Resonant enhancement of an oscillating electric field in an atom

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When an atom is placed into an oscillating electric field with frequency far from atomic resonances, the atomic electrons partly shield this field at the nucleus. It is conjectured that when the frequency of an electric field reaches an atomic resonance, the electric field at the nucleus may be significantly enhanced. In this paper we systematically study the mechanisms of this enhancement and show that it may reach five orders in magnitude in particular cases. As an application, we consider laser-assisted neutron capture in 139-lanthanum nucleus and screening and resonance enhancement of nuclear electromagnetic transitions by electrons.

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I. INTRODUCTION

It is well known that electron shells in an atom screen the atomic nucleus from an external electric field. As a consequence, a nuclear electric dipole moment (EDM) is practically unobservable due to this screening [1], and the atomic electrons partly shield the radiation to and from the nucleus [2]. This screening is a big obstacle in the study of CP-violating nuclear forces, which may provide valuable information about new physics beyond the Standard Model; see, e.g., [3–8] for reviews. Therefore, it is tempting to study not just small violations of the Schiff theorem, but to find large enhancements of nuclear electric moments.

To recall the basic idea of the Schiff theorem [1], let us consider a Hamiltonian H_0 of an atom with stationary states $|n\rangle$ and energy levels $\mathcal{E}_n$,

$$H_0|n\rangle = \mathcal{E}_n|n\rangle. \tag{1}$$

The electric field induced by the atomic electrons at the center of an atom is described by the operator

$$\mathbf{E}_e = -e\sum_i \frac{\mathbf{r}_i}{r_i^3} = \frac{i}{e\hbar Z}[H_0, \mathbf{p}], \tag{2}$$

where Z is the charge of the nucleus and $\mathbf{p}$ is the momentum operator for the electrons, $\mathbf{p} = \sum_i \mathbf{p}_i$ ($\mathbf{p}_i$ is the momentum of the ith electron; below we omit the summation symbol over the atomic electrons). When the atom is placed in a static electric field $\mathbf{E}_0$, according to the Schiff theorem [1], this electric field in the center of an atom is screened due to the electric field induced by atomic electrons,

$$\langle \mathbf{E}_e\rangle + \mathbf{E}_0 = 0. \tag{3}$$

It is instructive to recall a proof of this theorem. Let ψ be an exact wave function of the atom in the static electric field $H_E\psi = \mathcal{E}\psi$, where $H_E = H_0 - e\,\mathbf{E}_0\cdot\mathbf{r}$ is the full Hamiltonian and e is the electron charge ($e = -|e|$). Then the relation (3) immediately follows from the identity

$$0 = \langle\psi|\frac{i}{e\hbar}[H_E, \mathbf{p}]|\psi\rangle. \tag{4}$$

Physically, this means that the nuclear EDM is screened by the electron shells. For real atoms, however, this shielding is not complete due to effects of the finite size of the nucleus [1]. The shielding is incomplete in atomic [9] and molecular ions [10] and in atoms in a nonstable state [11].

Recently it was shown that an oscillating electric field also has only partial shielding inside the atom since the electrons respond to the changes in the electric field with a delay [2,12]. More specifically, when the frequency of an external electric field is far from atomic resonance, the field at the nucleus is proportional to the atomic dynamical polarizability, and the nuclear EDM is shielded only partly.

In the case when the frequency of an external field reaches the atomic resonance the situation changes drastically: the resulting electric field at the nucleus may be enhanced significantly. This case, however, requires a careful consideration since the solution at a resonance is very different from the off-resonance case. The aim of this paper is to provide an appropriate study of this case and to determine properties of an electric field inside an atom when the external electric field oscillates in a resonance with an atomic transition.

When the frequency of an external electric field is in resonance with an atomic transition between a ground state $|0\rangle$ and an excited state $|1\rangle$, it is possible to neglect other atomic states and treat the atom as a two-level system. In this case, there are temporal oscillations in the populations in the two-level system known as the Rabi oscillations [13].

For real atoms, however, it is necessary to take into account the spontaneous decay of the excited state with a rate Γ. Such an atom is in a mixed state, and the density matrix description is appropriate now. The evolution of such a system is governed by the optical Bloch equation [14]. The nonperturbative solution of these equations is studied in detail in [15]. We will apply this solution to find the electric field at the nucleus described by the operator (2).

It is pertinent to give an intuitive description of the Rabi oscillation when the spontaneous decay of the excited state is allowed. The behavior of this system is similar to the damped harmonic oscillator with a driving force in resonance with

the oscillator. At a sufficiently large time, when the transient oscillation may be neglected, the amplitude of the driven oscillation is independent of the amplitude of the applied force, and the phase of the resulting oscillation is shifted by $\pi/2$ with respect to the phase of the applied force. A similar situation is observed in atoms: as we will show in this paper, the amplitude of the electric field at the center of an atom is independent of the amplitude of an applied field (and, so, may be significantly enhanced) while the phase of this field is shifted by $\pi/2$ with respect to the phase of the applied electric field. This observation is one of the main results of this paper.

The rest of the paper is organized as follows. In Sec. II we start with a revision of the two-level atom description within the density matrix approach and apply this density matrix for deriving the shielding of the oscillating external electric field at the center of the atom when the frequency of this field is far from atomic resonances. In Sec. III we consider the density matrix near an atomic resonance and apply it to estimate the enhancement of the electric field at the atomic nucleus. In Sec. IV we give numerical estimates for the enhancement of the oscillating electric field in resonance with an E1 transition in a xenon atom and argue that the atomic resonance may give a significant enhancement of the amplitude of the process of the laser-induced neutron capture to the ^{139}La nucleus. We also consider qualitatively the shielding and the resonant enhancement of the photon capture and radiation with energy above the ionization potential in atoms. Section V is devoted to a summary of the results and an overview of their possible applications. In the Appendix, using the standard time-dependent perturbation theory based on the wave function approach, we re-derive the results of the screening of an electric field reported in Sec. III.

II. SHIELDING OF AN OSCILLATING ELECTRIC FIELD IN AN ATOM

We start this section with a short review of the density matrix description of a two-level quantum system with nonvanishing decay rate Γ from the excited state $|1\rangle$ to the ground state $|0\rangle$. Details of this approach may be found in many monographs, see, e.g., [16]. Then we apply the density matrix solution for determining the shielding of an oscillating electric field inside an atom when the frequency of the field is far from atomic resonances.

A. Free two-level atom

Let us consider a two-level atom with a ground state $|0\rangle$ and an excited state $|1\rangle$. The free Hamiltonian can be represented as

$$H_0 = \mathcal{E}_0|0\rangle\langle 0| + \mathcal{E}_1|1\rangle\langle 1|, \tag{5}$$

where $\mathcal{E}_0$ and $\mathcal{E}_1$ are the energies of the ground and excited states, respectively. If there is no spontaneous decay from the excited state to the ground state, the atom is in a pure state in any moment of time. However, if the excited state $|1\rangle$ has a finite lifetime, the atom is in a mixed state at any moment of time $t > 0$. In this case the state is described by a Hermitian density matrix $\rho = \begin{pmatrix}\rho_{11} & \rho_{10}\\ \rho_{01} & \rho_{00}\end{pmatrix}$, $\rho^\dagger = \rho$, with $\mathrm{tr}\,\rho = 1$. The density matrix obeys the von Neumann equation

$$\partial_t \rho = -\frac{i}{\hbar}[H_0, \rho] + (\partial_t \rho)_{\mathrm{spont}}, \tag{6}$$

where the term $(\partial_t \rho)_{\mathrm{spont}}$ describes the damping due to spontaneous emission:

$$(\partial_t \rho_{11})_{\mathrm{spont}} = -\Gamma \rho_{11}, \tag{7a}$$

$$(\partial_t \rho_{10})_{\mathrm{spont}} = -\frac{\Gamma}{2}\rho_{10}. \tag{7b}$$

Equations (7) have a clear interpretation: Eq. (7a) describes how the population of the excited state $|1\rangle$ decays because of the spontaneous emission with a rate Γ while Eq. (7b) shows that the damping of the coherence ρ_{10} between $|1\rangle$ and $|0\rangle$ appears with a rate twice smaller (see, e.g., [16] for details).

B. Two-level atom in an oscillating electric field

Let us consider an oscillating electric field

$$\mathbf{E}(t) = \mathbf{E}_0 \cos\omega t, \tag{8}$$

with amplitude $\mathbf{E}_0$ and frequency ω. The Hamiltonian of the two-level atom in this field reads

$$H = H_0 + V(t), \tag{9a}$$

$$V(t) = -\mathbf{D}\cdot\mathbf{E}\cos\omega t, \tag{9b}$$

where $\mathbf{D} = e\,\mathbf{r}$ is the electric dipole operator. In the basis of states $|0\rangle$ and $|1\rangle$ the interaction potential V can be rewritten as

$$V = \hbar\Omega(|0\rangle\langle 1| + |1\rangle\langle 0|)\cos\omega t, \tag{10}$$

where

$$\Omega = -\frac{1}{\hbar}\langle 0|\mathbf{D}\cdot\mathbf{E}|1\rangle \tag{11}$$

is the Rabi frequency.

The von Neumann equation

$$\partial_t \rho = -\frac{i}{\hbar}[H, \rho] + (\partial_t \rho)_{\mathrm{spont}} \tag{12}$$

implies the following equations for the components of the density matrix:

$$\partial_t \rho_{11} = -\Gamma\rho_{11} + i\Omega\cos\omega t(\rho_{10} - \rho_{01}), \tag{13a}$$

$$\partial_t \rho_{10} = -i\omega_{10}\rho_{10} - \frac{\Gamma}{2}\rho_{10} - i\Omega\cos\omega t(\rho_{00} - \rho_{11}), \tag{13b}$$

where $\omega_{10} \equiv \frac{1}{\hbar}(\mathcal{E}_1 - \mathcal{E}_0)$. Note that $\rho_{10} = (\rho_{01})^*$ and $\rho_{00} = 1 - \rho_{11}$.

We stress that Eqs. (13) describe exact evolution of the two-level atom in an oscillating electric field where the excited state $|1\rangle$ may decay spontaneously to the ground state $|0\rangle$ with the rate Γ.

It is suitable to introduce the following notations:

$$a(t) = \rho_{00}, \quad b(t) = \rho_{10} - \rho_{01}, \quad c(t) = \rho_{10} + \rho_{01}. \tag{14}$$

The system of equations (13) may be equivalently rewritten as

$$\dot{a} + \Gamma a = -\frac{\Omega}{\omega_{10}}\left(\dot{c} + \frac{\Gamma}{2}c\right)\cos\omega t, \tag{15a}$$

$$b = \frac{i}{\omega_{10}}\left(\dot{c} + \frac{\Gamma}{2}c\right), \tag{15b}$$

$$\ddot{c} + \Gamma\dot{c} + \left(\omega_{10}^2 + \frac{\Gamma^2}{4}\right)c = 2\Omega\omega_{10}(2a-1)\cos\omega t. \tag{15c}$$

Below we consider explicit solutions of these equations in a weak field and apply them for determining the electric field inside the atom.

C. Weak external electric field

According to Eq. (11), the Rabi frequency Ω is small when the external electric field is weak. More specifically, we consider the regime when $2\Omega^2 \ll \Gamma^2$. In this case, we can keep the leading in Ω terms in the function $a(t)$ and $c(t)$: $a(t) \propto \Omega^2$ and $c(t) \propto \Omega$. In this approximation Eq. (15c) acquires the form of the classical damped harmonic oscillator

$$\ddot{c} + \Gamma\dot{c} + \left(\omega_{10}^2 + \frac{\Gamma^2}{4}\right)c = -2\Omega\omega_{10}\cos\omega t. \tag{16}$$

The steady state solution of this equation reads

$$c(t) = \frac{2\Omega\omega_{10}}{\sqrt{(\omega^2 - \omega_{10}^2 - \Gamma^2/4)^2 + \Gamma^2\omega^2}}\cos(\omega t + \varphi), \tag{17}$$

where

$$\varphi = \arctan\frac{\Gamma\omega}{\omega^2 - \omega_{10}^2 - \Gamma^2/4}. \tag{18}$$

Given the solution (17) it is straightforward to find the functions $a(t)$ and $b(t)$ from Eqs. (15a) and (15b). However, we do not need these functions in our further considerations.

When the frequency of the external electric field is far from the atomic resonance, it is appropriate to apply the approximation

$$\omega^2 - \omega_{10}^2 - \Gamma^2/4 \approx \omega^2 - \omega_{10}^2. \tag{19}$$

Taking into account this approximation and considering Γ as a small parameter, we keep only the leading terms in Γ in the series expansion of the function (17):

$$c(t) = \frac{2\Omega\omega_{10}}{\omega^2 - \omega_{10}^2}\cos\omega t - \frac{2\Gamma\Omega\omega\omega_{10}}{(\omega^2 - \omega_{10}^2)^2}\sin\omega t. \tag{20}$$

Below we apply this solution to find the induced electric field at the center of the atom in the case when the applied electric field is weak.

D. Induced electric field at the center of the atom

The electric field at the center of the atom consists of two contributions: (i) the external electric field (8) and (ii) the field induced by the atomic electrons which we denote by $\mathbf{E}_e$,

$$\mathbf{E}_{\rm tot} = \mathbf{E}(t) + \langle\mathbf{E}_e\rangle. \tag{21}$$

The latter may be found from the relation

$$\langle\mathbf{E}_e\rangle = \frac{i}{e\hbar Z}\mathrm{tr}\,([H_0, \mathbf{p}]\rho) = -\frac{i}{e\hbar Z}\mathrm{tr}\,([H_0, \rho]\mathbf{p}), \tag{22}$$

where H_0 is the free Hamiltonian (5) and $\mathbf{p}$ is the momentum operator.

Without loss of generality we assume that the external electric field is directed along the z axis, $\mathbf{E}_0 = (0, 0, E_0)$. Then we need to consider only the z component of the electric field due to the atomic electrons $\langle\mathbf{E}_e\rangle_z \equiv \langle E_{e,z}\rangle = -\frac{i}{e\hbar Z}\mathrm{tr}\,([H_0, \rho]p_z)$ [17]. Assuming the nonrelativistic relation between the momentum and position operators $\mathbf{p} = i\frac{m_e}{\hbar}[H_0, \mathbf{r}]$, the operator p_z may be written in the basis of $|0\rangle$ and $|1\rangle$ states as

$$p_z = \frac{i}{e}m_e\omega_{10}\langle 1|D_z|0\rangle(|0\rangle\langle 1| - |1\rangle\langle 0|), \tag{23}$$

where $D_z = e\,z$ is the z component of the electric dipole operator.

Substituting (23) into (22) we find

$$\begin{aligned}\langle E_{e,z}\rangle &= -\frac{\omega_{10}^2 m_e}{Ze^2}\langle 1|D_z|0\rangle(\rho_{10} + \rho_{01}) \\ &= -\frac{\omega_{10}^2 m_e}{Ze^2}\langle 1|D_z|0\rangle c(t),\end{aligned} \tag{24}$$

where we employed the introduced above notation (14).

E. Partial shielding of an electric field off-resonance

When the frequency of the applied electric field is far from the atomic resonance, the function $c(t)$ is given by Eq. (20). Substituting this function into Eq. (24) we find the electric field at the center of the atom produced by the atomic electrons

$$\begin{aligned}\langle E_{e,z}\rangle = &\frac{2m_e}{\hbar e^2 Z}\frac{\omega_{10}^3|\langle 1|D_z|0\rangle|^2}{\omega^2 - \omega_{10}^2}E_0\cos\omega t \\ &- \frac{2m_e}{\hbar e^2 Z}\frac{\omega_{10}^3\omega\Gamma|\langle 1|D_z|0\rangle|^2}{(\omega^2 - \omega_{10}^2)^2}E_0\sin\omega t.\end{aligned} \tag{25}$$

This expression can be easily generalized to a real atom with a complete system of atomic states $|n\rangle$,

$$\langle E_{e,z}\rangle = E_1\cos\omega t + E_2\sin\omega t, \tag{26a}$$

$$E_1 = E_0\frac{2m_e}{\hbar e^2 Z}\sum_n\frac{\omega_{n0}^3|\langle n|D_z|0\rangle|^2}{\omega^2 - \omega_{n0}^2}, \tag{26b}$$

$$E_2 = -E_0\frac{2m_e\omega}{\hbar e^2 Z}\sum_n\frac{\omega_{n0}^3\Gamma_n|\langle n|D_z|0\rangle|^2}{(\omega^2 - \omega_{n0}^2)^2}, \tag{26c}$$

where $\omega_{n0} = \frac{1}{\hbar}(\mathcal{E}_n - \mathcal{E}_0)$.

Equations (26) are derived in the assumption that the widths Γ_n are constant while they may have strong dependence on energy in a general case. For example, the radiative widths scale on energy as $\Gamma_n^{(r)} \propto \omega_r^3$, and all $\Gamma_n^{(r)} \to 0$ if $\omega_r \to 0$, which is the case of an atom in the ground state.

Using the identity $\frac{\omega_{n0}^2}{\omega^2 - \omega_{n0}^2} = \frac{\omega^2}{\omega^2 - \omega_{n0}^2} - 1$ and completeness of the system of states $|n\rangle$ the amplitude of the electric field (26b) can be cast in the form

$$E_1 = -E_0 - E_0\alpha_{zz}(\omega)\frac{\omega^2 m_e}{e^2 Z}, \tag{27}$$

where

$$\alpha_{zz}(\omega) = \frac{2}{\hbar}\sum_n \frac{\omega_{n0}|\langle 0|D_z|n\rangle|^2}{\omega_{n0}^2 - \omega^2} \tag{28}$$

is the atomic dynamical polarizability.

The first term on the right-hand side in (27) cancels the external electric field when substituted into Eq. (21). The resulting electric field at the center of the atom is

$$E_{\rm tot} = -\alpha_{zz}(\omega)\frac{\omega^2 m_e}{e^2 Z}E_0\cos(\omega t) + E_2\sin(\omega t), \tag{29}$$

where E_2 is given in Eq. (26c). Thus, the shielding of an oscillating electric field by the atomic electrons is not complete.

The first term in (29) was first derived in [2] using the wave-function approach. The last term in (29) is a correction due to finite widths of the states. Although this correction is small, it dampens the shielding of the electric field by atomic electrons. This damping appears due to the finite lifetime of excited states of the atom.

Recall that expression (29) is obtained in the approximation when the applied electric field is weak. In this case, the standard time-dependent perturbation theory based on the wave function description is also applicable. In Appendix we demonstrate that the standard time-dependent perturbation theory yields the same result (29) for the electric field at the center of the atom.

III. ENHANCEMENT OF AN ELECTRIC FIELD NEAR RESONANCE

A. Density matrix near resonance

Let us consider the frequency of the external electric field near the atomic resonance,

$$\omega = \omega_{10} + \delta, \tag{30}$$

where δ is a small parameter. To obtain a solution of Eqs. (13) in this case, it is useful to apply the rotating wave approximation (RWA), see, e.g., [16]. This approximation is effectively taken into account by representing the cosine factor as $\cos\omega t = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$ and keeping only the following (resonant) terms in Eqs. (13):

$$\partial_t\rho_{11} = -\Gamma\rho_{11} + \frac{i}{2}\Omega(e^{i\omega t}\rho_{10} - e^{-i\omega t}\rho_{01}), \tag{31a}$$

$$\partial_t\rho_{10} = -i\omega_{10}\rho_{10} - \frac{\Gamma}{2}\rho_{10} - \frac{i}{2}\Omega e^{-i\omega t}(\rho_{00} - \rho_{11}). \tag{31b}$$

A steady state solution of these equations near the resonance has the simple form [16]

$$\rho = \begin{pmatrix} \frac{\Omega^2}{\Gamma^2+2\Omega^2+4\delta^2} & \frac{\Omega(2\delta-i\Gamma)e^{-i\omega t}}{\Gamma^2+2\Omega^2+4\delta^2} \\ \frac{\Omega(2\delta+i\Gamma)e^{i\omega t}}{\Gamma^2+2\Omega^2+4\delta^2} & \frac{\Gamma^2+\Omega^2+4\delta^2}{\Gamma^2+2\Omega^2+4\delta^2} \end{pmatrix}. \tag{32}$$

In particular, for $c(t) = \rho_{10} + \rho_{01}$ we have

$$c(t) = \frac{4\Omega\delta}{\Gamma^2+2\Omega^2+4\delta^2}\cos\omega t - \frac{2\Omega\Gamma}{\Gamma^2+2\Omega^2+4\delta^2}\sin\omega t. \tag{33}$$

We stress that (32) is a particular solution of Eqs. (31) remaining nonvanishing at large time. The general solution includes also the terms which are suppressed by the factor $e^{-\Gamma t}$. We neglect these terms assuming the time t sufficiently large.

B. Resonant enhancement of an electric field

Substituting the solution (33) into Eq. (24) we find the induced electric field at the center of an atom due to atomic electrons

$$\langle E_{e,z}\rangle = E_1\cos\omega t + E_2\sin\omega t, \tag{34}$$

where

$$E_1 = -\frac{m_e}{e^2 Z}\omega_{10}^2\langle 0|D_z|1\rangle\frac{4\Omega\delta}{\Gamma^2+2\Omega^2+4\delta^2}, \tag{35}$$

$$E_2 = \frac{m_e}{e^2 Z}\omega_{10}^2\langle 0|D_z|1\rangle\frac{2\Omega\Gamma}{\Gamma^2+2\Omega^2+4\delta^2}, \tag{36}$$

where δ is the detuning parameter.

The case when the applied electric field is in resonance with the atomic transition $\delta = 0$ is of special interest. In this case, the first term on the right-hand side in (34) vanishes, $E_1 = 0$, while the last one simplifies,

$$\langle E_{e,z}\rangle = \frac{m_e}{e^2 Z}\omega_{10}^2\langle 0|D_z|1\rangle\frac{2\Omega\Gamma}{\Gamma^2+2\Omega^2}\sin\omega t. \tag{37}$$

We stress that the electric field (37) produced by the atomic electrons at the center of the atom has the phase shift $\pi/2$ as compared with the applied electric field (8). Therefore, in the two-level approximation, the external electric field cannot be screened. Indeed, the total field at the center of the atom (21) reads

$$E_{\rm tot} \equiv E_0 + \langle E_{e,z}\rangle = E_{\rm t}\cos(\omega t - \alpha), \tag{38}$$

where

$$E_{\rm t} = \sqrt{E_0^2 + E_2^2}, \tag{39}$$

$$\alpha = \arctan\frac{E_2}{E_0}. \tag{40}$$

When the external electric field is weak, $2\Omega^2 \ll \Gamma^2$, the amplitude of the electric field (37) simplifies:

$$E_2 = -\frac{2m_e}{\hbar e^2 Z}\frac{\omega_{10}^2}{\Gamma}|\langle 0|D_z|1\rangle|^2 E_0. \tag{41}$$

Since the width Γ is typically small, this field is much stronger than the external electric field, $E_2 \gg E_0$, and, so, $E_{\rm t} \approx E_2$. In Appendix we demonstrate that formula (41) can be derived from the standard time-dependent perturbation theory which is valid for a weak external electric field. The derivation given in the Appendix shows that Eq. (41) is applicable in the general case of a multilevel atom in which the excited state may decay to any lower state with a rate Γ.

In real atoms it is necessary to take into account also off-resonance atomic levels, which provide partial shielding of the applied electric field in a similar way as is described in the previous section. This case is considered in detail in Appendix A 2, see Eqs. (A16).

It is important to note that the induced electric field (37) may be much larger than the applied electric field E_0. The

function $\frac{\Gamma\Omega}{\Gamma^2+2\Omega^2}$ reaches its maximum $\frac{1}{2\sqrt{2}}$ at

$$\Omega_{\max} = \Gamma/\sqrt{2}. \tag{42}$$

Thus, the maximum amplitude of the induced electric field is

$$E_{2,\max} = \frac{m_e}{\sqrt{2}e^2 Z}\omega^2 \langle 0|D_z|1\rangle. \tag{43}$$

As it follows from Eqs. (11) and (42), the maximal ratio of the field on the nucleus (43) to the applied external field E_0,

$$\left|\frac{E_{2,\max}}{E_0}\right| = \frac{m_e\omega^2|\langle 0|D_z|1\rangle|^2}{\hbar e^2 Z\,\Gamma} \tag{44}$$

is achieved for the applied field amplitude

$$E_0 = -\frac{\hbar\Gamma}{\sqrt{2}\langle 0|D_z|1\rangle}. \tag{45}$$

This ratio $E_{2,\max}/E_0$ may be very large due to a small linewidth Γ in the denominator. Below we illustrate this enhancement on a particular example of xenon atoms in a laser light.

IV. APPLICATIONS

A. Resonant enhancement of an electric field in a xenon atom

Let us consider a state $|1\rangle$ with the energy $\mathcal{E}_1 = \omega = 8.44$ eV in a xenon atom, $Z = 54$. The natural width of this state and the corresponding matrix element can be deduced, e.g., from [18]: $\Gamma \approx 2\times 10^{-7}$ eV, $\langle 1|z|0\rangle = -\frac{1}{\sqrt{3}}D_{\frac{3}{2}} = -0.66a_{\rm B}$, where $a_{\rm B}$ is the Bohr radius. According to Eq. (45), the amplitude of the applied electric field should be

$$E_0 \approx 40\ \text{V/cm}. \tag{46}$$

The maximum amplitude (43), however, is independent of the strength of the applied field,

$$E_{2,\max} \approx 4.3\times 10^6\ \text{V/cm}. \tag{47}$$

Thus, the ratio of the amplitudes of the resulting electric field on an atomic nucleus and the applied field is

$$\frac{E_{2,\max}}{E_0} \approx 10^5. \tag{48}$$

We conclude that the enhancement of an electric field inside an atom may be up to five orders of magnitude. However, it may be smaller if there is a collisional or Doppler broadening.

B. Laser-stimulated neutron capture in ^{139}La

It is predicted [19–22] that a laser electric field can stimulate the neutron capture in the ^{139}La nucleus, $Z = 57$. This laser field provides mixing of the s and p compound states and may enhance the capture of a neutron to the p-wave resonance. Indeed, a p-wave resonance is kinematically suppressed $\sim 10^6$ times at low neutron energy as compared with an s-wave resonance. The resonance of the p wave is found at energy $\mathcal{E} = 0.734$ eV. However, if one applies a laser field at this energy to excite a low-energy (thermal) neutron to the resonance, this field is significantly screened since it is far from atomic energy levels. According to (27), the electric field is suppressed by the factor

$$\alpha_{zz}\frac{\omega^2 m_e}{e^2 Z} \approx 0.003, \tag{49}$$

where we applied the atomic polarizability $\alpha_{zz} \approx 214\,a_{\rm B}^3 = 4.1\times 10^{-9}\,\text{eV}^{-3}$ calculated in [23].

In the experiment [24–26], a laser was used with the frequency $\omega = 1.165$ eV and strength $E_0 = 8700$ V/cm. The off-resonance suppression of this field gives the amplitude of the total field at the center of an atom

$$E_1 + E_0 = \alpha_{zz}\frac{\omega^2 m_e}{e^2 Z}E_0 \approx 44\ \text{V/cm}. \tag{50}$$

Such a weak field cannot give a significant enhancement of the neutron capture by the ^{139}La nucleus.

As we demonstrate in this paper, a significant enhancement of the electric field can be achieved when the laser field is in resonance with an atomic transition. We consider the excited state $|1\rangle = |4f6s^2\rangle$ in the La atom with energy $\mathcal{E}_1 = 1.88$ eV. The natural width of this state and the E1 matrix element for the $|0\rangle \leftrightarrow |1\rangle$ transition may be deduced from the NIST data [18]: $\Gamma \approx 6\times 10^{-9}$ eV, $\langle 0|z|1\rangle = -1.3a_{\rm B}$. Substituting these parameters into Eq. (45) we find that a relatively weak laser field is sufficient to saturate the atomic transition,

$$E_0 \approx 0.6\ \text{V/cm}. \tag{51}$$

The amplitude of the induced electric field is found from Eq. (43),

$$E_{2,\max} \approx 4\times 10^5\ \text{V/cm}. \tag{52}$$

Thus, the external field is enhanced by six orders of magnitude. However, the enhancement is smaller if there is a collisional or Doppler broadening.

C. Screening and resonance enhancement of nuclear electric dipole transitions by electrons

The excitation energies of nuclear states are typically higher than the ionization energies for atomic electrons. When the photon energy is higher than all atomic electron ionization energies, the screening of the electric field by atomic electrons is negligible, and the high-energy photons penetrate the atom and may be radiated or absorbed by the nucleus.

However, when the energy of a photon is in the region from the ionization energy of outer electrons to the ionization of the lowest $1s$ electron, there is an interesting interplay between the external photon field and induced electron field at the nucleus. In this section we consider this situation qualitatively, without specifying particular examples.

Let us start from off-resonance contributions to the induced electric field, which are described by Eq. (26b). When the photon energy ω is higher than the ionization potentials I_k for electrons in the outermost atomic shell, the contribution of this shell to E_e is suppressed by a small factor ω_{k0}^2/ω^2. However, the contribution of inner shells with $I_k > \omega$ is still significant. This case is analogous to an ion with a number of electrons N in a static field where the electric field on the nucleus is equal to $E_0(Z-N)/Z$ [9]. In the case of the oscillating field we also have the electron field on the nucleus $E_e \sim -E_0 N_{\rm eff}/Z$,

and the total field is $E(0) \sim E_0(Z - N_{\rm eff})/Z$, where $N_{\rm eff}$ is the number of atomic electrons with the ionization potential $I_k > \omega$.

Now let us consider the role of autoionization resonances. When the energy of the incident photon is close to one of the autoionization energy levels, we can apply the formula (41) to estimate the amplitude of the electric field at the center of the atom. Let us rewrite this formula identically as

$$E_{\rm tot} = \frac{2}{Z}\frac{\omega}{\Gamma_n}\tilde{\omega}\tilde{D}_{0n}^2 E_0 \sin\omega t, \tag{53}$$

where

$$\tilde{\omega} \equiv \omega\frac{\hbar a_{\rm B}}{e^2}, \quad \tilde{D}_{0n} \equiv \frac{\langle 0|D_z|n\rangle}{e a_{\rm B}} \tag{54}$$

are the energy and dipole transition matrix element in atomic units, respectively.

For typical E1 atomic transitions in outer electron shells, the factor $\tilde{\omega}\tilde{D}_{0n}^2$ may be estimated as

$$\tilde{\omega}\tilde{D}_{0n}^2 \sim 1. \tag{55}$$

When the photon energy is sufficient for ionizing deeper atomic electrons, this estimate is still applicable. Indeed, we can describe the situation using the effective charge $Z_{\rm eff}$ which a deep electron "sees." The ionization energy $\tilde{\omega}_{n0}$ scales as $Z_{\rm eff}^2$ while the electric dipole matrix element $\tilde{D}_{0n}$ scales as $1/Z_{\rm eff}$. Therefore, Eq. (55) may be used for rough estimates of the electric field near the resonance of autoionizing states.

The ratio ω/Γ_n is typically of order from 300 to 1000 (see, e.g., the data for the energies and lifetimes of hole states in $4p$ shell in xenon [27]). Assuming $\omega/\Gamma_n \sim 500$, we estimate the electric field (53) as

$$E_{\rm tot} \sim \frac{1000}{Z} E_0 \sin\omega t. \tag{56}$$

Thus, the amplitude of the electric field is enhanced by the factor of the order from 10 to 1000, depending on the atomic number Z.

We conclude that the interaction with atomic electrons may strongly affect nuclear electric dipole transitions for energies near autoionization levels of deep atomic shells.

V. CONCLUSIONS

In this paper we systematically studied the problems of shielding and enhancement of the oscillating electric field inside an atom. When the frequency of the external electric field is far from atomic resonances, the electric field at the nucleus is partly shielded. As was found in [2], the shielding coefficient is proportional to the atomic dynamical polarizability. Equation (29) shows that this screening is slightly suppressed when the widths of the states are taken into account.

Note that the electron shells partly screen not only the external electric field inside the atom, but also the nuclear radiation. Thus, the dipole radiation from nucleus may be observed, but it is suppressed by the same factor as in Eq. (29) when its energy is far from atomic resonances.

When the frequency of the external electric field approaches an atomic resonance, the atomic polarizability has a pole and the screening formula (29) is not applicable any more. In resonance, the oscillating electric field causes the atomic transition which may be considered using standard time-dependent perturbation theory. However, the perturbation theory should be applied with care since it gives a divergent result for the electric field induced by atomic electrons on the nucleus unless a width of the state is taken into account.

When the excited state in an atom is allowed to decay spontaneously to the ground state, the wave function description is not appropriate since the atom interacts with a photon and appears in a mixed quantum state. Thus, to describe the atom near resonance it is necessary to use the density matrix solution for a two-level atom [15] (see also [16] for a modern presentation). Recall that when the width of the excited state is small, the atom experiences the Rabi oscillations [13], but these oscillations are damped when the spontaneous decay is taken into account. We apply this solution for the density matrix to derive the resulting electric field at the center of an atom (38) when the external electric field is in resonance with an atomic transition. This is the main result of this paper.

It is important to note that at resonance a relatively weak external electric field is sufficient to saturate the atomic transition. In this case, the electric field induced by the atomic electrons at the center of an atom may be several orders in magnitude stronger than the applied electric field [see, e.g., Eq. (48)]. Another interesting feature is that the phase of the resulting electric field at the center of an atom is shifted approximately by $\pi/2$ with respect to the applied field. These facts should be taken into account when considering physical applications of these results.

As an application, we consider a laser-induced neutron capture in ^{139}La nucleus which was conjectured in [19–22]. However, the experiments [24–26] did not confirm a significant enhancement of the neutron capture process due to the laser field. We argue that one of the reasons for this negative result is the shielding of the electric field in atoms which was not taken into account. Indeed, the shielding factor for the electric field off the atomic resonances in the La atom may be as small as 0.003. However, when the electric field is in resonance with the E1 atomic transition, the electric field at the nucleus may reach 4×10^5 V/cm.

The screening and resonance enhancement of the photon field by electrons may strongly affect emission and absorption of photons by nuclei if the photon energy is smaller than the ionization potentials of deep atomic electrons.

In conclusion, we stress that the resonant enhancement of an electric field in an atom studied in Sec. III B may, in principle, have many further applications and generalizations. In particular, it would be interesting to develop a technique for measuring nuclear EDM using an oscillating electric field in resonance with an atomic or molecular transition. It is also tempting to study a similar enhancement of an oscillating magnetic field, as well as quadrupole and octupole waves due to atomic resonances. These issues deserve separate studies.

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APPENDIX: PERTURBATIVE COMPUTATION OF INDUCED ELECTRIC FIELD

Let us consider an atom with stationary states $|k\rangle$ and energies $\mathcal{E}_k$,

$$H_0|k\rangle = \mathcal{E}_k|k\rangle. \tag{A1}$$

When the atom is placed into a weak oscillating electric field (8), the evolution of the ground state may be described by the wave function $\psi_0(t)$, which in the leading order in the perturbation theory reads

$$\psi_0(t) = e^{-i\omega_0 t}\Bigg[|0\rangle - \frac{i}{\hbar}\sum_k \int_0^t d\tau\, e^{-i\omega_{k0}(t-\tau)-\frac{\Gamma_k}{2}(t-\tau)} \times |k\rangle\langle k|V(\tau)|0\rangle\Bigg], \tag{A2}$$

where $V(\tau)$ is given in Eq. (9b) and Γ_k are the widths of the excited states $|k\rangle$. In general, the widths are not just constants, but rather functions of energy of the system, which is $\mathcal{E} = \mathcal{E}_0 + \hbar\omega$. For example, dipole radiative widths have dependence $\propto \omega_\gamma^3$, where ω_γ is the energy of the radiated photon which is determined by the energy of the system.

Using this wave function we find the expectation value of the operator (2)

$$\begin{aligned}\langle \mathbf{E}_e\rangle = &-\frac{im_e}{\hbar Z}\sum_k e^{-i\omega_{k0}t-\Gamma_k t/2}\omega_{k0}^2 \\ &\times \int_0^t d\tau\, e^{i\omega_{k0}\tau+\Gamma_k\tau/2}\cos(\omega\tau)\langle 0|\mathbf{r}|k\rangle\langle k|\mathbf{E}_0\mathbf{r}|0\rangle \\ &+\frac{im_e}{\hbar Z}\sum_k e^{i\omega_{k0}t-\Gamma_k t/2}\omega_{k0}^2 \\ &\times \int_0^t d\tau\, e^{-i\omega_{k0}\tau+\Gamma_k\tau/2}\cos(\omega\tau)\langle 0|\mathbf{E}_0\mathbf{r}|k\rangle\langle k|\mathbf{r}|0\rangle.\end{aligned} \tag{A3}$$

Here we applied the nonrelativistic relation between the momentum and position operators $\mathbf{p} = i\frac{m_e}{\hbar}[H_0, \mathbf{r}]$.

Without loss of generality we assume further that the external electric field is along the z axis,

$$\mathbf{E}_0 = (0, 0, E_0). \tag{A4}$$

Then, it is sufficient to consider only the z component of the induced electric field (A3),

$$\begin{aligned}\langle E_{e,z}\rangle = &-\frac{im_e}{\hbar Z}E_0\sum_k |\langle 0|z|k\rangle|^2\omega_{k0}^2 \\ &\times\Bigg[e^{-i\omega_{k0}t-\Gamma_k t/2}\int_0^t d\tau\, e^{i\omega_{k0}\tau+\Gamma_k\tau/2}\cos(\omega\tau) \\ &-e^{i\omega_{k0}t-\Gamma_k t/2}\int_0^t d\tau\, e^{-i\omega_{k0}\tau+\Gamma_k\tau/2}\cos(\omega\tau)\Bigg].\end{aligned} \tag{A5}$$

Upon computation of the integrals we represent the induced electric field in the form of a sum of the dumped term $E_{\rm dump}$ and steady term $E_{\rm st}$,

$$\langle E_{e,z}\rangle = E_{\rm damp} + E_{\rm st}, \tag{A6}$$

$$\begin{aligned}E_{\rm damp} = &\frac{m_e}{2\hbar Z}E_0\Bigg[\sum_k \omega_{k0}^2\sin(\omega_{k0}t)e^{-\frac{\Gamma_k t}{2}}|\langle 0|z|k\rangle|^2 g_k^+(\omega) \\ &-\sum_k \omega_{k0}^2\cos(\omega_{k0}t)e^{-\frac{\Gamma_k t}{2}}|\langle 0|z|k\rangle|^2 f_k^-(\omega)\Bigg],\end{aligned} \tag{A7}$$

$$\begin{aligned}E_{\rm st} = &\frac{m_e}{2\hbar Z}E_0\sin(\omega t)\sum_k |\langle 0|z|k\rangle|^2\omega_{k0}^2\Gamma_k g_k^-(\omega) \\ &-\frac{m_e}{\hbar Z}E_0\cos(\omega t)\sum_k |\langle 0|z|k\rangle|^2\omega_{k0}^2 f_k^+(\omega),\end{aligned} \tag{A8}$$

where

$$f_k^\pm(\omega) = \frac{\omega_{k0}+\omega}{(\omega_{k0}+\omega)^2+\Gamma_k^2/4} \pm \frac{\omega_{k0}-\omega}{(\omega_{k0}-\omega)^2+\Gamma_k^2/4}, \tag{A9}$$

$$g_k^\pm(\omega) = \frac{1}{(\omega_{k0}+\omega)^2+\Gamma_k^2/4} \pm \frac{1}{(\omega_{k0}-\omega)^2+\Gamma_k^2/4}. \tag{A10}$$

In what follows, we will discard the terms $E_{\rm damp}$ which are suppressed by the factor $e^{-\Gamma t/2}$ at large time t. We will focus on the driven oscillations $E_{\rm st}$.

For large ω the functions (A9) and (A10) are vanishing. In this case the induced electric field (A6) vanishes, and there is no shielding of the external electric field. This simply means that the high energetic gamma quanta are not screened by atomic electrons and penetrate inside the atom.

1. Off-resonance case

Let us consider the external electric field with the frequency ω far from any atomic resonance,

$$(\omega_{k0}\pm\omega)^2 + \Gamma_k^2/4 \approx (\omega_{k0}\pm\omega)^2. \tag{A11}$$

Taking into account this approximation in Eqs. (A9) and (A10), for the induced electric field (A6) we find

$$\begin{aligned}\langle E_{e,z}\rangle = &-\frac{2m_e}{\hbar Z}E_0\sum_k |\langle 0|z|k\rangle|^2\frac{\omega_{k0}^3}{\left(\omega_{k0}^2-\omega^2\right)^2} \\ &\times\left[\Gamma_k\omega\sin(\omega t) + \left(\omega_{k0}^2-\omega^2\right)\cos(\omega t)\right].\end{aligned} \tag{A12}$$

Applying the identity $\frac{\omega_{k0}^2}{\omega_{k0}^2-\omega^2} = 1 + \frac{\omega^2}{\omega_{k0}^2-\omega^2}$ and completeness of the system of states $|k\rangle$, Eq. (A12) may be cast in the form

$$\begin{aligned}\langle E_{e,z}\rangle = &-E_0\cos\omega t - \frac{m_e\omega^2}{e^2 Z}\alpha_{zz}(\omega)E_0\cos\omega t \\ &-\frac{2m_e\omega}{\hbar Z}\sum_k |\langle 0|z|k\rangle|^2\frac{\omega_{k0}^3\Gamma_k}{\left(\omega_{k0}^2-\omega^2\right)^2}E_0\sin\omega t,\end{aligned} \tag{A13}$$

where $\alpha_{zz}(\omega)$ is the dynamical atomic polarizability (28).

In Eq. (A13), the first term cancels the external electric field (8). The second term in this equation, which is proportional to the dynamical atomic polarizability, was found in [2] as the residual electric field at the nucleus. The terms in the last line in Eq. (A13) represent the corrections to the electric field which appear when we take into account the spontaneous decay of the excited states.

Note that Eq. (A13) fully agrees with the expression for the electric field in the atom (29) which was derived using the solution for the density matrix for a weak external field.

2. Near-resonance case

When the external electric field is in resonance with an atomic level $|n\rangle$, $\omega = \omega_{n0}$, the functions (A9) and (A10) may be written as

$$f_k^+(\omega) = \begin{cases} \frac{1}{2\omega}, & k = n, \\ \frac{2\omega_{k0}}{\omega_{k0}^2 - \omega^2}, & k \neq n, \end{cases} \tag{A14}$$

$$g_k^-(\omega) = \begin{cases} -\frac{4}{\Gamma_n^2}, & k = n, \\ -4\frac{\omega_{k0}\omega}{(\omega_{k0}^2 - \omega^2)^2}, & k \neq n. \end{cases} \tag{A15}$$

Here we assume that all linewidths are small in comparison with energies, $\Gamma_n \ll \omega_{n0} = \omega$ and $(\omega_{k0} \pm \omega)^2 + \Gamma_k^2/4 \approx (\omega_{k0} \pm \omega)^2$ for $k \neq n$. Substituting these functions into (A6) we find

$$\langle E_{e,z} \rangle = -E_1 \cos\omega t - E_2 \sin\omega t, \tag{A16a}$$

$$E_1 = E_0\left[1 + \frac{m_e\omega^2}{e^2 Z}\beta_{zz}(\omega)\right], \tag{A16b}$$

$$E_2 = E_0\frac{2m_e\omega}{e^2\hbar Z}\sum_{k\neq n}|\langle 0|D_z|k\rangle|^2\frac{\omega_{k0}^3\Gamma_k}{\left(\omega_{k0}^2 - \omega^2\right)^2} + E_0\frac{2m_e\omega^2}{e^2\hbar Z\Gamma_n}|\langle 0|D_z|n\rangle|^2, \tag{A16c}$$

where

$$\beta_{zz}(\omega) = -\frac{3}{2\hbar\omega}|\langle 0|D_z|n\rangle|^2 + \frac{2}{\hbar}\sum_{k\neq n}\frac{\omega_{k0}}{\omega_{k0}^2 - \omega^2}|\langle 0|D_z|k\rangle|^2. \tag{A17}$$

This function differs from the atomic polarizability (28) only in the nth term.

The first term in (A16b) cancels the external electric field (8) while the second one represents the residual field after screening. The terms in the first line in (A16c) are analogous to the ones in the second line in (A13). The last term (A16c) appears much larger than the other terms owing to the small linewidth Γ_n in the denominator. Thus, the leading contribution to the total electric field (21) at the center of the atom reads

$$E_{\rm tot} \approx -\frac{2m_e\omega^2}{e^2\hbar Z\Gamma_n}|\langle 0|D_z|n\rangle|^2 E_0 \sin\omega t. \tag{A18}$$

We stress that the phase of this field is shifted by $\pi/2$ with respect to the applied field (8).

Naively, the field (A18) may be very large if the width of the state Γ_n is small enough. However, this is an artifact of the perturbation theory which is resolved in the nonperturbative solution (37).

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