Rapid Communications

Quantifying quantum coherence in experimentally observed neutrino oscillations

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Neutrino oscillation represents an intriguing physical phenomenon where the quantumness can be maintained and detected over a long distance. Previously, the nonclassical character of neutrino oscillation was tested with the Leggett-Garg inequality, where a clear violation of the classical bound was observed [J. A. Formaggio *et al.*, Phys. Rev. Lett. **117**, 050402 (2016)]. However, there are several limitations in testing neutrino oscillations with the Leggett-Garg inequality. In particular, the degree of violation of the Leggett-Garg inequality cannot be taken as a "measure of quantumness." Here, we focus on quantifying the quantumness of experimentally observed neutrino oscillation, using the tools of the recently developed quantum resource theory. We analyzed ensembles of reactor and accelerator neutrinos at distinct energies from a variety of neutrino sources, including Daya Bay (0.5 and 1.6 km), KamLAND (180 km), MINOS (735 km), and T2K (295 km). The quantumness of the three-flavored neutrino oscillation is characterized within a 3σ range relative to the theoretical prediction. It is found that the maximal coherence was observed in the neutrino source from the KamLAND reactor. However, even though the survival probability of the Daya Bay experiment did not vary significantly (it dropped about 10%), the coherence recorded can reach up to 40% of the maximal value. These results represent the longest distance over which quantumness was experimentally determined for quantum particles other than photons.

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Introduction. The phenomenon of neutrino oscillation was proposed more than half a century ago [1,2]. Since then, compelling experimental evidence of the transitions between different neutrino flavors has been obtained from different sources, including solar [3,4], atmosphere [5], reactor [6,7], and accelerator neutrinos [8–11]. In the three-generation neutrino framework, neutrinos and antineutrinos are produced simultaneously and detected in three different flavors, namely, electron e, muon μ, and tau τ leptons. The flavor states are a linear combination of the mass states [12,13]. Neutrino oscillation implies that a given flavor may change into another flavor during propagation, caused by nonzero neutrino mass and neutrino mixing. Recently, a number of refined measurements and analyses on the oscillations parameters have been presented [14–17]. Moreover, Stancil *et al.* [18] achieved the first ever transmission of information using a beam of neutrinos in the NuMI beam line and the MINERvA detector at Fermilab. However, the justification of neutrino oscillation is based on a crucial assumption that the different neutrino states are coherent during its propagation; this assumption of quantum coherence still needs to be verified carefully, as it leads to considerable constraints in ultrahigh-energy or astronomical scales [19,20]. Furthermore, to explore the possibility of utilizing neutrino oscillations for future applications in quantum information processing, an important step is to verify the "quantumness" in neutrino oscillations.

The question is, how might one test the quantumness of neutrino oscillations? In recent years, the idea of testing neutrino oscillations using the Leggett-Garg inequality (LGI) [21–23] has been considered [24–27]; it is suggested that experimentally observed neutrino oscillations can violate the classical limits imposed by the LGI. However, there are several fundamental problems associated with testing neutrino oscillations with LGI. (i) The LGI was originally designed to test the concept of macroscopic realism for macroscopic objects; violation of a LGI means that the system may not be a macroscopic reality nor can a noninvasive measurement be performed on it. However, these two conditions are not strictly satisfied for elementary particles probed in the current experimental settings. (ii) Experimental violation of LGI [25] assumed that there are only two neutrino states instead of three. (iii) Generally, the violation of LGI is not a good indicator for *quantifying* the amount of coherence. In some cases, the maximal violation depends on the channel parameter [28]. The situation is similar to the case where Bell's inequalities, in general, cannot be utilized for quantifying quantum entanglement [29–34].

In the context of quantum information, coherence is a fundamental concept that can be rigorously characterized in the context of quantum resource theory [35]. Similar to quantum entanglement, there are different measures for coherence (see Refs. [36,37] for a review). Among them, the l_1-norm of

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coherence,

$$\mathcal{C}(\rho) = \sum_{i\neq j} |\rho_{ij}| \geqslant 0, \quad (1)$$

is probably the most accessible one for the neutrino experiments performed; it is equal to a summation over the absolute values of all the off-diagonal elements ρ_{ij} of a given density matrix ρ. Moreover, it fulfills all the necessary requirements of a coherence measure: non-negativity, monotonicity, strong monotonicity, and convexity. The maximal possible value of $\mathcal{C}$ is bounded by $\mathcal{C}_{\max} = d - 1$, with d being a dimension of the density matrix ρ. In particular, $\mathcal{C}_{\max} = 2$ for three-flavored neutrino oscillations considered in this Rapid Communication.

There are many reasons for quantifying quantum coherence. For example, one can estimate the required copies for converting quantum states with different amounts of coherence through incoherent operations, which is similar to entanglement distillation [38]. Many results indicate that coherence can be regarded as resources for quantum algorithms [39], quantum channel discrimination [40], and quantum thermodynamics [36,41]. Furthermore, the tools of quantum information theory can be applied to solve many interesting problems in neutrino oscillations, such as the mass-degeneracy problem [42] and for distinguishing the nature of neutrinos between Dirac and Majorana fermions [43]. Recently, the correlation in terms of flavor transition probabilities of neutrino oscillations was studied by quantum information theoretic quantities, including concurrence [44], Svetlichny inequalities [45], quantum discord [46], and quantum estimation [47], where the three-flavor neutrino states are treated as a three-qubit system.

Here, we present a method for quantifying the quantumness of neutrino oscillations, with the use of a coherence measure developed in quantum resource theory. Through analyzing the experimental data from different sites, including Daya Bay, KamLAND, T2K, and MINOS, we study the coherence in the dynamics of the three-flavor neutrino oscillations. We conclude that a significant amount of quantum coherence exists from all four sources of neutrinos. In particular, the KamLAND Collaboration recorded the highest value of coherence; many events are close to the theoretical maximal value of 2. Furthermore, from the Daya Bay data, even though the transition probabilities from the electron neutrino to other flavors are less than 10%, the coherence can reach as much as 0.8 (40% of the maximal value). Currently, utilizing neutrino oscillations for practical applications remains a major technological challenge; most of the previous work is related to how neutrinos can be manipulated for the purpose of communications [18,48,49]. Quantification of the quantumness of neutrino oscillation represents a step towards the goal of quantum information processing using neutrinos. We aim at determining the energy of the neutrinos required to achieve maximal coherence for a given macroscopic distance [50].

The neutrino model. In the three-generation framework, a neutrino oscillation involves the mixing between the flavor states $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$ which are superpositions of the mass eigenstates $|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$ [here, (e, μ, τ) represents the neutrino flavor state and $(1, 2, 3)$ labels the neutrino mass state]. The explicit relation is given by a 3×3 unitary matrix U, i.e., the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [2]. Each flavor state is a linear superposition of the mass eigenstates, $|\nu_\alpha\rangle = \sum_k U_{\alpha k}|\nu_k\rangle$, where $\alpha = e, \mu, \tau$ and $k = 1, 2, 3$. In the standard parametrization, U is characterized by three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and a charge conjugation and parity (CP) violating phase δ_{CP},

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}, \quad (2)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ $(i, j = 1, 2, 3)$. Currently, there is little conclusive evidence about the CP phase, so we assume that it vanishes in the following discussion.

The massive neutrino states are eigenstates of the time-independent free Dirac Hamiltonian H with an energy E_k, and its time evolution satisfies the relativistic quantum mechanics dynamical equation. Explicitly, during the neutrino propagation, the wave-function solution is given by $|\nu_k(t)\rangle = e^{-iE_k t/\hbar}|\nu_k(0)\rangle$, which implies that the time evolution of the flavor neutrino states is given by $|\nu_\alpha(t)\rangle = a_{\alpha e}(t)|\nu_e\rangle + a_{\alpha\mu}(t)|\nu_\mu\rangle + a_{\alpha\tau}(t)|\nu_\tau\rangle$, where $a_{\alpha\beta}(t) \equiv \sum_k U_{\alpha k}e^{-iE_k t/\hbar}U^*_{\beta k}$. Finally, the probability for detecting β neutrino, given that the initial state is in the α neutrino state, is given by [51]

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{k>l} \mathrm{Re}(U^*_{\alpha k}U_{\beta k}U_{\alpha l}U^*_{\beta l}) \sin^2\left(\Delta m^2_{kl}\frac{Lc^3}{4\hbar E}\right) + 2\sum_{k>l} \mathrm{Im}(U^*_{\alpha k}U_{\beta k}U_{\alpha l}U^*_{\beta l}) \sin\left(\Delta m^2_{kl}\frac{Lc^3}{2\hbar E}\right), \quad (3)$$

where $\Delta m^2_{kl} \equiv m^2_k - m^2_l$, E is the energy of the neutrino which is different for different neutrino experiments, and $L = ct$ (with c being the speed of light) is the distance traveled by the neutrino particle. Note that in the neutrino experiments, one may vary the energy E, instead of time, for probing the variation of the transition probabilities (e.g., see Ref. [25]).

For analyzing the experimental data of neutrino oscillations, it is convenient to write the oscillatory term of Eq. (3), $\sin^2(\Delta m^2_{kl}\frac{Lc^3}{4\hbar E})$, in a simple form [52],

$$\sin^2\left(\Delta m^2_{kl}\frac{Lc^3}{4\hbar E}\right) = \sin^2\left(1.27\Delta m^2_{kl}(\mathrm{eV}^2)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right). \quad (4)$$

Note that the oscillation probabilities depend on seven independent parameters (three mixing angles, two mass-squared differences, distance, and energy) and five of them can be experimentally determined. For the following order of the neutrino mass spectrum, $m_1 < m_2 < m_3$, the best-fit values and the 3σ ranges of the three-flavor oscillation parameters

TABLE I. The neutrino mixing parameters in normal hierarchy from the global fit results [51].

Parameter	Best fit $\pm 1\sigma$	3σ range
Δm^2_{21} (10^{-5} eV^2)	$7.50^{+0.19}_{-0.17}$	$7.02 \rightarrow 8.09$
Δm^2_{31} (10^{-3} eV^2)	$2.457^{+0.047}_{-0.047}$	$2.317 \rightarrow 2.607$
θ_{12} (deg)	$33.48^{+0.78}_{-0.75}$	$31.29 \rightarrow 35.91$
θ_{23} (deg)	$42.3^{+3.0}_{-1.6}$	$38.2 \rightarrow 53.3$
θ_{13} (deg)	$8.50^{+0.20}_{-0.21}$	$7.85 \rightarrow 9.10$

are listed in Table I. Below, we shall consider separately the electron neutrino oscillations at Daya Bay (0.5 and 1.6 km) and KamLAND (180 km), and muon neutrino oscillations at MINOS (735 km) and T2K (295 km).

Coherence in electron antineutrino oscillations. If an electron neutrino is produced at the initial time $t = 0$, then its time evolution is given by $|\nu_e(t)\rangle = a_{ee}(t)|\nu_e\rangle + a_{e\mu}(t)|\nu_\mu\rangle + a_{e\tau}(t)|\nu_\tau\rangle$, where the probabilities for finding the neutrino in states $|\nu_e\rangle$, $|\nu_\mu\rangle$, and $|\nu_\tau\rangle$ are, respectively, $P_{ee}(t) = |a_{ee}(t)|^2$, $P_{e\mu}(t) = |a_{e\mu}(t)|^2$, and $P_{e\tau}(t) = |a_{e\tau}(t)|^2$. In Fig. 1, we plot the variations of the survival probabilities in neutrino oscillations from Daya Bay [14] and KamLAND [15], as a function of L/E; these sites make use of β decay to produce a source of the electron antineutrino, but with different baselines and energy, changing the ratio of L/E. The Daya Bay Collaboration used the fully constructed Daya Bay Reactor Neutrino Experiment as a new measurement of electron antineutrino disappearance, and it covers energy between 1 and 8 MeV, which signifies an effective ratio L/E in a range [0, 1] with dimension km/MeV. However, the KamLAND Collaboration demonstrated the oscillatory nature of the neutrino flavor transformation by observing electron antineutrinos with energies of a few MeV from nuclear reactors about 180 km away, corresponding to the range [0, 100] in terms of the ratio L/E.

For the short-range oscillations (with small L/E) at Daya Bay, shown in Fig. 1(a), the survival probability of ν_e is always higher than 0.9; the probabilities of detecting the other flavors are relatively small. It reaches the minimal point at around $L/E = 0.5$. On the other hand, in Fig. 1(b), the long-range neutrino oscillations in KamLAND involve significant contributions from all three flavors. Technically, the length of the baseline for the theoretical prediction of survival probability in the KamLAND Collaboration was taken as the average value, and it thus presents a relatively smoother manner than a cosine one from Eq. (3). It coincides with the data given by the KamLAND experiment.

To quantify the coherence, we shall focus on the off-diagonal elements of the density matrix $\rho(t) = |\nu_e(t)\rangle\langle\nu_e(t)|$, in the basis $\{|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle\}$,

$$\rho(t) = \begin{pmatrix} |a_{ee}(t)|^2 & a_{ee}(t)a^*_{e\mu}(t) & a_{ee}(t)a^*_{e\tau}(t) \\ a^*_{ee}(t)a_{e\mu}(t) & |a_{e\mu}(t)|^2 & a_{e\mu}(t)a^*_{e\tau}(t) \\ a^*_{ee}(t)a_{e\tau}(t) & a^*_{e\mu}(t)a_{e\tau}(t) & |a_{e\tau}(t)|^2 \end{pmatrix}, \quad (5)$$

where the coherence is given by $\mathcal{C} = 2|a_{ee}(t)a_{e\mu}(t)| + 2|a_{ee}(t)a_{e\tau}(t)| + 2|a_{e\mu}(t)a_{e\tau}(t)|$. Equivalently, it can be

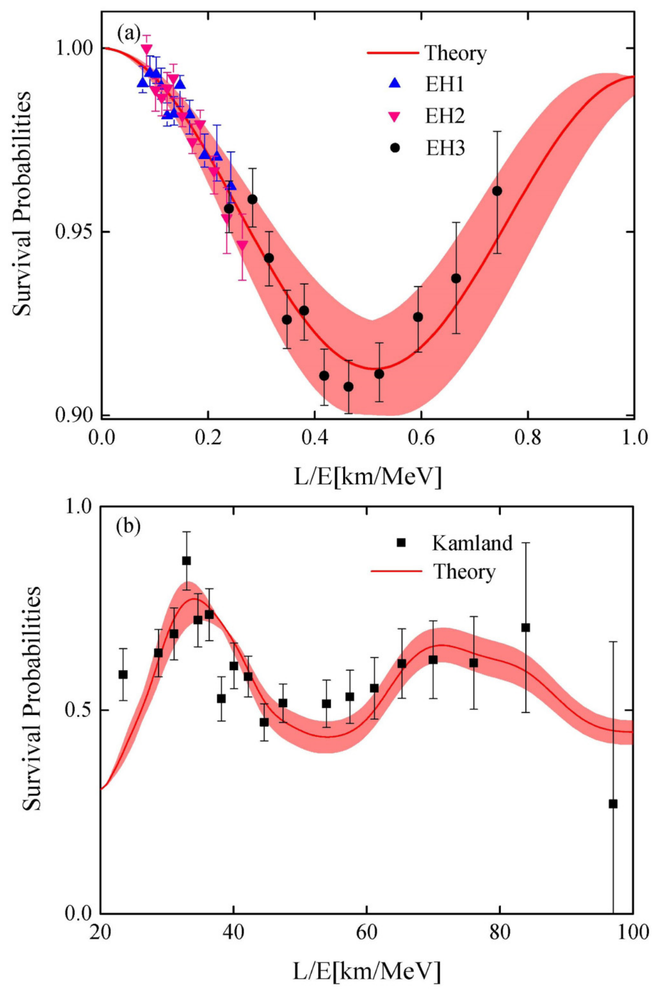

FIG. 1. (a) Short-range neutrino survival probabilities for the initial electron neutrino in theory (red, solid line) and the data of the Daya Bay Collaboration in three underground experimental halls (EH1: blue, upper triangle; EH2: pink, lower triangle; EH3: black, circle) taken from Ref. [14] (black, square) with ratio L/E changing are plotted. (b) Long-range neutrino survival probabilities for the initial electron neutrino in theory (red, solid line) and the data of the KamLAND Collaboration taken from Ref. [15] (black, square) with ratio L/E changing are shown. The red band indicates a 3σ confidence interval around the fitted prediction.

expressed in terms of the transition probabilities, i.e.,

$$\mathcal{C}_e = 2[\sqrt{P_{ee}(t)P_{e\mu}(t)} + \sqrt{P_{ee}(t)P_{e\tau}(t)} + \sqrt{P_{e\mu}(t)P_{e\tau}(t)}]. \quad (6)$$

Note that transition probabilities are subjected to the normalization constraint, $\sum_\alpha P_{\alpha\beta} = \sum_\beta P_{\alpha\beta} = 1$ $(\alpha, \beta = e, \mu, \tau)$. The amount of quantum coherence depends on the three transition probabilities. The maximal value of coherence for three-flavored neutrino oscillations is given by 2 when the transition probabilities are all equal to 1/3. A deviation of the probabilities yields a lower value of coherence. In particular, the coherence will be close to zero when the two oscillation probabilities are small compared with the survival probability. Moreover, a partial derivative of coherence with respect to survival probability presents that a tiny change of $P_{ee}(t)$ will

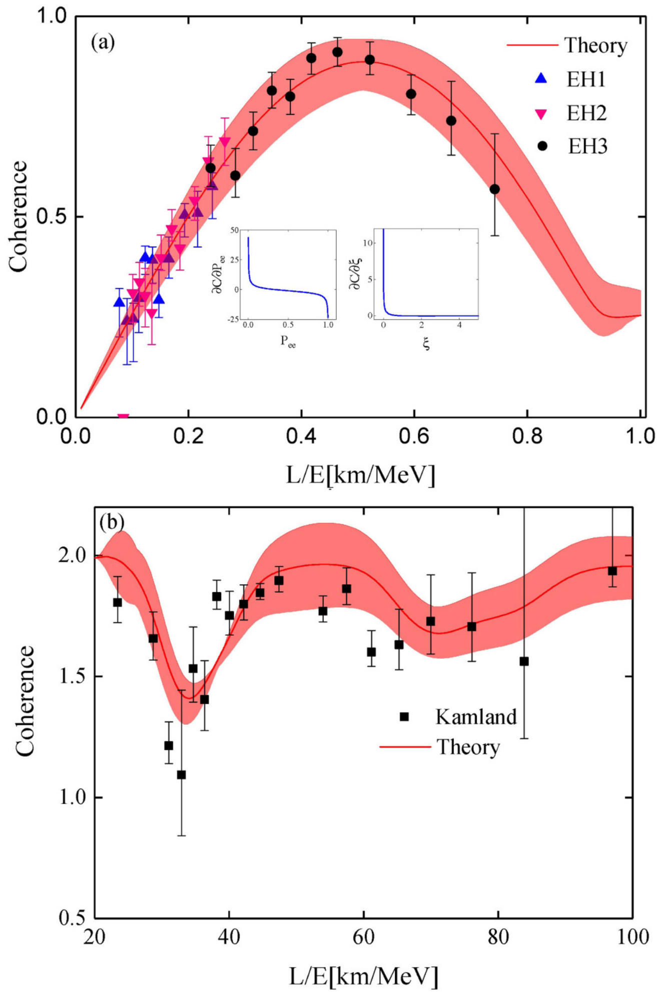

FIG. 2. The coherence in theory (red, solid line) with a 3σ confidence interval around the fitted prediction (red band) and experiment (black, square) with an error bar in (a) Daya Bay (black, circle) and (b) KamLAND Collaborations (black, square) for three-flavored neutrino oscillations as a function of ratio L/E between the traveled distance and neutrino energy are shown. The inset shows the derivative of coherence with respect to survival probability and the ratio ξ, respectively. Taking the error bar into consideration, the data are consistent with the theoretic 3σ range in the short-distance case.

lead to a relatively drastic variation of coherence when the survival probability takes a value either smaller than 0.1 or larger than 0.9 [see the inset of Fig. 2(a)]. Experimentally, only the survival probability is given, to quantify the measured coherence, and the ratio $\xi = P_{e\tau}(t)/P_{e\mu}(t)$ is determined from the theoretical prediction, while the coherence changes gently with the ratio ξ changing when $\xi > 0.5$, and there is a dramatic change of coherence for $\xi < 0.5$.

The coherence in theory and experiment for three-flavored neutrino oscillations as a function of ratio L/E is plotted in Fig. 2. For the Daya Bay Collaboration, the coherence may reach about 0.8 at $L/E = 0.5$, even though the transition probabilities to other flavors are less than 10%.

On the other hand, the coherence from the neutrino oscillation at KamLAND is in general higher [see Fig. 2(b)], even

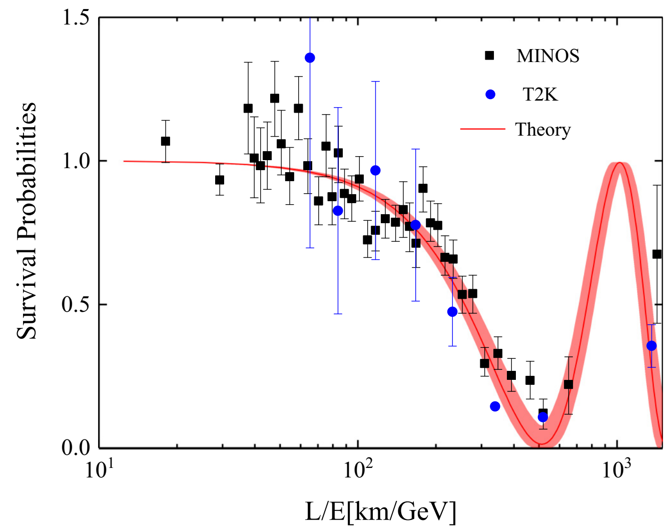

FIG. 3. Long-range neutrino survival probabilities for an initial muon neutrino in theory (red, solid line) and the data of MINOS (black, square) and T2K (blue, circle) Collaborations taken from Refs. [16,17] with ratio L/E changing are shown, respectively. The red band indicates a 3σ confidence interval around the fitted prediction.

reaching the maximum value of 2 for three-flavored neutrino oscillations. It implies more quantum resources can be used in the propagation of neutrinos in the KamLAND experiment.

Coherence in muon antineutrino oscillations. In the MINOS and T2K Collaboration, muon (anti)neutrinos are produced from the proton beams of the accelerators. In the MINOS experiment, its baseline takes a longer distance of 735 km and it covers the energy from 0.5 to 50 GeV [16], which reveals that the ratio L/E is in the range [15, 1500]. The T2K experiment demonstrates the oscillatory nature of neutrino flavor transformation by observing the muon antineutrino survival probability with energies of a few GeV from nuclear reactors about 295 km away [17].

When the muon flavor state is prepared at an initial time $t = 0$, the state of the time evolution for three-flavored neutrino oscillations is given by $|\nu_\mu(t)\rangle = a_{\mu e}(t)|\nu_e\rangle + a_{\mu\mu}(t)|\nu_\mu\rangle + a_{\mu\tau}(t)|\nu_\tau\rangle$. The probabilities of finding the neutrino in states $|\nu_e\rangle$, $|\nu_\mu\rangle$, and $|\nu_\tau\rangle$ are, respectively, $P_{\mu e}(t) = |a_{\mu e}(t)|^2$, $P_{\mu\mu}(t) = |a_{\mu\mu}(t)|^2$, and $P_{\mu\tau}(t) = |a_{\mu\tau}(t)|^2$. In Fig. 3, we show the survival probabilities of the muon neutrino oscillation as a function of L/E. Similar to the case of the electron neutrino, the coherence can be calculated as

$$\mathcal{C}_u = 2[\sqrt{P_{\mu e}(t)P_{\mu\mu}(t)}+\sqrt{P_{\mu e}(t)P_{\mu\tau}(t)}+\sqrt{P_{\mu\mu}(t)P_{\mu\tau}(t)}]. \tag{7}$$

Again, the coherence is determined primarily by the experimental data of the survival probability $P_{\mu\mu}$, and theoretical values of the ratio $\zeta = P_{\mu\tau}(t)/P_{\mu e}(t)$ are employed. The coherence of the neutrino oscillation from the MINOS and T2K Collaboration as a function of ratio L/E is plotted in Fig. 4. The theoretic coherence shows a more complicated behavior and the peaks appear at the points when the probabilities $P_{\mu\mu}(t)$ and $P_{\mu\tau}(t)$ are the same. It is nonvanishing in the neutrino propagation since two of the probabilities will be

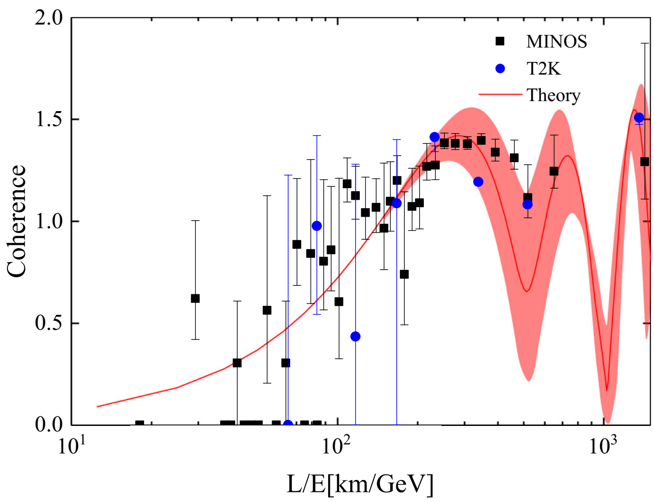

FIG. 4. The coherence in theory (red, solid line) with a 3σ confidence interval around the fitted prediction (red band), MINOS Collaboration (black, square), and T2K Collaboration (blue, circle) with an error bar for three-flavored neutrino oscillations as the function of ratio between the distance and neutrino energy L/E are shown. Taking the error bar into consideration, the data are consistent with the theoretic 3σ range in the long-distance case, too.

nonzero in spite of the trivial value of oscillation probability of the electron neutrino $P_{\mu e}(t)$ as compared to the survival probability $P_{\mu\mu}(t)$ and $P_{\mu\tau}(t)$. The experimental data for the coherence of MINOS are distributed around the theoretical prediction line, which is consistent with the theoretical prediction of coherence. On the other hand, the eight experimental data for coherence of the T2K Collaboration also give good agreement with theory.

Summary. In summary, we proposed a method for quantifying the quantumness of neutrino oscillations with the use of coherence measure developed in quantum resource theory. We compared the coherence in experimentally observed neutrino oscillations from different sources, including Daya Bay, KamLAND, MINOS, and T2K, all exhibiting good agreement with the theoretical predictions. In particular, we found a value close to the theoretical maximum of coherence in the case of the KamLAND Collaboration neutrinos. These results suggest that coherence can be a reliable tool for the quantification of superposition in the three-flavored neutrino oscillation over a macroscopic distance of thousands of kilometers, certifying an elementary particle suitable for quantum applications other than photons. While the manipulation of neutrinos is a huge challenge given the current technology, verifying the quantumness in neutrino oscillations represents an important step towards quantum information processing with neutrinos.

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