Quantum-state reconstruction of a mechanical mirror in a hybrid system

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We introduce a scheme to reconstruct the quantum state of a mechanical mirror in a hybrid optomechanical system. The scheme involves sending a beam of two-level atoms to interact with a quantized cavity field which is weakly coupled to the mechanical mirror. We show that the measured data of the excited-state probability of the atoms can be used directly to reconstruct the initial state of the mechanical mirror. If the mechanical mirror is initially a pure quantum state, we can reconstruct both the probability and the phase of the mechanical state. If it is initially a mixed state, we can reconstruct all the diagonal matrix elements and the first two diagonals above and below them.

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I. INTRODUCTION

Cavity optomechanics is a rapidly developing area of research [1] which explores the interaction between electromagnetic fields and mechanical states of motion via radiation-pressure forces [2–4]. Numerous research studies in cavity optomechanics cover a wide variety of problems such as ground-state cooling [5–13], generation of macroscopic quantum superposition in optomechancial systems [14–17], and creation and verification of quantum entanglement [18–31].

Quantum-state reconstruction of the mechanical states of motion plays a very crucial role in revealing and understanding various nonclassical properties and aspects in different cavity optomechanical systems [32]. Recently, several reconstruction schemes have been introduced to reconstruct motional mechanical states in cavity optomechanics. A mechanical state tomography scheme [33] is based on sending short optical pulse to enter an optomechanical cavity. The accumulated phase of the output pulse is measured to obtain information about the oscillator quadratures. Mechanical state tomography based on a back-action-evading interaction has been experimentally demonstrated to accurately measure the position of the mechanical oscillator [34].

Another interesting reconstruction scheme was introduced in which an optomechanical cavity is coupled to an outside continuous field [35]. Detection of a single photon emission and scattering spectrum [36] is used to measure the quantum state of the mechanical mirror in the system. Due to the coupling between the cavity field and the mechanical mirror, the mechanical motion can strongly modify the emission and scattering spectra of the photon. Therefore, the initial state of the mechanical mirror can be extracted from the emission or scattering spectrum. In this scheme, they can in principle reconstruct the whole information of a general quantum state. However, this method works well only when the optomechanical coupling strength is ultrastrong, which is not a common case [37]. In most cases, the optomechanical coupling strength is in the weak-coupling regime [1].

A number of studies investigate using atoms as a measurement tool for the quantum states of cavity fields [38–50]. A measurement scheme [39] was introduced to reconstruct the quantum state of a light field using a beam of two-level atoms initially prepared in a coherent superposition of their excited and ground states. In this scheme, a beam of two-level atoms is sent in an optical cavity to interact with a quantized electromagnetic field inside and the probability of detecting the atoms in the excited states is measured after the atoms exit the cavity. By controlling the phase difference between the two states of the atom and varying the interaction time, the complete quantum state of the cavity field can be reconstructed by solving a system of linear equations. Similarly, using atom as a detector, one can also measure the quantum state of a nanomechanical oscillator in an optomichanical system [51]. The atom is coupled to an optical field via a Raman transition and to the nanomechanical oscillator via a magnetic-sublevel-phonon interaction [51]. Since the atom is directly coupled to the nanomechanical oscillator, measurement of the probability of the atom to be in its ground state directly gives the Wigner function of the nanomechanical oscillator. This method is an interesting extension of the idea of nonlinear atom homodyne [40], which was developed to measure the quantum state of a single mode field.

In this paper, we propose a scheme to detect the quantum state of a mechanical oscillator in a hybrid optomechanical system. Our method is also based on the use of a beam of two-level atoms as detector for the state of the mechancial oscillator. Instead of using direct magnetic-sublevel-phonon coupling as in Ref. [51], the atoms can indirectly couple to the mechanical mirror in our system via the polariton-phonon coupling. In our method, the atom is initially prepared in the excited state and the cavity field is in the vacuum state. We show that by measuring the probability of the atoms being in the excited state for different interaction times after passing through the cavity, it is possible to reconstruct the initial state of the mechanical mirror by inverting a simple system of linear equations. Although stronger optomechanical coupling

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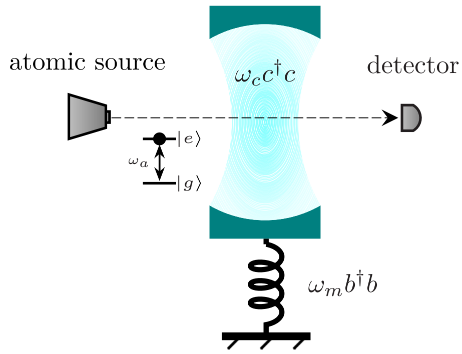

FIG. 1. Quantum-state reconstruction using atom as detector in the hybrid optomechanical system.

strength can have better reconstruction quality, our method does not require the coupling strength to be ultrastrong.

This paper is organized as follows. In Sec. II, we introduce the suggested model to measure the state of the mechanical mirror. In Sec. III, we derive the excited-state probability of the atom when the atom passes through the optomechanical cavity. In Sec. IV, we show how to measure the quantum state of the mechanical mirror. Finally, we summarize the results.

II. MODEL

We consider an optomechanical system consisting of an optical cavity with a fixed mirror on one side and a mechanical mirror on the other side of the cavity. The position of the mechanical mirror is described by $x = x_0(b + b^\dagger)$, where x_0 is the zero point position of the mechanical mirror and it is given by $x_0 = \sqrt{\hbar/2m\omega_m}$. Here $b(b^\dagger)$ is the mechanical mirror annihilation (creation) operator and m and ω_m are the mass and frequency of the mechanical mirror, respectively. The cavity field is weakly coupled to the mechanical mirror via the radiation pressure coupling. To detect the quantum state of the mechanical mirror, we consider a beam of two-level atoms entering the cavity to interact with the quantized field inside the cavity and the probability of finding the atom in the excited state is measured for different interaction times.

The total Hamiltonian describing the system depicted in Fig. 1 is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I, \tag{1}$$

where $\mathcal{H}_0$ and $\mathcal{H}_I$ are the free and interaction parts of the system's Hamiltonian, respectively. The free part of the Hamiltonian can be written as

$$\mathcal{H}_0 = \hbar\omega_c c^\dagger c + \hbar\frac{\omega_a}{2}\sigma_z + \hbar\omega_m b^\dagger b, \tag{2}$$

where the first term in Eq. (2) describes the Hamiltonian of the cavity field with frequency ω_c. The second and third terms in Eq. (2) represent the Hamiltonian of the two-level atom with transition frequency ω_a and the mechanical mirror with fundamental oscillation frequency ω_m, respectively. Here $c(c^\dagger)$ is the annihilation (creation) operator of the cavity field. The interaction part of the Hamiltonian in Eq. (1) is given by [52,53]

$$\mathcal{H}_I = -i\hbar g_c(\sigma_+ c - c^\dagger\sigma_-) - \hbar g_m c^\dagger c(b^\dagger + b). \tag{3}$$

The first term in Eq. (3) describes the interaction between the two-level atom and the cavity field. The second term describes the interaction between the cavity field and the mechanical mirror via the radiation pressure coupling. The coefficients g_c and g_m are the coupling strengths of the atom-field and the cavity field–mechanical mirror interaction, respectively. The coefficient g_m is defined as $g_m = (\omega_c/L)\sqrt{\hbar/2m\omega_m}$, where L is the length of the cavity and $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$ are the atomic raising and lowering operators, respectively.

In the weak mechanical coupling limit, the Hamiltonian of the system can be simplified in such a way that the mechanical mirror is coupled to a specific polariton state $|\pm, n\rangle$ in a Jaynes-Cummings (JC)-like coupling where $|+, n\rangle = \cos(\alpha_n/2)|e, n\rangle + i\,\sin(\alpha_n/2)|g, n+1\rangle$ and $|-, n\rangle = \sin(\alpha_n/2)|e, n\rangle - i\,\cos(\alpha_n/2)|g, n+1\rangle$ with $\tan\alpha_n = 2g_c\sqrt{n+1}/\delta$ and $\delta = \omega_a - \omega_c$ being the detuning between the atomic transition frequency and the cavity field frequency. The rotating-wave approximation under the condition $\omega_m + \Omega_n \gg g_{pn} \gg |\omega_m - \Omega_n|$ can be applied. This condition can be justified since ω_m can be in the range from kHz to GHz, while g_c is usually in the range from kHz to MHz. Since β is usually much less than 1, then the condition $g_{pn} \ll \omega_m + \Omega_n$ is not difficult to satisfy. To satisfy the other condition $g_{pn} \gg |\omega_m - \Omega_n|$, we can find a system where the atom-field coupling strength g_c is not far away from ω_m and then choose a suitable photon number n such that ω_m is very close to Ω_n. Under the rotating-wave approximation, the transformed Hamiltonian in the linear approximation is reduced to [52,53]

$$\mathcal{H}_\mathcal{T} = \sum_{n=1}^{\infty}\left[\hbar\frac{\Omega_n}{2}\sigma_z^{(n)} + \hbar g_{pn}\left(\sigma_-^{(n)}b^\dagger + \sigma_+^{(n)}b\right)\right] + \hbar\omega_m b^\dagger b, \tag{4}$$

where $\Omega_n = \sqrt{\delta^2 + 4g_c^2(n+1)}$ describes the energy of the polariton with photon number n and $g_{pn} = \beta g_c\sqrt{n+1}$ is the effective coupling between the polariton and the mechanical mirror. $\sigma_z^{(n)}$ and $\sigma_\mp^{(n)}$ are the polariton Pauli matrices for the polariton states $|\pm, n\rangle$. It is clearly seen that in the dressed-state picture the polariton effectively couples to the phonons. Using the effective Hamiltonian shown in Eq. (4), we can solve the dynamics of the system using the time-dependent Schrödinger's equation

$$|\dot{\psi}_\mathcal{T}\rangle = -\frac{i}{\hbar}\mathcal{H}_\mathcal{T}|\psi_\mathcal{T}\rangle, \tag{5}$$

where $|\psi_\mathcal{T}\rangle = \mathcal{T}|\psi_s\rangle$ is the transformed state of the total system with $\mathcal{T} = e^{-\beta c^\dagger c(b^\dagger - b)}$ and $|\psi_s\rangle$ the untransformed state of the system. By solving Eq. (5), we can obtain the population dynamics of the atom which encodes the information of the mechanical mirror. Therefore, it is possible to reconstruct the quantum state of the mechanical mirror by measuring the atomic population.

III. EXCITED-STATE PROBABILITY

When the atoms pass through the cavity, the population in the excited state can be modified by the cavity field and the mechanical mirror phonon. It is therefore possible to measure the quantum state of the mechanical mirror from the atomic excited-state probability. In this section, we calculate the evolution of the excited-state probability and show how we can measure the quantum state of the mechanical mirror.

We assume that the mechanical mirror is in an unknown superposition state $|\psi_{\rm mirror}\rangle = \sum_{l=0}^{\infty} u_l|l\rangle$, where u_l is an unknown amplitude to be determined and it satisfies that $\sum_{l=0}^{\infty}|u_l|^2 = 1$. We first consider the general case where the two-level atom is initially in a superposition of the excited state $|e\rangle$ and the ground state $|g\rangle$, i.e., $|\psi_{\rm atom}(0)\rangle = c_e|e\rangle + c_g|g\rangle$ with $|c_g|^2 + |c_e|^2 = 1$, and the cavity field is in the state $|\psi_{\rm cavity}(0)\rangle = \sum_{n=0}^{\infty} w_n|n\rangle$ with $\sum_{n=0}^{\infty}|w_n|^2 = 1$. The initial state of the total system can then be written as

$$|\psi_s(0)\rangle = \sum_{n=0}^{\infty}\sum_{l=0}^{\infty} w_n u_l (c_e|e,n\rangle + c_g|g,n\rangle)|l\rangle . \quad (6)$$

The initial state (6) in the transformed picture is given as $|\psi_T(0)\rangle = e^{-\beta n(b^\dagger - b)}|\psi_s(0)\rangle$. For $\beta n \ll 1$, the initial state $|\psi_T(0)\rangle \approx |\psi_s(0)\rangle$. Using the dressed-state bases of the atom-field subsystem, the system's initial state can be rewritten as

$$|\psi(0)\rangle = w_0 c_g \sum_{l=0}^{\infty} u_l|g,0\rangle|l\rangle + \sum_{n=0}^{\infty}\sum_{l=0}^{\infty} u_l\left[\left(c_e w_n\cos\left(\frac{\alpha_n}{2}\right) - i c_g w_{n+1}\sin\left(\frac{\alpha_n}{2}\right)\right)|+,n\rangle + \left(c_e w_n\sin\left(\frac{\alpha_n}{2}\right) + i c_g w_{n+1}\cos\left(\frac{\alpha_n}{2}\right)\right)|-,n\rangle\right]|l\rangle . \quad (7)$$

It is clear that the effective Hamiltonian (4) can lead to transitions such that $|+,n\rangle|l\rangle \longleftrightarrow |-,n\rangle|l+1\rangle$. The state of the system at time t is therefore given by

$$|\psi(t)\rangle = \sum_{l=0}^{\infty} C_{0,l}^{g}(t)|g,0\rangle|l\rangle + \sum_{n=0}^{\infty} C_{n,0}^{-}(t)|-,n\rangle|0\rangle + \sum_{n=0}^{\infty}\sum_{l=0}^{\infty}[C_{n,l}^{+}(t)|+,n\rangle|l\rangle + C_{n,l+1}^{-}(t)|-,n\rangle|l+1\rangle], \quad (8)$$

where $C_{0,l}^{g}(t)$ is the probability amplitude that the atom is in the ground state and the field is in the vacuum state with the mechanical mirror being in the state $|l\rangle$, and $C_{n,l}^{+}(t)$ $[C_{n,l}^{-}(t)]$ is the probability amplitude that the polariton is in the state $|+,n\rangle$ $(|-,n\rangle)$ and the mechanical mirror is in the state $|l\rangle$. The equations of motion for the probability amplitudes can be derived from the time-dependent Schrödinger's equation. A solution of these coupled equations can give the following expressions:

$$C_{0,l}^{g}(t) = 0, \quad (9a)$$

$$C_{n,l}^{+}(t) = e^{-i\omega_m(l+\frac{1}{2})t}\left[C_{n,l}^{+}(0)\cos\left(\frac{\omega_{nl}}{2}t\right) - i\frac{\Delta_n}{\omega_{nl}}C_{n,l}^{+}(0)\sin\left(\frac{\omega_{nl}}{2}t\right) - 2i\frac{g_{pn}\sqrt{l+1}}{\omega_{nl}}C_{n,l+1}^{-}(0)\sin\left(\frac{\omega_{nl}}{2}t\right)\right], \quad (9b)$$

$$C_{n,l+1}^{-}(t) = e^{-i\omega_m(l+\frac{1}{2})t}\left[C_{n,l+1}^{-}(0)\cos\left(\frac{\omega_{nl}}{2}t\right) + i\frac{\Delta_n}{\omega_{nl}}C_{n,l+1}^{-}(0)\sin\left(\frac{\omega_{nl}}{2}t\right) - 2i\frac{g_{pn}\sqrt{l+1}}{\omega_{nl}}C_{n,l}^{+}(0)\sin\left(\frac{\omega_{nl}}{2}t\right)\right], \quad (9c)$$

$$C_{n,0}^{-}(t) = e^{\frac{i\Omega_n}{2}t}C_{n,0}^{-}(0), \quad (9d)$$

where $\Delta_n = \Omega_n - \omega_m$ and $\omega_{nl} = \sqrt{\Delta_n^2 + 4g_{pn}^2(l+1)}$. For resonant atom-field interaction ($\delta = 0$), $\sin(\alpha_n/2) = \cos(\alpha_n/2) = 1/\sqrt{2}$. In this case, the probability for the atom to be in the excited state can be calculated from the following equation:

$$P_e(t) = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}|C_{n,l}^{+}(t) + C_{n,l}^{-}(t)|^2. \quad (10)$$

We consider the atom to be initially prepared in the excited state, i.e., $c_e = 1$ and $c_g = 0$, and the cavity field is in the vacuum state, i.e., $w_0 = 1$ and $w_n = 0$ for $n \neq 0$. The atomic population inversion defined by $W(t) = 2P_e(t) - 1$ follows directly by substituting the solutions of the probability amplitudes from Eqs. (8 a)–(8 d) into Eq. (10). The resulting expression for $W(t)$ is

$$\begin{aligned} W(t) = \sum_{l=0}^{\infty}[&\cos(g_{p0}\sqrt{l}t)\cos(g_{p0}\sqrt{l+1}t)\cos(\omega_m t)\,a_l \\ &+ \sin(g_{p0}\sqrt{l+1}t)\sin(g_{p0}\sqrt{l+2}t)(\mathrm{Re}\,(b_l)\cos(\omega_m t) - \mathrm{Im}\,(b_l)\sin(\omega_m t)) \\ &+ g(\cos(g_{p0}\sqrt{l+2}t) - \cos(g_{p0}\sqrt{l}t))\sin(g_{p0}\sqrt{l+1}t)(\mathrm{Re}\,(c_l)\sin(\omega_m t) + \mathrm{Im}\,(c_l)\cos(\omega_m t))], \end{aligned} \quad (11)$$

where, for simplicity's sake, we considered $\Omega_0 = 2g_c = \omega_m$. The coefficients a_l, b_l, and c_l are given by

$$a_l = |u_l|^2, \quad b_l = u_l u_{l+2}^*, \quad c_l = u_l u_{l+1}^*. \quad (12)$$

The coefficient a_l gives the probability distribution of the initial state of the mechanical mirror, while the phase information of the mirror's state is contained in either b_l or c_l depending on what the mirror's initial state is. In the next section, we show how the initial state of the mechanical mirror can be reconstructed using Eq. (11).

IV. QUANTUM-STATE RECONSTRUCTION OF THE MECHANICAL MIRROR

In order to reconstruct the initial state of the mechanical mirror, we need to find the coefficients a_l, b_l, and c_l in Eq. (11). From Eq. (11), we see that we have an infinite number of the coefficients a_l, b_l, and c_l. We can truncate the infinite summation over the number of phonons l in Eq. (11) to a maximum number $l_{\max}$ such that $l_{\max} \gg \bar{l}$, where $\bar{l}$ is the average number of phonons. Since a_l is a real number but both b_l and c_l are complex numbers, the unknown variable for each l is 5 (a_l, Re[b_l], Im[b_l], Re[c_l], Im[c_l]). For a cutoff $l_{\max}$, the total number of unknown variables is $5(l_{\max}+1)$. Therefore, to determine all the unknown variables, we need to measure $W(t)$ for at least $5(l_{\max}+1)$ interaction times. For each value of t_k [$k = 1, 2, \ldots, 5(l_{\max}+1)$], the expression for the inversion is given by

$$W(t_k) = \sum_{l=0}^{l_{\max}}[A_l(t_k)\, a_l + B_l(t_k)\,\mathrm{Re}\,(b_l) + C_l(t_k)\,\mathrm{Im}\,(b_l) + D_l(t_k)\,\mathrm{Re}\,(c_l) + E_l(t_k)\,\mathrm{Im}\,(c_l)], \tag{13}$$

where a_l, Re[b_l], Im[b_l], Re[c_l], Im[c_l] are unknown variables to be determined and

$$A_l(t_k) = \cos(g_{p0}\sqrt{l}t_k)\cos(g_{p0}\sqrt{l+1}t_k)\cos(\omega_m t_k), \tag{14a}$$

$$B_l(t_k) = \sin(g_{p0}\sqrt{l+1}t_k)\sin(g_{p0}\sqrt{l+2}t_k)\cos(\omega_m t_k), \tag{14b}$$

$$C_l(t_k) = -\sin(g_{p0}\sqrt{l+1}t_k)\sin(g_{p0}\sqrt{l+2}t_k)\sin(\omega_m t_k), \tag{14c}$$

$$D_l(t_k) = [\cos(g_{p0}\sqrt{l+2}t_k) - \cos(g_{p0}\sqrt{l}t_k)] \times \sin(g_{p0}\sqrt{l+1}t_k)\sin(\omega_m t_k), \tag{14d}$$

$$E_l(t_k) = [\cos(g_{p0}\sqrt{l+2}t_k) - \cos(g_{p0}\sqrt{l}t_k)] \times \sin(g_{p0}\sqrt{l+1}t_k)\cos(\omega_m t_k). \tag{14e}$$

Equation (12) can be further written as the matrix form

$$\mathbf{W} = \mathbf{M}\,\mathbf{X}, \tag{15}$$

where $\mathbf{W}$ is a $5(l_{\max}+1)$-dimensional vector which contains the experimental data of the population inversion, i.e., $\mathbf{W} = [W(t_1), W(t_2), \ldots, W(t_{5(l_{\max}+1)})]^T$. The vector $\mathbf{X}$ contains the unknown coefficients which need to be determined to reconstruct the initial state of the mechanical mirror and it is defined as $\mathbf{X} \equiv (\mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_{5(l_{\max}+1)})^T$ with $\mathbf{X}_l = (a_l,\ \mathrm{Re}\,(b_l),\ \mathrm{Im}\,(b_l),\ \mathrm{Re}\,(c_l),\ \mathrm{Im}\,(c_{l))^T}$. The matrix $\mathbf{M}$ is preknown and it is given by

$$\mathbf{M}_{k\,l} = A_l(t_k) + B_l(t_k) + C_l(t_k) + D_l(t_k) + E_l(t_k), \tag{16}$$

where the elements of this matrix are defined in Eqs. (13 a)–(13 d), and $k, l = 1, 2, \ldots, 5(l_{\max}+1)$. Having $\mathbf{W}$ and $\mathbf{M}$ we can either use matrix inversion or least-square fitting method to obtain the solutions for $\mathbf{X}$, which contains both the probability and phase information of the mirror's state.

Extracting the values of a_l from the vector $\mathbf{X}$ directly yields the phonon probability distribution of the mechanical mirror. The phase of the mechanical mirror can be obtained from other elements of the vector $\mathbf{X}$. In what follows we explain the procedure of reconstructing the phase of the mirror's state. When we look at Eq. (12), we see that it contains terms of the product of different amplitudes, i.e., $u_l u_{l+1}^*$ and $u_l u_{l+2}^*$. These terms contain the phase of the amplitude from which we can reconstruct the phase of the quantum state of the mechanical mirror. Since $a_l = |u_l|^2$ and $c_l = u_l u_{l+1}^*$, we have $c_l = \sqrt{a_l a_{l+1}}e^{-i(\varphi_{l+1}-\varphi_l)}$, where the amplitude $u_l = |u_l|e^{i\phi_l}$. Therefore, the phase difference between the probability amplitude u_l and its first neighbor u_{l+1} is given by

$$\Delta\varphi_{l+1} = \varphi_{l+1} - \varphi_l = \arctan\left[\frac{-\mathrm{Im}(c_l)}{\mathrm{Re}(c_l)}\right]. \tag{17}$$

Hence the phase can be determined from c_l. In certain cases some elements may be missing and the phase difference between neighboring elements is undefined. For example, in the squeezed vacuum, only even photon number has nonzero amplitude. In this case, $c_l = 0$ and the phase cannot be reconstructed. Fortunately, we can use a_l and b_l to determine the phase difference between the next-nearest-neighbor elements. We have $b_l = u_l u_{l+2}^* = \sqrt{a_l a_{l+2}}e^{-i(\varphi_{l+2}-\varphi_l)}$. Then the phase difference between two neighboring bases is given by

$$\Delta\varphi_{l+2} = \varphi_{l+2} - \varphi_l = \arctan\left[\frac{-\mathrm{Im}(b_l)}{\mathrm{Re}(b_l)}\right]. \tag{18}$$

By solving b_l, one can reconstruct the phase of squeezed vacuum.

In the following, we apply our scheme to three examples of the mechanical state and examine different conditions in which this scheme can work to reconstruct the initial state of the mechanical mirror.

A. Coherent state

Suppose that the initial state of the mechanical mirror is a coherent state and it can be written as

$$|\psi_{\mathrm{mirror}}(0)\rangle = e^{-\bar{l}/2}\sum_{l=0}^{\infty}\frac{\bar{l}^{l/2}}{\sqrt{l!}}|l\rangle, \tag{19}$$

where $\bar{l}$ is the average phonon number. The exact probability distribution of phonons is given by

$$P(l) = \frac{\bar{l}^l\, e^{-\bar{l}}}{l!}, \tag{20}$$

which is a Poisson distribution.

From Eq. (18), we know that $u_l = e^{-\bar{l}/2}\,\bar{l}^{l/2}/\sqrt{l!}$ and the population inversion can be calculated from Eq. (11) for an arbitrary time. To reconstruct the quantum state of the mechanical mirror, we need to choose a value of the cutoff $l_{\max}$ and measure $W(t)$ for $5(l_{\max}+1)$ discrete times. Then we use the least-square method to fit the unknown variables

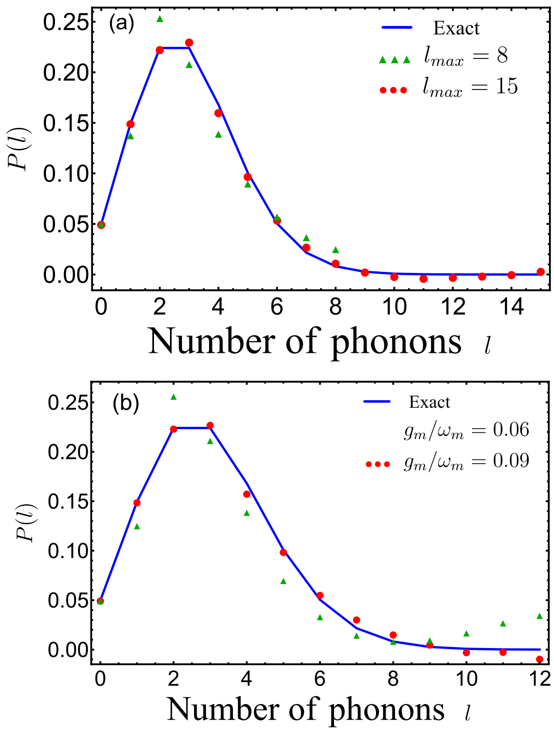

FIG. 2. Reconstruction of initial mechanical coherent state. (a) Comparison between the exact and reconstructed probability distribution of phonons for two different values of $l_{\max}$ with $g_m/\omega_m = 0.07$. (b) Comparison between the exact and reconstructed probability distribution of phonons for two different coupling strengths with $l_{\max} = 12$. Mean number of phonons $\bar{l} = 3$.

X from Eq. (14). In Fig. 2(a), we compare the reconstructed probability distribution of phonons with the exact distribution Eq. (20) with $\bar{l} = 3$ for two different cutoffs ($l_{\max} = 8$ and 15). In this example, we fix the coupling ratio to be $g_m/\omega_m = 0.07$. The solid curve represents the exact probability distribution of phonons, the triangles correspond to the reconstructed probability distribution when $l_{\max} = 8$, and the points correspond to the reconstructed probability distribution when $l_{\max} = 15$. It is clearly seen that in both cases ($l_{\max} = 8$ and 15) the reconstructed results of the probability distribution of phonons are close to the exact distribution. More importantly, when the cutoff $l_{\max}$ is increased, the quality of reconstruction apparently improves. In pratice, the cutoff $l_{\max}$ is chosen such that the phonon distribution is negligible when $l > l_{\max}$.

Figure 2(b) shows the effect of increasing the optomechanical coupling strength g_m on the quality of reconstruction while $l_{\max}$ is fixed. Here, the average phonon number is $\bar{l} = 3$ and $l_{\max}$ is set to be 12. When $g_m/\omega_m = 0.06$, the reconstructed probability distribution is close to the exact distribution but most points are still deviating from the exact probability distribution (the solid curve). However, when the optomechanical coupling strength is increased to $g_m/\omega_m = 0.09$, the result of reconstruction is clearly approaching the exact probability distribution. Therefore, increasing the optomechanical coupling strength leads to extracting more accurate information about the initial state of the mechanical mirror. We should emphasize that the optomechanical coupling strengths in the considered examples are in the weak-coupling regime. This is different from the method presented in [35], where strong optomechanical coupling is needed to reconstruct the initial state of the mechanical mirror, i.e., $g_m/\omega_m > 1$.

B. Pure quantum state with phase

In the previous subsection, we considered that the initial state has real amplitudes. In this subsection we show how to reconstruct a general quantum state when both probability and phase are included. A general quantum state can be expanded as a superposition of Fock state. Here, as an example, we consider that the initial state of the mechanical mirror is a superposition of four Fock states $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$, with a phase difference between neighboring Fock states:

$$|\psi_{\text{mirror}}(0)\rangle = \frac{1}{\sqrt{10}}(\sqrt{3}\;|0\rangle + e^{i\pi/5}\,|1\rangle + \sqrt{5}\,e^{i\pi/3}\,|2\rangle + e^{i\pi/4}\,|3\rangle). \tag{21}$$

In order to extract the quantum state of the mechanical mirror, we solve Eq. (14) for a truncated maximum number of phonons $l_{\max}$. In this example, we chose $l_{\max} = 4$, which means the atomic population inversion is measured for $5(l_{\max} + 1) = 25$ discrete interaction times. From Eq. (14), we can construct a linear system of equations using the measured data of the atomic population inversion $W(t_k)$ along with Eqs. (13 a)–(13 d). We solve this system of linear equations by simple matrix inversion to reconstruct the initial state of the mechanical mirror. The values of a_l which are contained in the vector **X** yield the probability of the mirror's quantum state, while the real and imaginary parts of c_l can be used to reconstruct the phase difference between the neighboring Fock states.

In Fig. 3(a), we compare the exact values of the phonon number distribution of the initial state of the mirror $1/10(3, 1, 5, 1)^T$ with the reconstructed values for two different values of the ratio between the optomechanical coupling and the oscillation frequency of the mechanical mirror, i.e., $g_m/\omega_m = 0.02$ and 0.05. The reconstructed values of the probability clearly converge to the exact values as g_m/ω_m increases. This is similar to the reconstruction of the coherent state and it is because stronger coupling strength can project more information from the mechanical mirror to the atom. The phase reconstruction is shown in Fig. 3(b), where we can see that the reconstructed values of the phase difference between two successive phonon numbers agree very well with the exact values for both coupling strengths. This indicates that the reconstruction of phase is not very sensitive to the fluctuation.

C. Squeezed vacuum state

The reconstruction method for the phase shown in subsection B fails when certain coefficients vanish. For example, in the squeezed vacuum state only the coefficients with even Fock number are nonzero. Then $c_l = u_l u_{l+1}^*$ is always zero and the reconstruction of phase is impossible. Fortunately, in our reconstruction method, we have the terms like $b_l = u_l u_{l+2}^*$ in addition to c_l [see Eq. (11)]. In the squeezed vacuum state, although c_l is zero, b_l is not equal to zero from which we

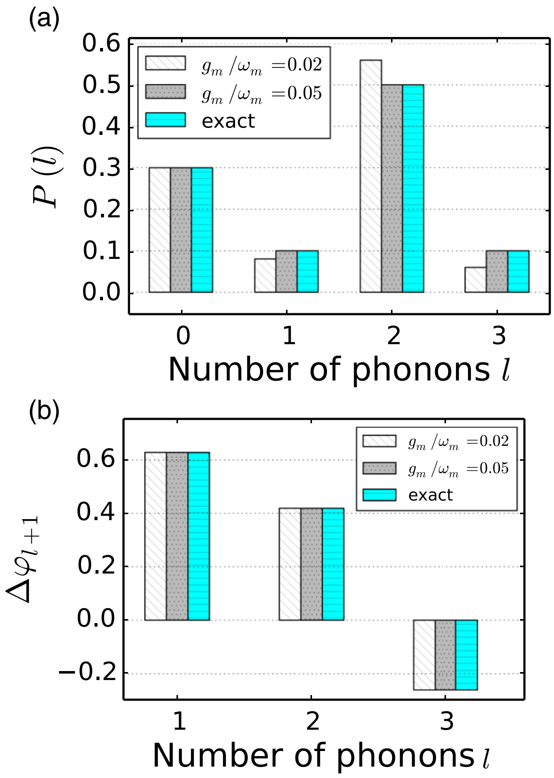

FIG. 3. Comparison between the exact and reconstructed probability distribution of phonons (a) and the phase difference $\Delta\varphi_{l+1}$ (b) where the initial state of the mechanical mirror is given by Eq. (21) for two different ratios between the optomechanical coupling g_m and the mechanical frequency ω_m.

can still extract the phase information of the squeezed vacuum state.

The initial state of the mechanical mirror in a squeezed vacuum state can be written as

$$|\psi_{\mathrm{mirror}}(0)\rangle = \sum_{l=0}^{\infty} C_{2l}\,|2l\rangle\,, \tag{22}$$

where the coefficients $C_{2\,l}$ are given by [54]

$$C_{2l} = \frac{(-1)^l}{\sqrt{\cosh r}}\frac{\sqrt{(2l)!}}{2^l\, l!}(e^{i\varphi}\tanh r)^l, \tag{23}$$

with r being the squeezing parameter. By solving a_l and b_l in Eq. (11), we can in principle reconstruct both the phonon number distribution and the phase of the squeezed vacuum state.

In Fig. 4(a), we plot the exact probability distribution of the mechanical mirror $P(2l) = |C_{2l}|^2$ along with the reconstructed distributions using two different values of g_m/ω_m. The reconstruction values are very close to the exact values and the quality of the probability distribution reconstruction improves as the mechanical coupling strength g_m is larger as in the previous example. In Fig. 4(b), we compare the exact phase differences $\Delta\varphi_{l+2}$ with the reconstructed ones using Eq. (17). Different from the phase reconstruction based on c_l, the reconstruction based on b_l is more sensitive to the coupling strength. Stronger coupling strength can give better phase reconstruction.

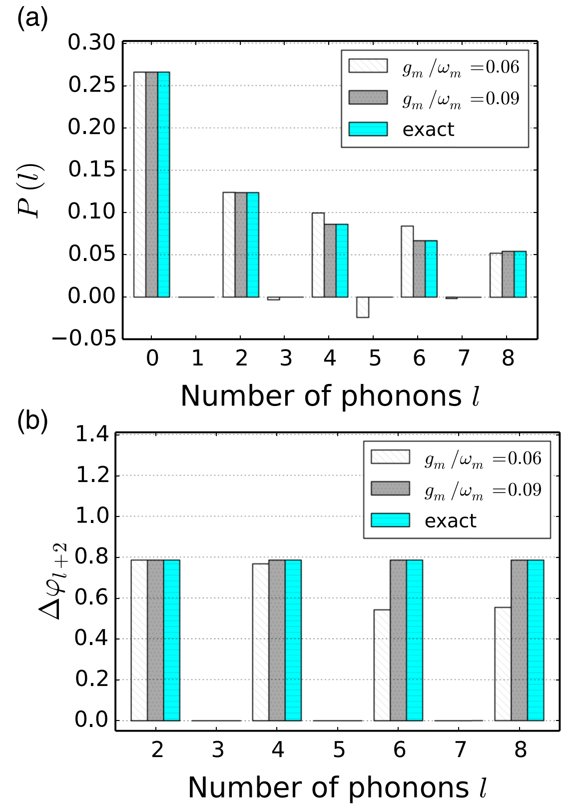

FIG. 4. Comparison between the exact and reconstructed probability distribution of phonons (a) and the phase difference $\Delta\varphi_{l+2}$ (b) when the initial state of the mechanical mirror is squeezed vacuum for two different ratios between the optomechanical coupling g_m and the mechanical frequency ω_m. $r = 2$ and $\varphi = \pi/8$.

D. Velocity fluctuation

In the previous subsections, we assume that interaction times are precisely determined. However, in practice the interaction times can vary a bit due to the uncertainty in the velocity of atoms. In this subsection, we show that even if the interaction times have small fluctuations, our method still works very well.

Suppose that each interaction time has an uncertainty. To reconstruct the quantum state of the mechanical mirror, we measure $W(t_k)$ for N_{runs} times for each t_k and their average $\overline{W}(t_k)$ is treated as $W(t_k)$ in Eq. (14). Then we reconstruct the quantum state of the mechanical mirror by using the same method presented in the previous section. We consider the case when the mechanical mirror is initially prepared in a coherent state with average number of phonons $\bar{l} = 3$ and each atom velocity has 4% uncertainty. In the numerical simulation, the sampling time is randomly chosen between $t_k - 0.02\Delta t$ and $t_k + 0.02\Delta t$ for each t_k and Δt is the gap between two successive interaction times (i.e., $\Delta t = t_{k+1} - t_k$). Using the least-square fitting method with 10^{-3} tolerance, we can obtain a solution for $\mathbf{X}$. The reconstructed results for two different iteration times are shown in Fig. 5, where

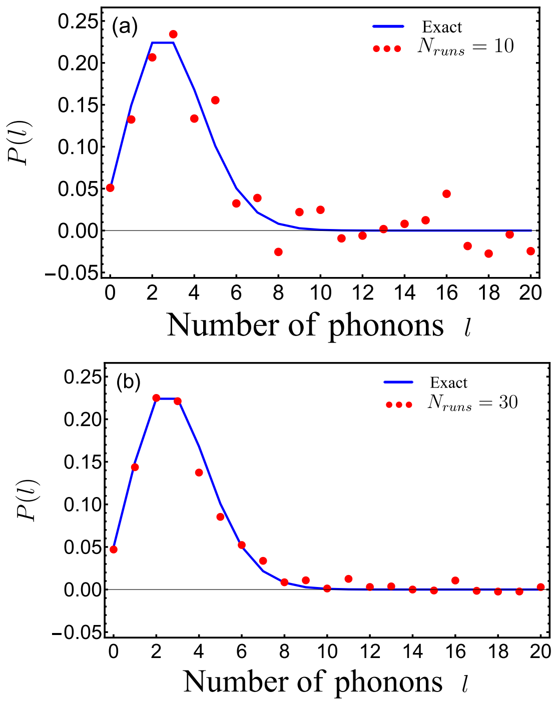

FIG. 5. Reconstruction of the initial state of the mechanical mirror when fluctuation in the interaction times between the atoms and the cavity field is considered. (a) Number of iterations is $N_{\mathrm{runs}} = 10$. (b) Number of iterations is $N_{\mathrm{runs}} = 30$. The average phonon number is $\bar{l} = 3$, $g_m/\omega_m = 0.9$, and the fluctuation in the interaction times is $\pm 2\%$.

Fig. 5(a) is the result of reconstruction using $N_{\mathrm{runs}} = 10$, while Fig. 5(b) is the result of reconstruction using $N_{\mathrm{runs}} = 30$. We see from both figures that, when $N_{\mathrm{runs}} = 10$, the reconstructed probability distribution of phonons deviates significantly from the exact distribution of phonons. However, when the iteration times are increased to $N_{\mathrm{runs}} = 30$, the reconstructed probability distribution of phonons converges to the exact distribution very well.

E. Reconstruction of thermal state

So far we considered quantum-state reconstruction of pure states of the mirror. Next we address the question about the reconstruction of mixed states. It turns out that, for arbitrary mixed states described by a density operator ρ, our proposed method can be applied to reconstruct only the diagonal elements ρ_{ll} and the off diagonal elements $\rho_{l,l+1}$ and $\rho_{l,l+2}$. It can be shown that, for mixed states, Eq. (11) can be obtained with a_l, b_l, and c_l replaced by ρ_{ll}, $\rho_{l,l+2}$, and $\rho_{l,l+1}$, respectively. We can then use the reconstruction method discussed in previous sections to obtain the partial information of the density matrix of the mechanical mirror.

One important state of the mirror that we can reconstruct using our method is the thermal state for which only the diagonal density matrix elements are nonvanishing. The state is given by

$$\rho^m(0) = \sum_{l=0}^{\infty} \rho_{ll} \left| l \right\rangle \left\langle l \right| , \tag{24}$$

where $\rho_{ll} = \frac{\bar{l}^l}{(\bar{l}+1)^{l+1}}$ is the phonon-number distribution with $\bar{l}$ being the average number of phonons. Since the thermal state has only the diagonal terms, the coefficients shown in Eq. (11) are $a_l = \rho_{ll}$ and $b_l = c_l = 0$. Therefore, the population inversion $W(t)$ for the thermal state is given by

$$W(t) = \sum_{l=0}^{\infty} \cos(g_{p0}\sqrt{l}t)\cos(g_{p0}\sqrt{l+1}t)\cos(\omega_m t)\rho_{ll}. \tag{25}$$

Using the same method discussed above, we can reconstruct the phonon distribution for the thermal state.

The numerical results are shown in Fig. 6 where we assume the average phonon number $\bar{l} = 2$. In Fig. 6(a) we compare the reconstruction results for two different cutoffs with the exact thermal distribution. We can see that, when $l_{\max}$ is not large enough, the reconstruction deviates from the exact distribution significantly. However, if we increase the cutoff $l_{\max}$, the reconstruction result matches the exact thermal distribution very well. In Fig. 6(b), we show the reconstruction results for two different coupling strengths. It shows that a larger coupling strength can give a better reconstruction result. In Fig. 6(c), we consider the case when the velocity of the atoms has certain uncertainty. In the figure, we compare the reconstruction results for two different velocity uncertainties, i.e., 2% and 4%. It is shown that smaller uncertainty can have

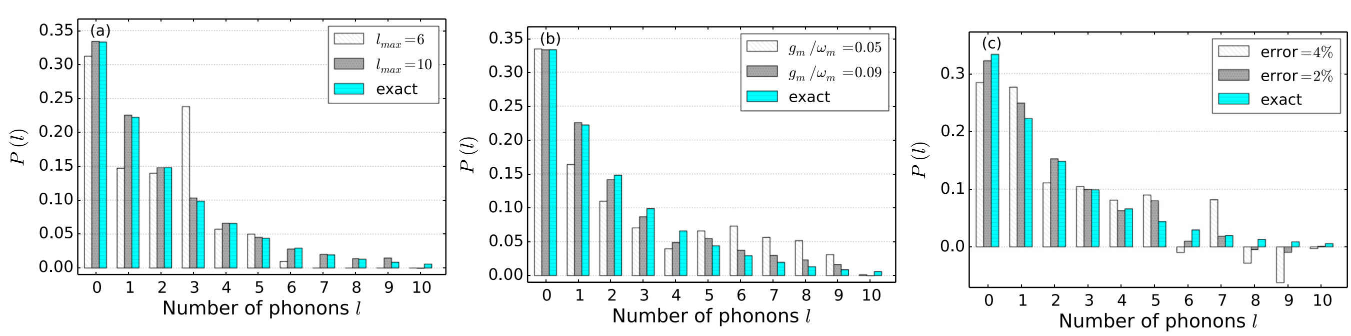

FIG. 6. Comparison between the exact and reconstructed phonon-number distributions for initially mechanical thermal state. (a) The comparison is made using $l_{\max} = 6$ and $l_{\max} = 10$ with $g_m/\omega_m = 0.07$. (b) The comparison is made for two different coupling strengths ($g_m/\omega_m = 0.05$ and 0.09) with $l_{\max} = 10$. (c) The comparison is made using two different values for the fluctuation in the interaction time, 4% and 2% with $g_m/\omega_m = 0.09$ and $l_{\max} = 10$. Mean number of phonons is $\bar{l} = 2$.

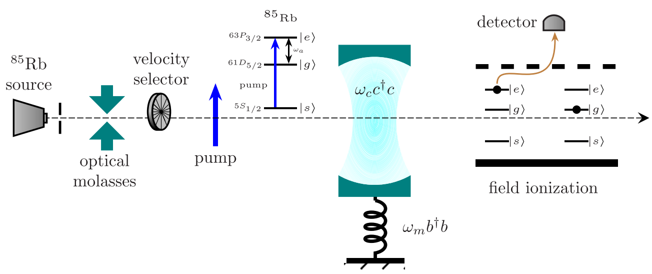

FIG. 7. Possible experimental realization with ^{85}Rb atoms. The transverse velocity of the atoms are collimated by a pinhole and optical molasses. The longitudinal velocity can be controlled by a velocity selector. The population in the excited state can be detected by the field-ionization technique.

better reconstruction results. Even if the uncertainty is 4%, the reconstructed phonon distribution cans still be very close to the exact thermal phonon distribution.

V. POSSIBLE EXPERIMENTAL REALIZATION

A possible experimental realization of our scheme is shown in Fig. 7, where the detection atom is the Rydberg Rubidium-85 (^{85}Rb) atom. Here, three energy levels of the ^{85}Rb atoms are involved, i.e., $5S_{1/2}$ ($|s\rangle$), $63P_{3/2}$ ($|e\rangle$), and $61D_{5/2}$ ($|g\rangle$). The ^{85}Rb atoms are ejected from the oven and they are collimated by a pinhole and an optical molasses. After the collimation, a velocity selector can be used to choose the atoms with certain longitudinal velocity to pass through [55–57]. Then a pumping field is applied to pump the atoms from the ground state $5S_{1/2}$ to the Rydberg excited state $63P_{3/2}$. Here, we can also use the Doppler pumping technique [58,59] to excite the atoms with special longitudinal velocity from the state $5S_{1/2}$ to the state $63P_{3/2}$ instead of using the velocity selector and a pumping field. The Rydberg atoms then pass through the optomechanical cavity and interact with the cavity field which is initally in the vacuum state. The transition $63P_{3/2} \leftrightarrow 61D_{5/2}$ has a transition frequency $\omega_a \cong 2\pi \times 21.5$ GHz [56], which is resonant with the cavity field frequency. The single photon coupling strength g_c can be about 44 kHz as in [56]. Since the longitudinal velocity is fixed, the interaction time is also fixed. After the interaction, the standard field-ionization technique can be applied to detect the atom in the state $|e\rangle$ or the state $|g\rangle$.

The experiment can then be repeated with the same interaction time t_{k_0} for a number of times to obtain the population inversion $W(t_{k_0})$. The above procedure can be repeated for different interaction times by controlling the velocity selector and obtain a set of values of $W(t_k)$ with $k = 1, 2, \ldots, 5(l_{\max} + 1)$. Finally, we use the method discussed in previous sections to reconstruct the quantum state of the mechanical resonator. Here, one should note that the measurement in our method is an ensemble measurement [60]. A single detection does not yield useful information about the state of the mirror. Instead, we have to prepare the system identically for each detection and repeat the measurement many times to obtain useful information for quantum-state reconstruction which is similar to Ref. [51]. The number of measurements depends on the required accuracy.

In this experimental proposal, the energy of the polariton with zero photon number is $\Omega_0 = 2g_c = 88$ kHz. We assume that the optomechanical system is in the weak-coupling regime such that $\beta = 0.1$. Then we have the effective coupling strength between the polariton and the mechanical mirror $g_{p0} = \beta g_c = 4.4$ kHz. To satisfy the condition for the rotating-wave approximation, we should have $\omega_m + \Omega_0 \gg g_{p0} \gg |\omega_m - \Omega_0|$. Thus the fundamental oscillation frequency of the mechanical mirror ω_m should be very close to Ω_0, which is about 88 kHz. This is realistic because the typical oscillation frequency of a mechanical mirror can range from kHz to GHz [1].

VI. CONCLUSION

In this paper, we proposed a scheme to detect the quantum state of the mechanical mirror. In this scheme, a beam of two-level atoms initially prepared in the excited state are sent through an optomechanical cavity to interact with the cavity field. The polariton formed by the atom and the cavity field can effectively couple to the phonon of the mechanical mirror. From this coupling, the quantum state of the mechanical mirror can imprint to the dynamics of the atom. By measuring the probability of the atoms to be in the excited state when exciting the optomechanical cavity, it is possible to reconstruct the quantum state of the mechanical mirror including both the phonon number distribution and the phase, even if the interaction times have a certain amount of uncertainty. We also show that a mechanical thermal state can be reconstructed with high fidelity. Our method does not require a strong optomechanical coupling and it may provide a useful tool in the quantum information processing based on optomechanical systems.

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[1] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014).
[2] C. K. Law, Phys. Rev. A **51**, 2537 (1995).
[3] T. J. Kippenberg and K. J. Vahala, Science **321**, 1172 (2008).
[4] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A **56**, 4175 (1997).
[5] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature (London) **475**, 359 (2011).
[6] J. Chan, T. P. Mayer Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter, Nature (London) **478**, 89 (2011).

[7] T. Rocheleau, T. Ndukum, C. Macklin, J. B. Hertzberg, A. A. Clerk, and K. C. Schwab, Nature (London) **463**, 72 (2010).

[8] A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, and A. N. Cleland, Nature (London) **464**, 697 (2010).

[9] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Phys. Rev. Lett. **99**, 093901 (2007).

[10] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. **99**, 093902 (2007).

[11] E. Verhagen, S. Deléglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Nature (London) **482**, 63 (2012).

[12] M. Karuza, C. Molinelli, M. Galassi, C. Biancofiore, R. Natali, P. Tombesi, G. Di Giuseppe, and D. Vitali, New J. Phys. **14**, 095015 (2012).

[13] S. Machnes, J. Cerrillo, M. Aspelmeyer, W. Wieczorek, M. B. Plenio, and A. Retzker, Phys. Rev. Lett. **108**, 153601 (2012).

[14] D. Kleckner, I. Pikovski, E. Jeffrey, L. Ament, E. Eliel, J. Brink, and D. Bouwmeester, New J. Phys. **10**, 095020 (2008).

[15] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).

[16] J. Q. Liao and L. Tian, Phys. Rev. Lett. **116**, 163602 (2016).

[17] W. Ge and M. S. Zubairy, Phys. Rev. A **91**, 013842 (2015).

[18] S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **88**, 120401 (2002).

[19] M. Pinard, A. Dantan, D. Vitali, O. Arcizet, T. Briant, and A. Heidmann, Europhys. Lett. **72**, 747 (2005).

[20] D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Phys. Rev. Lett. **98**, 030405 (2007).

[21] K. Børkje, A. Nunnenkamp, and S. M. Girvin, Phys. Rev. Lett. **107**, 123601 (2011).

[22] C. Genes, D. Vitali, and P. Tombesi, Phys. Rev. A **77**, 050307(R) (2008).

[23] W. Ge, M. Al-Amri, H. Nha, and M. S. Zubairy, Phys. Rev. A **88**, 022338 (2013).

[24] W. Ge, M. Al-Amri, H. Nha, and M. S. Zubairy, Phys. Rev. A **88**, 052301 (2013).

[25] W. Ge and M. S. Zubairy, Phys. Scr. **90**, 074015 (2015).

[26] Y.-D. Wang and A. A. Clerk, Phys. Rev. Lett. **110**, 253601 (2013).

[27] M. J. Woolley and A. A. Clerk, Phys. Rev. A **89**, 063805 (2014).

[28] J. Li, I. M. Haghighi, N. Malossi, S. Zippilli, and D. Vitali, New J. Phys. **17**, 103037 (2015).

[29] M. J. Hartmann and M. B. Plenio, Phys. Rev. Lett. **101**, 200503 (2008).

[30] R. Riedinger, S. Hong, R. A. Norte, J. A. Slater, J. Shang, A. G. Krause, V. Anant, M. Aspelmeyer, and S. Gröblacher, Nature (London) **530**, 313 (2016).

[31] C. F. Ockeloen-Korppi, E. Damskägg, J.-M. Pirkkalainen, M. Asjad, A. A. Clerk, F. Massel, M. J. Woolley, and M. A. Sillanpää, Nature (London) **556**, 478 (2018).

[32] M. R. Vanner, I. Pikovski, and M. S. Kim, Ann. Phys. (NY) **527**, 15 (2015).

[33] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, Č. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, Proc. Natl. Acad. Sci. USA **108**, 16182 (2011).

[34] M. R. Vanner, J. Hofer, G. D. Cole, and M. Aspelmeyer, Nat. Commun. **4**, 2295 (2013).

[35] J. Q. Liao and F. Nori, Phys. Rev. A **90**, 023851 (2014).

[36] J. Q. Liao, H. K. Cheung, and C. K. Law, Phys. Rev. A **85**, 025803 (2012).

[37] K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, Nat. Phys. **4**, 561 (2008).

[38] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45**, 5193 (1992).

[39] P. J. Bardroff, E. Mayr, and W. P. Schleich, Phys. Rev. A **51**, 4963 (1995).

[40] M. Wilkens and P. Meystre, Phys. Rev. A **43**, 3832 (1991).

[41] M. S. Zubairy, Phys. Lett. A **222**, 91 (1996).

[42] P. J. Bardroff, E. Mayr, W. P. Schleich, P. Domokos, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **53**, 2736 (1996).

[43] M. Freyberger and A. M. Herkommer, Phys. Rev. Lett. **72**, 1952 (1994).

[44] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **72**, 3339 (1994).

[45] K. Jacobs, P. L. Knight, and V. Vedral, J. Mod. Opt. **44**, 2427 (1997).

[46] M. S. Zubairy, Phys. Rev. A **57**, 2066 (1998).

[47] T. Azim and M. S. Zubairy, Phys. Lett. A **250**, 344 (1998).

[48] M. Mahmoudi, H. Tajalli, and M. S. Zubairy, J. Opt. B **2**, 315 (2000).

[49] M. Ahmad, S. Qamar, and M. S. Zubairy, Phys. Rev. A **64**, 023811 (2001).

[50] X. B. Zou, K. Pahlke, and W. Mathis, Phys. Rev. A **69**, 015802 (2004).

[51] S. Singh and P. Meystre, Phys. Rev. A **81**, 041804(R) (2010).

[52] J. Restrepo, C. Ciuti, and I. Favero, Phys. Rev. Lett. **112**, 013601 (2014).

[53] S. Asiri, W. Ge, and M. S. Zubairy, J. Phys. A: Math. Theor. **51**, 414017 (2018).

[54] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, New York, 1997).

[55] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).

[56] H. Walther, B. T. H. Varcoe, B.-G. Englert, and T. Becker, Rep. Prog. Phys. **69**, 1325 (2006).

[57] Z. Liao, M. Al-Amri, and M. S. Zubairy, Phys. Rev. A **88**, 053809 (2013).

[58] P. Thoumany, T. W. Hänsch, G. Stania, L. Urbonas, and Th. Becker, Opt. Lett. **34**, 1621 (2009).

[59] Z. Liao, M. Al-Amri, T. Becker, W. P. Schleich, M. O. Scully, and M. S. Zubairy, Phys. Rev. A **87**, 023405 (2013).

[60] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, New York, 2010).