncontextual if and only if it satisfies the following inequalities:

$$S_{KD1} = |\langle A_{11}B_{11}\rangle + \langle A_{12}B_{12}\rangle + \langle A_{21}B_{21}\rangle - \langle A_{22}B_{22}\rangle| \leqslant 2(1+\Delta_0), \tag{13}$$

$$S_{KD2} = |\langle A_{11}B_{11}\rangle + \langle A_{12}B_{12}\rangle - \langle A_{21}B_{21}\rangle + \langle A_{22}B_{22}\rangle| \leqslant 2(1+\Delta_0), \tag{14}$$

$$S_{KD3} = |\langle A_{11}B_{11}\rangle - \langle A_{12}B_{12}\rangle + \langle A_{21}B_{21}\rangle + \langle A_{22}B_{22}\rangle| \leqslant 2(1+\Delta_0), \tag{15}$$

$$S_{KD4} = |-\langle A_{11}B_{11}\rangle + \langle A_{12}B_{12}\rangle + \langle A_{21}B_{21}\rangle + \langle A_{22}B_{22}\rangle| \leqslant 2(1+\Delta_0), \tag{16}$$

where $\Delta_0$ is given by

$$\Delta_0 = \tfrac{1}{2}\,(|\langle A_{11}\rangle - \langle A_{12}\rangle| + |\langle A_{21}\rangle - \langle A_{22}\rangle| + |\langle B_{11}\rangle - \langle B_{21}\rangle| + |\langle B_{12}\rangle - \langle B_{22}\rangle|). \tag{17}$$

Notice that these inequalities are the same as CHSH if we set $A_{ij} = A_{ik}$, $B_{ij} = B_{ik}$, and they are a generalization of CHSH for a system that violates the no-signaling condition. We can see that the terms on the right-hand side of $\Delta_0$ are, intuitively, a measure of how different the expectations of each random variable are from one context to the other, and $\Delta_0$ is the cumulative sum of all such possible terms; the more no-signaling violation, the greater the value of $\Delta_0$. $\Delta_0$ has a very important role, because it changes the bounds of the inequalities that define the existence of a joint probability distribution under the assumption of a multimaximal coupling.

Considering a spin-orbit modes system, we can write the mean values of KD inequalities as a combination of normalized intensity $I_{\pm\pm}(\alpha_i, \beta_j)$ in the same way presented for CHSH inequality. For the calculation of $\Delta_0$, the average $\langle A_{i,j}\rangle$ and $\langle B_{i,j}\rangle$ are given by

$$\langle A_{i,j}\rangle = I_{++}(\alpha_i,\beta_j) - I_{--}(\alpha_i,\beta_j) + I_{+-}(\alpha_i,\beta_j) - I_{-+}(\alpha_i,\beta_j) \tag{18}$$

and

$$\langle B_{i,j}\rangle = I_{++}(\alpha_i,\beta_j) - I_{--}(\alpha_i,\beta_j) - I_{+-}(\alpha_i,\beta_j) + I_{-+}(\alpha_i,\beta_j), \tag{19}$$

respectively. The average of the joint measurements are obtained by

$$\langle A_{i,j}B_{i,j}\rangle = I_{++}(\alpha_i,\beta_j) + I_{--}(\alpha_i,\beta_j) - I_{+-}(\alpha_i,\beta_j) - I_{-+}(\alpha_i,\beta_j), \tag{20}$$

where $i = 1, 2$, and $j = 1, 2$ in Eqs. (18)–(20). It is worth mentioning that the construction of KD inequalities use the same basis that one uses to construct the CHSH inequality. In addition, for spin-orbit modes we can verify KD inequalities with the same normalized intensities used for a CHSH experiment. Then, the same apparatus can be used to study both CHSH and KD inequalities. The experimental investigation of both of them is presented in the next section.

## III. EXPERIMENTAL VIOLATION OF QUANTUM-LIKE INEQUALITIES WITH SPIN-ORBIT MODES

The experimental study of the inequalities discussed in the previous section by using spin-orbit modes was performed following the proposal presented in Ref. [27], by improving the mode generation. The two-qubit system was encoded in polarization and transverse mode of a laser beam. The first qubit was encoded in polarization, where horizontal (H) and vertical (V) polarizations represent the basis states $|0\rangle_P$ and

[1] Kaszlikowski and Kurzynski [68] point out that violation of the nonsignaling condition is already a form of context dependency, which they call *strong contextuality*, whereas the contextuality resulting from a lack of a joint they call *hidden contextuality*, or simply *contextuality*.

 

TABLE I. Codification of qubits on the polarization and transverse mode HG of a laser beam.

| Optical modes basis | Logical computational basis |
|---|---|
| $\hat{e}_H$ $\mathrm{HG}_{10} = \vert Hh\rangle$ | $\vert 00\rangle$ |
| $\hat{e}_H$ $\mathrm{HG}_{01} = \vert Hv\rangle$ | $\vert 01\rangle$ |
| $\hat{e}_V$ $\mathrm{HG}_{10} = \vert Vh\rangle$ | $\vert 10\rangle$ |
| $\hat{e}_V$ $\mathrm{HG}_{01} = \vert Vv\rangle$ | $\vert 11\rangle$ |

$|1\rangle_P$, respectively. The second qubit was encoded in first-order Hermite–Gauss modes. Thereby, we have the $\mathrm{HG}_{10}$, named $h$, and the $\mathrm{HG}_{01}$, named $v$, representing the basis states $|0\rangle_M$ and $|1\rangle_M$, respectively. Table I summarizes the codification. Note that we are using brackets notation to represent the laser beam modes due the analogy between laser modes and quantum states [24,27,34,35,63].

The experimental setup is illustrated in Fig. 2. The state preparation starts with a diode-pumped solid-state (DPSS) laser beam (532 mm, 1.5 mW, vertically polarized) illuminating an $S$-wave plate (SP). This device can be adjusted to produce directly the maximally nonseparable mode

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|Hh\rangle - |Vv\rangle), \tag{21}$$

which is an analog of a two-qubit entangled state; namely, a maximally nonseparable state [27]. By introducing the $\mathrm{PBS}_1$ in the laser path, the component $|Hh\rangle$ is transmitted, and we have the analog of a product state; namely, a separable mode [27]. The preparation stage finishes with a spatial filter (SF) used to perform a mode clean in order to improve the mode fidelity. The mode preparation is the main experimental improvement introduced here in comparison with Ref. [27].

The Bell-like measurement was performed by using a half-wave plate (HWP@ $\alpha_i/2$) and a Dove prism (DP@$\beta_j/2$) to chose the measurement basis ($\hat{e}_{\alpha\pm}$, $\mathrm{HG}_{\beta\pm}$), which are the orientations of the Bell basis measurement. The half-wave plate changes the polarization states and the Dove prism allowed us manipulate the transverse spatial mode. Therefore, after the combined action of these two devices, the spin-orbit mode can be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[\cos(\beta-\alpha)|Hh\rangle + \cos(\beta-\alpha)|Vv\rangle + \sin(\beta-\alpha)|Vh\rangle - \sin(\beta-\alpha)|Hv\rangle]. \tag{22}$$

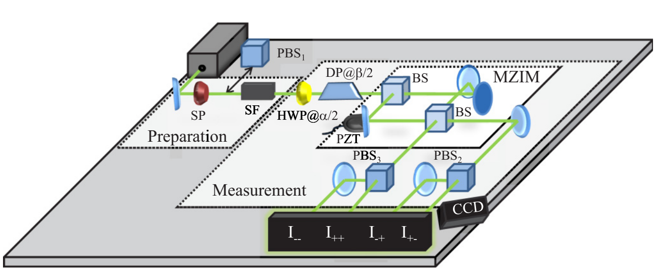


FIG. 2. Experimental setup for the CHSH and Kujala–Dzhafarov inequality. SP stands for $S$-wave plate, HWP for half-wave plates, DP for Dove prism, BS for beam splitter, and PBS for polarizing beam splitter.

The projective measurements start with a Mach–Zehnder interferometer with an additional mirror (MZIM) [69]. The MZIM performs a parity selection on the spin-orbit mode by adjusting the phase difference between interferometer arms, here implemented by a piezoelectric ceramic (PZT). Thereby, we have the even modes ($|Hh\rangle$ and $|Vv\rangle$) and the odd modes ($|Hv\rangle$ and $|Vh\rangle$) leaving the MZIM in different outputs. After the MZIM, output polarizing beam splitters ($\mathrm{PBS}_2$ and $\mathrm{PBS}_3$) project the four components in a bulkhead and the four intensities are registered simultaneously with a charge-coupled device (CCD) camera. Odd modes arrive in $\mathrm{PBS}_2$, the component with the horizontal polarization $|Hv\rangle$ (associated with the $\mathrm{I}_{+-}$ intensity) is transmitted, and the component with the vertical polarization $|Vh\rangle$ ($\mathrm{I}_{-+}$) is reflected. Even modes arrive in $\mathrm{PBS}_3$ and the component with the horizontal polarization $|Hh\rangle$ ($\mathrm{I}_{++}$) is transmitted while the component with the vertical polarization $|Vv\rangle$ ($\mathrm{I}_{--}$) is reflected. Note that, to calculate $\mathrm{M}(\alpha_i, \beta_j)$, $\langle A_{i,j}\rangle$, $\langle B_{i,j}\rangle$, and $\langle A_{i,j}B_{i,j}\rangle$ we take the normalized intensities $\mathrm{I}_{\pm\pm}$, obtained by integrating the intensity distribution of each outputs' images divided by their sum. To verify the violation of CHSH and Kujala–Dzhafarov inequalities we take the angle set that corresponds to the maximal violation of CHSH inequality: $\alpha_1 = \pi/8$, $\alpha_2 = 3\pi/8$, $\beta_1 = 0$, and $\beta_2 = \pi/4$.

## IV. RESULTS AND DISCUSSIONS

We performed the experiment by preparing two initial states: a maximally nonseparable mode and a separable mode, as described in Sec. III. By using the sets ($\alpha_i$, $\beta_j$) for maximal violation of the CHSH inequality, we register the four intensities for each combination. Figure 3 shows the resulting intensities for the maximally nonseparable mode. The outputs of the measurement setup are identified by the respective intensity labels $I_{++}$, $I_{+-}$, $I_{-+}$, and $I_{--}$. Each row is equivalent to the simultaneously captured images for a combination of ($\alpha_i$, $\beta_j$). We can observe a good visibility. We found 90% for

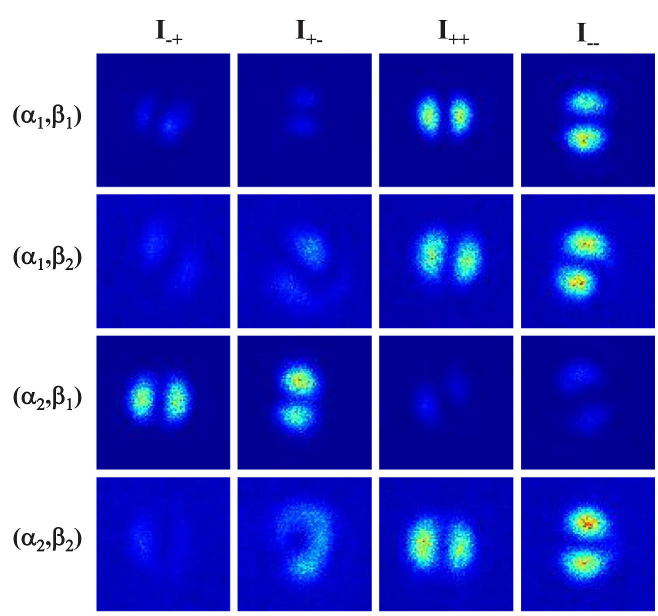


FIG. 3. Resulting images for maximally nonseparable mode.

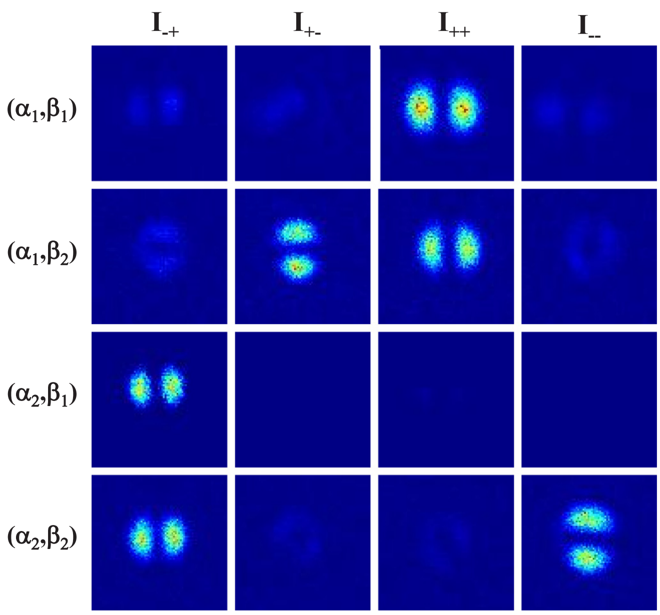


FIG. 4. Resulting images for separable modes.

the combinations with $\beta_1$. For the combinations containing $\beta_2$, where the DP is rotated, the visibility is worse (85%). The rotated DP slightly affects the polarization in a such way to transform a $|Hh\rangle$ into $|Vh\rangle$ and $|Vv\rangle$ into $|Hv\rangle$ mode components, making it hard to align the MZIM and consequently arrive at a better visibility.

Figure 4 shows the resulting intensities for the separable mode $|Hh\rangle$, obtained by including the $PBS_1$ in the laser path. The notations for intensities and angles are the same. The visibility of the MZIM for the separable mode is superior to the maximally nonseparable case once the mode has only the $|Hh\rangle$ component. Combinations with $\beta_2$ have also a worse visibility (around 88%) compared with those containing $\beta_1$ (95%). The normalized image intensities were used to calculate the inequalities described in the previous section.

### A. Clauser–Horne–Shimony–Holt inequality with spin-orbit modes

Table II presents both theoretical and experimental results for the calculation of $M(\alpha_i,\beta_j)$ and $S$. Theoretical values were obtained from the calculations of $M$ by using maximally entangled and product quantum states. Experimental values were obtained from the calculations of Eqs. (11) and (12). For the maximally nonseparable mode we observed a violation of $S = 2.503$, which is an important improvement compared with the result of Ref. [27]. By comparing theoretical and experimental $M(\alpha_i,\beta_j)$ values we observe a more accentuated difference for the combinations containing $\beta_2$ due the limited visibility of the MZIM.

TABLE II. Results for CHSH inequality measurement for maximally nonseparable and separable modes. Theoretical results were obtained from entangled and product quantum states. The experimental results were obtained from normalized intensity measurements of Figs. 3 and 4.

| | Maximally nonseparable | | Separable | |
|---|---|---|---|---|
| | Theory | Experiment | Theory | Experiment |
| $M(\alpha_1,\beta_1)$ | 0.707 | 0.679 | 0.707 | 0.665 |
| $M(\alpha_1,\beta_2)$ | 0.707 | 0.583 | 0.000 | 0.000 |
| $M(\alpha_2,\beta_1)$ | −0.707 | −0.679 | −0.707 | −0.661 |
| $M(\alpha_2,\beta_2)$ | 0.707 | 0.562 | 0.000 | 0.000 |
| $S$ | 2.828 | 2.503 | 1.414 | 1.326 |

TABLE III. Results of theoretical expected value of $S_{KD}$ for entangled state (Theory) and the obtained results for spin-orbit modes (Experiment).

| | Theory | Experiment |
|---|---|---|
| $S_{KD1}$ | 0.000 | 0.021 |
| $S_{KD2}$ | 2.828 | 2.503 |
| $S_{KD3}$ | 0.000 | 0.022 |
| $S_{KD4}$ | 0.000 | 0.213 |

For the separable mode the nonviolation of CHSH inequality were observed with $S = 1.326$. This result are more close to the theoretically calculated once we have a better visibility in the MZIM.

### B. Kujala–Dzahafarov inequality for spin-orbit modes

By using the normalized image intensities in Eq. (17) we obtained $\Delta_0 = 0.143$. Therefore, the bound value of the Kujala–Dzahafarov inequalities [Eqs. (13)–(16)] is $S_{KD} < 2(1+\Delta_0) = 2.286$. Table III presents the theoretical expected value of $S_{KD}$ for the entangled state and the obtained results for spin-orbit modes calculated from Eqs. (13)–(16). As can be seen, $S_{KD2} = 2.503 > 2.286$. This is a sufficient condition to infer the contextuality of spin-orbit modes. We notice a very good agreement between the quantum-theoretical prediction for the entangled state and the experimental results for nonseparable spin-orbit modes of a laser beam.

The contextuality in this scenario reveals the conflict between nonseparable modes and noncontextuality theory in correlations between these degrees of freedom of a laser beam. This is analogous to the conflict between quantum mechanics and noncontextual realism. This result reinforces the idea that we can use such systems to explore mathematical properties of quantum systems, such as entangled states. Once the spin-orbit mode of an intense laser beam presents an analogous mathematical structure to quantum systems it can be used to emulate an ensemble of quantum measurements with single-photon experiments [27,35–37,43,57].

The revisitation to the experiment of violation of the CHSH inequality for spin-orbit modes, beyond showing an improvement in the violation, provides evidence, from direct experimental measurements, of the relation between Bell inequalities and noncontextuality discussed in Ref. [70].

## V. CONCLUSIONS

We have experimentally demonstrated the violation of the Kujala–Dzhafarov noncontextuality inequalities for spin-orbit modes of an intense laser beam. A nonseparable mode was prepared by a radial polarization converter, and we also showed an improvement of the violation of the CHSH inequalities by the nonseparable spin-orbit mode. Considering the relevance of contextuality in different scenarios, such as universal quantum computation [71], spin-orbit modes were shown to be an appropriate platform for the experimental investigation of contextuality. We presented a quantum-like contextuality that can be directly compared with the quantum version that applies to single photons prepared in an entangled state of transverse and polarization degrees of freedom. In the genuine quantum case, photon detection gives dichotomic results, $H/V$ or $+/-$, and intensities were mapped to probabilities associated with these possible outcomes. Therefore, the results obtained in the classical framework are comparable to the ensemble of quantum measurements but not to the individual events with discrete outcomes, which belong to the realm of the quantum nature of light.

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