Photodetachment of the H^- ion in a quantum well with one expanding wall

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The standard closed orbit theory is extended to investigate the photodetachment of a negative ion near a moving wall. The photodetachment cross section of the H^- ion in a quantum well with one expanding wall is specifically put forward. In contrast to the photodetachment of the negative ion in a static quantum well, the returning kinetic energy of the detached electron is different from its initial value after collision with the moving wall, therefore an additional modulation factor appears in the oscillating cross section, which depends on the electron's initial outgoing and returning momentum. Four different types of closed orbits are found for the detached electron in a quantum well with one expanding wall, and their connections with the oscillating cross section are analyzed quantitatively. The calculation results suggest that the photodetachment cross section of this system depends on the speed of the moving wall and the initial distances from the ion to the walls sensitively. The method used in this paper is universal and can be extended to study the photodetachment of the nonhydrogenic negative ion near a moving wall, whether the wall is moving with constant velocity or oscillatory. The results provide insight into the behavior of photodetachment dynamics of the negative ion in the presence of a moving boundary and may have potential interest for future experimental researches.

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I. INTRODUCTION

Both experiments and theories have shown that the surface can affect the photodetachment process of negative ions [1–3]. In 2006, Yang *et al.* first studied the photodetachment of the H^- ion near an elastic surface on the basis of semiclassical closed orbit theory [4]. It is found that the photodetachment cross section of the H^- ion near a surface is oscillatory when the laser light polarization is perpendicular to the surface. The oscillations in the cross section can be attributed to the interference effect between the returning electron waves reflected by the surface and the source of the waves localized around the bound state of H^-, which was similar to the photodetachment of the H^- ion in a static electric field [5]. Afaq and Du put forward a theoretical imaging method to investigate the same system [6], and found the oscillations in the photodetachment cross section correspond to the case given by Yang *et al.* [4]. Further work for the photodetachment of the H^- ion near an inelastic surface has been reported subsequently [7]. Hansen *et al.* in 2006 [8] and Novick *et al.* in 2012 [9,10] investigated the escape of a quantum particle from an open vase-shaped cavity. Since the classical trajectory of the particle in a cavity is similar to that of the photodetached electron in a cavity, the investigation of the photodetachment dynamics of the H^- ion near a surface has been extended to a quantum well or a microcavity. For example, Yang *et al.* [11] and Zhao *et al.* [12] used both the closed orbit theory and the quantum mechanical method to study the photodetachment of the H^- ion in a quantum well. Zhao and Du investigated the photodetachment of the H^- ion in a wedge-shaped cavity [13]. Later, the photodetachment of the H^- ion inside a square, circular, or cubic microcavity was studied in great detail [14–17].

In these early studies, the surface in the quantum well or in the microcavity is static, and the detached electron's classical motion is relatively simple. The system is time independent. So, what will happen if the photodetachment process of the negative ion takes place near a moving surface? For the moving boundary problem, the photodetachment process is time dependent and its theoretical treatment becomes complicated and interesting. Until now, no reports have been given. Quantum systems with time-dependent boundary conditions are delicate to handle. Even the simplest system, such as a particle in a box with infinitely high but moving walls, remains the object of ongoing investigations. However, some works related to the moving boundary problem have been reported for several decades. In 1949, Fermi proposed a classical model for cosmic ray production with a particle moving in the inhomogeneous magnetic field, which was later called a Fermi accelerator [18]. A number of articles followed, studying the moving boundary problem from different aspects. For instance, Ulam introduced the Fermi accelerator with a ball bouncing back and forth between two oscillating walls [19]. A quantum mechanical treatment of a particle in an infinite square-well potential with a moving wall was given by Dosecher and Rice [20], who analyzed the case in which one of the walls is static while the other moves with a linear velocity. Furthermore, da Luz and Cheng used the semiclassical approximation to evaluate the propagators for the moving hard-wall potentials [21]. The time-dependent wave functions derived from their propagators are in agreement with those of the corresponding Schrödinger equation, which verifies the correctness of the semiclassical method. Di Martino *et al.* studied the quantum particle in a box with two moving walls [22]. They recast this problem into the equivalent one of a quantum particle in a fixed box the dynamics of which is governed by an appropriate time-dependent Schrödinger

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operator. Very recently, Matzkin investigated the issue of a single-particle nonlocality in a quantum system subjected to time-dependent boundary conditions [23]. From the above studies, one can find that the moving surface problem looks simple, but it can cause some interesting phenomena in reality.

In this paper, we investigate the photodetachment of the H^- ion in a quantum well with one wall moving with a constant velocity based on the semiclassical closed orbit theory. First, we discover all the closed orbits of the detached electron in the quantum well. Then we construct the electron wave function according to the semiclassical approximation. Finally, we put forward an analytical formula for the photodetachment cross section of this system, which can be written as a sum of a smooth background term plus many oscillating terms. The calculation results suggest that the photodetachment cross section of this system depends on the speed of the moving wall and the initial distances from the ion to the walls sensitively. In contrast to the photodetachment of the H^- ion in a quantum well with two fixed walls [11], due to the collision of the detached electron with the moving wall, some energy of the electron will be lost, which makes the returning momentum of the detached electron less than its initial value; as a consequence, the interference effect between the returning electron wave with the initial outgoing electron wave gets weakened. If the moving wall moves very fast, its effect on the photodetachment cross section vanishes. The semiclassical closed orbit theory used in this paper has many advantages: one is reflected in the construction of the wave function. We use the semiclassical approximation to construct the electron wave function instead of solving the time-dependent Schrödinger equation. It is well known that the Schrödinger equation is easy to formulate but its result is difficult to evaluate. However, in the semiclassical approximation, only the closed orbits contribute to the wave function and construction of the wave function becomes much simpler. Another advantage lies in its clear physical description and wide application, which can be extended to study the photodetachment of any nonhydrogenic negative ion near a moving surface for more general motions, such as an oscillatory motion. Therefore, our paper provides a very visual method to study the photodetachment dynamics of the negative ion near a moving boundary.

This paper is organized as follows: In Sec. II, we show a schematic representation of the theoretical model for the photodetachment of the H^- ion in a quantum well with one expanding wall and discuss the classical motion of the photodetached electron. The general formula for calculating the photodetachment cross section of this system has been put forward in Sec. III. In Sec. IV, we calculate the photodetachment cross section of H^- in a quantum well with one wall moving with a constant velocity. Some conclusions of this paper are presented in Sec. V. Atomic units are used in this paper unless indicated otherwise.

A. Theoretical model and the classical motion of the detached electron

Figure 1 shows a schematic representation of the theoretical model for the photodetachment of the H^- ion in a quantum well. The solid dark point at the origin denotes the negative-ion source. The quantum well is composed of two

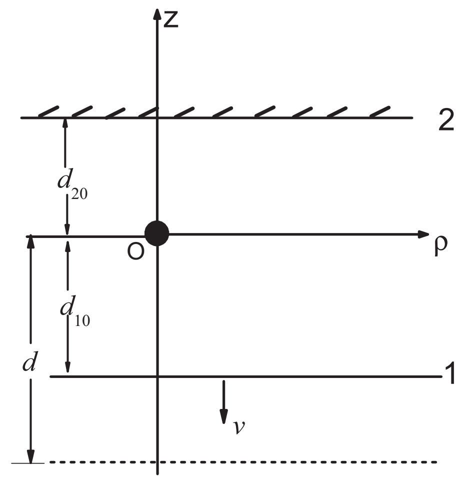

FIG. 1. Schematic representation of the theoretical model for the photodetachment of the H^- ion in a quantum well with one expanding wall.

parallel elastic walls. The moving wall is denoted by number "1" and the fixed wall is number "2." At first, the moving wall is located perpendicular to the $-z$ axis at the $z = -d_{10}$ plane, then it moves at a constant speed v along the $-z$ axis. At time t, the distance from the moving wall to the origin is d: $d = d_{10} + vt$. The second wall is fixed at the $z = d_{20}$ plane.

Following the physical picture description of the closed orbit theory, when a laser light is applied to the negative ion, it may absorb a photon. As the photon energy is larger than the binding energy of the negative ion, the bound electron will be detached. The photodetached electron can be considered as a free particle and moves freely in the quantum well, with the electron trajectories along straight lines inside the quantum well until they are reflected by the surfaces of the wall. After several reflections, the electron may return to the negative-ion source to form a closed orbit. The interference effect between the returning electron waves traveling along the closed orbits with the initial outgoing electron waves induces the oscillatory structures in the photodetachment cross section. In the quantum well, the detached electron has an infinite number of closed orbits which start from the origin and finally return to it after some time. In order to find out the closed orbit of the detached electron, we adopt a similar method as that given by da Luz and Cheng in Ref. [21]. The closed orbits can be classified by specifying which walls (the fixed or the moving) the particle collides with on the first and last collisions. We use two parameters n and m to distinguish different closed orbits, which are non-negative integers ($n = 0, 1, 2\ldots$; $m = 0,1,2\ldots$), where n denotes the number of collisions with the moving wall and m corresponds to the number of collisions with the fixed wall. According to the collision theory, after each collision with the moving wall, the speed of the detached electron will reduce $2v$. Assuming the initial momentum of the detached electron is k_0, then after n collisions with the moving wall the returning momentum of the detached electron becomes $k_{\rm ret} = k_0 - 2nv$. If the initial momentum of the detached electron $k_0 \leqslant 2nv$, it cannot be bounced back by the moving wall to the origin to form a closed orbit. The closed orbit for the detached electron in the

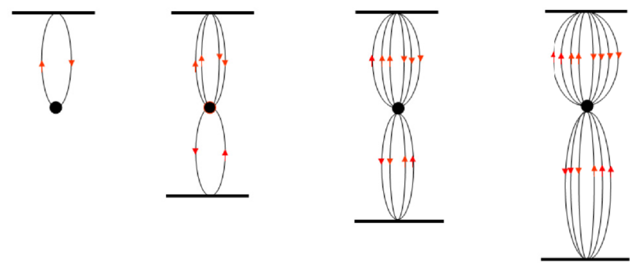

FIG. 2. The first type of closed orbit for the detached electron in the quantum well with one moving wall.

quantum well with one expanding wall can be classified into four types.

(1) Both the first and the last collisions with the fixed wall. For this kind of closed orbit, $m = n + 1$. The initial outgoing angle of the detached electron is $\theta_i = 0$, while the returning angle is $\theta_{\text{ret}} = \pi$. Some closed orbits belonging to this type are given in Fig. 2. Figure 2(a) shows the electron goes up along the $+z$ axis after collision with the fixed wall once and returns back to the origin. This orbit does not collide with the moving wall, so we denote it as the (1,0) closed orbit. Figure 2(b) shows the (2,1) closed orbit, which travels along the $+z$ direction, after being reflected by the fixed wall once, then travels toward the moving wall. If the initial speed of the electron is larger than the speed of the moving surface, the electron will hit the moving wall after a period of time. After being bounced back by the moving wall, it will continue traveling along the $+z$ axis, then hit the fixed wall for the second time and bounce back. Finally, it returns to the origin to form a closed orbit. Figure 2(c) shows the (3,2) closed orbit, which collides with the fixed wall three times and the moving wall twice before it returns to the origin. Figure 2(d) corresponds to the (4,3) closed orbit.

(2) Both the first and the last collisions with the moving wall. For this kind of closed orbit, $m = n - 1$. The initial outgoing angle of the detached electron is $\theta_i = \pi$, while the returning angle is $\theta_{\text{ret}} = 0$. The shape of this kind of closed orbit can be easily inferred from Fig. 2.

(3) The first collision with the fixed wall and the last collision with the moving wall. For this kind of closed orbit, $m = n$. Both the initial outgoing angle and the returning angle of the detached electron are equal to 0: $\theta_{\text{ret}} = \theta_i = 0$.

(4) The first collision with the moving wall and the last collision with the fixed wall. This kind of closed orbit is similar to the third type, but in reverse order. Both the initial outgoing angle and the returning angle of the detached electron are equal to π : $\theta_{\text{ret}} = \theta_i = \pi$.

The period of the above four types of closed orbit can be written as

$$T_j = \frac{2nd_{10} + 2md_{20}}{k_0 - 2nv}. \tag{1}$$

Here $j = 1, 2, 3, 4$ denotes the first, second, third, and fourth type of closed orbit, respectively.

The corresponding classical action of each closed orbit can be calculated using the formula $S_j = \int_0^{T_j} p_j dq_j$. For the above four types of closed orbits, the action can be described as follows:

$$S_1 = 2k_0(nd_{10} + md_{20}), \tag{2}$$

$$S_2 = 2k_0(nd_{10} + md_{20}), \tag{3}$$

$$S_3 = 2k_0 n(d_{10} + d_{20}) + 4mvd_{20}, \tag{4}$$

$$S_4 = 2k_0 n(d_{10} + d_{20}) - 4mvd_{20}. \tag{5}$$

II. DERIVATION OF THE PHOTODETACHMENT CROSS SECTION

As in the previous studies, the H^- ion can be considered as a one-electron system. Let the initial bound state of H^- be ψ_i: $\psi_i = B\exp(-k_b r)/r$. Here, $B = 0.31552$ is a normalization constant. k_b is related to the binding energy E_b of the H^- ion: $k_b = \sqrt{2E_b}$, $E_b = 0.754$ eV. Assume the laser light used for the photodetachment is polarized along the z axis. After photodetachment, the outgoing electron wave in the quantum well ψ_d satisfies the following Schrödinger equation [24]:

$$(E - H)\psi_d = D\psi_i, \tag{6}$$

where E is the initial kinetic energy of the detached electron, $E = k_0^2/2$, and k_0 is the initial momentum. H is the Hamiltonian governing the electron motion in the quantum well: $H = \frac{p^2}{2} + V_b(r) + V(r)$. $V_b(r)$ is a short-ranged potential between the neutral atom and the bound electron, which can be neglected after the photodetachment. $V(r)$ denotes the potential in the quantum well:

$$V(r) = \begin{cases} 0 & -d \leqslant z \leqslant d_{20} \\ +\infty & z < -d \text{or} z > d_{20} \end{cases}. \tag{7}$$

D is the dipole operator, for z-polarized laser light, $D = z$.

On the basis of the closed orbit theory, the photodetachment cross section of the H^- ion in the quantum well can be written as [16]

$$\sigma = \sigma_0 + \sigma_{\text{osc}}. \tag{8}$$

Here,σ_0 is the smooth background term in the photodetachment cross section without the quantum well:

$$\sigma_0 = \frac{16\sqrt{2}\pi^2 B^2 E^{3/2}}{3c(E + E_b)^3}.$$

σ_{osc} is an oscillatory term, which comes from the returning electron wave overlapping with the outgoing source wave:

$$\sigma_{\text{osc}} = -\frac{4\pi E_p}{c}\text{Im}\langle D\psi_i|\psi_{\text{ret}}\rangle. \tag{9}$$

In the above equation, E_p is the photon energy: $E_p = E + E_b$. ψ_{ret} is the returning electron wave function traveling along the closed orbit, which can be obtained by semiclassical approach. Drawing a small spherical surface centered at the origin with radius $R \approx 5.0a_0$, the initial outgoing wave on this sphere surface is [24]

$$\psi_{\text{out}}(R, \theta, \phi) = C(k_0)Y_{lm}(\theta, \phi)\frac{e^{ik_0 R}}{R}. \tag{10}$$

$C(k_0)$ is an energy-dependent factor, and $Y_{lm}(\theta, \phi)$ is the spherical harmonic function. For the H^- ion, the detached electron wave source is a p wave, and then $l = 1, m = 0$.

$$C(k_0) = \frac{4Bk_0 i}{(k_b^2 + k_0^2)^2}\sqrt{\frac{4\pi}{3}}.$$

The phase and amplitude will be changed as the outgoing wave propagates out from the surface. In the semiclassical approximation, the electron wave function in the quantum well can be written as

$$\psi_{SC}(r,\theta,\phi) = \sum_j \psi_{out}(R,\theta,\phi)A_j e^{i[S_j - \mu_j\cdot\pi/2]}, \tag{11}$$

where the sum includes all of the jth closed orbit. A_j is the amplitude of the wave function, $A_j = |\frac{J_j(\rho,z,0)}{J_j(\rho,z,t)}|^{1/2}$. $J(\rho,z,t)$ is the Jacobian: $J(\rho,z,t) = \rho(t)\frac{\partial(\rho,z)}{\partial(t,\theta)}$. Due to the free motion of the electron in the quantum well, we get

$$A_j = \frac{R}{R + k_0 T_j}\left|\frac{k_0}{k_{ret}^j}\right|^{1/2}. \tag{12}$$

S_j is the action along the closed orbit, which is given in Eqs. (2)–(5). μ_j is the Maslov index of the closed orbit, which equals to the number of collisions of the electron with two walls: $\mu_j = 2(n+m)$.

When the electron wave is bounced back by the walls of the quantum well to the negative-ion source along the closed orbit, the returning wave can be approximated by a sum of plane waves travelling along the z axis:

$$\psi_{ret} = \sum_j N_j C(k_0) Y_{lm}(\theta,\phi) e^{\pm i k_{ret}^j z}, \tag{13}$$

where N_j is a matching factor. By matching Eqs. (13) and (11) and letting $r \to R$, we get

$$N_j = g_0 \frac{A_j}{R} e^{i(S_j - \mu_j\cdot\pi/2)}. \tag{14}$$

Here g_0 is a sign factor. For a closed orbit with opposite outgoing and returning directions, such as the first and second type of closed orbits, $g_0 = -1$, while for the closed orbit with the same outgoing and returning directions, such as the third and fourth type of closed orbits, $g_0 = +1$.

Substituting the above equations into Eq. (9) and carrying out the overlap integral, we obtain the oscillatory part σ_{osc} in the photodetachment cross section:

$$\begin{aligned}\sigma_{osc} &= \sum_j 3g_0 C^*(k_0)C(k_{ret})\frac{2\pi E_p}{c} \times \frac{A_j}{R}\sin(S_j - \mu_j\pi/2)\\ &= \sigma_0 \sum_j 3g_0 \frac{C(k_{ret})}{C(k_0)}\frac{A_j}{k_0 R}\sin(S_j - \mu_j\pi/2).\end{aligned} \tag{15}$$

Finally, the total photodetachment cross section of the H^- ion in the quantum well with one expanding wall can be written as

$$\sigma = \sigma_0 + \sigma_{ret} = \sigma_0 H(k_0, v), \tag{16}$$

where $H(k_0, v)$ is a modulation function induced by the quantum well:

$$H(k_0,v) = 1 + \sum_j 3g_0 \frac{C(k_{ret})}{C(k_0)}\frac{A_j}{k_0 R}\sin(S_j - \mu_j\pi/2). \tag{17}$$

From Eqs. (16) and (17), we find the photodetachment cross section consists of a smooth background term plus many sinusoidal oscillating terms, which is similar to the photodetachment of H^- in a static quantum well [15]. However, there are some differences in comparison with the photodetachment of the H^- ion in a static quantum well.

(i) In the static quantum well, the detached electron can always collide with the down wall and guarantee one closed orbit; nevertheless, for the current moving wall, no closed orbit exists if the initial momentum of the detached electron $k_0 \leqslant 2nv$.

(ii) The detached electron's kinetic energy is conserved if it is bounced back to the ion source by a static wall. In contrast, when the electron is returned back by a moving wall, its returning kinetic energy is different from its initial value. Consequently, an extra coefficient related to the electron's initial and returning momentum along the closed orbit appears in the oscillating cross section.

(iii) Both the amplitude A and action S in the oscillating cross section are different from the case of the static quantum well.

III. CALCULATIONS AND DISCUSSIONS

Since the photodetachment cross section of H^- in the quantum well with one moving wall is related to positions of the two walls and the speed of the moving wall, in the following calculation, we consider the case with the fixed wall localized at $z_2 = 100$ a.u.

In Fig. 3, we show how the pattern of the photodetachment cross section varies with the electron energy and the speed of the moving wall. Suppose the initial distance from the ion to the moving wall is $d_{10} = 200$ a.u. It is found with the increase of the electron energy that oscillatory structures appear in the photodetachment cross section. As the moving wall moves very slowly, saw-tooth structures appear in the photodetachment cross section, which is similar to the photodetachment in the quantum well with fixed walls [11]. However, when the moving wall moves very fast, the saw-tooth structures in the photodetachment cross section nearly disappear, and the oscillatory structure becomes dampened.

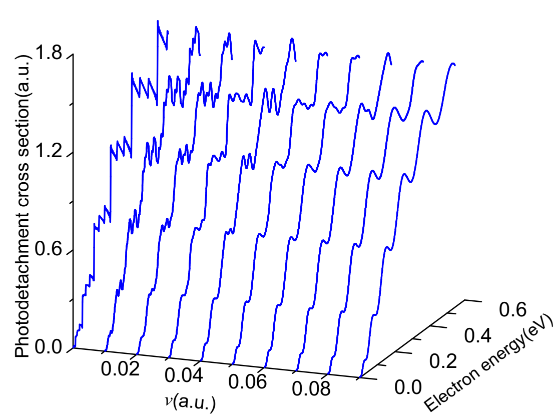

FIG. 3. Variation of the photodetachment cross section of the H^- ion in the quantum well with the speed of the moving wall and the electron energy. Assuming the fixed wall is localized at $z_2 = 100$ a.u., the initial distance from the ion to the moving wall is $d_{10} = 200$ a.u.

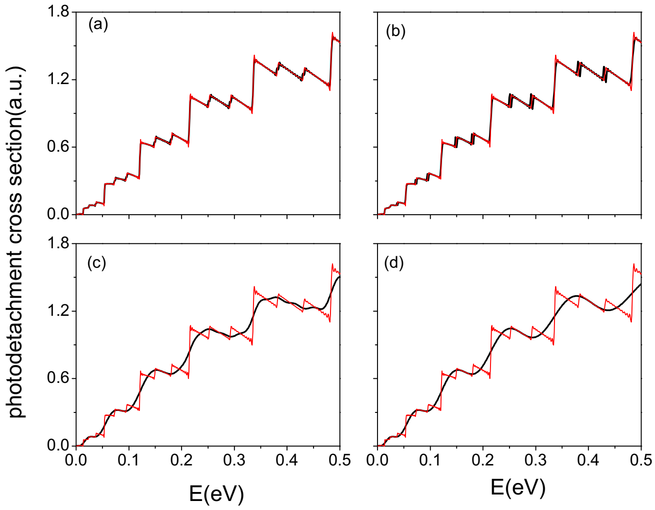

FIG. 4. Dependence of the photodetachment cross section of H$^-$ on the moving speed of the wall in the quantum well. The initial distance between H$^-$ and the wall is fixed to be $d_{10} = 200$ a.u. The red line is the photodetachment cross section of H$^-$ in a static quantum well and the black line is the case with one expanding wall. The speed of the moving wall is as follows: (a) $v = 0.0001$ a.u., (b) $v = 0.001$ a.u., (c) $v = 0.01$ a.u., and (d) $v = 0.1$ a.u.

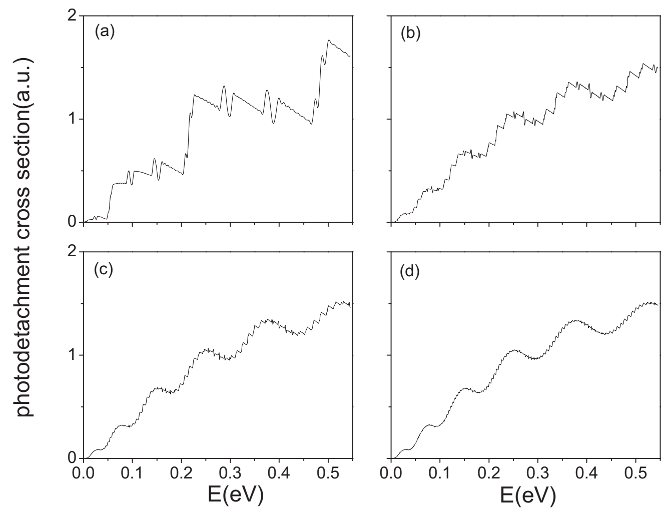

FIG. 5. Dependence of the photodetachment cross section of the H$^-$ ion on the initial position d_{10} of the moving wall in the quantum well. The moving speed of the wall is fixed to $v = 0.002$ a.u. The initial distance between H$^-$ and the moving wall is (a) $d_{10} = 50$ a.u., (b) $d_{10} = 500$ a.u., (c) $d_{10} = 1000$ a.u., and (d) $d_{10} = 2000$ a.u.

In order to show the influence of the moving wall in the quantum well on the photodetachment cross section of the H$^-$ ion clearly, we plot the photodetachment cross section for different speed of the moving wall. We assume the moving wall is localized at $z = -200$ a.u. The result is shown in Fig. 4. The red line is the photodetachment cross section of the H$^-$ ion in the quantum well with two static walls, which is given for comparison. We find the photodetachment cross section exhibits saw-tooth oscillatory structures, which is caused by the interference of the returning electron wave with the initial outgoing wave. As the down wall in the quantum well is moving, the oscillatory pattern in the photodetachment cross section will change. The black line in each plot denotes the photodetachment cross section with one expanding wall. Figure 4(a) shows the photodetachment cross section with the moving wall moves very slowly, $v = 0.0001$ a.u. Under this condition, the effect of the moving wall on the cross section is not obvious. As we increase the speed of the moving wall, the moving wall effect on the photodetachment cross section becomes apparent. Figure 4(b) shows the photodetachment cross section with the speed of the moving wall $v = 0.001$ a.u. The oscillatory structures in the cross section become irregular. As we further increase the moving speed, $v = 0.01$ a.u., the saw-tooth oscillatory structures nearly disappear, and the oscillating cross section becomes dampened, as we show in Fig. 4(c). When the moving wall moves very fast, $v = 0.1$ a.u., the influence of this wall on the photodetachment cross section can be totally omitted [Fig. 4(d)], and the photodetachment cross section approaches to the case near one elastic surface [4], which can be considered as a smooth background term plus a sinusoidal oscillating term. The reason can be analyzed on the basis of the classical motion of the detached electron in the quantum well. As we show in Sec. II, if the initial momentum of the detached electron $k_0 > 2nv$, it can be bounced back by the moving wall to the origin to form a closed orbit. In Fig. 4, the initial momentum of the detached electron $0 < k_0 < 0.2$ a.u.; as the moving speed of the wall $v = 0.1$ a.u., the detached electron cannot be bounced back by the moving wall to the origin, therefore all the other closed orbits disappear except the one shown in Fig. 2(a). Under this condition, the effect of the moving wall on the photodetachment cross section disappears and the cross section is similar to the case near one elastic surface.

Next, we let the wall move at a given speed $v = 0.002$ a.u., then we show how the pattern of the photodetachment cross section varies with the initial distance from the moving wall to the origin. The results are given in Fig. 5. It is found that, as the moving wall is very close to the origin, its influence on the photodetachment cross section is significant. For example, in Fig. 5(a), the initial distance from the moving wall to the origin is $d_{20} = 50$ a.u. After the electron is emitted from the origin, it will hit the fixed and the moving walls in a short period of time and return back to the origin to form a closed orbit. The interference between the returning electron waves with the outgoing waves traveling along the closed orbits causes the oscillatory structure in the cross section. Four types of closed orbits described in Sec. II all contribute to the photodetachment cross section, thus the oscillating amplitude in the cross section is very large. With the increase of the initial distance from the ion to the moving wall, the oscillating amplitude in the cross section becomes decreased, but the oscillating frequency gets increased, as shown in Figs. 5(b) and 5(c). If the initial distance from the moving surface to the origin is very large, $d_{10} \geqslant 2000$ a.u., the contribution of the closed orbit to the photodetachment cross section caused by the collision with the moving wall becomes dampened. Only the closed orbit shown in Fig. 2(a) has a great influence on the photodetachment cross section. As a consequence, the oscillatory structure caused by the moving wall nearly

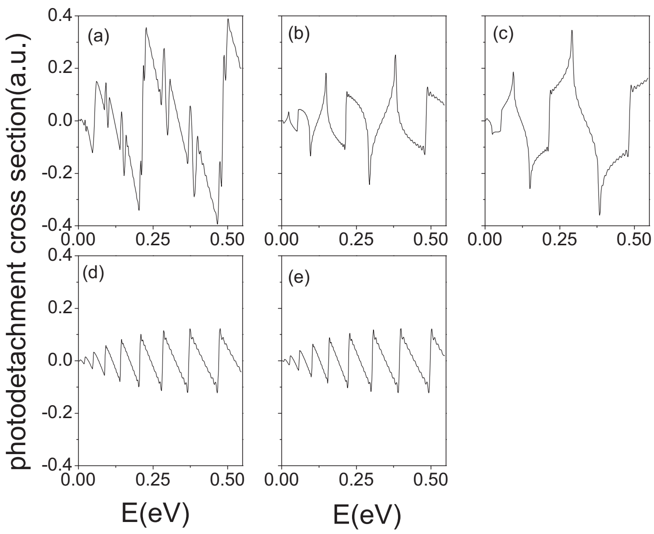

FIG. 6. Connection of each kind of closed orbit to the photodetachment cross section in the quantum well with one moving wall. The wall moves with a speed $v = 0.002$ a.u. The initial distance from the moving wall to the origin is $d_{20} = 50$ a.u. (a) The total oscillating cross section induced by all the closed orbits. (b, c, d, e)The oscillating cross section induced by the first, second, third, and fourth type of closed orbit, respectively.

disappears, and the cross section approaches to the case near one elastic surface [4].

Finally, in order to see the contribution of each kind of closed orbit to the photodetachment cross section, we calculate the oscillating cross section connected with the four types of closed orbits. The results are given in Fig. 6. The moving wall moves with a speed $v = 0.002$ a.u., and the initial distance from the moving wall to the origin is $d_{20} = 50$ a.u. Figure 6(a) is the total oscillating cross section induced by all the closed orbits. Figures 6(b)–6(e) show some typical oscillatory patterns caused by the four kinds of the detached electron's closed orbits, respectively. We can see that the contribution of the second kind of closed orbit to the cross section is significant, followed by the first kind of closed orbit. These two kinds of closed orbits make a strong modulation for the oscillating cross section. The oscillating amplitude in the cross section is relatively large and the oscillatory structure is irregular, as we can see from Figs. 6(b) and 6(c). However, the oscillating amplitude in the cross section caused by the third and fourth types of closed orbit is relatively small, but its oscillating frequency is relatively large. They only give a weak modulation for the oscillating pattern in the cross section. In addition, the oscillatory structures in the cross section induced by these closed orbits are regular.

IV. CONCLUSIONS

In summary, we have investigated the photodetachment dynamics of negative ions in the quantum well with one expanding wall for the first time. Four different types of closed orbits have been found for the detached electron in the quantum well, which depend on the order of the collisions with the fixed wall or the moving wall. An analytical formula for the photodetachment cross section of the H^- ion has been put forward based on the semiclassical closed orbit theory. It is found that the photodetachment cross section can be written as a summation of a smooth background term plus many oscillating terms induced by the two walls. Due to the collision of the detached electron with the moving wall, some of the electron's kinetic energy is lost; as a result, the returning kinetic energy of the detached electron is different from its initial value after bouncing back from the moving wall. In order to solve this problem, we introduce a modulation factor which depends on the electron's initial and returning momentum in the oscillating cross section in contrast to the case in the static quantum well. The calculation results suggest that the photodetachment cross section depends on the electron's energy and the position and the speed of the moving wall sensitively. As the wall moves slowly, the cross section approximates to the case in the static quantum well. However, as the wall moves fast, the moving wall can affect the photodetachment cross section obviously. It can weaken the oscillatory structure in the cross section significantly.

In this paper, we only deal with the simplest case of an expanding wall with constant velocity. For more general surface motions, the method used in this paper is still suitable. An immediate application of the current method would be the photodetachment of the negative ion in the presence of an oscillatory surface, for example, the surface moves with the equation $l(t) = l_0 + l_0 \sin(wt)$. Under this condition, more interesting physics can happen. The detached electron's movement will become irregular and oscillatory, and an infinite number of the electron's closed orbits would appear, which depends on the oscillating frequency w in the equation. For the oscillatory surface, we will use the Taylor series expansion of the motion equation in order to find the closed orbit of the detached electron. In addition, the photodetachment cross section will oscillate with time. So we can adopt the perturbation theory and calculate the average photodetachment cross section. A study of the case in which the wall moves oscillating with time is in progress and will be reported in our future studies. We hope that our paper will provide a useful guide for the future experimental study of the photodetachment dynamics of the negative ion near a moving surface.

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[1] P. J. Rou, Phys. Rev. Lett **83**, 5086 (1999).

[2] D. M. Hartley and P. J. Rou, Chem. Phys. **201**, 427 (1995).

[3] D. M. Hartley and P. J. Rou, Surf. Sci. **341**, 213 (1995).

[4] G. C. Yang, Y. Z. Zheng, and X. X. Chi, J. Phys. B **39**, 1855 (2006).

[5] M. L. Du, Phys. Rev. A **70**, 055402 (2004).

[6] A. Afaq and M. L. Du, J. Phys. B **40**, 1309 (2007).
[7] G. C. Yang, Y. Z. Zheng, and X. X. Chi, J. Theor. Comput. Chem. **6**, 353 (2007).
[8] P. Hansen, K. A. Mitchell, and J. B. Delos, Phys. Rev. E **73**, 066226 (2006).
[9] J. Novick, M. L. Keeler, J. Giefer, and J. B. Delos, Phys. Rev. E **85**, 016205 (2012).
[10] J. Novick and J. B. Delos, Phys. Rev. E **85**, 016206 (2012).
[11] G. C. Yang, K. K. Rui, and Y. Z. Zheng, Physica B: Condens. Matter **404**, 1576 (2009).
[12] H. J. Zhao, Z. J. Ma, and M. L. Du, Physica B: Condens. Matter **54**, 466 (2015).
[13] H. J. Zhao and M. L. Du, Phys. Rev. E, **84**, 016217 (2011).
[14] D. H. Wang, S. S. Li, Y. H. Wang, and H. F. Mu, J. Phys. Soc. Jpn. **81**, 114301 (2012).
[15] D. H. Wang, Chin. J. Phys. **52**, 138 (2014).
[16] H. J. Zhao and M. L. Du, Physica B: Condens. Matter **530**, 121 (2017).
[17] D. H. Wang, Z. H. Pang, K. Z. Zhuang, Y. F. Li, and L. Xie, Prama. J. Phys. **89**, 71 (2017).
[18] E. Fermi, Phys. Rev. **75**, 1169 (1949).
[19] S. M. Ulam, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (University of California, Berkeley, CA, 1961), Chap. 3, p. 315.
[20] S. W. Dosecher and M. H. Rice, Am. J. Phys. **37**, 1246 (1969).
[21] M. G. E. da Luz and B. K. Cheng, J. Phys. A: Math. Gen. **25**, L1043 (1992).
[22] S. Di Martino, F. Anza, P. Facchi, A. Kossakowski, G. Marmo, A. Messina, B. Militello, and S. Pascazio, J. Phys. A: Math. Theor. **46**, 365301 (2013).
[23] A. Matzkin, J. Phys. A: Math. Theor. **51**, 095303 (2018).
[24] B. C. Yang, J. B. Delos, and M. L. Du, Phys. Rev. A **89**, 013417 (2014).