Optomechanical quantum Cavendish experiment

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An open question in experimental physics is the characterization of gravitational effects in quantum regimes. We propose an experimental setup that uses well-tested techniques in cavity optomechanics to observe the effects of the gravitational interaction between two micromechanical oscillators on the interference of the cavity photons through the shifts in the visibility of interfering photons. The gravitational coupling leads to a shift in the period and magnitude of the visibility whose observability is within reach of current technology. We discuss the feasibility of the setup as well as the effects on entanglement due to gravitational interaction.

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I. INTRODUCTION

One of the biggest difficulties in constructing a theory of quantum gravity is the lack of experimental data. Unavailability of clean data from regimes where both quantum and gravitational effects are present have cast a long shadow on the fundamental conceptual problems that a theory of quantum gravity is expected to solve [1,2]. Although both theories have been successfully tested to extremely high degrees in their respective domains of validity, the disparities between them (i.e., large distances and massive bodies for general relativity versus short distances and small masses for quantum mechanics), which stem from the weakness of gravity and the decoherence of quantum states, have led to the yet-unsurmounted task of designing experiments that can access regimes where both theories predict effects of comparable degrees of observability.

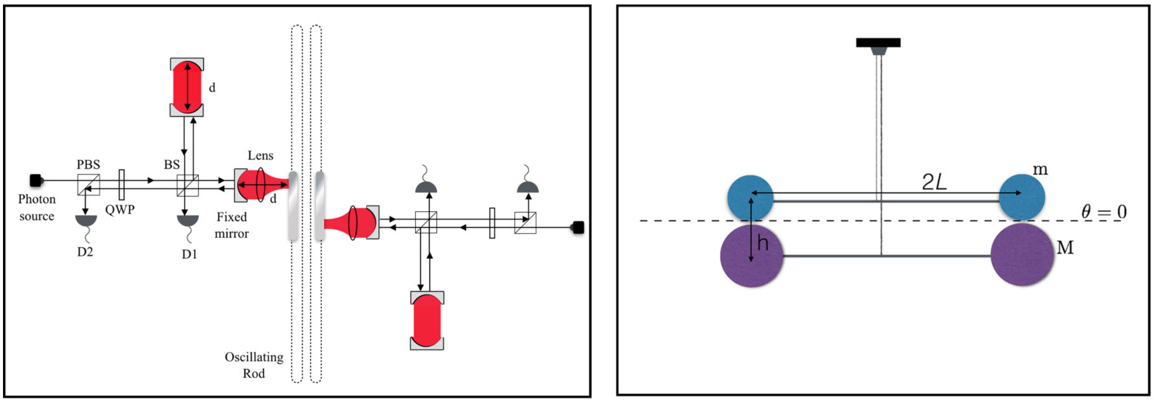
These experiments are of two types: (1) those where the goal is only to construct a measurement apparatus sensitive enough to provide information about cosmological and astrophysical phenomena, or (2) those experiments where both the source of observations and the measurement apparatus need to be constructed. The former include observations of the primordial cosmic microwave background (CMB) for information about the very early universe (i.e., a rare example of a natural quantum gravity regime) and sensitive detection of gravitational waves from black hole mergers as a possible source of information about the quantum degrees of freedom inside black holes [3]. The latter approach was first proposed by Feynman [4], where he suggested putting a massive object in superposition to test whether its gravitational field can also be put in superposition (i.e., is quantum in nature) or whether a "gravitational collapse" would prevent this from happening.

Advances in optomechanics [5] and atom interferometry [6] have made the possibility of measuring the effects of gravity in table-top quantum systems closer than ever. Another promising route exploits advanced satellite technologies that will allow quantum protocols to be tested over large length scales where the effects of gravity and spacetime curvature are expected to be nontrivial [7].

Proposals to observe the effects of models of gravity that modify quantum mechanics, such as gravitational decoherence and semiclassical gravity, in optomechanical settings have been considered before [8]. Experiments to date have also been able to demonstrate the effect of Earth's (classical) background gravitational field on quantum particles, whether as single states [9,10] or in superposition [11].

What is lacking are experiments that probe the *mutual* gravitational interaction between two quantum systems. In this paper, we address this by considering the effect of the gravitational interaction between two quantum systems in an optomechanical setting. In particular, we investigate the question, given the Newtonian gravitational interaction between two quantized systems, how can we experimentally observe the effects of this interaction? To this end we propose an optomechanical setup to observe the effect of the gravitational interaction between two quantum micromechanical oscillators. A setup involving superposing mirrors of order 10^{14} atoms was proposed in [12], and its application in observing the effects of gravitational decoherence models was considered in [13]. Here we assume that the gravitational interaction is Newtonian gravity $GMm/|\hat{r}_1 - \hat{r}_2|$, where $\hat{r}_1$ and $\hat{r}_2$ are position operators of the gravitating masses, and calculate its effect on the visibility pattern of interfering photons in an optomechanical setup perturbatively. We find that the gravitational coupling leads to an observable shift in the period and magnitude of the visibility of photons that is within reach of today's technologies.

Our paper is organized as follows. We first discuss the setup to be used to search for the model's signatures and the parameters that will optimize between their strength and experimental feasibility. The nature and magnitude of the signatures is then discussed, as well as the requirements to deal with environmental decoherence and systematic errors. We sum up our results in a concluding section.



(a) upper view of the set-up (b) front view of the oscillators

FIG. 1. The proposed setup consists of two freely moving angular oscillators suspended with vertical displacement h between them and moving angularly in the horizontal plane to which they are fixed. At the center of each oscillator is a mirror that forms the oscillating part of a cavity system, whose other part is a fixed mirror at distance d away. A focusing lens is used to reduce leakage of cavity photons due to reflections from angularly oscillating mirrors. Photons with high radiation pressure are put in a superposition of either entering the cavity with the movable end mirror or an empty cavity with the same unperturbed length. The beams exiting each cavity are then recombined and the resulting visibility pattern analyzed, as in shown in (a).

II. EXPERIMENTAL PROPOSAL

Figure 1 shows the experimental setup. It consists of a mechanical component [Fig. 2(b)] formed by two oscillating rods with end masses, as well as an optical component [Fig. 2(a)]. For the mechanical component, two microrods of length $2L$ each are suspended from their center with a relative vertical separation h. Masses of mass M and m, respectively, are fixed at the ends of each rod and mirrors are attached to the center of each of the rods. The mirrors will form the end mirrors, which act as mechanical oscillators, of high-finesse optical cavities.

The optical component of the experiment has two similar parts, one for each of the oscillating mirrors. Each part follows the scheme of Ref. [14], which makes use of a Michelson interferometer to prepare a microscopic oscillator in a superposed state. For each part, an input pulse will be generated using a high-radiation-pressure photon source. The input pulse will be split using a beam splitter into two paths, one going into the cavity with the movable end mirror attached to the oscillating rod while the other passes through an empty cavity. A lens will be placed in the cavity to focus the incoming beam onto an edge of the mirror, so that the time needed to cross the length d of the empty cavity is much smaller than the period of the rod. Individually, the visibility pattern of the photon from each part of the setup can reveal that the associated mirror is in a superposed state, as explained in Ref. [14]. However, the gravitational interaction between the two oscillating rods will lead to a shift in the visibility patterns observed.

The Hamiltonian describing the interaction between the cavity modes with the mirrors is given by [15]

$$\begin{aligned}H_1 &= \hbar\omega_c(c_1^\dagger c_1 + c_2^\dagger c_2) + \hbar\Omega_a a^\dagger a - \Lambda_m \hbar\Omega_a c_1^\dagger c_1(a^\dagger + a)\\ &\quad + \hbar\omega_d(d_1^\dagger d_1 + d_2^\dagger d_2) + \hbar\Omega_b b^\dagger b - \Lambda_M \hbar\Omega_b d_1^\dagger d_1(b^\dagger + b),\end{aligned} \tag{1}$$

where a and $a^\dagger$ (respectively b and $b^\dagger$) are the creation and annihilation operators of the mechanical modes of rod m (M), c_1 and $c_1^\dagger$ (d_1 and $d_1^\dagger$) are the creation and annihilation operators of photons in the path entering the cavity containing the mirror attached on rod m (M), while c_2 and $c_2^\dagger$ (d_2 and $d_2^\dagger$) are those of photons in the path not entering the cavity. In addition, ω_c and ω_d are the frequencies of the two input pulses, Ω_a and Ω_b are the natural frequencies of the two rods of masses m and M, respectively, and

$$\Lambda_m = \frac{\omega_c}{2d\ \Omega_a}\sqrt{\frac{\hbar}{m\Omega_a}}, \quad \Lambda_M = \frac{\omega_d}{2d\ \Omega_b}\sqrt{\frac{\hbar}{M\Omega_b}} \tag{2}$$

are the optomechanical coupling constants [14]. The rods are assumed initially to be in coherent oscillatory states

$$|\beta_j\rangle = \sum_{n=0}^{\infty}\frac{\beta_j^n}{\sqrt{n!}}|n\rangle, \quad j \in \{m, M\}, \tag{3}$$

where $|n\rangle$ are the Fock eigenstates of the harmonic oscillator. The initial state of the total system is

$$\begin{aligned}|\psi(0)\rangle &= \frac{1}{\sqrt{2}}(|0,1\rangle_c + |1,0\rangle_c)|\beta_m\rangle\\ &\quad \times \otimes \frac{1}{\sqrt{2}}(|0,1\rangle_d + |1,0\rangle_d)|\beta_M\rangle,\end{aligned} \tag{4}$$

where $|1,0\rangle_\chi = \chi_1^\dagger|0\rangle$, $|0,1\rangle_\chi = \chi_2^\dagger|0\rangle$ for $\chi = c, d$ and where $|0\rangle$ is the vacuum state of the cavity modes. Under the action of H_1, this state evolves to [16]

$$\begin{aligned}|\psi(t)\rangle &= e^{-iH_1 t}|\psi(0)\rangle\\ &= \frac{e^{-i\omega_c t}}{\sqrt{2}}(|0,1\rangle_c|\Phi_{0,m}(t)\rangle + e^{i\phi_m(t)}|1,0\rangle_c|\Phi_{1,m}(t)\rangle)\\ &\quad \otimes\frac{e^{-i\omega_d t}}{\sqrt{2}}(|0,1\rangle_d|\Phi_{0,M}(t)\rangle + e^{i\phi_M(t)}|1,0\rangle_d|\Phi_{1,M}(t)\rangle),\end{aligned} \tag{5}$$

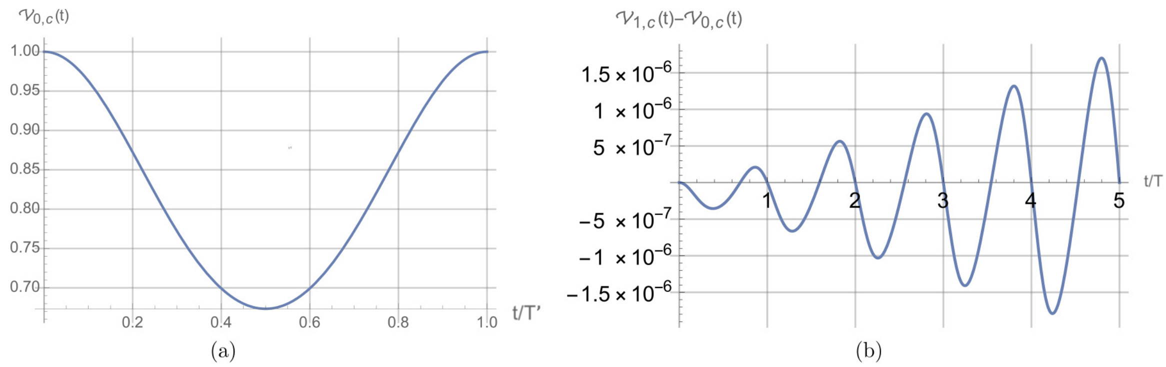


FIG. 2. (a) The visibility pattern of the photon field in the cavity system of rod m before coupling it to rod M, showing periodic behavior whose period is determined by that of the oscillator $T' = 2\pi/\Omega_a$, and the strength of its drop at every half-period depends on the optomechanical coupling between the rod and the photon field. (b) The shift in the magnitude of visibility from the case with no gravitational coupling as a function of time due to the combined effect of the modified period of the oscillator, $2\pi/\Omega_a \to 2\pi/\omega_a = T$, and the action of the coupled Hamiltonian on the state of the system and calculated perturbatively in Eq. (19).

where

$$\Phi_{0,j}(t) = \beta_j e^{-i\Omega_k t}$$
$$\Phi_{1,j}(t) = \beta_j e^{-i\Omega_k t} + \Lambda_j(1 - e^{-i\Omega_k t})$$
$$\phi_j(t) = \Lambda_j^2(\Omega_k t - \sin\Omega_k t) + \Lambda_j \mathrm{Im}[\beta_j(1 - e^{-i\Omega_k t})] \quad (6)$$

for $(j,k) \in \{(m,a),(M,b)\}$. The interferometric visibility pattern is directly measurable from the statistics of photon detection and, therefore, it provides an important source of information about the cavity system. If the system evolves only according to H_1, then the visibility pattern on the photons of the two cavities will be

$$\mathcal{V}_{0,c}(t) = e^{-\Lambda_m^2(1-\cos\Omega_a t)}$$
$$\mathcal{V}_{0,d}(t) = e^{-\Lambda_M^2(1-\cos\Omega_b t)}, \quad (7)$$

which shows the independence of each cavity system from the other and that the timescale of oscillation of the visibility pattern is set by the frequency of the oscillating rod. In this case, the visibility is given by twice the absolute value of one of the off-diagonal terms in the photon density matrix so that, for instance, if ρ_c is the reduced density matrix of the photon coupled to rod m, then $\mathcal{V}_{0,c}(t) = 2|\,\mathrm{Tr}[\rho_{0,c}(t)\,|0,1\rangle_c\,\langle 1,0|_c]|$.

Our setup is designed so as to maximize the effect of the gravitational interaction between the two oscillators. Assuming Newtonian gravity, the total quantized Hamiltonian of the system of interacting oscillators, up to a constant term, is (see Appendix A)

$$H = \hbar\omega_c(c_1^\dagger c_1 + c_2^\dagger c_2) + \hbar\omega_a a^\dagger a - \lambda_m\hbar\omega_a c_1^\dagger c_1(a^\dagger + a)$$
$$+ \hbar\omega_d(d_1^\dagger d_1 + d_2^\dagger d_2) + \hbar\omega_b b^\dagger b - \lambda_M\hbar\omega_b d_1^\dagger d_1(b^\dagger + b)$$
$$+ \hbar\gamma(a^\dagger + a)(b^\dagger + b). \quad (8)$$

We will denote

$$H_g := \hbar\gamma(a^\dagger + a)(b^\dagger + b), \quad (9)$$

where

$$\gamma := -\frac{G}{2h^3}\sqrt{\frac{Mm}{\omega_a\omega_b}} \quad (10)$$

is the gravitational coupling constant between the two oscillators. We note also that the frequencies of the oscillators and the optomechanical coupling constant is modified from the old Hamiltonian in Eq. (1) according to

$$\Omega_a \to \omega_a = \sqrt{\Omega_a^2 + \frac{GM}{h^3}}, \quad \Omega_b \to \omega_b = \sqrt{\Omega_b^2 + \frac{Gm}{h^3}}, \quad (11)$$

$$\Lambda_m \to \lambda_m = \frac{\omega_c}{2\omega_a d}\sqrt{\frac{\hbar}{m\omega_a}}, \quad \Lambda_M \to \lambda_M = \frac{\omega_d}{2\omega_b d}\sqrt{\frac{\hbar}{M\omega_b}}. \quad (12)$$

The visibility pattern of photons in the coupled system will be different from that of the uncoupled system given in Eq. (8). To calculate this shift, we switch to the interaction picture in which the density matrix of the total system is

$$\rho_I(t) = U(t)\rho_I(0)U^\dagger(t), \quad (13)$$

where $\rho_I(0) = |\psi(0)\rangle\,\langle\psi(0)|$,

$$U(t) = \mathcal{T}\exp\left[-\frac{i}{\hbar}\int_0^t dt' H_I(t')\right], \quad (14)$$

with $\mathcal{T}$ being the time-ordering operator, and

$$H_I(t) = e^{iH_0 t/\hbar} H_g e^{-iH_0 t/\hbar}$$
$$= \hbar\gamma[a^\dagger e^{i\omega_a t} + a e^{-i\omega_a t} + 2\lambda_m c_1^\dagger c_1(1 - \cos\omega_a t)]$$
$$\times [b^\dagger e^{i\omega_b t} + b e^{-i\omega_b t} + 2\lambda_M d_1^\dagger d_1(1 - \cos\omega_b t)], \quad (15)$$

(see Appendix B) where H_0 is comprised of the first two lines of Eq. (8). The expectation value of any operator O is independent of the picture used to calculate it. In the interaction picture, this is equal to

$$\langle O(t)\rangle = \mathrm{Tr}[\rho_I(t)O_I(t)]$$
$$= \mathrm{Tr}[U(t)\rho_I(0)U^\dagger(t)e^{iH_0 t/\hbar}O_S e^{-iH_0 t/\hbar}]$$
$$= \mathrm{Tr}[e^{-iH_0 t/\hbar}U(t)\rho_I(0)U^\dagger(t)e^{iH_0 t/\hbar}O_S], \quad (16)$$

where O_S is the operator in the Schrödinger picture.

The visibility of photons in the cavity of rod m due to the full Hamiltonian is therefore

$$\mathcal{V}_{1,c}(t) = 2|\,\mathrm{Tr}[\rho_{1,c}(t)\,|0,1\rangle_c\,\langle 1,0|_c]|, \tag{17}$$

where

$$\rho_{1,c}(t) = \mathrm{Tr}_{m,M,d}[e^{-iH_0t/\hbar}U(t)\rho_I(0)U^\dagger(t)e^{iH_0t/\hbar}] \tag{18}$$

is the partial state of the photons in the cavity of rod m in the Schrödinger picture after tracing out the two oscillators and the photons in the cavity of rod M. The visibility (17) of photons in the cavity of rod m is

$$\begin{aligned}\mathcal{V}_{1,c}(t) &\approx e^{-\lambda_m^2(1-\cos\omega_a t)}\\ &\quad\times\left|1+i2\gamma\int_0^t dt'\lambda_m[1-\cos\omega_a(t'-t)]\right.\\ &\quad\left.\times[2\beta_M\cos\omega_b t'+\lambda_M(1-\cos\omega_b t')]\right|\\ &= e^{-\lambda_m^2(1-\cos\omega_a t)}\times\left|1+i2\gamma\lambda_m\left[(2\beta_M-\lambda_M)\right.\right.\\ &\quad\times\left(\frac{\sin\omega_b t}{\omega_b}-\frac{\omega_a\sin\omega_a t-\omega_b\sin\omega_b t}{\omega_a^2-\omega_b^2}\right)\\ &\quad\left.\left.+\lambda_M\left(t-\frac{\sin\omega_a t}{\omega_a}\right)\right]\right|\end{aligned} \tag{19}$$

to first order in γ (see Appendix C).

Quantum optomechanics allows coherent quantum control over massive mechanical objects ranging from nanosized devices of 10^{-20} kg, to micromechanical structures of masses 10^{-11} kg, up to centimeter-sized suspended mirrors of several kilograms in mass for gravitational wave detectors [5]. The first breakthroughs in quantum optomechanics were with 10^{-16}-kg masses [17,18], followed recently by room-temperature regimes with masses around 10^{-12} kg [19], and proposals for future experiments reaching 10^{-6}-kg masses [20]. We assume the masses attached to the end of the rods to be micromechanical structures with masses $M = m = 10^{-13}$ kg and to be separated by a vertical distance $h = 10^{-8}$ m, each mounted on an oscillator with frequencies $\Omega_a = 3\times 10^3$ Hz, and $\Omega_b = \alpha\Omega_a$ for $\alpha = 0.9$. The oscillators are assumed to be cooled down to near their ground states so that $\beta_M = \beta_m = 1$. We propose to use light of frequency $\omega_c = \omega_d = 450\times 10^{12}$ Hz in both cavities, each with cavity length $d = 10$ cm.

The precoupling visibility pattern, $\mathcal{V}_{0,c}(t)$, and the shift in visibility induced by the gravitational interaction, $\mathcal{V}_{1,c}(t) - \mathcal{V}_{0,c}(t)$, for photons of the cavity system of m are both shown in Fig. 2. In Fig. 2(a), we see that the visibility pattern of cavity photons in the noninteracting system has the same period $2\pi/\Omega_a$ as the oscillator, and at half that period it reaches its minimum point at $e^{-2\Lambda_m^2}$. The drop in visibility in the middle of the period is because oscillations of the rod contain which-path information about the position of the superposed photons, dependent on the coupling strength λ_m between the photon field and the oscillator. When the oscillator returns to its original position after a full period of oscillation, this which-path information is deleted and the visibility is restored to its original value.

Figure 2(b) shows the shift in visibility as a function of time when the two oscillators are coupled to each other via Newtonian gravity. The sources of this shift are twofold. The first is due to the difference in frequencies between the coupled oscillators and their idealized uncoupled state. This is observable as a shift in the frequency of the visibility pattern of photons of magnitude $\omega_k - \Omega_k \approx \frac{GMm}{2jh^3\Omega_k} \sim O(\gamma)$, for $(j,k)\in\{(m,a),(M,b)\}$. The second kind of shift is due to the second term in Eq. (19), which oscillates around $(\gamma\lambda_m\lambda_M)^2t^2 \sim O(\gamma^2)$ and is observable as a growing variation in the shape of the visibility pattern from the one in $e^{-\lambda_m^2(1-\cos\omega_a t)}$. Recall also that λ_M is the coupling parameter between the mirror in the cavity of rod M and its cavity mode. From Eq. (19), we see that when this coupling is turned off ($\lambda_M = 0$), the shift in visibility is still that of Fig. 2(b) for small times. However, the effect of the coupling is an increase in the shift with time due to the $\lambda_M(t - \frac{\sin(\omega_a t)}{\omega_a})$ term. The linear behavior of the shift in visibility with t is predicted from perturbation theory for timescales below $\gamma^{-1} \sim 853$ s for the parameters used above. Maintaining the coherence of the state for long enough times will therefore lead to more observable effects.

III. DISCUSSION

Reminiscent of the experiment done by Cavendish [21] using suspended masses to measure the gravitational interaction between them, a quantum Cavendish experiment is one that uses suspended masses in a quantized state to detect and measure gravitational effects in quantum regimes so that the effects of Earth's gravity cancel out. Such types of experiments have been used before in sensitive verification of Newton's inverse-square law at scales below the dark-energy length scale [22], and have been first incorporated in an optomechanical setup to approach the quantum limit of mechanical sensing in [23]. Recently, such a setup was used to measure the gravitational force of milligram masses [20], and proposals have considered its application in testing gravitational decoherence models [24] and its implementation using optically levitated nanodumbbells [25]. Quantization of suspended linearly moving mirrors whose dynamics is dominated by the radiation pressure of cavity photons has been achieved with masses ranging from 40 kg [26] to milligrams [27].

Our setup requires forming coherent states of torsional mirrors of nanogram masses by cooling them to their ground states, surpassing the standard quantum limit of detection [28]. The suspended masses are coupled to a cavity field inside an optomechanical setup, and the effect of the mutual gravitational interaction between the masses is calculated on the visibility pattern of cavity photons, whose observation is based on robust and well-tested experimental techniques.

We found that the effects on the visibility are of two types: a shift in the period of revival by an amount $\delta T = \frac{2\pi}{\Omega_a} - \frac{2\pi}{\omega_a}$, and a change in the shape of the visibility pattern from the functional form $e^{-\lambda_m^2(1-\cos\omega_a t)}$ that is of order $O(\gamma^2)$ for timescale $t \lesssim \gamma^{-1}$. In practice, it is easier to detect δT, which corresponds to $\delta T \approx 0.78$ ns for the parameters used above, than the shift in vertical magnitude that is of order 10^{-6} in Fig. 2(b).

To illustrate, suppose that the visibility at some time t is drawn from an *a priori* Gaussian distribution of variance σ^2. Then the error on the estimate of the visibility at time t obtained by averaging over N data points is $\sigma_{\text{error}} = \frac{\sigma}{\sqrt{N}}$. If $\sigma_{\text{error}} \sim 10^{-6}$, then $N \sim 10^{12}\sigma^2$, which is difficult to achieve. On the other hand, the accuracy of measuring δT is dependent only on the time resolution available.

In practice, an oscillator in a coherent state $|\beta\rangle\langle\beta|$ will be in a thermal mixture

$$\frac{1}{\pi\bar{n}}\int d^2\beta e^{-|\beta|^2/\bar{n}}\,|\beta\rangle\langle\beta|\,, \tag{20}$$

where $\bar{n} = (e^{\hbar\omega_a/k_B T} - 1)^{-1}$ is the mean thermal number of phonon excitations at temperature T. This will modify the visibility according to [12]

$$e^{-\lambda_m^2(1-\cos\omega_a t)} \rightarrow e^{-\lambda_m^2(2\bar{n}+1)(1-\cos\omega_a t)}, \tag{21}$$

which causes a fast decay in visibility that is revived only after a full period. The width of the visibility's revived peak scales according to $\sim \frac{1}{\lambda_m\sqrt{\frac{4k_BT}{\hbar\omega_a}+2}}$. Increasing this width constitutes one of the main experimental challenges to realize this proposal and requires a method to cool down the center-of-mass mode of the oscillator to very near their ground state [29]. Another experimental challenge is due to decoherence from the mechanical damping of the oscillator and from dephasing with the environment, which lowers the revived peak of visibility. In order to observe the shift in the period of the oscillators, we need at least to be able to resolve one full period of the oscillation. If the environment is modeled as an Ohmic thermal bath of harmonic oscillators and the damping rate of oscillators is Γ_a, then the dephasing rate due to the environment at temperature T is $\Gamma_D = \Gamma_a k_B T m(\Delta x)^2/\hbar^2$, where $\Delta x \sim \sqrt{\frac{\hbar}{m\omega_a}}$ is the uncertainty in position of the oscillator [30]. The condition for observing the shift in period mentioned above is then $\Gamma_D \lesssim \omega_a$, which corresponds to

$$Q \gtrsim \frac{k_B T}{\hbar\omega_a} \sim \bar{n}, \tag{22}$$

where $Q := \omega_a/\Gamma_a$ is the quality factor of the oscillator. Values of $Q \sim 10^7$ have been achieved for suspended nanoparticles [31], which corresponds to $T \lesssim 0.23$ K for the parameters of the setup considered here.

Another source of systematic errors in the setup proposed is the effect of gravitational interaction with surrounding objects in the laboratory. If the rods are in a plane at half the height of a cylinder with all surrounding objects having a mass distribution of cylindrical symmetry, then the overall contribution would be a constant to the Hamiltonian that does not affect the dynamics described above. Earth's gravity would give an overall phase to the states of the oscillators that does not affect their visibility patterns. If we suppose that an inhomogeneity in the mass distribution surrounding the oscillators was due to a mass $\mathcal{M}$ located at distance $\vec{R}$ from the center of the two rods, then this adds terms of the form $\frac{2G\mathcal{M}m}{|\vec{R}-\vec{r}_a|} + \frac{2G\mathcal{M}M}{|\vec{R}-\vec{r}_b|}$ to the classical Hamiltonian of the system,

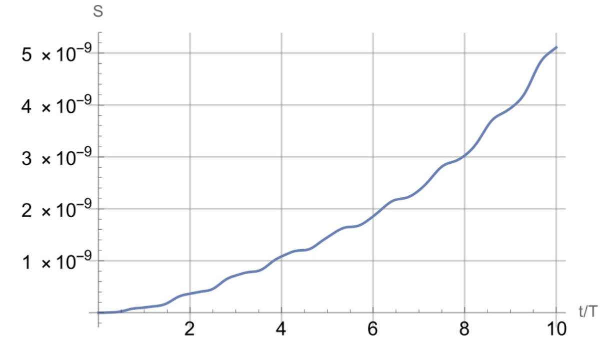


FIG. 3. Plot of linear entropy S against t/T.

where $\vec{r}_a$ and $\vec{r}_b$ are the position vectors of the masses from the center. Expanding these terms to first order and quantizing, this will add to the quantized Hamiltonian $-\frac{G\mathcal{M}}{R^2}\sqrt{\frac{\hbar m}{\omega_a}}(a^\dagger + a) - \frac{G\mathcal{M}}{R^2}\sqrt{\frac{\hbar M}{\omega_b}}(b^\dagger + b)$ up to constant terms. Comparing with terms proportional to $(a^\dagger + a)$ and $(b^\dagger + b)$ in Eq. (8), we see that the condition for this inhomogeneity to have negligible effects on the dynamics is to have $N_a\lambda_a\hbar\omega_a \gg \frac{G\mathcal{M}}{R^2}\sqrt{\frac{\hbar m}{\omega_a}}$, where N_a is the number of photons in cavity of mass m, and similarly for M. For the parameters of our setup, this corresponds to $\frac{\mathcal{M}}{N_a R^2} \ll 2.2 \times 10^5\,\text{kg}\,\text{m}^{-2}$. Satisfying this condition means that the systematic errors due to surrounding mass distribution is negligible.

As mentioned in the Introduction, the main feature of our proposed scheme is the observation of effects arising from gravitationally interacting quantum systems (whereas most previous studies are for a quantum test mass in the background gravitational field of the Earth). It is also interesting to note that entanglement, albeit quite weak, is generated due to this gravitational interaction. Denoting for convenience the system associated with m system 1 (consisting of the oscillating mirror and the cavity modes) and that of M as system 2, we see that the initial state $|\psi(0)\rangle\langle\psi(0)|$ in Eq. (4) is separable between the two systems. Since the only coupling between systems 1 and 2 in the proposed scheme is gravity, any resulting entanglement between the two systems can be attributed to the gravitational force. To quantify the amount of entanglement, we can use the linear entropy, defined as $S = 1 - \text{Tr}(\rho_1^2)$, where ρ_1 is the partial state of system 1. The calculation of linear entropy is carried out using the same set of parameters above to second order in γ and is given in Appendix D. Figure 3 shows the result. Even though the amount of entanglement generated for the period shown is small, we observe an increase with time, similar to the shift in the visibility pattern. Since the visibility is related to how much which-path information the position of the oscillator can reveal, which in turn is dependent on the amount of entanglement between the oscillator and the cavity photons, the increase in the amount of entanglement due to gravity shown in Fig. 3 means that, by monogamy of entanglement, the correlations between the oscillator and the cavity photons will correspondingly decay. This causes the visibility pattern to have a growth term as given in Eq. (19). We expect that an exact calculation will give a linear entropy and visibility that are bounded from above. Given the recent interest in observing entanglement due to gravity [32,33], it will be desirable to obtain an entanglement witness that can experimentally verify the entanglement generated for this scheme. One may also consider whether observable steady-state entanglement due to gravity may be obtained similar to other optomechanical settings, for example, in [34].

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APPENDIX A: TOTAL HAMILTONIAN WITH NEWTONIAN GRAVITY

Two classical harmonic oscillators will couple via gravity according to

$$H = \frac{p_m^2}{2m} + \frac{1}{2} I_m \Omega_a^2 \theta_m^2 + \frac{p_M^2}{2M} + \frac{1}{2} I_M \Omega_b^2 \theta_M^2 + H_g, \tag{A1}$$

where $I_m = 2mL^2$ and $I_M = 2ML^2$ are the two moments of inertia for the two rods. For two angular oscillators with masses at each end of a rod of length L and suspended with vertical displacement h, the classical gravitational interaction will be

$$\begin{aligned} H_g &= 2\frac{-GMm}{\left(h^2 + \left[2L\sin\left(\frac{\theta_M-\theta_m}{2}\right)\right]^2\right)^{1/2}} \\ &\approx 2\frac{-GMm}{h(1+[L(\theta_M-\theta_m)/h]^2)^{1/2}} \\ &\approx 2\frac{-GMm}{h}\left[1 - \frac{1}{2}\left(\frac{L(\theta_M-\theta_m)}{h}\right)^2\right] \\ &= \frac{-2GMm}{h} + \frac{GMm}{h^3}[L(\theta_M-\theta_m)]^2 \\ &= \frac{-2GMm}{h} + \frac{GMmL^2}{h^3}\left(\theta_M^2 + \theta_m^2 - \theta_M\theta_m - \theta_m\theta_M\right). \end{aligned} \tag{A2}$$

Therefore, up to a constant term, the total Hamiltonian can be written as

$$\begin{aligned} H = &\frac{p_m^2}{2m} + \frac{1}{2} m\omega_a^2\theta_m^2 + \frac{p_M^2}{2M} \\ &+ \frac{1}{2} M\omega_b^2\theta_M^2 - \frac{2GMmL^2}{h^3}\theta_m\theta_M, \end{aligned} \tag{A3}$$

where $\omega_a = \sqrt{\Omega_a^2 + \frac{GM}{h^3}}$ and $\omega_b = \sqrt{\Omega_b^2 + \frac{Gm}{h^3}}$. The frequency of a photon inside a cavity of length d is

$$\omega_c = 2\pi\frac{nc}{2d} = \frac{n\pi c}{d}, \tag{A4}$$

where $n = 1, 2, 3, \ldots$ and c is the speed of light. When it couples to an angular oscillator with displacement θ, the length of the cavity varies $d \to d + \delta$ for $\delta << d$, so that

$$\omega_c = \frac{n\pi c}{d+\delta} = \frac{n\pi c}{d(1+\delta/d)} \approx \frac{n\pi c}{d}(1 - \delta/d). \tag{A5}$$

In our case, $\delta = L\sin\theta \approx L\theta$. So,

$$\omega_c \to \omega_c - \omega_c\frac{L\theta}{d}. \tag{A6}$$

Introducing the annihilation operators for the two oscillators,

$$a = \sqrt{\frac{I_m\omega_a}{2\hbar}}\left(\theta_m + \frac{i}{I_m\omega_a}p_m\right), \tag{A7}$$

$$b = \sqrt{\frac{I_M\omega_b}{2\hbar}}\left(\theta_M + \frac{i}{I_M\omega_b}p_M\right), \tag{A8}$$

and substituting back in the total Hamiltonian to rewrite it in terms of the creation and/or annihilation operators, including the photon cavity terms, gives the quantized Hamiltonian of the total system as

$$\begin{aligned} H = &\hbar\omega_c(c_1^\dagger c_1 + c_2^\dagger c_2) + \hbar\omega_a a^\dagger a - \lambda_m\hbar\omega_a c_1^\dagger c_1(a^\dagger + a) \\ &+ \hbar\omega_d(d_1^\dagger d_1 + d_2^\dagger d_2) + \hbar\omega_b b^\dagger b - \lambda_M\hbar\omega_b d_1^\dagger d_1(b^\dagger + b) \\ &+ \hbar\gamma(a^\dagger + a)(b^\dagger + b), \end{aligned} \tag{A9}$$

exactly as given in Eq. (8).

APPENDIX B: INTERACTION HAMILTONIAN

We will derive here the expression in Eq. (16) for the interaction Hamiltonian $H_I(t) = e^{iH_0t/\hbar}H_g e^{-iH_0t/\hbar}$. Given operators A and B, the Baker-Campbell-Hausdorff (BCH) formula is

$$e^A B e^{-A} = B + [A, B] + \tfrac{1}{2}[A, [A, B]] + \cdots. \tag{B1}$$

The operator $e^{-iH_0t/\hbar}$ was calculated to be [35]

$$\begin{aligned} e^{-iH_0t/\hbar} = &e^{-i\omega_c t(c_1^\dagger c_1 + c_2^\dagger c_2)} e^{i(\lambda_m c_1^\dagger c_1)^2(\omega_a t - \sin(\omega_a t))} \\ &\times e^{\lambda_m c_1^\dagger c_1(a^+\alpha - a\alpha^*)} e^{-i\omega_a t a^+ a} \times [M], \end{aligned} \tag{B2}$$

where $\alpha = (1 - e^{-i\omega_a t})$, and $[M]$ here and below denotes the same part of the term as on its left but under the isomorphic transformations

$$(.)_{a,c,m} \to (.)_{b,d,M}$$
$$a, c \to b, d.$$

Using the BCH formula, the interaction Hamiltonian can be written as

$$\begin{aligned} H_I(t) &= e^{iH_0t/\hbar}\hbar\gamma(a^\dagger + a)(b^\dagger + b)e^{-iH_0t/\hbar} \\ &= \hbar\gamma e^{i\omega_a t a^\dagger a} e^{\lambda_m c_1^\dagger c_1(a\alpha^* - a^\dagger\alpha)}(a^\dagger + a) \\ &\quad \times e^{-\lambda_m c_1^\dagger c_1(a\alpha^* - a^\dagger\alpha)} e^{-i\omega_a t a^\dagger a} \times [M] \\ &= \hbar\gamma e^{i\omega_a t a^\dagger a}[a^\dagger + a + \lambda_m c_1^\dagger c_1(\alpha + \alpha^*)]e^{-i\omega_a t a^\dagger a} \times [M] \\ &= \hbar\gamma[a^\dagger e^{i\omega_a t} + a e^{-i\omega_a t} + \lambda_m c_1^\dagger c_1(\alpha + \alpha^*)] \times [M] \\ &= \hbar\gamma[a^\dagger e^{i\omega_a t} + a e^{-i\omega_a t} + 2\lambda_m c_1^\dagger c_1(1 - \cos\omega_a t)] \times [M], \end{aligned} \tag{B3}$$

which is Eq. (16).

APPENDIX C: VISIBILITY IN THE COUPLED SYSTEM

To calculate the visibility from Eq. (18), we need to know what the action of $e^{-iH_0t\hbar}U(t)$ on $\rho_I(0)$ is. First, note that

$$\begin{aligned}
e^{-iH_0t/\hbar}a^\dagger &= e^{-iH_0t/\hbar}a^\dagger e^{iH_0t/\hbar}e^{-iH_0t/\hbar}\\
&= e^{-\lambda_m c_1^\dagger c_1(a\alpha^*-a^\dagger\alpha)}e^{-i\omega_a t a^\dagger a}a^\dagger e^{i\omega_a t a^\dagger a}e^{\lambda_m c_1^\dagger c_1(a\alpha^*-a^\dagger\alpha)}e^{-iH_0t/\hbar}\\
&= e^{-\lambda_m c_1^\dagger c_1(a\alpha^*-a^\dagger\alpha)}a^\dagger e^{-i\omega_a t}e^{\lambda_m c_1^\dagger c_1(a\alpha^*-a^\dagger\alpha)}e^{-iH_0t/\hbar}\\
&= (a^\dagger - \lambda_m c_1^\dagger c_1\alpha^*)e^{-i\omega_a t}e^{-iH_0t/\hbar},
\end{aligned} \tag{C1}$$

and similarly,

$$e^{-iH_0t/\hbar}a = (a-\lambda_m c_1^\dagger c_1\alpha)e^{i\omega_a t}e^{-iH_0t/\hbar}. \tag{C2}$$

This allows us to write, using the BCH formula, up to first order

$$\begin{aligned}
e^{-iH_0t/\hbar}U(t) &\approx e^{-iH_0t/\hbar}\left(1-\frac{i}{\hbar}\int_0^t dt' H_I(t')\right) = e^{-iH_0t/\hbar}\left(1 - i\gamma\int_0^t dt'[a^\dagger e^{i\omega_a t'} + ae^{-i\omega_a t'} + 2\lambda_m c_1^\dagger c_1(1-\cos\omega_a t')]\times[M]\right)\\
&= e^{-iH_0t/\hbar} - \frac{i\gamma}{\hbar}\int_0^t dt'[(a^\dagger-\lambda_m c_1^\dagger c_1\alpha^*)e^{i\omega_a(t'-t)} + (a-\lambda_m c_1^\dagger c_1\alpha)e^{-i\omega_a(t'-t)}\\
&\quad + 2\lambda_m c_1^\dagger c_1(1-\cos\omega_a t')]\times[M]e^{-iH_0t/\hbar}\\
&= 1 - \frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + ae^{-i\omega_a(t'-t)} + \lambda_m c_1^\dagger c_1(2-2\cos\omega_a t' - \alpha^* e^{i\omega_a(t'-t)} - \alpha e^{-i\omega_a(t'-t)})]\times[M]e^{-iH_0t/\hbar}\\
&= 1 - \frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + ae^{-i\omega_a(t'-t)} + \lambda_m c_1^\dagger c_1(2-2\cos\omega_a t' - 2\cos\omega_a(t'-t) + 2\cos\omega_a t')]\times[M]e^{-iH_0t/\hbar}\\
&= 1 - \frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + ae^{-i\omega_a(t'-t)} + 2\lambda_m c_1^\dagger c_1(1-\cos\omega_a(t'-t))]\times[M]e^{-iH_0t/\hbar}.
\end{aligned} \tag{C3}$$

Using this relation, we can calculate the action of this operator on the initial state $|\psi(0)\rangle_I$ of the total system given in Eq. (5) perturbatively to be

$$\begin{aligned}
&e^{-iH_0t/\hbar}U(t)|\psi(0)\rangle_I = |\psi(t)\rangle - \frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + ae^{-i\omega_a(t'-t)} + 2\lambda_m c_1^\dagger c_1(1-\cos\omega_a(t'-t))]\times[M]|\psi(t)\rangle\\
&= |\psi(t)\rangle - \frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + ae^{-i\omega_a(t'-t)} + 2\lambda_m c_1^\dagger c_1(1-\cos\omega_a(t'-t))]\\
&\quad\times[b^\dagger e^{i\omega_b(t'-t)} + be^{-i\omega_b(t'-t)} + 2\lambda_M d_1^\dagger d_1(1-\cos\omega_b(t'-t))]|\psi(t)\rangle\\
&= \frac{e^{-i\omega_c t - i\omega_d t}}{2}\Bigg[\left(1-\frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + \Phi_{0,m}(t)e^{-i\omega_a(t'-t)}][b^\dagger e^{i\omega_b(t'-t)} + \Phi_{0,M}(t)e^{-i\omega_b(t'-t)}]\right)\\
&\quad\times|0,1\rangle_c|0,1\rangle_d|\Phi_{0,m}\rangle|\Phi_{0,M}\rangle + \left(1-\frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + \Phi_{0,m}(t)e^{-i\omega_a(t'-t)}]\right.\\
&\quad\left.\times[b^\dagger e^{i\omega_b(t'-t)} + \Phi_{1,M}(t)e^{-i\omega_b(t'-t)} + 2\lambda_M(1-\cos\omega_b(t'-t))]\right)e^{i\phi_M(t)}|0,1\rangle_c|1,0\rangle_d|\Phi_{0,m}\rangle|\Phi_{1,M}\rangle\\
&\quad+\left(1-\frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + \Phi_{1,m}(t) + 2\lambda_m(1-\cos\omega_a(t'-t))][b^\dagger e^{i\omega_b(t'-t)} + \Phi_{0,M}(t)e^{-i\omega_b(t'-t)}]\right)\\
&\quad\times e^{i\phi_m(t)}|1,0\rangle_c|0,1\rangle_d|\Phi_{1,m}\rangle|\Phi_{0,M}\rangle + \left(1-\frac{i\gamma}{\hbar}\int_0^t dt'[a^\dagger e^{i\omega_a(t'-t)} + \Phi_{1,m}(t)e^{-i\omega_a(t'-t)+2\lambda_m(1-\cos\omega_a(t'-t))}]\right.\\
&\quad\left.\times[b^\dagger e^{i\omega_b(t'-t)} + \Phi_{1,M}(t)e^{-i\omega_b(t'-t)} + 2\lambda_M(1-\cos\omega_b(t'-t))]\right)e^{i\phi_m(t)}e^{i\phi_M(t)}|1,0\rangle_c|1,0\rangle_d|\Phi_{1,m}\rangle|\Phi_{1,M}\rangle\Bigg].
\end{aligned} \tag{C4}$$

Tracing out the two oscillators and the photons in the cavity of rod M from the density matrix formed by this state, keeping terms only of order $O(\gamma)$, and calculating twice the absolute value of one of the off-diagonal terms will give the expression $\mathcal{V}_{1,c}(t)$ in Eq. (19).

APPENDIX D: LINEAR ENTROPY

If we define $A := \frac{-1}{\gamma\hbar}\int_0^t dt' e^{-iH_0 t/\hbar} H_I(t') e^{iH_0 t/\hbar}$, then we note that it is Hermitian, and from Eqs. (C1) and (36) that it can be written as $A = \frac{-1}{\gamma\hbar}\int_0^t dt' H_I(t'-t)$. The density matrix of the two systems: system 1 for oscillator of mass m with its cavity photons and system 2 for oscillator of mass M with its cavity photons, can be written as a separable pure bipartite state $\rho = \rho_1 \otimes \rho_2 =: |\psi_1\rangle\langle\psi_1| \otimes |\psi_2\rangle\langle\psi_2|$ so that $|\psi_1\rangle|\psi_2\rangle = |\psi(t)\rangle$, as given in Eq. (5). Further defining

$$A_1{}^2 := (\langle\psi_2| A |\psi_2\rangle)^2, \tag{D1}$$

$$A^2{}_1 := \langle\psi_2| A^2 |\psi_2\rangle, \tag{D2}$$

if we use the BCH formula in Eq. (B1) and calculate up to second order in γ, then under the action of the unitary $U = e^{i\gamma A}$, the state in the Schrödinger picture evolves according to

$$\begin{aligned}\rho' &= U\rho U^\dagger \\ &= e^{i\gamma A}\rho e^{-i\gamma A} \\ &= \rho + i\gamma[A,\rho] + \frac{1}{2}[i\gamma A,[i\gamma A,\rho]] + \cdots \\ &= \rho + i\gamma[A,\rho] - \frac{\gamma^2}{2}(A^2\rho + \rho A^2 - 2A\rho A).\end{aligned} \tag{D3}$$

Tracing out system 2 will give

$$\rho_1' = \rho_1 + i\gamma[A_1,\rho_1] - \frac{\gamma^2}{2}\left(A_1^2\rho_1 + \rho_1 A_1^2 - 2A_1\rho_1 A_1\right). \tag{D4}$$

Squaring this and keeping terms only up to second order in γ will give

$$\begin{aligned}\rho_1'^2 = \rho_1^2 + i\gamma\left[A_1,\rho_1^2\right] - \frac{\gamma^2}{2}\big(&2A_1\rho_1A_1\rho_1 \\ &+ 2\rho_1A_1\rho_1A_1 - 2A_1\rho_1^2A_1 - 2\rho_1A_1^2\rho_1 \\ &+ \rho_1A_1^2\rho_1 + \rho_1^2A_1^2 - 2\rho_1A_1\rho_1A_1 \\ &+ A_1^2\rho_1^2 + \rho_1A_1^2\rho_1 - 2A_1\rho_1A_1\rho_1\big).\end{aligned} \tag{D5}$$

Finally, taking the trace of this gives

$$\begin{aligned}\mathrm{Tr}\,\rho_1'^2 &= 1 - \frac{\gamma^2}{2}\left[4\,\mathrm{Tr}\left(A_1^2\rho_1\right) - 4\,\mathrm{Tr}\left(A_1^2\rho_1\right)\right] \\ &= 1 - 2\gamma^2\left[\mathrm{Tr}\left(A_1^2\rho_1\right) - \mathrm{Tr}\left(A_1^2\rho_1\right)\right],\end{aligned} \tag{D6}$$

so that the linear entropy will now be

$$\begin{aligned}S &:= 1 - \mathrm{Tr}\,\rho_1'^2 \\ &= 2\gamma^2\left[\mathrm{Tr}\left(A_1^2\rho_1\right) - \mathrm{Tr}\left(A_1^2\rho_1\right)\right].\end{aligned} \tag{D7}$$

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