Realizing an adiabatic quantum search algorithm with shortcuts to adiabaticity in an ion-trap system

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(Received 24 May 2018; published 19 November 2018)

Adiabatic quantum computing (AQC) is an approach for solving optimization problems and has advantages against environmental decoherence and certain kinds of random unitary perturbations; however, long coherence time of quantum systems is required to meet the adiabatic criteria for the implementation of AQC. To break the dilemma of evolution time in AQC, the method of shortcuts to adiabaticity (STA) is employed to speed up the evolution process and suppress the nonadiabatic transitions. In this work we demonstrate the application of STA in AQC by implementing a fast two-bit Grover's algorithm with STA in an ion trap system, and the method here is also applicable for realizing Grover's algorithm with more entries.

DOI: 10.1103/PhysRevA.98.052323

I. INTRODUCTION

Adiabatic quantum computing (AQC) is one of the good models for building quantum computers [1–3]. In AQC the solution of a computational problem is encoded in the ground state of a Hamiltonian, which can be acquired by slowly evolving an easily prepared Hamiltonian as long as the adiabatic condition holds to guarantee the system follows the instantaneous ground state. Since only ground state relates to the computation, AQC is robust against environmental decoherence and a certain class of unitary perturbations [4]. However, the evolution time requirement of adiabatic passage is the obstacle for realizing AQC in physical systems. There are two solutions. One is prolonging coherence time of the quantum systems so that the evolution time can be long enough to meet the adiabatic condition. But long coherence time is not easily obtained since the quantum systems may suffer from decoherence, noise, or losses. The other is speeding evolution process up while suppressing the nonadiabatic excitations, which may be the ideal way to actually implement AQC experimentally. To speed up the adiabatic process and avoid the nonadiabatic transitions, Demirplak and Rice [5] and Berry [6] proposed the shortcuts to adiabaticity (STA) method, in which a time-dependent compensating term can be added into the original Hamiltonian to suppress nonadiabatic transitions; therefore, the adiabaticity of the original Hamiltonian holds without following adiabatic condition [7,8].

Grover's search algorithm [9] is one of the great examples to prove the power of quantum computers, by which searching marked one out of N entries can be quadratically faster than classic computers; in other words searching attempts are only on the order of $\sqrt{N}$ on quantum computers. For implementation of the algorithm on quantum systems, an initial uniform superposition of all possible states is prepared as the database, and the target state is obtained by rotating the initial state step by step with appropriate unitary logic gates. The operated state is checked by an oracle function which returns for instance 1 if the state is target state and returns zero otherwise. The original Grover's algorithm is designed for a quantum circuit model. For the past years, there were experiments that had successfully demonstrated the search algorithm with two qubits ($N = 4$) in quantum systems of NMR [10,11], optics [12,13], and trapped ions [14]. There also had been proposals employing cavity QED to realize the quantum gates dynamics in search algorithm [15,16], and continuous version of Grover's algorithm [17,18], in which the Hamiltonian is continuously driven to rotate the initial state to the energy marked target state. Adiabatic quantum computation versions of Grover's algorithm [2,19], which has been proven to be equivalent to the quantum circuit versions [3,20,21], is implemented by continuously driving the designed Hamiltonian in time. They connect initial superposition state $|w\rangle$ and the target state $|m\rangle$ via the Hamiltonian $H_0 = [1 - u(t)]H_i + u(t)H_f$, where $H_i = I - |w\rangle\langle w|$, $H_f = I - |m\rangle\langle m|$, and $u(t)$ is a monotonic function of time. The search process starts from $u(t = 0) = 0$ and the target state $|m\rangle$ is reached at $u(t = t_f) = 1$. In this representation, the marked state and all other computational basis states can be described as the eigenstates of Pauli operator σ_z; therefore, Grover's algorithm can be simplified as a two-dimensional problem regardless of the number of the entries.

In this paper we follow the proposal in Ref. [22] and demonstrate the application of STA in AQC by implementing an $N = 2$ adiabatic search algorithm using STA with a single trapped ion. We show that the adiabaticity of the searching process is retained by applying the STA, the searching time is far less than the requirement of conventional versions of adiabatic evolution, and the target state is found with fidelity over 98%. Though we only demonstrate the two entries case with a single ion, the method we use in this paper can be extended to the multiple ion qubits.

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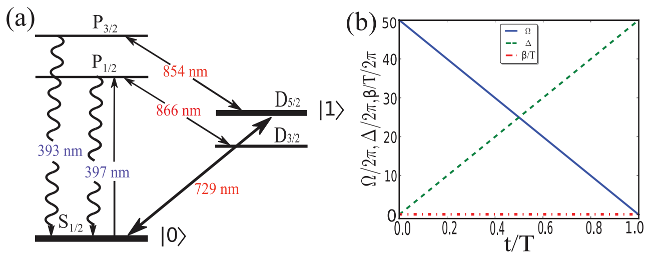

FIG. 1. (a) Relevant energy levels of $^{40}Ca^+$. The qubit states are $S_{1/2}(m=-1/2)$ and $D_{5/2}(m=-5/2)$ with lifetime of about 1 s. The narrow linewidth 729 nm laser beam is used to coherently couple the qubit states. Laser 397 nm is for Doppler cooling, optical pumping, and quantum state detection. 854 nm and 866 nm laser beams repump ion out of D states. (b) The experimental parameters for realizing the quantum adiabatic search algorithm, $\Omega_1 = 2\pi\times 50$ kHz, $\Delta_1 = 2\pi\times 50$ kHz, and $\Omega = \Omega_1[1-u(t)]$, $\Delta = \Delta_1 u(t)$, $u(t) = t/T$, and counteradiabatic term β/T.

II. EXPERIMENTAL HAMILTONIAN AND SETUP

In our experiment, a single $^{40}Ca^+$ is loaded in a blade-shaped linear Paul trap working at radio frequency $2\pi\times 13.3$ MHz. The secular frequencies used in this work are $2\pi\times 1.6$ MHz and $2\pi\times 1.4$ MHz in the radial and axial direction, respectively. A Zeeman splitting of about 8.8 MHz is created by a magnetic field between the Zeeman levels in $S_{1/2}$ and $D_{5/2}$, and the sublevels $S_{1/2}(m_j=-1/2)$ and $D_{5/2}(m_j=-5/2)$ are chosen as the qubit states $|0\rangle$ and $|1\rangle$, respectively, as shown in Fig. 1(a). Initially the ion is Doppler cooled with laser light at 397 nm yielding average phonon number at round 19; afterward the axial motional mode is prepared to its ground state ($\bar{n} \approx 0.015$) by resolved sideband cooling [23]. A 397 nm laser beam with right-circular polarization is shined on the ion for 20 μs to pump the ion in electronic state $S_{1/2}(m=-1/2)$. State manipulation of qubit is achieved by laser pulses from a stabilized diode laser with wavelength at 729 nm and linewidth about 20 Hz. The quantum operations and laser beam modulation are realized by sending rf signals generated by signal generators or an arbitrary waveform generator (AWG) to an acousto-optical modulator (AOM). The ion's electronic state is detected by using an electron shelving technique [23] and the states ($S_{1/2}$ and $D_{5/2}$) can be distinguished with almost perfect fidelity within detection time of 600 μs.

We realize the adiabatic searching Hamiltonian using the carrier $[S_{1/2}(m_j=-1/2) \leftrightarrow D_{5/2}(m_j=-5/2)]$ transition. When the 729 nm laser beam interacts with ion, the interaction Hamiltonian in the rotating reference frame of laser can be written as

$$H = \hbar/2[\Delta\sigma_z + \Omega\sigma_x\cos(\phi) + \Omega\sigma_y\sin(\phi)], \qquad (1)$$

where detuning $\Delta = \omega_L - \omega_0$, ω_L is the frequency of 729 nm laser, and ω_0 is the frequency between the energy levels $S_{1/2}(m_j=-1/2)$ and $D_{5/2}(m_j=-5/2)$. Ω is the Rabi frequency of carrier transition and ϕ is the initial phase of 729 nm laser beam; $\sigma_\alpha(\alpha = x, y, z)$ are the Pauli spin matrices. By precisely controlling the Rabi frequency, detuning, and laser phase, we can realize an arbitrary Hamiltonian for a single ion qubit.

In the $N=2$ case, the database of search algorithm is $(|0\rangle + |1\rangle)/\sqrt{2}$, which is the eigenstate of σ_x, and the target state we set is state $|0\rangle$, an eigenstate of σ_z. Therefore, the adiabatic Hamiltonian connecting the initial superposition state and the target state can be written as $H_0 = [1-u(t)]\sigma_x + u(t)\sigma_z$. Since all the coherent operations in ion experiments take the initial phase of the laser beam as reference, we can take the laser initial phase as zero, and the interaction Hamiltonian can be rewritten as $H_0 = \hbar/2[\Delta\sigma_z + \Omega\sigma_x]$. To realize the Hamiltonian for implementing the adiabatic search algorithm, we need to modulate the Rabi frequency and detuning simultaneously. Hence we implement the equivalent Hamiltonian $H_0 = \hbar/2\{\Omega_1[1-u(t)]\sigma_x + \Delta_1 u(t)\sigma_z\}$ by setting the detuning $\Delta = \Delta_1 u(t)$, Rabi frequency $\Omega = \Omega_1[1-u(t)]$, monotonic function $u(t) = t/T$, and we choose $\Delta_1 = \Omega_1 = 2\pi\times 50$ kHz. The Rabi frequency and detuning modulation are controlled by the rf signals generated by AWG and an AOM with double pass configuration. A sequence starting with a $\pi/2$ pulse prepares the qubit in ground superposition state $(|0\rangle + |1\rangle)/\sqrt{2}$. Afterwards the predesigned 729 nm laser pulses are sent to implement the adiabatic Hamiltonian and quantum state tomography [24]; finally a 397 nm laser pulse is applied to detect the population of the ion's internal state.

To realize the adiabatic evolution of the quantum system, the adiabatic criteria [7] has to be met; therefore, long coherence time is normally required for the quantum system; usually the evolution time could be as long as tens times that of Rabi oscillation periods. However, due to limited coherence time in our system, this method is hardly applicable in practice. To finish the searching process within the coherence time, we use the STA method, in which a time-dependent counter or transitionless driving term $H_D(t)$ added to the original Hamiltonian H_0 makes a quantum system evolution follow the adiabatic path exactly with arbitrarily short time theoretically. According to the STA theory, the transitionless tracking term for adiabatic Grover's algorithm should be $H_D = \frac{-\hbar}{2T}\frac{\sigma_y}{[1-2u(t)]^2+2u(t)[1-u(t)]}$; however, this term makes the overall Hamiltonian too complicated; in practice we can take the approximation of this term as a constant [22]. As shown in Fig. 1(b), in addition to the Rabi frequency and detuning modulations, the time-independent counteradiabatic term $H_D = -\frac{\hbar}{2}\frac{\beta}{T}\sigma_y$ with constant amplitude instead of a time-dependent term is applied to suppress the nonadiabatic transition; the constant parameter β can be found by numerical simulation. Now the Hamiltonian we need to realize is

$$H = H_0 + H_D = \hbar/2[\Delta\sigma_z + \Omega\sigma_x - \beta/T\sigma_y]. \qquad (2)$$

To realize the Hamiltonian Eq. (2) in an ion trap system, we need to write this Hamiltonian similar to Eq. (1) in the new form

$$H = \hbar/2[\Delta\sigma_z + \Omega'\sigma_x\cos(\phi') + \Omega'\sigma_y\sin(\phi')], \qquad (3)$$

where the effective Rabi frequency $\Omega' = \sqrt{\Omega^2 + (\beta/T)^2}$, phase $\phi' = \arccos(\Omega/\Omega')$. With AWG modulating the detuning, Rabi frequency, and laser phase, we can implement the Hamiltonian Eq. (3) with arbitrarily short searching time. Since we have the Rabi frequency modulation during the searching process, the time-dependent ac Stark shift induced

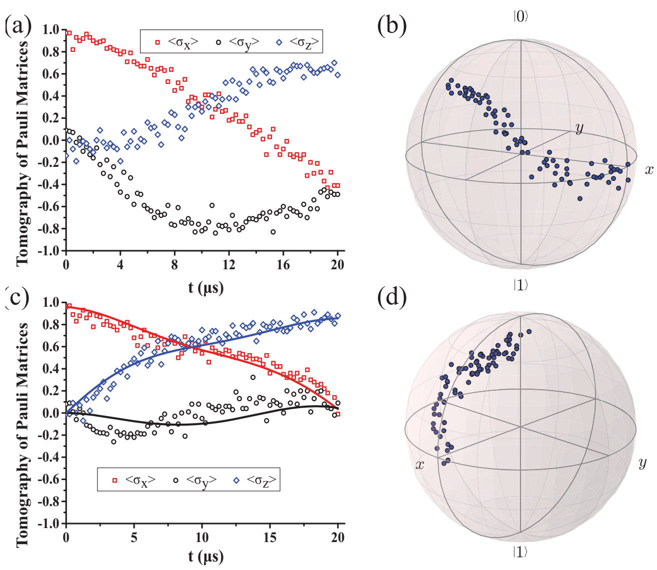

FIG. 2. (a) Quantum adiabatic search algorithm implemented with running time $T = 20\ \mu$s. (b) The quantum state evolution of (a) is plotted on the Bloch sphere. (c) Quantum adiabatic search algorithm implemented with STA and running time $T = 20\ \mu$s, β set at 2.5. (d) The quantum state evolution of (c) is plotted on the Bloch sphere. Note that all the markers are for experimental data points and the solid curves are the simulations without adjusting fitting parameters to the data points.

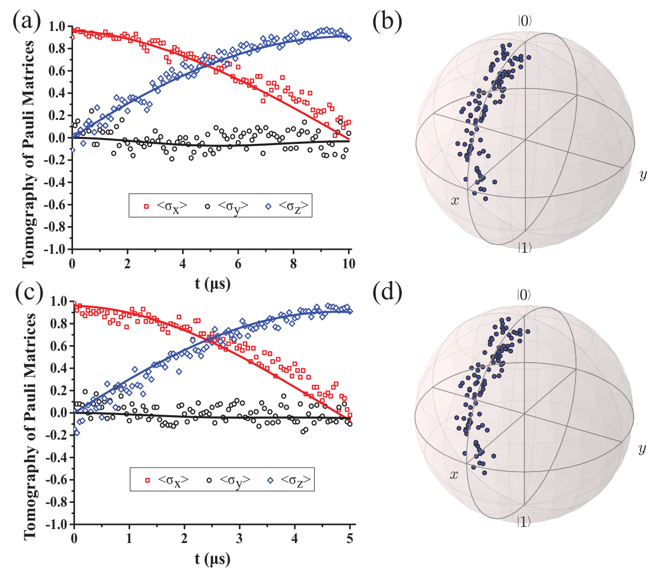

FIG. 3. (a) STA assisted quantum adiabatic search algorithm implemented with running time $T = 10\ \mu$s, and counteradiabatic term coefficient β at 1.5. (b) The quantum state evolution of (a) is plotted on the Bloch sphere. (c) Quantum adiabatic search algorithm implemented with STA, running time $T = 5\ \mu$s, and β at 1.2. (d) The quantum state evolution of (c) is plotted on the Bloch sphere. Note that all the markers are for experimental data points, and the solid curves are the simulations without adjusting fitting parameters to the data points.

by the laser-atom interaction cannot be neglected. To compensate the ac Stark shift, instead of using another far off-resonant laser beam as in Ref. [25], we simply track the time-dependent carrier transition resonance. The ac Stark shifts as a function of Rabi frequencies are determined by measuring ac Stark shifts under different Rabi frequencies. Based on the predetermined Rabi frequency in the experiment, the ac Stark shift can be compensated by setting the laser frequency to shifted carrier resonance.

III. EXPERIMENTAL RESULTS AND ANALYSIS

With the experimental setup in the previous section, we implement the quantum adiabatic search algorithm by first preparing the ion in ground superposition state of all possible states. Here the state $(|0\rangle + |1\rangle)/\sqrt{2}$ is prepared using a $\pi/2$ pulse with fidelity better than 98%. Afterwards the adiabatic search process sweeps from $t = 0$ to $t = T$, and quantum state tomography [24] at truncated time is applied to acquire the state evolution. At each truncated time, the experimental sequence is repeated by 200 times to get the probability of $D_{5/2}$ state and also maximum likelihood estimation [24] is utilized in the postprocess to reduce the statistical and systematic errors.

In the first experiment we implement the quantum adiabatic search algorithm with running time $T = 20\ \mu$s, Fig. 2(a) shows the experimental results without STA, and Fig. 2(b) shows the state evolution of Fig. 2(a) on the Bloch sphere. Since the evolution time 20 μs is far from the requirements of adiabatic criteria and we can see that the Hamiltonian does not evolve adiabatically, therefore, the final state totally deviates from the desired state $|0\rangle$. As a comparison, in the second experiment, we apply STA by adding a counteradiabatic field with β setting at 2.5; the corresponding quantum tomography results, simulations [26,27], and state evolution on the Bloch sphere are shown in Figs. 2(c) and 2(d). It can be seen that the searching process almost perfectly follows the adiabatic path indicated by solid lines and the target state $|0\rangle$ can be searched with fidelity better than 95%. Since the counteradiabatic term is used to suppress the nonadiabatic transitions, the constant β should be properly chosen; incorrect value of β could cause insufficient suppression or induce extra nonadiabatic transitions to the system.

To further investigate the speed of the search algorithm with STA, we also implement the experiments at the π pulse time ($T = 10\ \mu$s) and $\pi/2$ pulse time ($T = 5\ \mu$s); the experimental results and simulations are shown in Fig. 3. In those two experiments the coefficients of counteradiabatic term β are set at 1.5 and 1.2, respectively. With the help of STA, the evolution of the quantum system still follows an adiabatic path very well with shorter running time, and the target state is searched with fidelity better than 98%. As the Rabi frequency we set is 50 kHz, the π pulse time for the system should be 10 μs; here in Fig. 3(c) we show that the time for searching out the target state with high fidelity can be as short as 5 μs ($\pi/2$ pulse time), which is far shorter than the requirement of the adiabatic condition.

In adiabatic quantum search algorithm, the searching process shown in Figs. 2 and 3 is similar to the evolution of a $\pi/2$ pulse of Rabi oscillation, in which Rabi frequency has to be adjusted to achieve state transfer in different time, while in the STA method state evolution is an adiabatic process and can be implemented in arbitrary time only by adjusting the compensating term.

Theoretically the searching time can be arbitrarily short by applying the STA; however, there are some potential limitations in practice that prohibit arbitrary searching time. The AWG has a limited sample rate but precise control of laser pulse requires a higher sample rate and AOM has limited time resolution. At the same time as the searching time T becomes very short, the counteradiabatic term and the corresponding effective Rabi frequency Ω' would be very large; finally, the maximum Rabi frequency sets the searching time limit as well.

IV. CONCLUSION

In this paper, we demonstrated the application of STA by implementing the quantum adiabatic search algorithm with the STA method. We compared searching experiments with and without STA to verify that the STA method is a appropriate tool for suppressing the nonadiabatic transitions. In our work the counteradiabatic term is found by using numerical simulation, but in experiments it can always be determined by iterative attempts based on the experimental results. Although we just realized the quantum adiabatic search algorithm with a single ion qubit, this method can be applied to multiple ion qubits with the single ion addressing technique; it would be interesting to demonstrate the STA assisted adiabatic search algorithm with multiple qubits in the future.

ACKNOWLEDGMENTS

This work is supported by the National Basic Research Program of China (Grant No. 2016YFA0301903), the National Natural Science Foundation of China (Grants No. 11174370, No. 11304387, No. 61632021, No. 11305262, No. 11574398, and No. 61205108), and the Research Plan Project of National University of Defense Technology (Grant No. ZK16-03-04).

[1] T. Albash and D. A. Lidar, Rev. Mod. Phys. **90**, 015002 (2018).
[2] E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, arXiv:quant-ph/0001106.
[3] W. Van Dam, M. Mosca, and U. Vazirani, How powerful is adiabatic quantum computation? in *Proceedings of the 42nd IEEE Symposium on Foundations of Computer Science 2001* (IEEE, New York, 2001), pp. 279–287.
[4] A. M. Childs, E. Farhi, and J. Preskill, Phys. Rev. A **65**, 012322 (2001).
[5] M. Demirplak, J. Phys. Chem. A **107**, 9937 (2003).
[6] M. Berry, J. Phys. A: Math. Theor. **42**, 365303 (2009).
[7] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, Science **292**, 472 (2001).
[8] X. Chen, E. Torrontegui, and J. G. Muga, Phys. Rev. A **83**, 062116 (2011).
[9] L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
[10] I. L. Chuang, N. Gershenfeld, and M. Kubinec, Phys. Rev. Lett. **80**, 3408 (1998).
[11] J. A. Jones, M. Mosca, and R. H. Hansen, Nature (London) **393**, 344 (1998).
[12] N. Bhattacharya, H. B. van Linden van den Heuvell, and R. J. C. Spreeuw, Phys. Rev. Lett. **88**, 137901 (2002).
[13] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, Nature (London) **434**, 169 (2005).
[14] K.-A. Brickman, P. C. Haljan, P. J. Lee, M. Acton, L. Deslauriers, and C. Monroe, Phys. Rev. A **72**, 050306 (2005).
[15] D. Daems and S. Guérin, Phys. Rev. Lett. **99**, 170503 (2007).
[16] F. Yamaguchi, P. Milman, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **66**, 010302 (2002).
[17] E. Farhi and S. Gutmann, Phys. Rev. A **57**, 2403 (1998).
[18] V. L. Ermakov and B. M. Fung, Phys. Rev. A **66**, 042310 (2002).
[19] J. Roland and N. J. Cerf, Phys. Rev. A **65**, 042308 (2002).
[20] D. Aharonov, W. Van Dam, J. Kempe, Z. Landau, S. Lloyd, and O. Regev, SIAM Rev. **50**, 755 (2008).
[21] J. Roland and N. J. Cerf, Phys. Rev. A **68**, 062311 (2003).
[22] S. Oh and S. Kais, J. Chem. Phys. **141**, 224108 (2014).
[23] C. Roos, T. Zeiger, H. Rohde, H. C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **83**, 4713 (1999).
[24] J. B. Altepeter, E. R. Jeffrey, and P. G. Kwiat, Adv. At. Mol. Opt. Phys. **52**, 105 (2005).
[25] H. Häffner, S. Gulde, M. Riebe, G. Lancaster, C. Becher, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **90**, 143602 (2003).
[26] J. R. Johansson, P. D. Nation, and F. Nori, Comput. Phys. Commun. **184**, 1234 (2013).
[27] J. R. Johansson, P. D. Nation, and F. Nori, Comput. Phys. Commun. **183**, 1760 (2012).