# Sub-shot-noise interferometric timing measurement with a squeezed frequency comb

Shaofeng Wang,[1,2] Xiao Xiang,[1,2] Nicolas Treps,[3] Claude Fabre,[3,*] Tao Liu,[1,2] Shougang Zhang,[1,2] and Ruifang Dong[1,2,†]
[1] *Key Laboratory of Time and Frequency Primary Standards, National Time Service Center, Chinese Academy of Sciences, Xi'an 710600, China*
[2] *University of Chinese Academy of Sciences, Beijing 100049, China*
[3] *Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-PSL Research University, Collège de France, 4 Place Jussieu, 75252 Paris, France*

(Received 1 May 2018; published 13 November 2018)

Accurate time-delay measurement has become a crucial issue for space positioning experiments which require increasing precision over large distances. The femtosecond frequency comb has been found to be a good source for its advantages of a large nonambiguity range and coherence length up to tens of kilometers or more. A scheme combining homodyne detection and frequency combs was proposed [B. Lamine *et al.*, Phys. Rev. Lett. **101**, 123601 (2008)] that leads to a timing sensitivity reaching the shot-noise limit; with a squeezed frequency comb as the input field, the measured timing sensitivity can be further improved to the level of the sub-shot-noise limit. Based on this scheme and a squeezed frequency comb that was measured to have a phase squeezing of 1.5 dB at an analyzing frequency of 2 MHz, a narrow-band interferometric timing measurement is implemented. The minimum measurable timing variation modulation at 2 MHz is reduced from a shot-noise-limited value of $(2.8 \pm 0.1) \times 10^{-20}$ s to $(2.4 \pm 0.1) \times 10^{-20}$ s.

DOI: 10.1103/PhysRevA.98.053821

## I. INTRODUCTION

Accurate time-delay measurement between remote clocks is fundamental to many areas ranging from navigation and global positioning to tests of general relativity theory, long baseline interferometry in radio astronomy, gravitational wave observation, and future quantum telecommunications. According to Einstein's theory of time synchronization [1,2], time-delay measurement between two observers A and B consists in repeatedly exchanging light pulses and measuring their arrival times. The measurement sensitivity of timing fluctuation $\Delta u$ fundamentally determines the achievable accuracy of time delay. The conventional method is performed by measuring the arrival time of the maximum of the pulse envelope, which is called incoherent time-of-flight (TOF) measurement [3]. The standard quantum limit (SQL) for the measurable timing sensitivity of the TOF measurement is inversely proportional to the frequency bandwidth $\Delta\omega$, i.e., $(\Delta u)^{\mathrm{TOF}} \geqslant (\Delta u)^{\mathrm{TOF}}_{\mathrm{SQL}} = 1/2\Delta\omega\sqrt{N}$, where $N$ is the mean total number of photons measured in the experiment during the detection time. With the help of optical interference, the second method of measuring the arrival time is determined by the phase shift on the signal arriving from A relative to a local oscillator (LO) derived from the local clock in B after transferring a given distance and is referred to as a coherent phase [4,5] measurement. Such measurement has a SQL which is inversely proportional to the center frequency of the signal $\omega_0$, that is, $(\Delta u)^{\mathrm{ph}} \geqslant (\Delta u)^{\mathrm{ph}}_{\mathrm{SQL}} = 1/2\omega_0\sqrt{N}$. As $\omega_0 > \Delta\omega$, the phase method has a better ultimate sensitivity than the TOF technique. However, it is limited to a very short ambiguity range that equals half the laser wavelength. The superiority of a phase-stabilized optical frequency comb over the cw laser source is its large nonambiguity range in addition to an equivalent long coherence length. These dual characteristics have found great utility in characterizing the temporal jitter of the sources themselves for applications that include high-fidelity optical time-delay measurement [6–8]. For example, as proposed by Ye [7] and demonstrated by Cui *et al.* [8], by combining time-of-flight measurement with optical coherence detection, ultrafast pulse trains allow determination of an absolute distance with a precision far beyond an optical wavelength. In accordance with the time region, a time delay with attosecond precision can be attained, which ultimately depends on how excellent detection can be applied to the interference.

As homodyne detection is the optimal optical coherence detection strategy for achieving a SQL precision [9], a different scheme combining femtosecond optical frequency combs and homodyne detection was proposed [10]. Furthermore, by properly shaping the LO pulses [11–13] in the homodyne detection setup, a SQL with a better timing sensitivity of $(\Delta u)_{\mathrm{SQL}} = 1/(2\sqrt{\omega_0^2 + \Delta\omega^2}\sqrt{N})$ can be obtained. It has been further shown [14] that one can directly access the desired parameter while being insensitive to fluctuations induced by parameters of the environment such as pressure, temperature, humidity, and carbon dioxide content by applying appropriate temporal shaping to the LO pulses. Recently, an experiment has been accomplished to demonstrate a quantum-limited measurement of the delay between two laser pulses [15].

*claude.fabre@lkb.upmc.fr
†dongruifang@ntsc.ac.cn

 

Recently, the SQL sensitivity has been overcome by using the cw squeezed states which have been implemented in the proposal of application to gravitational wave measurement [4], a quantum laser pointer [16], and sub-shot-noise spectroscopy [17]. In a similar way, the SQL for timing measurement is promising to be beaten with the quantum noise squeezing of optical frequency combs [18–22]. Benefiting from the large number of photons and from the optimal choice of both the detection strategy and the quantum resource, the proposed scheme represents a significant potential improvement in space-time positioning [10]. In this paper we build upon the experimentally generated quantum optical frequency comb to realize quantum improved measurement of timing fluctuation. The applied squeezed optical frequency comb was generated by a synchronously pumped optical parametric oscillator (SPOPO) and measured to have a phase-quadrature quantum noise reduction of 1.5 dB at an analyzing frequency of 2 MHz. Through a narrow-band homodyne detection, the minimum measurable timing variation of the signal field pulses relative to the LO pulses was reduced from a shot-noise-limited value of $(2.8 \pm 0.1) \times 10^{-20}$ s to $(2.4 \pm 0.1) \times 10^{-20}$ s. By combining this sub-shot-noise interferometric timing measurement setup with the proposal provided by Ye [7], an ultraprecise absolute distance or time-delay measurement can be expected.

## II. THEORETICAL MODEL

Without loss of generality, the electric field emitted by a frequency comb A can be decomposed in different order orthonormal modes $\nu_k(u)$ which have the shape of the well-known Hermite-Gaussian modes in the temporal domain. They are a function of the general space-time light cone variable $u$ [10],

$$\nu_k(u) = \frac{1}{\sqrt{2^n n!}} H_k\left(\frac{u\Delta\omega}{\sqrt{2}}\right) e^{-(u\Delta\omega)^2/4} e^{i\omega_0 u}, \tag{1}$$

with the frequency bandwidth $\Delta\omega$ and the center frequency $\omega_0$. Let us consider the specific case when the train of signal pulses emitted by A have the Gaussian shape of the Hermite-Gaussian mode of zeroth order $\nu_0(u)$ given by Eq. (1) with $k = 0$ and a mean photon number of $N$ during the measurement time. A small timing variation $\Delta u$ leads to a modification of the field received in B. One can write

$$\nu_0(u - \Delta u) \approx \nu_0(u) + \frac{\Delta u}{u_0} w_1(u), \tag{2}$$

with $w_1(u) = \frac{1}{\sqrt{\alpha^2+1}}[i\alpha\nu_0(u) + \nu_1(u)]$, $\alpha = \frac{\omega_0}{\Delta\omega}$, and $u_0 = 1/\sqrt{\omega_0^2 + \Delta\omega^2}$. The information about the timing signal $\Delta u$ is therefore contained in the mode $w_1(u)$, which we will call the timing mode. The timing variation $\Delta u$ can then be retrieved by shaping the LO to be in the timing mode $w_1(u)$ [7–10] and using the balanced homodyne detection (BHD) scheme which projects the input field on the LO mode. The photocurrent $\hat{D}$ generated by the BHD has a mean value which is given by

$$\langle\hat{D}\rangle \propto 2\sqrt{NN_{\mathrm{LO}}}\left(\frac{\Delta u}{u_0}\cos(\theta_s - \theta_{\mathrm{LO}}) + \frac{\alpha}{\alpha^2+1}\sin(\theta_s - \theta_{\mathrm{LO}})\right), \tag{3}$$

where $N_{\mathrm{LO}}$ is the mean number of photon of the local field and $\theta_{\mathrm{LO}}$ and $\theta_s$ are the phase of the local field and of the signal field, respectively. When $\theta_s - \theta_{\mathrm{LO}} = 0$, the optimal detection is implemented. Under this condition, the standard deviation of the BHD signal is given by

$$\Delta\hat{D} = \sqrt{\langle\delta^2\hat{D}\rangle} \propto \sqrt{\frac{N_{\mathrm{LO}}}{1+\alpha^2}(\alpha^2\Delta^2\hat{P}_0 + \Delta^2\hat{Q}_1)}, \tag{4}$$

where $\Delta^2\hat{P}_0$ and $\Delta^2\hat{Q}_1$ are the phase-quadrature fluctuation of the Hermite-Gaussian mode $\nu_0(u)$ and amplitude-quadrature fluctuation of the Hermite-Gaussian mode $\nu_1(u)$, respectively. Consider a signal-to-noise ratio (SNR) of 1 ($\langle\hat{D}\rangle = \Delta\hat{D}$); the minimum resolvable time variation is thus deduced

$$(\Delta u)_{\min} = \frac{1}{2\sqrt{N}(\omega_0^2 + \Delta\omega^2)}\sqrt{\omega_0^2\Delta^2\hat{P}_0 + \Delta\omega^2\Delta^2\hat{Q}_1}. \tag{5}$$

When $\Delta^2\hat{P}_0 = \Delta^2\hat{Q}_1 = 1$, the SQL given by Ref. [7] is then retrieved. It is worth noting that, when a narrow bandwidth ($\Delta\omega \ll \omega_0$) optical frequency comb is used, it is easy to see that $\Delta^2\hat{Q}_1$ almost has no contribution to $(\Delta u)_{\min}$. The minimum achievable timing sensitivity is then reduced to

$$(\Delta u)_{\min} = \frac{\sqrt{\Delta^2\hat{P}_0}}{2\omega_0\sqrt{N}}. \tag{6}$$

In addition, if the input mode $\nu_0(u)$ is a phase-squeezed quantum optical frequency comb, i.e., $\Delta^2\hat{P}_0 < 1$, a sensitivity of $(\Delta u)_{\min} < (\Delta u)_{\mathrm{SQL}}$ can be realized.

Such a quantum optical frequency comb is conveniently generated by a SPOPO, which is a type-I optical parametric oscillator pumped by a train of ultrashort pulses that are synchronized with the pulses making round-trips inside the optical cavity. As long as the pump power $P$ is close to but below the SPOPO oscillation threshold $P_{\mathrm{thr}}$, the orthonormal modes are highly squeezed in alternative quadratures. The phase-quadrature fluctuation of the Hermite-Gaussian mode $\nu_0(u)$ projected into the BHD setup can be shown to be

$$\Delta^2\hat{P}_0(\Omega) = 1 - \varsigma\eta_{\mathrm{tot}}\frac{4\gamma_s^2 r}{\gamma_s^2(1+r)^2 + \Omega^2}, \tag{7}$$

where $\Omega$ is the analysis frequency; $\varsigma$ and $\gamma_s$ are the escape efficiency and decay rate of the SPOPO, respectively; $\eta_{\mathrm{tot}} = \rho\eta\xi^2$ is the total detection efficiency of the BHD setup, where $\rho$ is the quantum efficiency of the balanced photodetectors pair, $\eta$ is the propagation loss of the signal, and $\xi$ is the mode matching efficiency between the signal and LO; and $r$ is the normalized amplitude pumping rate, defined as $r = \sqrt{P/P_{\mathrm{thr}}}$.

## III. EXPERIMENTAL SETUP

The simplified experimental setup for quantum optical frequency comb generation and sub-shot-noise timing measurement is shown in Fig. 1. The utilized laser source was a commercial Ti:sapphire oscillator (Fusion 100-1200, Femtolasers), which produced nearly-Fourier-transform-limited pulses with a duration of 130 fs at 815 nm with an average power of 1.4 W and a repetition rate of 75 MHz. Approximately 400 mW of the laser output was focused on a 0.5-mm-long $BiB_3O_6$ (BiBO) crystal for frequency doubling, and a frequency-doubling efficiency of 23% was achieved,

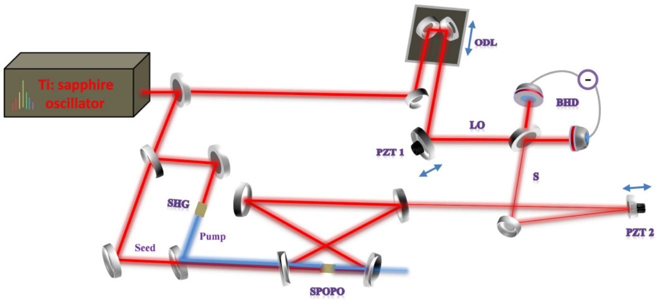


FIG. 1. Simplified experimental layout for quantum optical frequency comb generation and sub-shot-noise timing measurement. The Ti:sapphire oscillator produced nearly Fourier-transform-limited pulses with a duration of 130 fs at 815 nm and a repetition rate of 75 MHz. The following denotations are used: optical delay line, ODL; SHG, second harmonic generation, where a 0.5-mm-long BIBO crystal was used for frequency doubling; SPOPO, synchronously pumped optical parametric oscillator, a 4-m-long multifolded 13-mirror ring cavity with a 2-mm-long BIBO crystal located in the cavity beam waist for parametric down-conversion; PZT, piezoelectric transducer; BHD, balanced homodyne detection, with a pair of homemade low-noise photodiodes; LO, local arm with 2.5-mW laser power; and S, signal arm with 2-$\mu$W coherent pulses or phase-quadrature squeezed pulses.

which was enough for pumping the subsequent SPOPO. The SPOPO is a multifolded 13-mirror ring cavity, and the cavity length is about 4 m, which is determined by the pulse repetition rate. For the sake of simplicity, a four-mirror ring cavity is drafted in Fig. 1 for substitution of the complex cavity structure. The cavity was designed to be singly resonant for the signal field at 815 nm and transparent for the pump at 407.5 nm. The reflectivities of the input coupler and the output coupler are 99% and 79.5%, respectively. All the other cavity mirrors are highly reflective at 815 nm. A 2-mm-long BiBO crystal was located in the cavity beam waist for parametric down-conversion. By comparing the measured cavity finesse, which was about 24, with the theoretical value of 27, the escape efficiency $\varsigma$ was estimated to be about 0.814 and the pump threshold was measured to be about 55 mW.

With the help of the Pound-Drever-Hall technique [23], the SPOPO cavity length was stabilized with a reference beam (not shown in Fig. 1) propagating in the opposite direction to the seed path. Then the pump beam, together with an additional weak seed beam, was injected into the SPOPO cavity synchronously. When scanning the relative phase between the pump and the seed beam, phase-sensitive (de)amplification of the seed was observed.

In order to characterize the generated quantum optical frequency comb at the output of the SPOPO, a Mach-Zehnder-like interferometer closed by a BHD setup was built. The SPOPO was placed in the signal arm and about 2.5-mW laser power was used as the LO for BHD. Then the signal and LO were combined on a 50:50 beam splitter (BS) and the two BS outputs then focused onto a pair of homemade low-noise photodiodes [24] (Hamamatsu, S3883, quantum efficiency of $\rho = 0.93$ for BHD). A variable optical delay line in the LO arm was used to ensure the temporal overlap between the two interferometer arms. The relative phase between the LO and signal arm was scanned by a sawtooth modulation applied to piezoelectric transducer (PZT1) in the LO arm. By scanning and locking the relative phase between the LO and the signal arm to be $\pi/2$, the phase-quadrature squeezing of the seed can then be measured.

Furthermore, to perform the sub-shot-noise measurement of timing variation, another piezoelectric transducer actuator (PZT2) attached to a highly reflective mirror with a small angle of incidence (less than 1°) was introduced to the signal arm. The modulation on PZT2 would induce a longitudinal optical path change, which is equivalent to a timing variation between the two interferometer arms. Since the phase fluctuation of the utilized coherent frequency comb source in our experiment is not shot-noise limited until the analyzing frequency of 2 MHz, the timing variation modulation frequency applied on PZT2 is chosen at 2 MHz and the narrow-band homodyne detection around 2 MHz is implemented. Using the method outlined in Ref. [15], the conversion coefficient between the moving distance and the modulation voltage driven at 2 MHz can be calibrated to be $4.96 \pm 0.03$ pm/V, which corresponds to $(1.65 \pm 0.01) \times 10^{-20}$ s/V in timing variation for our system.

## IV. EXPERIMENTAL RESULTS

### A. Quantum optical frequency comb generation

The first step of the experiment is to characterize the quadrature fluctuation of the quantum optical frequency comb generated by the SPOPO. By setting the pump power to 38 mW and locking the relative phase between the pump and the seed beam to the parametric amplification regime, a phase-quadrature squeezed optical frequency comb was generated. The relative phase between the LO and the SPOPO output was scanned by a sawtooth modulation applied to PZT1 in the LO arm. When the 2-$\mu$W beam was measured at the output of the SPOPO, the quadrature variances of the squeezed pulses which are projected onto at analyzing frequency of 2 MHz were then measured and the results are shown in Fig. 2. The black dashed line and red solid line are the shot-noise limit (SNL) and the experimental data, respectively. One can see that, without any loss correction, a quantum optical frequency comb with $2 \pm 0.1$ dB of phase squeezing ($3 \pm 0.1$ dB of antisqueezing) was achieved. The magenta dash-dotted line is the data locking to the phase quadrature. The about 0.5-dB degradation from the scanned phase-quadrature squeezing can be attributed to the excess noise introduced by the phase-locking system. The data acquisition was implemented by using a spectrum analyzer with a resolution bandwidth (RBW) of 100 kHz and a video bandwidth (VBW) of 30 Hz.

The system loss was further carefully evaluated. The propagation efficiency of the squeezed state through optical components was measured to be $\eta = 0.98$. The mode-matching efficiency between the LO and the signal was optimized to be $\xi = 0.865$. By substituting the experimental parameters into Eq. (7), the theoretical calculation of the squeezing degree yields the blue dotted line. The very good agreement between theory and experiments implies that the escape efficiency of SPOPO and the efficiency of detection are the two main restricting factors for the measured squeezing.

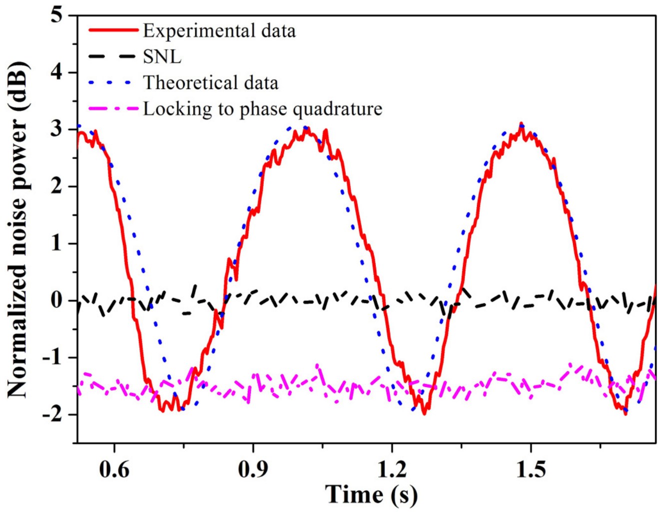


FIG. 2. Measured quadrature variance as a function of the relative phase between the signal and the LO arm. The black dashed line is the shot-noise level, the red solid line is the experimental data, and the blue dotted line is the theoretical curve. The magenta dash-dotted line is the data locking to the phase quadrature. The data acquisition was implemented by using a spectrum analyzer with a RBW of 100 kHz and a VBW of 30 Hz.

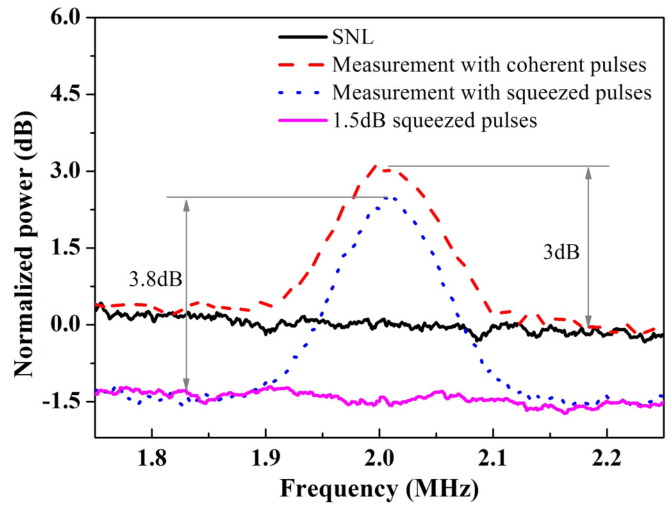


FIG. 3. Measured BHD signals with coherent pulses and with squeezed pulses as the signal arm. The applied modulation voltage on PZT2 at the modulation frequency of 2 MHz was set to 1.7 V. The data acquisition was implemented by using a spectrum analyzer with a RBW of 100 kHz and a VBW of 30 Hz.

### B. Sub-shot-noise measurement of the timing variation

Based on the theory, the minimum resolvable timing variation occurs for a signal that is equal in magnitude to the background noise, which corresponds to a SNR of 1. In order to calibrate the timing measurement sensitivity, a sine amplitude modulation at the modulation frequency of 2 MHz was applied on PZT2 for a certain timing variation. Considering the fact that the noise power measured by BHD is the sum of the signal power and background noise power, a 3 dB of the spectrum analyzer measured signal power above the noise floor implies a SNR of 1.

First, a 2-$\mu$W coherent light produced by blocking the pump was adopted. As shown by the red dashed line in Fig. 3, we observed $3 \pm 0.1$ dB of the spectrum analyzer measured signal above the noise floor at an analyzing frequency of 2 MHz when the applied modulation voltage on PZT2 was 1.7 V, which corresponds to a timing variation of $(2.8 \pm 0.1) \times 10^{-20}$ s at a modulation frequency of 2 MHz, thus $(\Delta u)_{\mathrm{min,coh}} = (8.9 \pm 0.2) \times 10^{-23}$ s$/\sqrt{\mathrm{Hz}}$ was deduced for a RBW of the spectrum analyzer equal to 100 kHz. Comparing with the SQL calculation based on Eq. (6) [$(\Delta u)_{\mathrm{SQL}} = 9.15 \times 10^{-23}$ s$/\sqrt{\mathrm{Hz}}$ with an optical power of 2 $\mu$W and a total detection efficiency of 0.68], very good agreement was achieved, which means that the measurement is indeed shot-noise limited.

From the above-mentioned theoretical model, to demonstrate the sub-shot-noise measurement of the timing variation, signal pulses with phase-quadrature squeezing in mode $v_0(u)$ are needed, while the LO should be maintained in the mode of $i v_0(u)$. In the experiment, the LO was close to the Hermite-Gaussian mode $i v_0(u)$, therefore the relative phase between the signal and LO should be locked to $\pi/2$ in order to satisfy the optimal detection condition. Under this condition, the $1.5 \pm 0.1$ dB phase-quadrature squeezing (magenta solid line) was maintained below the SNL, which is shown in Fig. 3 by the black solid line.

Subsequently, a timing variation measurement was performed using the above 2-$\mu$W, 1.5-dB phase-quadrature-squeezed pulses in mode $v_0(u)$. From the blue dotted line in Fig. 3 we observed $3.8 \pm 0.1$ dB of the spectrum analyzer measured signal above the noise floor at an analyzing frequency of 2 MHz. Subtracting the noise floor contribution, it is clear that the improvement of the SNR is 1.5 dB, which corresponds to the amount of squeezing. Thus a narrow-band sub-shot-noise interferometric timing measurement has been implemented. The minimum detectable timing variation was deduced to be $(2.4 \pm 0.1) \times 10^{-20}$ s, corresponding to $(\Delta u)_{\mathrm{min,sqz}} = (7.5 \pm 0.2) \times 10^{-23}$ s$/\sqrt{\mathrm{Hz}}$, for a RBW of 100 kHz. The observed ratio between $(\Delta u)_{\mathrm{min,sqz}}$ and $(\Delta u)_{\mathrm{min,coh}}$ is indeed compatible with the measured squeezing.

## V. CONCLUSION

Using a 130-fs Ti:sapphire mode-locked laser and SPOPO, we have successfully generated a phase-squeezed quantum optical frequency comb, which has a measured squeezing of 1.5 dB at an analyzing frequency of 2 MHz. Further applying the phase-quadrature squeezing, a narrow-band sub-shot-noise interferometric timing measurement at a modulation frequency of 2 MHz has been implemented. The result shows that the minimum measurable time variation was reduced from $(2.8 \pm 0.1) \times 10^{-20}$ s to $(2.4 \pm 0.1) \times 10^{-20}$ s. Such a demonstration verifies that a quantum optical frequency comb can be used to effectively improve the timing sensitivity beyond the SQL. Our implementation is an important step in the direction of a real sub-shot-noise timing measurement, though the true timing measurements need a setup that can provide sub-shot-noise measurement at lower frequencies. To go further, the amount of squeezing and the attainability of the phase squeezing at low frequency are necessary and should be technically solvable.

## ACKNOWLEDGMENTS

We wish to acknowledge support from National Natural Science Foundation of China (Grants No. 61875205 and No. 91836301), Research Equipment Development Project of Chinese Academy of Sciences (Quantum optimization time transfer experiment system based on femtosecond optical frequency combs), the National Youth Talent Support Program of China (Grant No. [2013]33), Frontier Science Key Research Project of Chinese Academy of Sciences (Grant No. QYZDB-SSWSLH007), European Union Grant QCUMbER (No. 665148), and the French National Research Agency project SPOCQ.

[1] A. Einstein, Ann. Phys. (Leipzig) **322**, 891 (1905).
[2] A. Einstein, *The Collected Papers of Albert Einstein, Volume 2: The Swiss Years: Writings, 1900–1909* (Princeton University Press, Princeton, 1989).
[3] V. Giovannetti, S. Lloyd, and L. Maccone, Nature (London) **412**, 417 (2001).
[4] C. Caves, Phys. Rev. D **23**, 1693 (1981).
[5] M. T. Jaekel and S. Reynaud, Europhys. Lett. **13**, 301 (1990).
[6] F. R. Giorgetta, W. C. Swann, L. C. Sinclair, E. Baumann, I. Coddington, and N. R. Newbury, Nat. Photon. **7**, 434 (2013).
[7] J. Ye, Opt. Lett. **29**, 1153 (2004).
[8] M. Cui, M. G. Zeitouny, N. Bhattacharya, S. A. van den Berg, H. P. Urbach, and J. J. M. Braat, Opt. Lett. **34**, 1982 (2009).
[9] V. Delaubert, N. Treps, C. Fabre, H. Bachor, and P. Réfrégier, Europhys. Lett. **81**, 44001 (2008).
[10] B. Lamine, C. Fabre, and N. Treps, Phys. Rev. Lett. **101**, 123601 (2008).
[11] G. Labroille, O. Pinel, N. Treps, and M. Joffre, Opt. Express **21**, 21889 (2013).
[12] C. Zhou, B. Li, X. Xiang, S. Wang, R. Dong, T. Liu, and S. Zhang, Opt. Express **25**, 4038 (2017).
[13] M. Sato, M. Suzuki, M. Shiozawa, T. Tanabe, K. Ohno, and F. Kannariet, Jpn. J. Appl. Phys. **41**, 3704 (2002).
[14] P. Jian, O. Pinel, C. Fabre, B. Lamine, and N. Treps, Opt. Express **20**, 27133 (2012).
[15] V. Thiel, J. Roslund, P. Jian, C. Fabre, and N. Treps, Quantum Sci. Technol. **2**, 034008 (2017).
[16] N. Treps, N. Grosse, W. P. Bowen, C. Fabre, H. Bachor, and P. K. Lam, Science, **301**, 940 (2003).
[17] P. H. Souto Ribeiro, C. Schwob, A. Maître, and C. Fabre, Opt. Lett. **22**, 1893 (1997).
[18] G. J. de Valcárcel, G. Patera, N. Treps, and C. Fabre, Phys. Rev. A **74**, 061801(R) (2006).
[19] G. Patera, N. Treps, C. Fabre, and G. de Valcárcel, Eur. Phys. J. D **56**, 123 (2010).
[20] O. Pinel, P. Jian, R. M. de Araujo, J. Feng, B. Chalopin, C. Fabre, and N. Treps, Phys. Rev. Lett. **108**, 083601 (2012).
[21] L. Lopez, B. Chalopin, A. R. Rivière de la Souchère, C. Fabre, A. Maître, and N. Treps, Phys. Rev. A **80**, 043816 (2009).
[22] O. Pinel, J. Fade, D. Braun, P. Jian, N. Treps, and C. Fabre, Phys. Rev. A **85**, 010101(R) (2012).
[23] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, Appl. Phys. B **31**, 97 (1983).
[24] S. Wang, X. Xiang, C. Zhou, Y. Zhai, R. Quan, M. Wang, F. Hou, S. Zhang, R. Dong, and T. Liu, Rev. Sci. Instrum. **88**, 013107 (2017).