Effective quantum Zeno dynamics in dissipative quantum systems

Vladislav Popkov,[1,2,3] Simon Essink,[1] Carlo Presilla,[4,5] and Gunter Schütz[6]
[1]*HISKP, University of Bonn, Nussallee 14-16, 53115 Bonn, Germany*
[2]*Institut für Teoretische Physik, Universität zu Köln, Zülpicher str. 77, 50937 Köln, Germany*
[3]*Centro Interdipartimentale per lo studio di Dinamiche Complesse, Università di Firenze, via G. Sansone 1, 50019 Sesto Fiorentino, Italy*
[4]*Dipartimento di Fisica, Sapienza Università di Roma, Piazzale Aldo Moro 2, Rome 00185, Italy*
[5]*Istituto Nazionale di Fisica Nucleare, Sezione di Roma 1, Rome 00185, Italy*
[6]*ICS, Forschungszentrum Jüelich GmbH Wilhelm-Johnen-Straße 52428 Jüelich, Germany*

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We investigate the time evolution of an open quantum system described by a Lindblad master equation with dissipation acting only on a part of the degrees of freedom $\mathcal{H}_0$ of the system, and targeting a unique dark state in $\mathcal{H}_0$. We show that, in the Zeno limit of large dissipation, the density matrix of the system traced over the dissipative subspace $\mathcal{H}_0$, evolves according to another Lindblad dynamics, with renormalized effective Hamiltonian and weak effective dissipation. This behavior is explicitly checked in the case of Heisenberg spin chains with one or both boundary spins strongly coupled to a magnetic reservoir. Moreover, the populations of the eigenstates of the renormalized effective Hamiltonian evolve in time according to a classical Markov dynamics. As a direct application of this result, we propose a computationally efficient exact method to evaluate the nonequilibrium steady state of a general system in the limit of strong dissipation.

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I. INTRODUCTION

A quantum system interacting with an environment is, under a Markov assumption, well described by a Lindblad master equation (LME) [1,2]. It follows that the reduced density matrix (RDM) of the system undergoes a coherent and dissipative evolution [3,4]. If the coherent and dissipative parts of LME do not depend on time, then, after a transient, the system reaches a (unique) nonequilibrium steady state (NESS), which is independent of the initial conditions. Even if the NESS is trivial, the relaxation dynamics may not be: specially if a large dissipation-free subspace exists, the NESS can be approached through a complicated multistage evolution.

If the dissipation time scales are short in comparison with the coherent evolution times, then the so-called quantum Zeno regime occurs. Quantum Zeno effect [5,6] predicts an inhibition of quantum transitions in a quantum system subjected to frequent measurements. It has been observed experimentally, in various setups [7–11]. Applications of Zeno effect include dissipation-protected realization of quantum gates [12], engineering of nontrivial quantum states, and implementation of universal quantum computations [13–16] creating quantum simulators [17], localization of a single atom in a lattice [18], realization of exotic effective dynamics [19,20], development of theoretical tools for a real-time observation of quantum many-body dynamics [21].

It is well recognized that the evolution of a system near the Zeno limit is not frozen but can proceed via Raman-like processes involving virtual levels, which couple states within a given Zeno subspace [22,23], while the occupation of the virtual levels remains negligible.

In more detail, one can distinguish three stages of relaxation, occurring at different time scales. On the shortest time scale, only the degrees of freedom directly affected by the dissipation relax to their stationary values. On the second, intermediate time scale, an effective coherent evolution takes place, governed by a dissipation-projected Hamiltonian [24]. Finally, on the longest time scale, all system characteristics relax to their stationary values.

In this paper, we focus on the third stage of evolution and derive an effective dynamics of the system in the decoherence-free subspace. It happens that, in the assumed Zeno regime, and under the nondegeneracy assumption for the local kernel of the dissipator (2), this dynamics is also of Lindblad type. As an application, we demonstrate that the spectrum of the reduced density matrix, which does not change on the intermediate time scale, on the longest time scale evolves according to a classical Markov process, with generator F computable from the LME entries.

II. MAIN RESULTS

Consider an open quantum system, with finite Hilbert space $\mathcal{H}$, under strong dissipation acting only on a subspace $\mathcal{H}_0$ of the degrees of freedom, described by the Lindblad master equation,

$$\frac{\partial \rho(\tau)}{\partial \tau} = -\frac{i}{\hbar}[H, \rho(\tau)] + \Gamma \mathcal{D}[\rho(\tau)]. \tag{1}$$

Let the dissipation-free subspace be $\mathcal{H}_1$, $\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_1$, and denote by $\mathrm{tr}_{\mathcal{H}_0}$ and $\mathrm{tr}_{\mathcal{H}_1}$ the trace over $\mathcal{H}_0$ and $\mathcal{H}_1$, having dimensions d_0 and d_1, respectively. We assume the Lindblad dissipator $\mathcal{D}$ to target a unique state $\psi_0 \in \mathcal{H}_0$, namely,

$$(\mathrm{tr}_{\mathcal{H}_1} \mathcal{D})\psi_0 = 0. \tag{2}$$

The aim of this paper is to show that, in the Zeno limit, when the effective dissipation strength Γ is much stronger than the unitary part of the evolution, the solution of the problem (1) for all times $\tau > O(1)$ has the approximate form

$$\rho(\tau) = \psi_0 \otimes R(\tau),$$

where $R(\tau) = \mathrm{tr}_{\mathcal{H}_0}\, \rho(\tau)$ evolves according to another LME

$$\frac{\partial R(\tau)}{\partial \tau} = -\frac{i}{\hbar}[\tilde{H}, R(\tau)] + \frac{1}{\Gamma}\tilde{\mathcal{D}}[R(\tau)]. \tag{3}$$

More precisely, we demonstrate that

$$\|\rho(\tau) - \psi_0 \otimes R(\tau)\| = O\left(\frac{1}{\Gamma}\right), \tag{4}$$

for $\Gamma \to \infty$ and for all times $\tau \gg 1/\Gamma$. The choice of the norm $\|\cdot\|$ is rather arbitrary. Note that the LMEs (1) and (3), besides being defined in terms of different Hamiltonians and dissipators, have dissipation strength Γ and $1/\Gamma$, respectively.

Using $1/\Gamma \ll 1$ as a small parameter, we obtain the above result by writing the Dyson series for the Liouvillian dynamics associated to the LME (1). We start rescaling the time $\Gamma\tau = t$ in the original LME. In the limit of strong dissipation $\Gamma \gg 1$, we obtain an equation with a perturbative term,

$$\frac{\partial \rho}{\partial t} = \mathcal{D}[\rho] - \frac{i}{\Gamma}[H, \rho] = (\mathcal{L}_0 + K)\rho = \mathcal{L}\rho, \tag{5}$$

where $\mathcal{L} = \mathcal{L}_0 + K$ and the linear operators $\mathcal{L}_0$ and $K = -(i/\Gamma)[H, \cdot]$ denote the dissipator and the commutator, respectively. The formal solution of Eq. (5) is

$$\rho(t) = e^{\mathcal{L}t}\rho(0) = \mathcal{E}(t)\rho(0), \tag{6}$$

where the propagator $\mathcal{E}(t)$ satisfies

$$\mathcal{E}(t) = e^{\mathcal{L}_0 t}\left(1 + \int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K \mathcal{E}(t_1)\right). \tag{7}$$

Iterating Eq. (7) we get the Dyson expansion. Up to the second order we obtain

$$\mathcal{E}(t) = e^{\mathcal{L}_0 t}\left(1 + \int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1} + \int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1} \int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2} K e^{\mathcal{L}_0 t_2} + \dots\right). \tag{8}$$

Introduce the spectral projection $\mathcal{P}_0$ onto the kernel of the dissipator $\mathcal{L}_0$, namely, $\mathcal{P}_0 = \lim_{t\to\infty} \exp(\mathcal{L}_0 t)$. Define also its complement $\mathcal{Q}_0 = I_{\mathcal{H}} - \mathcal{P}_0$, where $I_{\mathcal{H}}$ is the identity operator in the space $\mathcal{H}$. Obviously, $\mathcal{P}_0\mathcal{Q}_0 = 0$. If $1/\Gamma$ is small, the dissipative part of the Lindbladian constrains the system to a decoherence-free subspace. In fact, the leakage outside of decoherence-free subspace (defined as the subspace belonging to the dissipator Kernel) can be rigorously proven to be negligible, see Ref. [24]. Therefore, we shall only consider an evolution inside the decoherence-free subspace, which is given by the propagator $\mathcal{P}_0\mathcal{E}(t)\mathcal{P}_0$. Performing the calculations as indicated in Appendix, we obtain

$$\mathcal{P}_0\mathcal{E}(t)\mathcal{P}_0 = \mathcal{P}_0 + t\mathcal{P}_0 K\mathcal{P}_0 + \frac{t^2}{2!}(\mathcal{P}_0 K\mathcal{P}_0)^2 - t\mathcal{P}_0 K\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0 + \dots, \tag{9}$$

where ... is the contribution from the remaining orders of the Dyson expansion, and $\mathcal{S}$ is the pseudoinverse of the dissipator,

$$\mathcal{L}_0\mathcal{S} = \mathcal{S}\mathcal{L}_0 = \mathcal{Q}_0. \tag{10}$$

Note that the first three terms in Eq. (9) can be exponentiated, as $\mathcal{P}_0\exp(t\mathcal{P}_0K\mathcal{P}_0)$. They all describe a unitary dynamics within the decoherence-free subspace, as is seen by applying the propagator $\mathcal{P}_0K\mathcal{P}_0$ on a state $\rho = \psi_0 \otimes R$,

$$\begin{aligned}\mathcal{P}_0K\mathcal{P}_0\rho &= -\frac{i}{\Gamma}\mathcal{P}_0[H, \psi_0 \otimes R]\\ &= -\frac{i}{\Gamma}\psi_0 \otimes \mathrm{tr}_{\mathcal{H}_0}(H(\psi_0 \otimes R) - (\psi_0 \otimes R)H)\\ &= -\frac{i}{\Gamma}\psi_0 \otimes [h_D, R],\end{aligned} \tag{11}$$

where h_D is the dissipation-projected Hamiltonian

$$h_D = \mathrm{tr}_{\mathcal{H}_0}[(\psi_0 \otimes I_{\mathcal{H}_1})H]. \tag{12}$$

Since the operator K is proportional to the small parameter $1/\Gamma$, we conclude that the terms $t\mathcal{P}_0K\mathcal{P}_0$ and $\frac{t^2}{2!}(\mathcal{P}_0K\mathcal{P}_0)^2$ give a contribution $O(1)$ to the propagator for times $t \sim O(\Gamma)$, while the last term $-t\mathcal{P}_0K\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0$ contributes $O(1)$ changes to the propagator for $t \sim O(\Gamma^2)$. The physical interpretation of Eq. (9) is thus as follows. One observes three different processes, taking place at different time scales $\tau = t/\Gamma$: (i) at short times $\tau \sim 1/\Gamma$, the system is projected onto the decoherence-free subspace; (ii) at intermediate times $\tau \sim 1$, the evolution inside the decoherence-free subspace is unitary $\mathcal{P}_0K\mathcal{P}_0 \sim -i\psi_0 \otimes [h_D, \cdot]$; (iii) at large times $\tau \sim \Gamma$ the term $t\mathcal{P}_0K\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0$ sets in. Note that the slowest part of the evolution, taking place at the longest time scale, cannot by any means be ignored since it is the only part containing a relaxation towards the NESS. In fact, the unitary evolution alone governed by the effective Hamiltonian (12), does not lead to any relaxation.

To derive the evolution equation from the Dyson expansion, assume the system to start in the dissipation-free subspace, i.e., $\rho(0) = \mathcal{P}_0\rho(0)$. This is equivalent to assuming the factorized initial state $\rho(0) = \psi_0 \otimes R(0)$. The time evolution inside the decoherence-free subspace is given by $\mathcal{P}_0\mathcal{E}(t)\mathcal{P}_0[\psi_0 \otimes R(0)] = \psi_0 \otimes R(t)$. We obtain the evolution equation in differential form considering $\lim_{t\to 0}[\rho(t) - \rho(0)]/t = \partial\rho/\partial t$. Using the Dyson expansion, tracing over $\mathcal{H}_0$, and rescaling the time $t/\Gamma = \tau$, we obtain

$$\frac{\partial R}{\partial \tau} = -i[h_D, R(\tau)] + \frac{1}{\Gamma}W \tag{13}$$

$$W = -\Gamma^2\, \mathrm{tr}_{\mathcal{H}_0}(\mathcal{P}_0K\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0\rho). \tag{14}$$

Equation (13) is valid for time scales beyond the shortest one, i.e., $\tau \gg 1/\Gamma$. The total error of the effective description (13) of the evolution $\rho(0) \to \rho(\tau) \approx \psi_0 \otimes R(\tau)$ for large Γ results from two contributions: a leakage outside the dissipation-free subspace and higher-order dissipation terms, both contributions being generically of order $1/\Gamma$, see also Fig. 1.

To evaluate W from Eq. (14), we make two assumptions: (i) the kernel of $\mathcal{L}_0$ is one dimensional, i.e., the eigenvalue 0 of the dissipator is nondegenerate,

$$\mathcal{L}_0\psi_0 = 0; \tag{15}$$

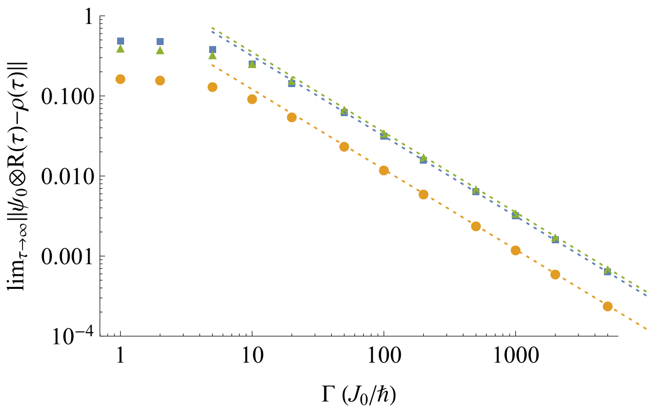

FIG. 1. Asymptotic error (Euclidean norm) $\lim_{\tau\to\infty}\|\psi_0\otimes R(\tau)-\rho(\tau)\|$ as a function of the dissipation strength Γ for the XYZ Heisenberg spin chain with dissipation acting on the first and last spins. Here, $R(\tau)$ is the solution of Eq. (13) whereas $\rho(\tau)$ is the solution of Eq. (1). Parameters: $N=4$, $J_x=J_0$, $J_y=2.2J_0$, $J_z=0.77J_0$ for all data points. Triangles: $\theta_L=\varphi_L=\theta_R=\varphi_R=0$, $\mu_L=0.9$, $\mu_R=0.7$. Squares: $\theta_L=\pi/3$, $\varphi_L=\pi/4$, $\theta_R=3\pi/7$, $\varphi_R=4\pi/15$, $\mu_L=0.9$, $\mu_R=0.7$. Circles: $\theta_L=\pi/3$, $\varphi_L=\pi/4$, $\theta_R=3\pi/7$, $\varphi_R=4\pi/15$, $\mu_L=0.5$, $\mu_R=-0.3$. The straight lines with slope $1/\Gamma$ are guides to the eye.

(ii) $\mathcal{L}_0$ is diagonalizable, i.e., a basis ψ_k (not necessarily orthogonal) exists,

$$\mathcal{L}_0\psi_k=\xi_k\psi_k. \tag{16}$$

Note that $\psi_k^\dagger$ are also eigenvectors of the dissipator, with eigenvalues ξ_k^*, namely, $\mathcal{L}_0\psi_k^\dagger=\xi_k^*\psi_k^\dagger$. We also introduce a complementary basis φ_k, trace orthonormal to the basis ψ_j,

$$\mathrm{tr}(\varphi_k\psi_j)=\delta_{k,j}. \tag{17}$$

Hereafter, we work in the representation in which ψ_k,φ_k are square matrices.

First, we note that the action of $\mathcal{P}_0$ on the arbitrary element $X\in\mathcal{H}$ is

$$\mathcal{P}_0X=\psi_0\otimes\mathrm{tr}_{\mathcal{H}_0}X. \tag{18}$$

In fact, due to the definition of $\mathcal{P}_0$ we have

$$\begin{aligned}\mathcal{P}_0X&=\lim_{t\to\infty}e^{\mathcal{L}_0t}X=\lim_{t\to\infty}e^{\mathcal{L}_0t}\sum_k\psi_k\otimes x_k\\&=\sum_k\lim_{t\to\infty}e^{\xi_kt}\psi_k\otimes x_k=\psi_0\otimes x_0,\end{aligned} \tag{19}$$

since the real part of all ξ_k for $k>0$ is strictly negative. In the decomposition $X=\sum_k\psi_k\otimes x_k$, the element x_0 can be found using the trace-orthonormal basis φ_k as $x_0=\mathrm{tr}_{\mathcal{H}_0}(\varphi_0\otimes I_{\mathcal{H}_1})X$. The element φ_0 of this basis, satisfying $\mathrm{tr}(\varphi_0\psi_k)=\delta_{k,0}$, can always be chosen as the unit matrix, $\varphi_0=I_{\mathcal{H}_0}$, since all the eigenfunctions of the dissipator with nonzero eigenvalues are traceless, and $\mathrm{tr}\,\psi_0=1$. Substituting $x_0=\mathrm{tr}_{\mathcal{H}_0}X$ in Eq. (19), we obtain Eq. (18).

It is convenient to define the Hamiltonian decomposition

$$H=\sum_n\varphi_n\otimes g_n=\sum_n\varphi_n^\dagger\otimes g_n^\dagger, \tag{20}$$

$$g_k=\mathrm{tr}_{\mathcal{H}_0}[(\psi_k\otimes I_{\mathcal{H}_1})H]. \tag{21}$$

We have, step by step,

$$\begin{aligned}\mathcal{P}_0\rho(0)&=\rho(0),\\(\Gamma K)\mathcal{P}_0\rho(0)&=-i[H,\rho(0)]\\&=-i\sum_{m,n}[C_{mn}\psi_m^\dagger\otimes(g_nR)-\text{H.c.}],\\\mathcal{Q}_0S(\Gamma K)\mathcal{P}_0\rho(0)&=-i\sum_{m>0,n}\frac{1}{\xi_m^*}[C_{mn}\psi_m^\dagger\otimes(g_nR)-\text{H.c.}],\end{aligned}$$

where

$$C_{mn}=\mathrm{tr}(\varphi_m^\dagger\varphi_n\psi_0). \tag{22}$$

Since $\varphi_0=I_{\mathcal{H}_0}$, the coefficients C_{mn} satisfy

$$C_{0n}=C_{n0}=\delta_{0,n}. \tag{23}$$

In the last step, using Eqs. (17) and (18), we arrive at

$$W=\sum_{m>0,n>0}\left(\frac{C_{mn}}{-\xi_m^*}(-g_m^\dagger g_nR+g_nRg_m^\dagger)+\text{H.c.}\right). \tag{24}$$

Note that the term $n=0$ does not appear in the sum (24) because of Eq. (23). Using the substitution $-C_{mn}/\xi_m^*=Y_{mn}=A_{mn}/2+iB_{mn}$ with $A_{mn}=Y_{mn}+Y_{nm}^*$ positive matrix and $B_{mn}=(Y_{mn}-Y_{nm}^*)/(2i)$ Hermitian matrix, and changing the order of summation in the H.c. term in (24), we can put W in the general Lindbladian form,

$$W=-i[\tilde{H}_a,R]+\tilde{\mathcal{D}}R, \tag{25}$$

$$\tilde{H}_a=\sum_{m>0,n>0}B_{mn}g_m^\dagger g_n, \tag{26}$$

$$\tilde{\mathcal{D}}R=\sum_{m>0,n>0}A_{mn}\left(g_nRg_m^\dagger-\frac{1}{2}g_m^\dagger g_nR-\frac{1}{2}Rg_m^\dagger g_n\right). \tag{27}$$

According to Eq. (13), from the above expression of W we conclude that the effective time evolution of the system in the dissipation-free subspace has the standard Lindblad form of Eq. (3), with $\tilde{H}=h_D+\tilde{H}_a/\Gamma$ and the dissipator $\tilde{\mathcal{D}}/\Gamma$ with $\tilde{\mathcal{D}}$ given by Eq. (27). Note that the stronger is the dissipation Γ in the original system, the weaker is the effective dissipation (of order $1/\Gamma$) in the effective dynamics [25].

III. HEISENBERG SPIN CHAIN WITH THE FIRST SPIN IN A TARGET STATE

To illustrate our findings, we consider a system of interacting spins, with one spin strongly dissipatively coupled to an environment, which targets an arbitrary mixed state ψ_0 of that spin. In the Lindblad formalism, this is achieved via the application of two Lindblad operators [26],

$$L_1=\sqrt{\frac{1+\mu}{2}}|0^\perp\rangle\langle0|,\quad L_2=\sqrt{\frac{1-\mu}{2}}|0\rangle\langle0^\perp|, \tag{28}$$

where $|0\rangle$ is an arbitrary normalized state in $\mathcal{H}_0\equiv\mathbb{C}_2$, $\langle0^\perp|0\rangle=0$ and μ real parameter with $0\leqslant\mu\leqslant1$. The resulting dissipator $\mathcal{L}_0=\mathcal{D}_{L_1}+\mathcal{D}_{L_2}$, where

$$\mathcal{D}_LX=LXL^\dagger-\frac{1}{2}(L^\dagger LX+XL^\dagger L), \tag{29}$$

targets the arbitrary mixed state of a single spin

$$\psi_0 = \frac{1+\mu}{2}|0\rangle\langle 0| + \frac{1-\mu}{2}|0^{\perp}\rangle\langle 0^{\perp}|. \tag{30}$$

In fact, ψ_0 is an eigenvector of the dissipator $\mathcal{L}_0$ with eigenvalue $\xi_0 = 0$, namely, $\mathcal{L}_0\psi_0 = 0$. The other eigenvectors and the corresponding eigenvalues of $\mathcal{L}_0$ are

$$\psi_1 = |0\rangle\langle 0^{\perp}|, \quad \xi_1 = -\frac{1}{2}, \tag{31}$$

$$\psi_2 = |0^{\perp}\rangle\langle 0|, \quad \xi_2 = -\frac{1}{2}, \tag{32}$$

$$\psi_3 = |0\rangle\langle 0| - |0^{\perp}\rangle\langle 0^{\perp}|, \quad \xi_3 = -1. \tag{33}$$

The trace-orthonormal basis φ_k satisfying $\mathrm{tr}(\varphi_k\psi_m) = \delta_{k,m}$ is given by

$$\varphi_0 = I_{\mathbb{C}_2}, \tag{34}$$

$$\varphi_1 = |0^{\perp}\rangle\langle 0|, \tag{35}$$

$$\varphi_2 = |0\rangle\langle 0^{\perp}|, \tag{36}$$

$$\varphi_3 = \frac{1-\mu}{2}|0\rangle\langle 0| - \frac{1+\mu}{2}|0^{\perp}\rangle\langle 0^{\perp}|. \tag{37}$$

Given the explicit form of φ_k, ψ_k, we readily compute the coefficients C_{mn} from Eq. (22). The only nonzero coefficients C_{mn} are the diagonal ones: $C_{00} = 1$, $C_{11} = (1+\mu)/2$, $C_{22} = (1-\mu)/2$, $C_{33} = (1-\mu^2)/4$. Substituting them into Eq. (24) and using Eq. (25), we obtain $\tilde{H}_a = 0$ and

$$\tilde{\mathcal{D}} = 2(1+\mu)\mathcal{D}_{g_1} + 2(1-\mu)\mathcal{D}_{g_1^{\dagger}} + \frac{1}{2}(1-\mu^2)\mathcal{D}_{g_3}. \tag{38}$$

The operators g_k, given by Eq. (21), can be evaluated afterward the Hamiltonian H of the system is specified.

For definiteness, we consider the coherent part of the dynamics to be given by an open anisotropic XYZ Heisenberg spin chain, with Hamiltonian

$$H = \sum_{n=1}^{N-1} \vec{\sigma}_n \cdot (J\vec{\sigma}_{n+1}), \tag{39}$$

where $\vec{\sigma}_n = (\sigma_n^x, \sigma_n^y, \sigma_n^z)$ and $J = \mathrm{diag}(J_x, J_y, J_z)$ is the anisotropy tensor of the exchange interaction. We parametrize the state $|0\rangle$ via spherical coordinates θ, φ,

$$|0\rangle = \begin{pmatrix} \cos(\theta/2)e^{-i\varphi/2} \\ \sin(\theta/2)e^{i\varphi/2} \end{pmatrix}. \tag{40}$$

Introducing a standard unit vector in polar coordinates,

$$\vec{n}(\theta, \varphi) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$

and other two unit vectors defined as $\vec{n}' = \vec{n}(\frac{\pi}{2} - \theta, \varphi + \pi)$, $\vec{n}'' = \vec{n}(\frac{\pi}{2}, \varphi + \frac{\pi}{2})$, in such a way that the triplet $\vec{n}, \vec{n}', \vec{n}''$ forms an orthonormal basis in the three-dimensional space, we find

$$g_1 = (J\vec{n}')\cdot\vec{\sigma}_1 - i(J\vec{n}'')\cdot\vec{\sigma}_1, \tag{41}$$

$$g_3 = 2(J\vec{n})\cdot\vec{\sigma}_1. \tag{42}$$

Note that, after tracing over the spin space of the first site as indicated in (21), in the above expressions we renumerate the $N-1$ sites not directly affected by the dissipation as $1, 2, \ldots, M = N-1$. With this convention, the dissipation-projected Hamiltonian is still an anisotropic XYZ Heisenberg Hamiltonian as H but with M sites and a boundary field

$$h_D = \sum_{j=1}^{M-1} \vec{\sigma}_j \cdot (J\vec{\sigma}_{j+1}) + (J\vec{n})\cdot\vec{\sigma}_1. \tag{43}$$

The Hamiltonian (43) and the dissipator defined by Eq. (38) determine the effective LME, which governs the time evolution of the reduced density matrix $R(\tau)$ in the Zeno limit.

IV. HEISENBERG SPIN CHAIN WITH THE FIRST AND THE LAST SPINS IN A TARGET STATE

Previous results straightforwardly extend to more general setups. As an example, consider the same spin chain discussed above with dissipation acting only at the boundary spins 1 and N. Within this setup, and by tuning of the Hamiltonian parameters, one can generate, in the Zeno limit, a bulk NESS ranging from a maximally mixed state [27] to a pure spin-helix state carrying ballistic current of magnetization [28,29]. Here we assume the dissipation to target generic spin-1/2 mixed states, ψ_0^L and ψ_0^R, at the sites 1 and N, respectively,

$$\psi_0^L = \frac{1+\mu_L}{2}|0_L\rangle\langle 0_L| + \frac{1-\mu_L}{2}|0_L^{\perp}\rangle\langle 0_L^{\perp}|, \tag{44}$$

$$\psi_0^R = \frac{1+\mu_R}{2}|0_R\rangle\langle 0_R| + \frac{1-\mu_R}{2}|0_R^{\perp}\rangle\langle 0_R^{\perp}|. \tag{45}$$

As discussed above, this is realized by applying two Lindblad operators, of the form (28), at each end of the chain with parameters μ_L and μ_R, respectively.

Overall the dissipation targets a state, which is the product of the states targeted at the left and right boundaries, $\psi_0 = \psi_0^L \otimes \psi_0^R$. The eigenvalues of the full dissipator are the sum of the eigenvalues of the left and right boundary dissipators separately, $\xi_{m_L} + \xi_{m_R}$, and the respective eigenvectors are $\psi_{m_L,m_R} = \psi_{m_L}^L \otimes \psi_{m_L}^R$, where the individual $\psi_m^{L,R}$ have the form (33). The Hamiltonian decomposition in terms of the trace-orthonormal basis for the left and right dissipators, $\varphi_{n_L}^L, \varphi_{n_R}^R$, now reads

$$H = \sum_{n_L,n_R} \varphi_{n_L}^L \otimes g_{n_L,n_R} \otimes \varphi_{n_R}^R, \tag{46}$$

$$g_{n_L,n_R} = \mathrm{tr}_{1,N}\left[\left(\psi_{n_L}^L \otimes I^{2^{N-1}}\right)H\left(I^{2^{N-1}} \otimes \psi_{n_R}^R\right)\right]. \tag{47}$$

We can therefore apply the general formula (24), with $\xi_m \to \xi_{m_L} + \xi_{m_R}$ and $g_n \to g_{n_L,n_R}$. Note that, due to the locality of the interactions, $g_{n_L,n_R} = 0$ if $n_L n_R \neq 0$. After some algebra, and using Eq. (23), we obtain that Eq. (24) splits into the sum of two contributions, associated to the left and right ends of the chain,

$$W = \tilde{\mathcal{D}}_L R + \tilde{\mathcal{D}}_R R, \tag{48}$$

where, according to (38),

$$\tilde{\mathcal{D}}_L = 2(1+\mu_L)\mathcal{D}_{g_{10}} + 2(1-\mu_L)\mathcal{D}_{g_{10}^{\dagger}} + \tfrac{1}{2}\left(1-\mu_L^2\right)\mathcal{D}_{g_{30}},$$

$$\tilde{\mathcal{D}}_R = 2(1+\mu_R)\mathcal{D}_{g_{01}} + 2(1-\mu_R)\mathcal{D}_{g_{01}^{\dagger}} + \tfrac{1}{2}\left(1-\mu_R^2\right)\mathcal{D}_{g_{03}}.$$

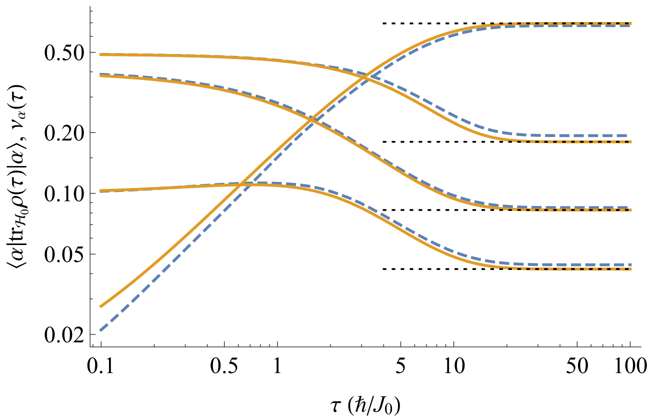

FIG. 2. Populations of the eigenstates of h_D, $\langle\alpha|\,\mathrm{tr}_{\mathcal{H}_0}\,\rho(\tau)|\alpha\rangle$ (dashed lines) and solutions $\nu_\alpha(\tau)$ of the classical Markov equation (50) (solid lines) as a function of time τ for the XYZ Heisenberg spin chain with dissipation acting on the first and last spins. We set $\Gamma = 50J_0/\hbar$ and all the other parameters are as in Fig. 1, case of squares. The initial condition is $\rho(0) = \psi_0^L \otimes R(0) \otimes \psi_0^R$, where $R(0)$ is a diagonal matrix with entries 0.01, 0.4, 0.1, 0.49 in the h_D basis. The straight dotted lines indicate the exact eigenvalues of $\mathrm{tr}_{\mathcal{H}_0}\,\rho(\tau)$ for $\tau \to \infty$ in Zeno limit, computed from the Markov process with the rates (51).

Also in the present case, W does not have coherent contributions of the kind (26).

The operators g_{k0}, g_{0k}, as well as the dissipation-projected Hamiltonian h_D, can be evaluated exactly as in the previous case of a single spin directly affected by the dissipation. The result is expressed in terms of the parameters μ_L, μ_R and of the polar coordinates θ_L, φ_L and θ_R, φ_R, which define the states $|0_L\rangle$ and $|0_R\rangle$. In particular, the Hamiltonian h_D is again a XYZ Hamiltonian with $M = N - 2$ spins, namely, those not directly affected by the dissipation, with two boundary terms relative to the spins 1 and M. Explicit formulas will be given elsewhere. In Figs. 1, 2, and 3 we illustrate the behavior of the resulting effective LME in comparison with the exact dynamics of the system.

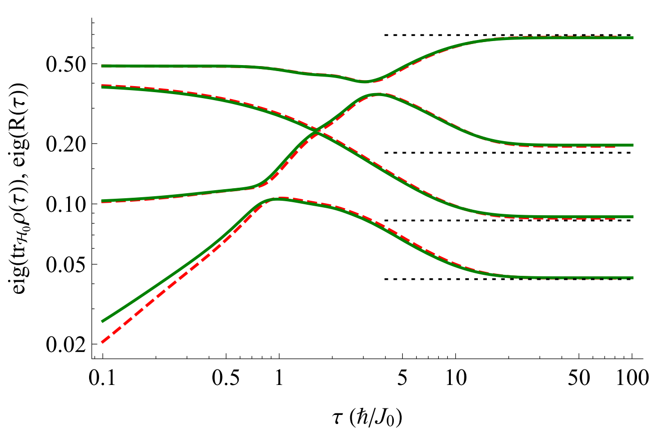

FIG. 3. Comparison of the LME and LME effective dynamics: eigenvalues of $\mathrm{tr}_{\mathcal{H}_0}\,\rho(\tau)$ (dashed lines) and of $R(\tau)$ (solid lines) as a function of time τ. Same parameters and same straight dotted lines as in Fig. 2. Note the similarity with Fig. 2 except for the avoided level crossings.

V. EVALUATION OF THE NESS IN THE ZENO LIMIT

As a direct application of our findings, we can compute the NESS in the Zeno limit, bypassing the solution of the LME. Denote $R_\infty = \lim_{\Gamma\to\infty,\tau\to\infty} R(\tau)$. From the LME (3) we have

$$[R_\infty, h_D] = 0.$$

If the spectrum of the dissipation projected Hamiltonian h_D is nondegenerate, then h_D and R_∞ share the same set of eigenvectors $|\alpha\rangle$. It follows that

$$R_\infty = \sum_\alpha \nu_\alpha^\infty |\alpha\rangle\langle\alpha|. \tag{49}$$

Deriving from (3) an evolution equation for the populations of the eigenstates of h_D, $\nu_\alpha(\tau) = \langle\alpha|R(\tau)|\alpha\rangle$, assuming that the effective dissipator has the canonical form $\tilde{\mathcal{D}}\cdot = \sum_k A_k(\tilde{L}_k \cdot \tilde{L}_k^\dagger - \frac{1}{2}\{\cdot, \tilde{L}_k^\dagger \tilde{L}_k\})$ starting from the state $R(\tau) = \sum_\alpha \nu_\alpha(\tau)|\alpha\rangle\langle\alpha|$, we obtain in the Zeno limit

$$\frac{\partial \nu_\alpha(\tau)}{\partial\tau} = \sum_{\beta\neq\alpha} w_{\beta\alpha}\nu_\beta - \nu_\alpha \sum_{\beta\neq\alpha} w_{\alpha\beta}, \tag{50}$$

$$w_{\beta\alpha} = \frac{1}{\Gamma}\sum_k A_k \left|\langle\alpha|\tilde{L}_k|\beta\rangle\right|^2. \tag{51}$$

We recognize Eq. (50) as the classical master equation of a Markov process with transition rates $w_{\alpha\beta}$. This is a manifestation of the well-known fact that a part of the degrees of freedom of the LME evolves in time via a classical Markov process [3,30]. Perron-Frobenius theorem guarantees an existence of a time-independent steady-state solution of Eq. (50), with non-negative entries ν_α^∞. After normalization $\sum_\alpha \nu_\alpha^\infty = 1$, the coefficients ν_α^∞ acquire the double meaning of eigenvalues of the reduced NESS (49), and steady-state probabilities in the associated classical Markov process, see Fig. 2 for an illustration. Note that by diagonalizing h_D one gets both the eigenvectors $|\alpha\rangle$ of R_∞ and the transition rates $w_{\alpha\beta}$ (and, therefore, the eigenvalues ν_α^∞). Thus, the problem of finding the NESS, which generically requires the diagonalization of the full Lindbladian, represented by a non-Hermitian matrix of size $d^2 \times d^2$, reduces, in the Zeno limit, to the diagonalization of the Hermitian matrix h_D, of size $d_1 \times d_1$ with $d_1 < d$. In the example discussed in the Sec. IV, we have $d = 2^N$ and $d_1 = d/4$.

VI. CONCLUSIONS

One might be concerned that, since our results hold in the Zeno limit, an impractically strong dissipation must be provided. However, one-dimensional quantum many-body systems with dissipation acting on a few degrees of freedom are well suited for an effective Zeno description whenever their size is sufficiently large. To see this fact, consider a one-dimensional system of size N with local interactions and dissipation acting near the edges. Let Γ be the finite strength of the dissipation. A perturbation spreads with finite speed (see, e.g. Lieb-Robinson bound [31]), so that the relaxation time of the system toward the global steady state increases at least linearly with the system size, $\tau_{\mathrm{bulk}} \sim N\hbar/J_0$, see, e.g., Ref. [32], while the relaxation of the edges takes a time of the order $\tau_{\rm diss} \sim 1/\Gamma$. Here, J_0 is a factor which fixes the energy scale associated to the Hamiltonian of the system. For arbitrary Γ and sufficiently large N, that is

$$\frac{\hbar\Gamma}{J_0} \gg \frac{1}{N}, \tag{52}$$

the system enters an effective Zeno regime $\tau_{\rm diss} \ll \tau_{\rm bulk}$, so the NESS of the system should be well approximated by the NESS computed in the Zeno limit $\Gamma \to \infty$. For a few cases for which exact results are known, validity of the (52) can be demonstrated, see, e.g., Refs. [26,33]. However, if the Zeno NESS is protected by extra symmetries, singular NESS behavior can happen.

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APPENDIX: PROOF OF EQ. (9)

Here, we compute the Dyson series up to the second order of the perturbation K. The calculation follows in part Ref. [24] and uses a similar notation.

The time evolution of the state $\rho = \psi_0 \otimes R$ is be defined via a Dyson series for $\mathcal{E}(t)\mathcal{P}_0$. Up to the second order of the Dyson series, we have

$$\begin{aligned}\mathcal{E}(t)\mathcal{P}_0 &= e^{\mathcal{L}_0 t}\left(1 + \int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1} + \int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\right.\\ &\quad \left.\times \int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2} K e^{\mathcal{L}_0 t_2} + \dots\right)\mathcal{P}_0\\ &= \mathcal{P}_0 + e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\mathcal{P}_0\\ &\quad + e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2} K e^{\mathcal{L}_0 t_2}\mathcal{P}_0.\end{aligned} \tag{A1}$$

In passing from the first to the second line we have used the obvious relation

$$e^{\mathcal{L}_0 t}\mathcal{P}_0 = \mathcal{P}_0 e^{\mathcal{L}_0 t} = \mathcal{P}_0. \tag{A2}$$

Let us focus on the second term of Eq. (A1) and insert the identity decomposition $I = \mathcal{Q}_0 + \mathcal{P}_0$:

$$\begin{aligned}&e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\mathcal{P}_0\\ &\quad = e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1}(\mathcal{P}_0 + \mathcal{Q}_0) K e^{\mathcal{L}_0 t_1}\mathcal{P}_0\\ &\quad = t\mathcal{P}_0 K \mathcal{P}_0 + e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1}\mathcal{Q}_0 K \mathcal{P}_0.\end{aligned} \tag{A3}$$

In the second term of Eq. (A3), we split the integral

$$\begin{aligned}&e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1}\mathcal{Q}_0 K \mathcal{P}_0\\ &\quad = e^{\mathcal{L}_0 t}\left(\int_0^{-\infty} dt_1 \dots + \int_{-\infty}^t dt_1 \dots\right)\\ &\quad = e^{\mathcal{L}_0 t}\int_0^{-\infty} dt_1 e^{-\mathcal{L}_0 t_1}\mathcal{Q}_0 K \mathcal{P}_0\\ &\qquad - \int_t^{-\infty} dt_1 e^{\mathcal{L}_0 (t-t_1)}\mathcal{Q}_0 K \mathcal{P}_0,\end{aligned} \tag{A4}$$

and, after the substitutions $t_1 \to -\tilde{t}_1$, $dt_1 \to -d\tilde{t}_1$, we obtain

$$\begin{aligned}&e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1}\mathcal{Q}_0 K \mathcal{P}_0\\ &\quad = -e^{\mathcal{L}_0 t}\int_0^{\infty} d\tilde{t}_1 e^{\mathcal{L}_0 \tilde{t}_1}\mathcal{Q}_0 K \mathcal{P}_0\\ &\qquad + \int_{-t}^{\infty} d\tilde{t}_1 e^{\mathcal{L}_0 (t+\tilde{t}_1)}\mathcal{Q}_0 K \mathcal{P}_0.\end{aligned} \tag{A5}$$

Next, we make the change of variable $t + \tilde{t}_1 \to u$, $d\tilde{t}_1 \to du$ in the second integral of Eq. (A5) and obtain

$$\begin{aligned}&e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1}\mathcal{Q}_0 K \mathcal{P}_0\\ &\quad = -e^{\mathcal{L}_0 t}\int_0^{\infty} d\tilde{t}_1 e^{\mathcal{L}_0 \tilde{t}_1}\mathcal{Q}_0 K \mathcal{P}_0 + \int_0^{\infty} du e^{\mathcal{L}_0 u}\mathcal{Q}_0 K \mathcal{P}_0.\end{aligned} \tag{A6}$$

Renaming $\tilde{t}_1, u \to t$, we can write

$$e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1}\mathcal{Q}_0 K \mathcal{P}_0 = (e^{\mathcal{L}_0 t} - I)\mathcal{S} K \mathcal{P}_0, \tag{A7}$$

where

$$\mathcal{S} = -\int_0^{\infty} dt e^{\mathcal{L}_0 t}\mathcal{Q}_0 \tag{A8}$$

is the pseudoinverse of the dissipator, namely,

$$\mathcal{L}_0\mathcal{S} = \mathcal{S}\mathcal{L}_0 = \mathcal{Q}_0. \tag{A9}$$

The operator $\mathcal{S}$ is bounded, since the eigenvalues of $\mathcal{L}_0$ (apart from the nondegenerate 0 eigenvalue which is excluded by the multiplication with $\mathcal{Q}_0$) are nonzero and finite. Combining Eqs. (A3) and (A7), we conclude

$$\mathcal{E}(t)\mathcal{P}_0 = \mathcal{P}_0 + t\mathcal{P}_0 K \mathcal{P}_0 + (e^{\mathcal{L}_0 t} - I)\mathcal{S} K \mathcal{P}_0 + \dots \tag{A10}$$

(... denoting contributions from second and higher orders), which retrieves the result reported in Ref. [1]. Equation (A10) shows, in particular, that the leaking outside the dissipation-free subspace for times $t > 1/\Gamma$ is of order $1/\Gamma$, namely,

$$\|\rho(t) - \psi_0 \otimes \mathrm{tr}_{\mathcal{H}_0}\rho(t)\| = O(\Gamma^{-1}). \tag{A11}$$

The evolution inside the decoherence-free subspace is given by $\mathcal{P}_0\mathcal{E}(t)\mathcal{P}_0$. Making use of Eq. (A2), up to the second-order Dyson term we thus obtain

$$\begin{aligned}\mathcal{P}_0\mathcal{E}(t)\mathcal{P}_0 &= \mathcal{P}_0 + t\mathcal{P}_0 K \mathcal{P}_0 + \mathcal{P}_0 e^{\mathcal{L}_0 t}\\ &\quad \times \int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2} K e^{\mathcal{L}_0 t_2}\mathcal{P}_0.\end{aligned} \tag{A12}$$

Now we estimate the $O(K^2)$ contribution to $\mathcal{P}_0\mathcal{E}(t)\mathcal{P}_0$:

$$\begin{aligned}
&\mathcal{P}_0 e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2} K e^{\mathcal{L}_0 t_2}\mathcal{P}_0\\
&= \mathcal{P}_0\int_0^t dt_1\int_0^{t_1} dt_2 K e^{\mathcal{L}_0 t_1-\mathcal{L}_0 t_2} K\mathcal{P}_0\\
&= \mathcal{P}_0\int_0^t dt_1\int_0^{t_1} dt_2 K e^{\mathcal{L}_0 t_1-\mathcal{L}_0 t_2}(\mathcal{P}_0+\mathcal{Q}_0) K\mathcal{P}_0\\
&= \frac{t^2}{2}(\mathcal{P}_0 K\mathcal{P}_0)^2 + \mathcal{P}_0 K\int_0^t dt_1 e^{\mathcal{L}_0 t_1}\int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2}\mathcal{Q}_0 K\mathcal{P}_0\\
&= \frac{t^2}{2}(\mathcal{P}_0 K\mathcal{P}_0)^2 + \mathcal{P}_0 K\int_0^t dt_1 (e^{\mathcal{L}_0 t}-I)\mathcal{S}K\mathcal{P}_0\\
&= \frac{t^2}{2}(\mathcal{P}_0 K\mathcal{P}_0)^2 - t\mathcal{P}_0 K(\mathcal{P}_0+\mathcal{Q}_0)\mathcal{S}K\mathcal{P}_0\\
&\quad + \mathcal{P}_0 K\int_0^t dt_1 e^{\mathcal{L}_0 t}\mathcal{S}K\mathcal{P}_0. \qquad \text{(A13)}
\end{aligned}$$

Let us concentrate on the last term of the above expression. Inserting the identity decomposition $I = \mathcal{Q}_0 + \mathcal{P}_0$ and using Eq. (A2), we have

$$\begin{aligned}
&\mathcal{P}_0 K\int_0^t dt_1 e^{\mathcal{L}_0 t}\mathcal{S}K\mathcal{P}_0\\
&= \mathcal{P}_0 K\int_0^t dt_1 e^{\mathcal{L}_0 t}(\mathcal{P}_0+\mathcal{Q}_0)\mathcal{S}K\mathcal{P}_0\\
&= t\mathcal{P}_0 K\mathcal{P}_0\mathcal{S}K\mathcal{P}_0 + \mathcal{P}_0 K\int_0^t dt_1 e^{\mathcal{L}_0 t}\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0. \qquad \text{(A14)}
\end{aligned}$$

Gathering all terms of order K^2, we conclude

$$\begin{aligned}
&\mathcal{P}_0 e^{\mathcal{L}_0 t}\int_0^t dt_1 e^{-\mathcal{L}_0 t_1} K e^{\mathcal{L}_0 t_1}\int_0^{t_1} dt_2 e^{-\mathcal{L}_0 t_2} K e^{\mathcal{L}_0 t_2}\mathcal{P}_0\\
&= \frac{t^2}{2}(\mathcal{P}_0 K\mathcal{P}_0)^2 - t\mathcal{P}_0 K\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0\\
&\quad + \mathcal{P}_0 K\int_0^t dt_1 e^{\mathcal{L}_0 t}\mathcal{Q}_0\mathcal{S}K\mathcal{P}_0. \qquad \text{(A15)}
\end{aligned}$$

In the last term of Eq. (A15), the integral over time converges, thus this term is of order $\|K^2\| = O(1/\Gamma^2)$ and can be neglected. Bringing together Eqs. (A12) and (A15), we obtain Eq. (9).

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