Decoherence dynamics of qubits coupled to systems at quantum transitions

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We study the decoherence properties of a two-level (qubit) system homogeneously coupled to an environmental many-body system at a quantum transition, considering both continuous and first-order quantum transitions. In particular, we consider a d-dimensional quantum Ising model as environment system. We study the dynamic of the qubit decoherence along the global quantum evolution starting from pure states of the qubit and the ground state of the environment system. This issue is discussed within dynamic finite-size scaling frameworks. We analyze the dynamic finite-size scaling of appropriate qubit-decoherence functions. At continuous quantum transitions, they develop power laws of the size of the environment system, with a substantial enhancement of the growth rate of the qubit decoherence with respect to the case in which the environment system is in normal noncritical conditions. The enhancement of the qubit decoherence growth rate appears much larger at first-order quantum transitions, leading to exponential laws when increasing the size of the environment system.

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I. INTRODUCTION

Decoherence generally arises when a quantum system interacts with an environmental many-body system S. This issue is crucially related to the emergence of classical behavior in quantum systems [1,2], quantum effects such as interference and entanglement [3,4], and it is particularly relevant for the efficiency of quantum information protocols [5]. The decoherence dynamics has been investigated in some paradigmatic models, such as two-level (qubit) systems interacting with many-body systems, in particular the so-called central spin models; see, e.g., Refs. [6–17], where the qubit is globally, or partially, coupled to the environmental system S.

A typical problem concerns the coherence loss of the qubit during the entangled quantum evolution of the global system, starting from pure states of the qubit and the ground state of S. The decoherence rate may significantly depend on the quantum phase of S, and, in particular, whether S develops critical behavior arising from quantum transitions. Indeed, the response of many-body systems at quantum transitions is generally amplified by *critical* quantum fluctuations. At quantum transitions, small variations of the driving parameter give rise to significant changes of the ground state and low-excitation properties of many-body systems [18]. At first-order quantum transitions the ground-state properties appear discontinuous in the infinite-volume limit, generally arising from level crossings. Continuous quantum transitions show continuous change of the ground state at the transition point, and correlation functions develop a divergent length scale.

Environmental systems at quantum transitions may significantly drive the dynamics of the qubit decoherence. An enhanced quantum decoherence has been put forward [10] in the case of continuous quantum transitions. In this paper we return to this issue, providing a quantitative scaling framework to support the enhancement of the growth rate of the quantum decoherence and extend the analysis to the case in which the environmental system is at a first-order quantum transition.

We consider a qubit homogeneously coupled to a d-dimensional many-body system S of size L (or equivalently with $N \sim L^d$ degrees of freedom). In particular, as environmental systems we consider the paradigmatic d-dimensional quantum Ising models, whose quantum phase diagrams present both continuous and first-order quantum transitions [18]. The two-level qubit system is equally coupled to all L^d spins of S. We consider the standard out-of-equilibrium protocol in which the initial global state is a product of pure states of the qubit and S. We study the quantum decoherence dynamics during the quantum evolution of the global system, as measured by its density matrix, which is obtained by tracing out the S states.

We investigate the quantum decoherence dynamics when the environmental Ising system experiences a quantum transition. The decoherence properties are analyzed within dynamic finite-size scaling frameworks. At both continuous and first-order quantum transitions, dynamic finite-size scaling behavior arises from the interplay between the coupling of the qubit with S, the Hamiltonian parameters of S close to the quantum transition, and the size L of S. We show that the *critical* conditions of the environmental system at quantum transitions give rise to a substantial enhancement of the growth rate of the decoherence dynamics with respect to noncritical systems. In particular, the decoherence growth rate at continuous quantum transitions turns out to be characterized by power laws L^ζ of the size L, with exponents ζ that are larger than that of the volume (L^d) law expected for systems in normal conditions. The rate enhancement of the qubit coherence loss is even more substantial at first-order quantum transitions. Indeed, the corresponding dynamic finite-size scaling theory predicts an exponentially large decoherence growth rate, related to the exponentially suppressed difference of the lowest levels in finite-size many-body systems at first-order quantum transitions.

The paper is organized as follows: In Sec. II we present the general setting of the out-of-equilibrium problem that we consider. In Sec. III we discuss the decoherence properties when the environmental system is critical at a continuous quantum transition, within a dynamic finite-size scaling framework, and show the enhanced growth rate of decoherence with respect to normal conditions. Section IV extends this analysis to first-order quantum transitions, showing that the decoherence growth-rate enhancement is even more pronounced, leading to exponential laws. Finally, in Sec. V, we summarize and draw our conclusions.

II. GENERAL SETTING OF THE PROBLEM

We consider a d-dimensional quantum many-body system S of size L^d with Hamiltonian

$$H_S(v) = H_c + vP_v, \tag{1}$$

where P_v is the spatial integral of local operators, and $[H_c, P_v] \neq 0$ and the parameter v drives the quantum transition located at $v = 0$. Then we consider a further two-level system globally coupled to the many-body system by the Hamiltonian term

$$H_q = w\Sigma^{(3)}P_v, \tag{2}$$

where the Pauli operator $\Sigma^{(3)}$ is associated with the two states $|\pm\rangle$ of the qubit, so that $\Sigma^{(3)}|\pm\rangle = \pm|\pm\rangle$. Therefore the global Hamiltonian reads

$$H_{qS}(v, w) = H_c + (v + w\Sigma^{(3)})P_v. \tag{3}$$

We are interested in the quantum evolution of the global system starting from the initial $t = 0$ condition:

$$|\Psi_{qS}(t = 0)\rangle = |q_0\rangle \otimes |G_v\rangle, \tag{4}$$

where $|q_0\rangle$ is a generic pure state of the qubit,

$$|q_0\rangle = c_+|+\rangle + c_-|-\rangle, \quad |c_+|^2 + |c_-|^2 = 1, \tag{5}$$

and $|G_v\rangle$ is the ground state of the system with Hamiltonian $H_S(v)$. Then the global wave function describing the quantum evolution for $t > 0$ must be a solution of the Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi_{qS}(t)\rangle = H_{qS}(v, w)|\Psi_{qS}(t)\rangle. \tag{6}$$

It can be written as

$$|\Psi_{qS}(t)\rangle = c_+|+\rangle \otimes |\phi_{v+w}(t)\rangle + c_-|-\rangle \otimes |\phi_{v-w}(t)\rangle, \tag{7}$$

where

$$|\phi_{v\pm w}(t)\rangle = e^{-iH_S(v\pm w)t}|G_v\rangle, \tag{8}$$

i.e., they are solutions of the Schrödinger equation for the system S only,

$$i\frac{\partial}{\partial t}|\phi_{v\pm w}(t)\rangle = H_S(v \pm w)|\phi_{v\pm w}(t)\rangle, \tag{9}$$

with $|\phi_{v\pm w}(t = 0)\rangle = |G_v\rangle$. Note that the expectation value $\langle\Psi_{qS}(t)|\Sigma^{(3)}|\Psi_{qS}(t)\rangle = |c_+|^2 - |c_-|^2$ does not change along the quantum evolution, thus it is fixed by the initial condition of the qubit.

The quantum decoherence behavior can be inferred from the qubit density matrix,

$$\rho_q(t) = \mathrm{Tr}_S\rho_{qS}(t), \quad \rho_{qS}(t) = |\Psi_{qS}(t)\rangle\langle\Psi_{qS}(t)|, \tag{10}$$

where Tr_S is the trace over the S states. The *purity* of the qubit during its quantum evolution can be quantified by the trace of the square density matrix ρ_q, i.e.,

$$\mathrm{Tr}\rho_q(t)^2 = 1 - 2|c_+|^2|c_-|^2F_D(t), \tag{11}$$

where

$$F_D(t) = 1 - |\langle\phi_{v-w}(t)|\phi_{v+w}(t)\rangle|^2, \tag{12}$$

and $0 \leqslant F_D(t) \leqslant 1$. The function F_D measures the quantum decoherence, quantifying the departure from a pure state. Indeed $F_D(t) = 0$ implies that the qubit is in a pure state, while $F_D(t) = 1$ indicates that the qubit is maximally entangled, corresponding to a diagonal density matrix

$$\rho_q = \mathrm{diag}[|c_+|^2, |c_-|^2]. \tag{13}$$

Of course, the time evolution of the decoherence function $F_D(t) \equiv F_D(w, v, L, t)$ depends on the parameters of the global system, i.e., the v that measures the distance of the many-body system from the quantum transition, the coupling w between the qubit and the system, and the size L of the system.

Note that the overlap

$$L_D(t) \equiv |\langle\phi_{v-w}(t)|\phi_{v+w}(t)\rangle| \tag{14}$$

entering the definition of F_D can be interpreted as the fidelity or Loschmidt echo (see, e.g., Ref. [10]) of the S states associated with two different quench protocols involving the isolated system S. For both of them the system S starts from the ground state of the Hamiltonian $H_S(v)$ as $t = 0$; then one considers, and compares using L_D, the quantum evolutions at the same t, arising from the sudden change of the Hamiltonian parameter v to $v - w$ and to $v + w$.

Noting that

$$\langle\phi_{v-w}(t)|\phi_{v+w}(t)\rangle = \langle G_v|e^{iH_S(v-w)t}e^{-iH_S(v+w)t}|G_v\rangle, \tag{15}$$

one can easily show that F_D is an even function of w. Therefore, since $F_D(0, v, L, t) = 0$, and assuming an analytical behavior around $\mu = 0$ (at finite L and t), we expect

$$F_D(w, v, L, t) = \frac{w^2}{2}C_D(v, L, t) + O(w^4) \tag{16}$$

for small values of w. Thus the growth rate of the decoherence in the limit of small qubit-S coupling w is described by the growth-rate function

$$C_D(v, L, t) = \partial^2F_D/\partial w^2|_{w=0}, \tag{17}$$

for a given value v of H_S. It measures the sensitivity of the coherence properties of the subsystems to the qubit-S coupling w.

The above setting can be straightforwardly extended to n-level systems coupled to an environmental many-body system. The scaling arguments we will report herein can be extended as well.

We also note that the above considerations can be straightforwardly extended to the case in which the initial qubit state is not pure, but a mixed state, and is thus described by a nontrivial density matrix. Of course, the calculations become more cumbersome; however, the function F_D maintains its crucial role to describe the coherence properties during the evolution of the global system.

As concrete examples of environmental systems S, we consider the paradigmatic d-dimensional quantum Ising model defined on an L^d lattice,

$$H_I = -J \sum_{\langle \mathbf{x},\mathbf{y}\rangle} \sigma_{\mathbf{x}}^{(3)} \sigma_{\mathbf{y}}^{(3)} - g \sum_{\mathbf{x}} \sigma_{\mathbf{x}}^{(1)}, \tag{18}$$

where $\sigma^{(k)}$ are the Pauli matrices, the first sum is over all bonds connecting nearest-neighbor sites $\langle \mathbf{x}, \mathbf{y}\rangle$, while the other sums are over the sites. We assume $\hbar = 1$, $J = 1$, the lattice spacing $a = 1$, and $g > 0$.

At $g = g_c > 0$ (for one-dimensional quantum Ising systems $g_c = 1$), the model undergoes a continuous quantum transition belonging to the $(d+1)$-dimensional Ising universality class [18–20], separating a disordered phase ($g > g_c$) from an ordered ($g < g_c$) one. For any $g < g_c$, the presence of a longitudinal external field v coupled to

$$P_\ell = -\sum_{\mathbf{x}} \sigma_{\mathbf{x}}^{(3)} \tag{19}$$

drives first-order quantum transitions along the $v = 0$ line.

Then we consider a two-level qubit system, described by the Pauli operator $\Sigma^{(3)}$ globally coupled to the Ising system by the Hamiltonian term

$$H_q = w \Sigma^{(3)} P_\ell. \tag{20}$$

We are interested in the coherence properties of the qubit when the system S is a d-dimensional Ising model with Hamiltonian

$$H_S(v) = H_I + v P_\ell, \tag{21}$$

cf. Eqs. (18) and (19), and the qubit coupling is described by H_q given in Eq. (20).

In the following sections we show that, at both continuous and first-order quantum transitions of the environmental Ising system S, the interplay between the coupling with the qubit, the Hamiltonian parameters, and the size L gives rise to dynamic scaling behavior of the decoherence function $F_D(w, v, L, t)$, and correspondingly of its growth-rate function $C_D(v, L, t)$. For this purpose we consider dynamic finite-size scaling frameworks, which allows us to characterize the decoherence dynamics at both continuous and first-order quantum transitions. We derive the general features of the dynamic finite-size scaling of F_D and C_D, evidencing the differences between continuous and first-order quantum transitions.

III. THE DECOHERENCE DYNAMICS WITH A CRITICAL ENVIRONMENTAL SYSTEM

The theory of finite-size scaling at quantum transitions is well established; see, e.g., Refs. [21–24] and references therein. The continuous quantum transition of the Ising model (18) is characterized by two relevant parameters, $r \equiv g - g_c$ and v (such that they vanish at the critical point), with renormalization-group dimension y_r and y_h, respectively. The relevant finite-size scaling variables are

$$\kappa_r = L^{y_r} r, \quad \kappa_v = L^{y_h} v. \tag{22}$$

The finite-size scaling limit is obtained by taking $L \to \infty$ keeping κ_r and κ_v fixed.

The equilibrium critical exponents y_r and y_h are those of the $(d+1)$-dimensional Ising universality class [18–20]. Therefore, for one-dimensional systems they are $y_r = 1/\nu = 1$ and $y_h = (d+3-\eta)/2 = (4-\eta)/2$ with $\eta = 1/4$. For two-dimensional models the critical exponents are not known exactly, but there are very accurate estimates; see, e.g., Refs. [25–29], and in particular [28] $y_r = 1/\nu$ with $\nu = 0.629\,971(4)$ and $y_h = (5-\eta)/2$ with $\eta = 0.036\,298(2)$. For three-dimensional systems they assume mean-field values, $y_r = 2$ and $y_h = 3$, apart from logarithms. The temperature T gives rise to a relevant perturbation at continuous quantum transitions, associated with the scaling variable $\tau = L^z T$, where $z = 1$ (for any spatial dimension) is the dynamic exponent characterizing the behavior of the energy differences of the lowest-energy states and, in particular, the gap $\Delta \sim L^{-z}$. In the following we assume $T = 0$.

A generic observable O in the finite-size scaling limit behaves as

$$O(r, v, L) \approx L^{y_o} \mathcal{O}(\kappa_r, \kappa_v), \tag{23}$$

where the exponent y_o is the renormalization-group dimension associated with O, and $\mathcal{O}$ is a universal finite-size scaling function. The approach to such an asymptotic behavior is characterized by power-law corrections, typically controlled by irrelevant perturbations at the corresponding fixed point [21]. The equilibrium finite-size scaling at quantum transitions has been also extended to quantum-information concepts [3,30–32], such as the ground-state fidelity and its susceptibility, which measure the change of the ground state when varying the Hamiltonian parameters around a quantum transition [33].

Out-of-equilibrium time-dependent processes require also an appropriate rescaling of the time t, encoded by the scaling variable

$$\theta = L^{-z} t \sim \Delta(L) t. \tag{24}$$

For example, we may consider the dynamic behavior of an isolated system after a quench associated with a sudden change of the parameter v, from v to $v + w$ at $t = 0$ (keeping g fixed), starting from the ground state $|G_v\rangle$. The resulting quantum evolution of the state is

$$|\phi(t)\rangle = e^{-i H_S(v+w) t} |G_v\rangle. \tag{25}$$

This problem can be studied within a dynamic finite-size scaling framework [34]. The dynamic finite-size scaling limit is defined as the infinite-volume $L \to \infty$ limit, keeping the scaling variables θ, κ_r, κ_v, and

$$\kappa_w = L^{y_h} w \tag{26}$$

fixed. Then a generic observable O in the dynamic finite-size scaling limit is expected to behave as [34]

$$O(r, v, w, L, t) \approx L^{y_o} \mathcal{O}(\kappa_r, \kappa_v, \kappa_w, \theta), \tag{27}$$

where again y_o is the renormalization-group dimension of O, and $\mathcal{O}$ is a dynamic finite-size scaling function. The equilibrium finite-size scaling behavior is recovered in the limit $w \to 0$.

An analogous dynamic finite-size scaling is developed by the Loschmidt echo L_e associated with quench protocols, when suddenly changing the driving parameter from v to $v + w$. The Loschmidt echo, defined as

$$L_e(w, v, L, t) = |\langle G_v|e^{-iH_S(v+w)t}|G_v\rangle|, \tag{28}$$

quantifies the deviation of the postquench state at time $t > 0$ from the initial $t = 0$ ground state $|G_v\rangle$ associated with the Hamiltonian $H_S(v)$. It is expected to approach the asymptotic dynamic finite-size scaling [34]

$$L_e(r, w, v, L, t) \approx \mathcal{L}_e(\kappa_r, \kappa_w, \kappa_v, \theta). \tag{29}$$

This has been confirmed by numerical calculations within the one-dimensional quantum Ising model around its continuous quantum transition at $g_c = 1$ [34]. We also mention that the dynamic finite-size scaling framework has been exploited to study the scaling properties of work fluctuations after quenches at quantum transitions [35].

To derive the dynamic finite-size scaling behavior of the decoherence function F_D [cf. Eq. (11)], we exploit its close relation with the Loschmidt echo L_D defined in Eq. (14), between quantum states of S, along the quantum evolutions arising from two different quench protocols of the isolated system S, starting from the same state $|G_v\rangle$ [cf. Eq. (15)]. Therefore, we expect that F_D develops a dynamic finite-size scaling analogous to that of the Loschmidt echo in Eq. (28) associated with standard quench protocols, as reported in Eq. (29).

To begin with, we consider quenches at the critical point $g = g_c$, corresponding to $r = 0$, driven by the parameter v. According to the above scaling arguments, we expect that the decoherence function F_D [cf. Eq. (12)] develops the asymptotic dynamic finite-size scaling

$$F_D(r = 0, w, v, L, t) \approx \mathcal{F}_D(\kappa_w, \kappa_v, \theta), \tag{30}$$

with

$$\mathcal{F}_D(\kappa_w = 0, \kappa_v, \theta) = 0, \quad \mathcal{F}_D(\kappa_w, \kappa_v, \theta = 0) = 0. \tag{31}$$

Note that the above dynamic finite-size scaling requires that also the coupling w between the qubit and S is sufficiently small, indeed the dynamic finite-size scaling limit requires that $\kappa_w = L^{y_h} w$ must be kept constant in the large-L limit. We do not expect universal finite-size scaling behavior without such a rescaling, i.e., for generic finite values of w.

Moreover, Eq. (30) implies that the decoherence growth-rate function C_D [cf. Eqs. (16) and (17)] behaves as

$$C_D(r = 0, v, L, t) \approx L^{2y_h}\mathcal{C}_D(\kappa_v, \theta). \tag{32}$$

This scaling equation characterizes the amplified $O(L^{2y_h})$ rate of departure from coherence of the qubit when the environment system S is at a continuous quantum transition. Indeed, in the case of systems out of criticality one generally expects $C_D \sim L^d$, and

$$2y_h = d + 3 - \eta > d. \tag{33}$$

We may also consider the more general case when the parameter $r = g - g_c$ is not zero, but sufficiently small to keep the system within the critical region. The effects of a nonvanishing parameter r can be taken into account by adding a further dependence on κ_r [cf. Eq. (22)] in the scaling function $\mathcal{C}_D(\kappa_v, \theta)$; i.e., we expect

$$C_D(r, v, L, t) \approx L^{2y_h}\mathcal{C}_D(\kappa_r, \kappa_v, \theta). \tag{34}$$

The scaling behavior in the thermodynamic limit can be formally obtained by considering the limit $L \to \infty$ keeping fixed the scaling variables

$$\rho_v = \kappa_v \kappa_r^{-y_h/y_r} \equiv \xi_r^{y_h} v, \tag{35}$$

$$\theta_v = \theta \kappa_r^{z/y_r} \equiv \xi_r^{-z} t, \tag{36}$$

where $\xi_r \sim r^{-1/y_r}$ is related to the diverging length scale when approaching the critical point $r = 0$ in the thermodynamic limit. Therefore, simple manipulations of the finite-size scaling equation (34) lead to the following scaling behavior in the thermodynamic limit

$$C_D(r, v, L \to \infty, t) \approx \xi_r^{2y_h}\mathcal{C}_\infty(\rho_v, \theta_v), \tag{37}$$

which is obtained by replacing the scaling variables κ_r, κ_v, and θ with ρ_v, θ_v, and L/ξ_r, and considering the thermodynamic limit $L/\xi_r \to \infty$.

The asymptotic behavior described by the dynamic finite-size scaling at continuous quantum transitions is expected to be universal, i.e., independent of the microscopic features of the system S. Its main features only depend on the universality class of the continuous quantum transition of S and the general properties of the coupling between the qubit and the system S. In the case at hand the qubit is coupled to the order parameter of the magnetic transition. Note that the dynamic finite-size scaling functions generally depend on the boundary conditions and the geometry of the system, while the power laws of the observables and the scaling variables remain unchanged. The approach to the dynamic finite-size scaling is expected to be generally characterized by power-law-suppressed corrections, as it generally occurs at continuous quantum transitions [21].

We may also consider the case in which the qubit is homogeneously coupled to the transverse spin operators, i.e., we replace P_ℓ [cf. Eq. (19)] with $P_t = -\sum_{\mathbf{x}} \sigma_{\mathbf{x}}^{(1)}$, and the qubit-$S$ coupling (20) with

$$H_{q,t} = u\Sigma^{(1)}P_t. \tag{38}$$

For simplicity we assume that S is initially prepared in the ground state for $v = 0$ and a given $r = g - g_c$. By using scaling arguments analogous to those leading to Eq. (30), we arrive at the dynamic finite-size scaling

$$F_D(u, r, L, t) \approx \mathcal{F}_D(\kappa_u, \kappa_r, \theta), \tag{39}$$

with κ_r defined in Eq. (22), and $\kappa_u = L^{y_r}u$. This also implies

$$C_D(r, L, t) \approx L^{2y_r}\mathcal{C}_D(\kappa_r, \theta) \tag{40}$$

for the corresponding decoherence growth-rate function. Note again the enhancement of the decoherence dynamics, because $2y_r > d$. The decoherence dynamics of this central spin model, with the qubit homogeneously coupled to the transverse spin variables of a one-dimension Ising model, was also considered in Ref. [10]; the scaling behavior of its numerical results appears consistent with the dynamic finite-size scaling prediction (39).

IV. QUBIT DECOHERENCE WITH ENVIRONMENTAL SYSTEMS AT FIRST-ORDER QUANTUM TRANSITIONS

In this section we extend the dynamic finite-size scaling of the decoherence dynamics to the case in which the environmental system S is at a first-order quantum transition, i.e., along the line $g < g_c$ of the phase diagram of a d-dimensional Ising model. We again consider the quantum evolution of the global system starting from pure states of both the qubit and the environmental Ising system S.

As shown by earlier works [22,36–38], the finite-size scaling behavior of isolated many-body systems at first-order quantum transitions significantly depend on the type of boundary condition; in particular, whether they favor one of the phases or are neutral, giving rise to finite-size scaling characterized by exponential or power-law behavior. In the following we consider Ising systems with boundary conditions that do not favor any of the two magnetized phases, such as periodic and open boundary conditions, which generally lead to exponential finite-size scaling laws.

The first-order quantum transition line for $g < g_c$ are related to the level crossing of the two lowest states $|\uparrow\rangle$ and $|\downarrow\rangle$ for $v = 0$, such that $\langle\uparrow|\sigma_{\mathbf{x}}^{(3)}|\uparrow\rangle = m_0$ and $\langle\downarrow|\sigma_{\mathbf{x}}^{(3)}|\downarrow\rangle = -m_0$ (independently of $\mathbf{x}$) with $m_0 > 0$. The degeneracy of these states at $v = 0$ is lifted by the longitudinal field v. Therefore, $v = 0$ is a first-order quantum transition point, where the longitudinal magnetization $M = L^{-d}\sum_{\mathbf{x}} M_{\mathbf{x}}$, with $M_{\mathbf{x}} \equiv \langle\sigma_{\mathbf{x}}^{(3)}\rangle$, becomes discontinuous in the infinite-volume limit. The first-order quantum transition separates two different phases characterized by opposite values of the magnetization m_0, i.e.,

$$\lim_{v\to 0^{\pm}} \lim_{L\to\infty} M = \pm m_0. \tag{41}$$

For one-dimensional systems [39] $m_0 = (1 - g^2)^{1/8}$.

In a finite system of size L, the two lowest states are superpositions of two magnetized states $|+\rangle$ and $|-\rangle$ such that $\langle\pm|\sigma_{\mathbf{x}}^{(3)}|\pm\rangle = \pm m_0$ for all sites $\mathbf{x}$. Due to tunneling effects, the energy gap Δ at $v = 0$ vanishes exponentially as L increases [22,40],

$$\Delta(L) \sim e^{-cL^d}, \tag{42}$$

apart from powers of L. In particular, the energy gap $\Delta(L)$ of the one-dimensional Ising system (18) for $g < 1$ is exponentially suppressed as follows [39,41]:

$$\Delta(L) = 2(1 - g^2)g^L[1 + O(g^{2L})] \tag{43}$$

for open boundary conditions, and

$$\Delta(L) \approx 2(\pi L)^{-1/2}(1 - g^2)g^L \tag{44}$$

for periodic boundary conditions. The differences $\Delta_i \equiv E_i - E_0$ for the higher excited states ($i > 1$) are finite for $L \to \infty$.

The emergence of a dynamic finite-size scaling after a quench protocol is also expected along the first-order quantum transition line for $g < g_c$ [34], which is associated with a sudden change of the parameter v from v to $v + w$ at $t = 0$, starting from the ground state $|G_v\rangle$. Extending to generic dimensions the arguments of Refs. [34,42], we identify the following scaling variables:

$$\kappa_v = \frac{2m_0 v L^d}{\Delta(L)}, \quad \kappa_w = \frac{2m_0 w L^d}{\Delta(L)}, \quad \theta = t\Delta(L). \tag{45}$$

In particular, the scaling variables κ_v and κ_w are the ratios between the energy associated with the corresponding longitudinal-field perturbations, which are approximately given by $2m_0 v L^d$ and $2m_0 w L^d$, respectively, and the energy difference $\Delta(L)$ of the two lowest states at $v = 0$. Then, the expected dynamic finite-size scaling of the magnetization is [34]

$$M(w, v, L, t) = m_0 \mathcal{M}(\kappa_w, \kappa_v, \theta). \tag{46}$$

This dynamic finite-size scaling is expected to hold for any $g < g_c$. The scaling function $\mathcal{M}$ is independent of g, apart from trivial normalizations of the arguments. The dynamic finite-size scaling at first-order quantum transitions has been numerically confirmed in the case of the one-dimensional Ising model [34]. The approach to the asymptotic dynamic finite-size scaling is expected to be exponential when increasing the size of the system. An analogous dynamic finite-size scaling applies to the Loschmidt echo defined as in Eq. (28), we expect $L_e(w, v, L, t) \approx \mathcal{L}_e(\kappa_w, \kappa_v, \theta)$, which is formally identical to Eq. (29).

Then, using the same arguments of the previous section, i.e., noting that the decoherence function F_D can be written in terms of quench-like amplitudes related to the environmental system only, we conjecture an analogous dynamic finite-size scaling for the decoherence function

$$F_D(w, v, L, t) \approx \mathcal{F}_D(\kappa_w, \kappa_v, \theta). \tag{47}$$

Correspondingly, matching the expansion of the F_D in powers of w and that of $\mathcal{F}_D$ in powers of κ_w, we obtain the decoherence growth-rate function

$$C_D(v, L, t) \approx \frac{4m_0^2 L^{2d}}{\Delta(L)^2}\mathcal{C}_D(\kappa_v, \theta). \tag{48}$$

Therefore, when the environment system S is at a first-order quantum transition, the decoherence growth rate gets significantly enhanced, increasing exponentially with L. Indeed the prefactor of Eq. (48) behaves as

$$\frac{4m_0^2 L^{2d}}{\Delta(L)^2} \sim \exp(bL^d), \tag{49}$$

apart from powers of L.

In the case of the quantum Ising systems with periodic or open boundary conditions, the dynamic finite-size scaling functions can be exactly computed, exploiting a two-level truncation of the spectrum [22,42]. As shown in Ref. [42], in the long-time limit and for large systems, the scaling properties in a small interval around $v = 0$, more precisely for $m_0|v| \ll \Delta_2 = O(1)$, are captured by a two-level truncation, which only takes into account the two nearly degenerate lowest-energy states. The effective evolution is determined by the Schrödinger equation [42]

$$i\frac{d}{dt}\Psi(t) = H_2(v)\Psi(t), \tag{50}$$

where $\Psi(t)$ is a two-component wave function whose components correspond to the states $|+\rangle$ and $|-\rangle$, and

$$H_2(v) = -\beta\sigma^{(3)} + \delta\sigma^{(1)},$$
$$\beta = m_0 v L^d, \quad \delta = \frac{\Delta(L)}{2}, \quad \kappa_v = \frac{\beta}{\delta}, \tag{51}$$

where $\sigma^{(k)}$ are the Pauli matrices. The initial condition is given by the ground state of $H_2(v)$, i.e., by

$$|\Psi(w, v, L, t = 0)\rangle = \sin(\alpha_v/2)|-\rangle - \cos(\alpha_v/2)|+\rangle, \tag{52}$$

with $\tan\alpha_v = \kappa_v^{-1}$ and $\alpha_v \in (0, \pi)$. The quantum evolution after quenching from v to $v + w$ can be easily obtained by diagonalizing $H_2(v + w)$, whose eigenstates are

$$|0\rangle = \sin(\alpha_{v+w}/2)|-\rangle - \cos(\alpha_{v+w}/2)|+\rangle, \tag{53}$$

$$|1\rangle = \cos(\alpha_{v+w}/2)|-\rangle + \sin(\alpha_{v+w}/2)|+\rangle, \tag{54}$$

where $\tan\alpha_{v+w} = (\kappa_v + \kappa_w)^{-1}$ with $\alpha_{v+w} \in (0, \pi)$. Their eigenvalue difference is given by

$$E_1 - E_0 = \Delta(L)\sqrt{1 + (\kappa_v + \kappa_w)^2}. \tag{55}$$

Then, apart from an irrelevant phase, the time-dependent state evolves as

$$|\Psi(w, v, L, t)\rangle = \cos\left(\frac{\alpha_v - \alpha_{v+w}}{2}\right)|0\rangle + e^{-i\theta\sqrt{1+(\kappa_v+\kappa_w)^2}} \times \sin\left(\frac{\alpha_v - \alpha_{v+w}}{2}\right)|1\rangle. \tag{56}$$

Note that the time-dependent wave function is written in terms of scaling variables only. The dynamic finite-size scaling of the magnetization can be easily obtained [34] by computing the expectation value of the operator $\sigma^{(3)}$ over the state $|\Psi(w, v, L, t)\rangle$.

The decoherence function F_D can be straightforwardly obtained by computing

$$F_D(w, v, L, t) = 1 - |\langle\Psi(-w, v, L, t)|\Psi(w, v, L, t)\rangle|^2. \tag{57}$$

Using Eq. (56), one can immediately see that $F_D(w, v, L, t)$ is a function of κ_w, κ_v, and θ only, confirming the dynamic finite-size scaling (47). The resulting expression is quite cumbersome; some plots are shown in Fig. 1. The curves are also characterized by revivals, which are typical of two-level systems.

The dynamic finite-size scaling of the decoherence growth-rate function C_D is obtained by computing

$$C_D(v, L, t) = \left.\frac{\partial^2 F_D}{\partial w^2}\right|_{w=0} = \left(\frac{\partial\kappa_w}{\partial w}\right)^2 \mathcal{C}_D(\kappa_v, \theta), \tag{58}$$

which leads to the analytical result

$$\mathcal{C}_D(\kappa_v, \theta) = \frac{2[1 - \cos(\theta\sqrt{1 + \kappa_v^2})]}{(1 + \kappa_v^2)^2}. \tag{59}$$

Note the simple result for $\kappa_v = 0$:

$$\mathcal{C}_D(0, \theta) = 2(1 - \cos\theta), \tag{60}$$

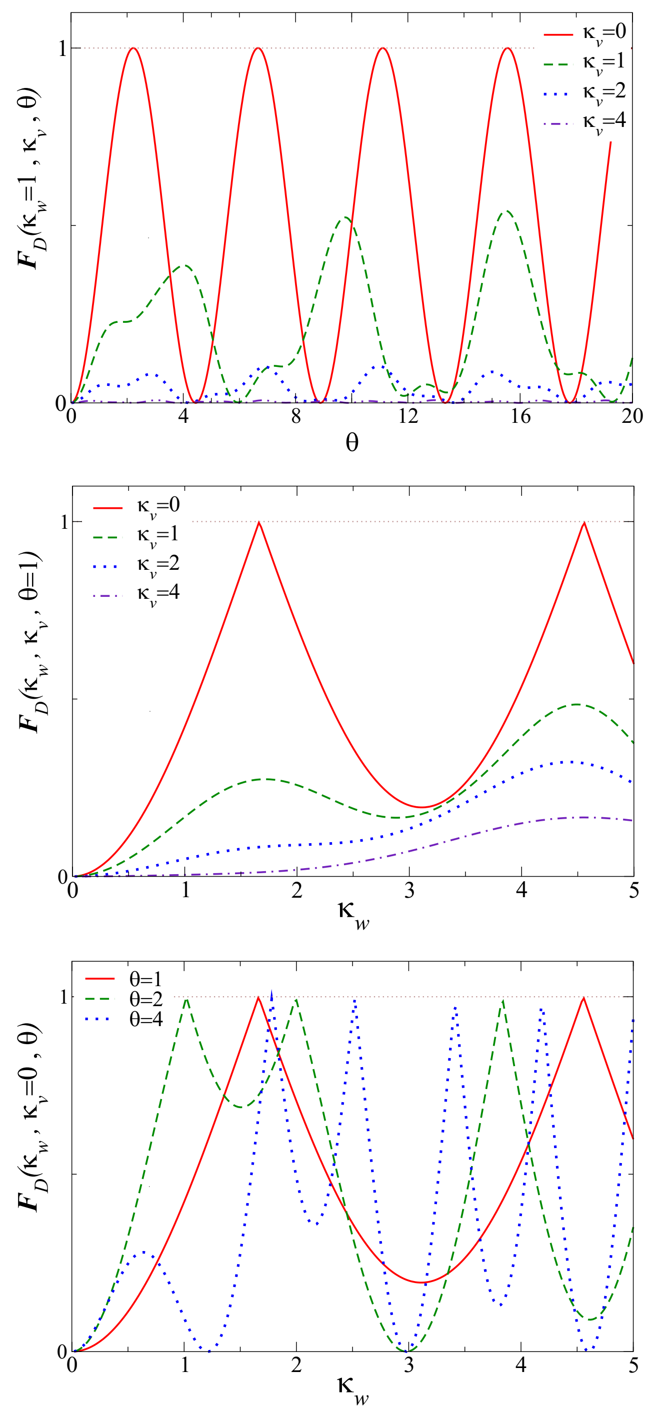

FIG. 1. Some plots of the scaling function $\mathcal{F}_D(\kappa_w, \kappa_v, \theta)$ associated with the decoherence dynamics at first-order quantum transitions [cf. Eq. (47)]. The top figure shows plots versus θ for $\kappa_w = 1$ and some values of κ_v; the middle figure shows plots versus κ_w for $\theta = 1$ and some values of κ_v; the bottom figure shows plots versus κ_w for $\kappa_v = 0$ and some values of θ. The units of the scaling variables can be easily inferred by taking into account that we set $\hbar = 1$, $J = 1$ and the lattice spacing $a = 1$.

and that $\mathcal{C}_D(\kappa_v, \theta)$ vanishes for $\kappa_v \to \infty$. We stress that the above dynamic finite-size scaling functions are expected to be independent of $g < g_c$ along the first-order transition line, apart from trivial g-dependent normalizations of the scaling variables.

We finally mention that a notable feature of one-dimensional quantum Ising systems at first-order quantum transitions, with neutral boundary conditions such as periodic and open boundary conditions, is their rigidity with respect to external perturbations [22,42], i.e., their response to global or local longitudinal perturbations is analogous. Therefore, an analogous quantum decoherence dynamics at the first-order transition line is expected in the case of a local coupling between the longitudinal parameter v, the qubit, and the Ising chain; for example when replacing P_ℓ [cf Eqs. (19) and (20)] with

$$p_\ell = -\sigma^{(3)}_{x_c}, \quad h_q = w\Sigma^{(3)} p_\ell, \tag{61}$$

respectively, where x_c is one of the sites of the chain (sufficiently far from the boundaries). The only difference is that the relevant scaling variables turn into $\kappa_v = 2m_0 v/\Delta(L)$ and $\kappa_w = 2m_0 w/\Delta(L)$ instead of those reported in Eq. (45). They give rise to a two-level scenario as well, with the same dynamic finite-size scaling functions.

V. CONCLUSIONS

We have investigated the decoherence dynamics of a two-level qubit system globally and homogeneously coupled to a many-body spin system S, such as a d-dimensional quantum Ising system, at a quantum transition. In particular, we have considered the out-of-equilibrium quantum evolution of the global system starting from pure states of both the qubit and S. The decoherence dynamics of the qubit is described by the time evolution of its density matrix, obtained tracing out the states of S. Its behavior can be characterized by the decoherence function F_D defined in Eq. (11), which quantifies the departure of the qubit from a pure state, independently of its initial pure state. The sensitivity to the qubit-S coupling w is measured by the decoherence growth-rate function $C_D = \partial^2 F_D/\partial w^2|_{w=0}$ [cf. Eq. (16)].

We have shown that the rate of the quantum decoherence gets enhanced when the environmental system S experiences a quantum transition. At both continuous and first-order quantum transitions of S, the interplay among the coupling between the qubit and S, the Hamiltonian parameters and the size of S, during the quantum evolution gives rise to scaling behavior of the decoherence function $F_D(w, v, L, t)$ [cf. Eq. (12)] and the corresponding decoherence growth-rate function $C_D(v, L, t)$ [cf. Eq. (17)] in the limit of large size L of S. This is shown within dynamic finite-size scaling frameworks, which allow us to determine the behavior of the decoherence functions at both continuous and first-order quantum transitions of the environmental system S, in appropriate dynamic finite-size scaling limits.

We derive the general properties of the dynamic finite-size scaling of the decoherence functions F_D and C_D, providing evidence of the differences between continuous and first-order quantum transitions. We show that they are characterized by power laws of the size L at continuous quantum transitions, while exponential laws generally emerge at first-order quantum transitions. This behavior represents a substantial enhancement of the rate of the decoherence dynamics. For example, at continuous quantum transitions, when the qubit couples longitudinally to the Ising model, the rate function turns out to increase as $C_D \sim L^\zeta$ where $\zeta = 2y_h = 15/8$ for $d = 1$, $\zeta = 2y_h \approx 4.96$ for $d = 2$, and $\zeta = 2y_h = 6$ for $d = 3$ (apart from logarithms). Therefore they show a significant enhancement of the decoherence growth rate, when compared with the general volume L^d law expected for systems in normal conditions. The decoherence growth-rate enhancement appears even more substantial at first-order quantum transitions, where $C_D \sim \exp(bL^d)$ increases exponentially.

Note that the main features of the dynamic finite-size scaling, such as the general size dependence and the scaling functions, are expected to be universal, i.e., they are expected not to depend on the microscopic details of the models. Therefore, their predictions can be extended to all continuous quantum transitions belonging to the same Ising universality class with analogous coupling between qubit and system. An analogous statement holds for the dynamic finite-size scaling with environmental systems at first-order quantum transitions. In particular, the dynamic finite-size scaling with environmental Ising systems is expected to be the same, apart from normalizations, along the first-order transition line for $g < g_c$, and in any system sharing the same global properties, such as first-order quantum transitions arising from an avoided two-level crossing phenomenon in the large-L limit.

Finally, we would like to stress that the dynamic finite-size scaling frameworks, which was exploited to study the decoherence dynamics of qubit coupled longitudinally and transversally to Ising systems at quantum transitions, can be straightforwardly extended to general continuous and first-order quantum transitions, and generic couplings of the qubit to its environmental system.

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